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SYSTEM CONTROL IN CONDITIONS OF FUZZY DYNAMIC PROCESSES

UPRAVLJANJE SISTEMA V POGOJIH MEHKIH DINAMIČNIH PROCESOV

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Keywords: dynamic system, control, random process, fuzzy reasoning, neuro-fuzzy training

Abstract

In this article, a mathematical model of controlling the system in conditions of fuzzy processes is presented. Such a system can also be a power supply system. Analytical approaches that can be used to describe the mutual impact of output and stocks (additional capacities) on hierarchically distributed occurrence/usage/variation or demand already exist. We add dynamics to the system with the use of continuous and discrete dynamic processes, which are of a random (stochastic) form. The dynamic discrete model of control for this system is built with a system of difference equations, and the dynamic continuous model is built with a system of differential equations. These systems of equations can be solved with a one-part z-transform in discrete situations in with Laplace transform in the continuous systems. Fuzzy system control follows a continuous and discrete stochastic mathematical closed-loop model of control of stocks (additional capacities) in production systems. The fuzzy model is demonstrated with a numerical example.

Povzetek

V članku je predstavljeno upravljanja sistema v pogojih mehkih dinamičnih procesov. Takšen sistem je lahko tudi energetski sistem. Razviti so analitični pristopi, s katerimi opišemo medsebojni vpliv proizvodnje ter zalog (dodatnih kapacitet) na hierarhično porazdeljeno prostorsko dogajanje/ porabo/spremembo oziroma povpraševanje. V sistem vpeljemo dinamiko, kar storimo z uporabo zveznih in diskretnih dinamičnih procesov, ki so zaradi zahteve po čim tesnejšem približku opisova-

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nja dejanskega sistema, slučajnostne (stohastične) narave. Dinamični diskretni model upravljanja takšnega sistema izgradimo s sistemom diferenčnih enačb, zvezni model pa opišemo s sistemom diferencialnih enačb. Za reševanje uporabimo enostrano z-transformacijo pri diskretnih sistemih in Laplaceovo transformacijo pri zveznih sistemih. Mehko upravljanje sistema izhaja iz zveznega in diskretnega slučajnostnega matematičnega modela upravljanja zalog v proizvodnem sistemu. V članku je prikazan mehki pristop, ki ga vpeljemo z uporabo dvofaznega sistema mehkega sklepanja in pomeni približek zaprtozančnemu sistemu upravljanja. Mehki algoritem je ilustriran z numeričnim primerom.

1 INTRODUCTION

A production system (power supply, logistics, traffic, etc.) is a complex dynamic system. If we could create a theoretical mathematical dynamic model of it, we would have to take into consideration a great many variables and their interrelationships. However, with methods of logical and methodological decomposition, every system may be divided into a finite set of simpler subsystems, which are then studied and analysed separately, [1].

A model of optimal control is determined with a system, its input/output variables, and the optimality criterion function. The system represents a regulation circle, which generally consists of a regulator, a control process, a feed-back loop, and input and output information. In this article, dynamic systems will be studied. The optimality criterion is the standard against which the control quality is evaluated. The term 'control quality' means the optimal and synchronized balancing of planned and actual output functions, [2, 3].

Let us consider a production model in a linear stationary dynamic system in which the input variables indicate the demand for products manufactured by a company. These variables, i.e. the demand, in this case, can either be a one-dimensional or multi-dimensional vector functions or they can be deterministic, stochastic or fuzzy. In this article, a system with fuzzy variables is presented.

2 DEFINING THE PROBLEM

Demand for a product should be met, if possible, by the current production. The difference between the current production and demand is the input function for the control process; the output function is the current stock/additional capacities. When the difference is positive, the surplus will be stocked, and when it is negative, the demand will also be covered by stock. In the case of a power supplier, stock in the usual sense does not exist (such as cars or computers, etc.); energy cannot be produced in advance for a known customer, nor can stock be built up for unknown customers. The demand for energy services is neither uniform in time nor known in advance. It varies, has its ups (maxima) and downs (minima), and can only be met by installing and activating additional proper technological capacities. Because of this, the function of stock in the energy supply process is held by all the additional technological potential/capacities large enough to meet periods of extra demand. The demand for energy services is not given and precisely known in advance. Demand is not given with explicitly expressed mathematical functions; it is a random process for which the statistical indicators are known. The system input is the demand for the products/services that a given subject offers. Any given demand should be met with current production. The difference between the current

capacity of production/services and demand is the input function for the object of control. The output function measures the amount of unsatisfied customers or unsatisfied demand in general. When this difference is positive, i.e. when the power supply capacity exceeds the demand, a surplus of energy will be made. When the difference is negative, i.e. when the demand surpasses the capacities, extra capacities will have to be added or, if they are not sufficient, extra external purchasing will have to be done. Otherwise, there will be delays, queues, etc. In the new cycle, there will be a system regulator, which will contain all the necessary data about the true state and that will, according to given demand, provide basic information for the production process. In this way, the regulation circuit is closed. With optimal control, we will understand the situation in which all customers are satisfied with the minimum involvement of additional facilities. On the basis of the described regulation circuit, we can establish a mathematical model of power supply control, i.e. a system of difference equations for discrete systems or a system or differential equations for continuous systems, [2]. For this model, the regulation circuit is given in Figure 1, [4]. The task is to determine the optimum production and stock/capacities so that the total cost will be as low as possible, [3].



Figure1: Regulation circuit of the power supply system

3. A MATHEMATICAL MODEL OF THE SYSTEM CONTROL

In the building of the model, we will restrict ourselves to a dynamic linear system, in which the input is a random process with known statistical properties. The system provides the output, which is, due to the condition of linearity, also a random process. These processes could be continuous or discrete. The model and its solution for continuous processes is obtained in [1] and for discrete processes in [5].

3.1 Continuous processes

Notations for $t \ge 0$ are as follows:

Z(t) - additional capacities (stocks) at a given time t,

u(t) - production at time t,

d(t) - demand for product at time t,

 $\lambda\,$ - lead time,

v(t) - delivery to storehouse at time t,

Q(t) - criterion function, complete costs.

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Let Z(t), u(t) and d(t) be stationary continuous random variables/functions; they are characteristics of continuous stationary random processes.

Now the system will be modelled with the known equations [1]:

$$\frac{dZ(t)}{dt} = v(t) - d(t) \tag{3.1}$$

$$v(t) = u(t - \lambda) \tag{3.2}$$

$$u(t) = -\int_{0}^{t} G(\tau) Z(t-\tau) d\tau$$
(3.3)

In the last equation, the function G(t) is the weight of the regulation that must be determined at optimum control, so that the criterion of the minimum total cost is satisfied. The lead time λ is the time period needed to activate the additional capacities in the power supply process. Assuming that the input variable demand is a stationary random process, we can also consider production and stock/additional capacities to be stationary random processes for reasons of the linearity of the system.

Let us consider the functions Z(t), u(t) and d(t) to be continuous stationary random processes. From this point of view, let us express the total cost, the minimum of which we are trying to define, with the mathematical expectation of the square of the random variables Z(t) and u(t):

$$Q(t) = K_z E(Z^2(t)) + K_u E(u^2(t))$$
(3.4)

Equations represent a linear model of control in which the minimum of the mean square error has to be determined.

 K_z and K_u are positive constant factors, attributing greater or smaller weight to individual costs. Both factors have been determined empirically for the product and are therefore in the separate plant [9]:

- K_z constant coefficient, dependent on activated resources, derived empirically,
- K_{u} constant coefficient, dependent on performed services and derived empirically.

3.2 Discrete processes

Similar to the continuous system, we have in similar notations in the discrete system.

Let us denote:

- Z(k) activated facilities (resources, stocks) at given moment (output),
- u(k) the amount of services performed (production) at a given moment,
- d(k) the demand for services at a given moment (input),
- κ time elapsed between the moment the data are received and the carrying out of a service,
- Q(k) criterion function, complete costs,
- K_z constant coefficient, dependent on activated resources, derived empirically,
- *K*_u constant coefficient, dependent on performed services and derived empirically,
- G(k) operator (weight) of regulation.

 $k \in \{0, 1, 2, ...\}$

The dynamic linear system will then be modelled with the following difference equations:

$$Z(k) - Z(k-1) = \psi \left[v(k) - d(k) \right], \ \psi \in \mathbf{R}^+$$
(3.5)

$$v(k) = u(k - \kappa), \ \kappa \in \mathbb{N}$$
(3.6)

$$u(k) = -\sum_{\kappa=0}^{\infty} G(\kappa) Z(k-\kappa)$$
(3.7)

$$Q(k) = K_{z}E\left\{\left(Z^{2}(k)\right)\right\} + K_{u}E\left\{\left(u^{2}(k-1)\right)\right\} minimum$$
(3.8)

4 FUZZY SYSTEM

Construction of a fuzzy system takes several steps [6], [7]: selection of decision variables and their fuzzification, establishing the goal and the construction of the algorithm (base of rules of fuzzy reasoning), inference and defuzzification of the results of fuzzy inference. A graphic presentation of a fuzzy system is given in Figure 2, [5].

The entire system demonstrates the course of inference from input variables against output; it is built on the basis of 'if-then' fuzzy rules. The fuzzy inference consists of three phases:

- 1. Fuzzification,
- 2. Fuzzy inference,
- 3. Defuzzification.



Figure 2: The fuzzy system

In our closed-loop model we designed a two-phase fuzzy system, given in Figure 3.



Figure 3: The two-phased fuzzy system

Let us assume that the demand *d* depends on [1]:

- the market area,
- the density of the area,
- the price,
- the season, and
- the uncertainty.

The demand is, in fact, the basic variable, on which the behaviour of all retailers depends. We assume that all expressions are fuzzy variables, market area, density of the area, price, season and uncertainty are input fuzzy variables, and demand is an output fuzzy variable in the first phase and in the same time also an input fuzzy variable for the second phase, Figure 4.



Figure 4: The two-phased fuzzy system with fuzzy demand in the first phase

4.1 Fuzzification

In the fuzzification phase, fuzzy sets for all fuzzy variables (input and output) must be defined, as well as their membership functions. Every fuzzy variable is presented by more terms/fuzzy sets. In this system, there are eight fuzzy variables: the market area, the density of the area, the price, the season, the uncertainty and the demand in the first phase and the demand, and the production and the capacity in the second phase.

The fuzzy variable demand is the output of the first rules block while simultaneously being the input for the second phase (i.e. rules block 2).

Fuzzy sets are given by terms below.

- In the first rules block:
 - a) the input fuzzy variable MARKET AREA is represented by: SMALL, BIG,
 - b) the input fuzzy variable DENSITY OF THE AREA is represented by: WEAK, MEDIUM, STRONG,
 - c) the input fuzzy variable PRICE is represented by: LOW, MEDIUM, HIGH,
 - d) the input fuzzy variable SEASON is represented by: LOW, HIGH,
 - e) the input fuzzy variable UNCERTAINTY is represented by: SMALL, MEDIUM, BIG, VERY_BIG,
 - f) the output fuzzy variable DEMAND is represented by: VERY_LOW, LOW, MEDIUM, HIGH, EXTREMELY_HIGH.

- In the second rule block:
 - g) the input fuzzy variable DEMAND is represented by: VERY_LOW, LOW, MEDIUM, HIGH, EXTREMELY_HIGH.
 - h) the input fuzzy variable PRODUCTION is represented by: LOW, MEDIUM, HIGH,
 - i) the output fuzzy variable CAPACITY is represented by: VERY_LOW, LOW, MEDIUM, HIGH, EXTREMELY_HIGH.

This fuzzy system is a two-phased system. The final output is CAPACITY (i.e. STOCKS) which depends on inputs DEMAND and PRODUCTION. This means that the control system is, in fact, the closed-loop system.

For every fuzzy set and for every fuzzy variable, we have to create membership functions, see Figures 5 to 12.

On the x-axis, the measures are given in units such as the number of customers, EUR, EUR/ kWh, MWh and so on, depending on the data. On the y-axis, membership is measured for every possible fuzzy variable and for every fuzzy set.

Due to the simplicity in this model, we suppose that all units for all fuzzy variables are given in relative measure, i.e. percentages from 0 to 100. Of course, the expert knows what, for example, 30% for 'market area' or 80 % of the 'price' etc. means.



Figure 5: MBF of 'MARKET'

Figure 6: MBF of 'DENSITY'



Figure 7: MBF of 'PRICE'

Figure 8: MBF of 'SEASON'



Figure 9: MBF of 'UNCERTAINTY'

Figure 10: MBF of 'DEMAND'



Figure 11: MBF of ' PRODUCTION '



Figure 12: MBF of 'CAPACITY'

4.2 Fuzzy inference

Fuzzy inference is a process in which a certain conclusion is derived from a set of fuzzy statements. In addition to linguistic variables, there are basic widgets of a fuzzy logic system as well as sets of rules that define the behaviour of a system. A single fuzzy rule (implication) assumes the form: *if x is A, then y is B,* where *A* and *B* are linguistic values defined by fuzzy sets on the universes of discourse *X* and *Y*, respectively. The *if* part of the rule is called the antecedent or premise, while the *then* part is called the consequent or conclusion. Variables *x* and *y* are defined by the sets *X* and *Y*.

With the assembly of a base of rules, the question always appears of how to obtain the rules. Usually, this is written down as a base of knowledge within the framework of 'if-then' rules by an expert for a definite system based on his own knowledge and experiences. An expert must also define entry and exit fuzzy functions, as well as their shape and position. However, it often occurs that the expert's knowledge is not sufficient, and he cannot define an adequate number of rules. Therefore, the procedures of forming or supplementation to the base of rules based on available numerical data were developed.

With fuzzy inference, we must put all values and facts in a definite order and connect them to the procedure of inference execution, so that will be feasible do so with a computer. This order is given as a list or system of rules.

In our work, we applied FuzzyTech software (FuzzyTech, 2001), [8]. In accordance with this software tool, 144 rules in the first phase (Rule block 1) and 15 rules in the second phase (Rule block 2) were automatically created. Some of them are represented in Tables 1 and 2.

IF					THEN	
DENSITY	MARKET	PRICE	SEASON	UNCER- TAINTY	DoS	DEMAND
WEAK	SMALL	LOW	LOW	SMALL	0.97	LOW
WEAK	SMALL	MEDIUM	HIGH	SMALL	1.00	LOW
WEAK	SMALL	HIGH	LOW	MEDIUM	0.64	VERY_LOW
WEAK	SMALL	HIGH	HIGH	VERY_BIG	1.00	LOW
WEAK	BIG	MEDIUM	LOW	BIG	1.00	LOW
WEAK	BIG	HIGH	HIGH	BIG	1.00	MEDIUM
MEDIUM	SMALL	LOW	LOW	SMALL	1.00	LOW
MEDIUM	SMALL	MEDIUM	HIGH	BIG	1.00	MEDIUM
MEDIUM	SMALL	HIGH	LOW	MEDIUM	0.75	LOW
MEDIUM	BIG	LOW	LOW	VERY_BIG	1.00	HIGH
MEDIUM	BIG	HIGH	HIGH	MEDIUM	0.63	MEDIUM
MEDIUM	BIG	HIGH	HIGH	VERY_BIG	0.20	HIGH
STRONG	SMALL	LOW	LOW	SMALL	1.00	MEDIUM

Table 1: Some rules Rules of the Rule Block 'RB1'

IF					THEN	
STRONG	SMALL	LOW	HIGH	MEDIUM	1.00	HIGH
STRONG	SMALL	HIGH	LOW	MEDIUM	1.00	LOW
STRONG	SMALL	HIGH	LOW	VERY_BIG	1.00	MEDIUM
STRONG	BIG	MEDIUM	HIGH	MEDIUM	0.98	HIGH
STRONG	BIG	MEDIUM	HIGH	VERY_BIG	0.73	EXTR_HIGH

IF		THEN		
DEMAND	PRODUCTION	DoS	CAPACITY	
VERY_LOW	LOW	1.00	VERY_LOW	
VERY_LOW	MEDIUM	1.00	LOW	
VERY_LOW	HIGH	1.00	LOW	
LOW	LOW	1.00	LOW	
LOW	MEDIUM	1.00	LOW	
LOW	HIGH	1.00	MEDIUM	
MEDIUM	LOW	1.00	LOW	
MEDIUM	MEDIUM	1.00	MEDIUM	
MEDIUM	HIGH	1.00	HIGH	
HIGH	LOW	1.00	MEDIUM	
HIGH	MEDIUM	1.00	HIGH	
HIGH	HIGH	1.00	HIGH	
EXTR_HIGH	LOW	1.00	HIGH	
EXTR_HIGH	MEDIUM	1.00	HIGH	
EXTR_HIGH	HIGH	1.00	EXTR_HIGH	

Table 2: Rules of the Rule Block 'RB2'

4.3 Defuzzification

Results from the evaluation of fuzzy rules is fuzzy. Defuzzification is the conversion of a given fuzzy quantity to a precise, crisp quantity. In the procedure of defuzzification, fuzzy output variables are changed into crisp numerical values. There are many procedures for defuzzification, which give different results.

The most frequently method used in praxis is CoM-defuzzification (the Centre of Maximum). As more than one output term can be accepted as valid, the defuzzification method should be a compromise between different results. The CoM method does this by computing the crisp output as a weighted average of the term membership maxima, weighted by the inference results, [6]. CoM is a type of compromise between the aggregated results of different terms *j* of a linguistic output variable, and is based on the maximum *Yj* of each term *j*.

As already mentioned, there are many methods of defuzzification that generally give various results. In our example, our model is created by FuzzyTech 5.55i software, and we use the Centre of Maximum (CoM) defuzzification method.

4.4 Optimisation

When the system structure is set and all elements of the system are defined, the model must also be tested and checked for its fit to data and for whether it produces the desired results. In our case, we have tasks with relatively simple optimization, because we have limited the problem to concrete conditions. We simplified the system so that it is well defined and gives the desired results. During optimization, we verify the entire definition area of input data. For each point of the definition area, we check whether the system is giving the desired result and if this result is logical. If we are not satisfied with the results, we can change any of the membership functions or any of the fuzzy inference rules.

For optimisation, there are various methods, such as trial and error, or using graphic tools that can visually demonstrate system activity. Such a graphic demonstration shows us the response to a change of data or change in the definition of the system elements, [8]. One of the most efficient methods is using neural nets during the neuro-fuzzy training to obtain good and regular results.

4.5 Neuro-fuzzy training

To optimise our results and to obtain a stable and robust fuzzy model, we have to perform neuro-fuzzy training, [9], [10]. At this point, help from an expert who knows the system very well is required. Suppose that we have a base of knowledge and we can start our neuro-fuzzy procedure. We have used FuzzyTech software's option for neuro-fuzzy learning in the first phase, [1]. Making 500 iterations in the phase of training (35 samples) and 500 iterations in the phase of checking (also 35 samples), we have changed the shapes of the membership functions for all fuzzy variables and also changed the weights (DoS) for some rules for fuzzy inference in Rules block 1. When comparing expert and fuzzy results, the statistical data are the following: the average deviation (expert results vs. fuzzy results) is 1.74%, 16 data points (samples) between 0 and 1%, 9 data points between 1 and 2%, 5 between 2 and 4% and 5 data points between 4 and 8%.

5 NUMERICAL EXAMPLE

When we have a robust fuzzy system, we can start numerical simulations. Using FuzzyTech software, we can simulate all possible situations interactively. Some results in Phase 1 are given in Table 3. The first five columns represent input fuzzy variables; the last column 'demand' as output of the fuzzy system is divided into two sub-columns. In the first, we can see crisp values of demand before neuro-fuzzy training and, in the second, values after neuro-fuzzy training.

					Demand	
Density	Market	Price	Season	Uncertainty	before training	after train- ing
0	0	100	0	0	0	1
50	50	50	50	50	60	50
100	100	1	100	100	100	100
30	30	80	80	50	50	42
70	50	90	20	90	45	58
60	100	40	50	50	77	73
60	60	30	50	50	71	67
70	70	90	90	70	72	76
90	50	100	100	100	74	78
50	30	80	20	20	31	28
19	33	49	66	66	53	52
39	70	60	60	50	52	54
78	78	39	78	78	88	80

Table 3: Some numerical results in phase 1

Of course, in the table, we merely have some results, but with the interactive simulation that is possible with FuzzyTech software, we can simulate every situation. The quality of the results depends on the expert who prepares a data file for the neuro-training procedure.

After optimization of the first subsystem (Phase 1), we can also run the fuzzy system in Phase 2. Some numerical results are presented in Table 4, in which the fuzzy variables of density, market, price, season, uncertainty and production are inputs, and the fuzzy variable capacity is the output of a two-phased fuzzy system. The fuzzy variable demand is an output in the first subsystem while simultaneously being an input to the second subsystem, i.e. the second phase in the entire fuzzy system.

Density	Market	Price	Season	Uncertainty	Production	Capacity
90	98	10	100	95	90	116
50	50	50	50	50	50	50
100	100	1	100	100	100	90
30	30	80	80	50	50	34
70	50	90	20	90	90	68
60	100	40	50	50	80	90
60	60	30	50	50	80	60
70	70	90	90	70	70	72
90	50	100	100	100	70	82
50	30	80	20	20	30	23
20	40	40	60	50	80	65
40	70	60	60	50	100	70
78	78	39	78	78	10	41

Table 4: Some numerical results in two-phased fuzzy system

6 CONCLUSION

A theoretical mathematical model of system control can also be used in an energy technology system and in all its subsystems. Input-output signals are discrete or continuous functions. For operations, many conditions have to be fulfilled. During the control process, a great deal of information must be processed, which can only be done if a transparent and properly developed information system is available. The solution, i.e. optimal control, depends on many numerical parameters. All data and numerical analysis can only be processed into information for control if high quality and sophisticated software and powerful hardware are available.

For the study of the structure, interrelationships and operation of a phenomenon with system characteristics, the best method is the general systems theory. When we refer to system technology as a synthesis of organization, information technology and operations, we have to consider its dynamic dimension when creating a mathematical model. As each such complex phenomenon makes up a system, the technology in this article is again dealt with as a dynamic system. Elements of the technological system compose an ordered entity of interrelationships and thus allow the system to perform production functions. During the control process, a great deal of information must be processed, which can only be done if a transparent and properly developed information system is available. Models of optimum control can also be used in the power station system.

The fuzzy approach in creating the mathematical model with which we are describing the system can be successful in the case that we have a good robust base of expert knowledge. With appropriate computer tools, an algorithm can be used for concrete numerical examples.

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