

Mutually orthogonal cycle systems*

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Abstract

An ℓ -cycle system \mathcal{F} of a graph Γ is a set of ℓ -cycles which partition the edge set of Γ . Two such cycle systems \mathcal{F} and \mathcal{F}' are said to be *orthogonal* if no two distinct cycles from $\mathcal{F} \cup \mathcal{F}'$ share more than one edge. Orthogonal cycle systems naturally arise from face 2-colourable polyhedra and in higher genus from Heffter arrays with certain orderings. A set of pairwise orthogonal ℓ -cycle systems of Γ is said to be a set of mutually orthogonal cycle systems of Γ .

Let $\mu(\ell, n)$ (respectively, $\mu'(\ell, n)$) be the maximum integer μ such that there exists a set of μ mutually orthogonal (cyclic) ℓ -cycle systems of the complete graph K_n . We show that if $\ell \geq 4$ is even and $n \equiv 1 \pmod{2\ell}$, then $\mu'(\ell, n)$, and hence $\mu(\ell, n)$, is bounded below by a constant multiple of n/ℓ^2 . In contrast, we obtain the following upper bounds: $\mu(\ell, n) \leq n - 2$; $\mu(\ell, n) \leq (n - 2)(n - 3)/(2(\ell - 3))$ when $\ell \geq 4$; $\mu(\ell, n) \leq 1$ when $\ell > n/\sqrt{2}$; and $\mu'(\ell, n) \leq n - 3$ when $n \geq 4$. We also obtain computational results for small values of n and ℓ .

Keywords: Orthogonal cycle decompositions, cyclic cycle systems, Heffter arrays, completely-reducible, super-simple.

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Medsebojno pravokotni ciklični sistemi*

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Povzetek

ℓ -ciklični sistem \mathcal{F} grafa Γ je množica ℓ -ciklov, ki razdelijo množico povezav grafa Γ . Dva takšna ciklična sistema \mathcal{F} in \mathcal{F}' sta medsebojno pravokotna, če si nobena dva različna cikla iz $\mathcal{F} \cup \mathcal{F}'$ ne delita več kot ene povezave. Pravokotni sistemi ciklov nastanejo naravno iz poliedrov z 2-barvnim barvanjem lic, pri ploskvah višjega rodu pa iz Heffterjevih polj, ki zadoščajo določenim pogojem. Množica paroma pravokotnih ℓ -cikličnih sistemov grafa Γ je množica medsebojno pravokotnih cikličnih sistemov grafa Γ .

Naj bo $\mu(\ell, n)$ (oziroma, $\mu'(\ell, n)$) maksimalno celo število μ , pri katerem obstaja množica μ medsebojno pravokotnih (cikličnih) sistemov ℓ -ciklov polnega grafa K_n . Dokažemo: če je $\ell \geq 4$ sod in $n \equiv 1 \pmod{2\ell}$, potem je $\mu'(\ell, n)$, in torej tudi $\mu(\ell, n)$, omejen navzdol s konstantnim večkratnikom števila n/ℓ^2 . Dobimo tudi naslednje zgornje meje: $\mu(\ell, n) \leq n - 2$; $\mu(\ell, n) \leq (n - 2)(n - 3)/(2(\ell - 3))$, če je $\ell \geq 4$; $\mu(\ell, n) \leq 1$, če je $\ell > n/\sqrt{2}$; in $\mu'(\ell, n) \leq n - 3$, če je $n \geq 4$. Predstavimo tudi računске rezultate za majhne vrednosti n in ℓ .

Ključne besede: Pravokotne ciklične dekompozicije, ciklični sistemi ciklov, Heffterjeva polja, popolnoma reducibilni, superenostaven.

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