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Motion of rock masses on slopes

Gibanje skalnih gmot po pobočjih

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Abstract

This paper shows the different ways of how rock masses (stones, rocks, and blocks) move along slopes and for each different way of motion (free fall, bouncing, rolling, sliding, slowing down, lubrication, fluidization) adequate dynamic equations are given. Knowing the kinematics and dynamics of travelling rock masses is necessary for mathematical modeling of motion and by this an assessment of maximal possible rockfall runout distances as an example of a sudden and hazardeous natural phenomenon, threatening man and his property, especially in the natural environment.

Kratka vsebina

V prispevku prikazujemo različne načine gibanja skalnih gmot (kamnov, skal in blokov) po pobočjih in za vsak možni način gibanja (prosti pad, poskakovanje, kotaljenje, drsenje, ustavljanje, lubrikacija, utekočinjenje) podajamo ustrezne enačbe gibanja. Poznavanje kinematike in dinamike premikanja skalnih gmot je nujno za matematično modeliranje gibanja in s tem ocene maksimalnega možnega dosega podorov kot primera naglega in nevarnega naravnega pojava, ki ogroža človeka in njegovo imetje, predvsem v naravnem okolju.

Introduction

In analysis of risk related to rock falls and other large forms of rock slides and rock avalanches, including large rock falls, for a detailed mathematical modelling of the phenomena the knowledge of mathematical description (equations) of all the different ways of disintegrated rock mass motion is necessary (stones, rocks, and blocks). Usually, the aim of mathematical modelling is to estimate the maximum runout distance in space and, which is a part of risk analysis, providing hazard assessment against these dangerous phenomena (Petje, 2005). There are also empirical equations for the runout estimate of these phenomena (Petje et al., 2005a), which can be used at the regional scale (from 1:5,000 to 1:25,000). In considering the risk of rock falls at more detailed scales (from 1:500 to 1:2,000) it is usually necessary to make the use of more detailed mathematical models of rock fall motion (Petje et al., 2005b), which are based on the equations discussed in this paper.

As an introduction, let us discuss the difference between the motion of coherent and disintegrated material, which is analogous to the difference between the motion of a rigid body and motion of fluids (Erismann & Abele, 2001). Gravity is the governing principle in the motion of disintegrated material. It creates the vertical compression stress which increases from top to bottom. The tendency of the material to spread horizontally is due to the stress acting constantly or during collisions upon the surface of particles of the material. This creates the horizontal stress that is usually a function of the vertical stress, thus creating resistance against the vertical shear.

The resistance against shear deformations depends on the depth. Thus, the disintegrated material (rock mass) is neither isotropic nor homogeneous. The anisotropy of disintegrated material is shown when the material is under extension stress (strain). While the fluid stays a continuum as long as the static stress holds it together, the rock debris is much less capable of filling in the voids. The disintegrated material can be almost considered as coherent under stress, and as incoherent if subjected to extension forces. This is valid for all directions. When the material moves along the slope with decreasing inclination, it moves as if it were coherent. On a slope with increasing inclination, the material loses its cohesion and tends to be split into parts. In such cases the analogy with fluids given above is a rough estimate.

The main differences between the coherent and the disintegrated material regarding their reach on a slope can be summarised as follows:

- When the material is longitudinally compressed (on slopes with decreasing inclination), the difference between the motion of the coherent and disintegrated material is minimum.
- The dissipation of energy between the moving particles in disintegrated material occurs mainly by friction or collision between the particles at the expense of the potential (gravitational) energy. Such loss of internal kinetic energy does not occur in coherent material, which is why the coherent material moves further than the disintegrated one.
- In motion of the disintegrated material along undulated and inclined terrain, a part of the gravitational energy is lost due to the internal relative motion. The coherent material bridges the undulations on the inclined terrain and travels a larger distance.
- The disintegrated material loses its potential energy (due to reduction of the thickness of the material that moves) by lateral spreading of material on unconfined terrain and thus reducing the reach when compared to the coherent material.
- The disintegrated material moves easier through narrow cross sections like gorges or in sharp curves than the coherent one.
- The disintegrated material may form a fan.
- The coherent material has a tendency to destroy local barriers, while the disintegrated material rather tends to flow over them.

In the time of quantitative research in rockfalls and rocks slides, the motion velocity was based on the single events, that is, based on the time that elapsed between the start of rock mass motion and its stopping point. By scientific approach, Heim (1932) first started to study the velocity in rock fall motion.

Figure 1 shows the vertical cross section along the motion trajectory of the centre of gravity of rock mass from its release to its stop. Line E is the energy line between the centres of gravity. The slope $\tan \beta_e$ represents the average of the slope on which the centre of gravity has moved. If we presuppose the constant Coulomb's friction coefficient $\mu_e = \tan \beta_e$, this means that the mass has moved on the energy line and not on the actual slope, and thus has a constant velocity. The entire energy is released and transformed into heat. In reality, the centre of gravity is at a distance from the energy line by dz and the potential energy is transformed into kinetic energy. The velocity is calculated from the equation $v = \sqrt{2g \cdot dz}$. The velocity vector has the motion direction of the centre of gravity in a given moment. Velocity is thus not identical to its horizontal component (except in horizontal motion). Due to spreading and thus thinning it can be expected that all mass does not move with the same velocity, however, for the worst case scenario the velocity of the centre of gravity can be used. The weakest point of this method is also its biggest advantage: the simplicity of use. With the position of the centres of gravity we can determine the average coeffici-

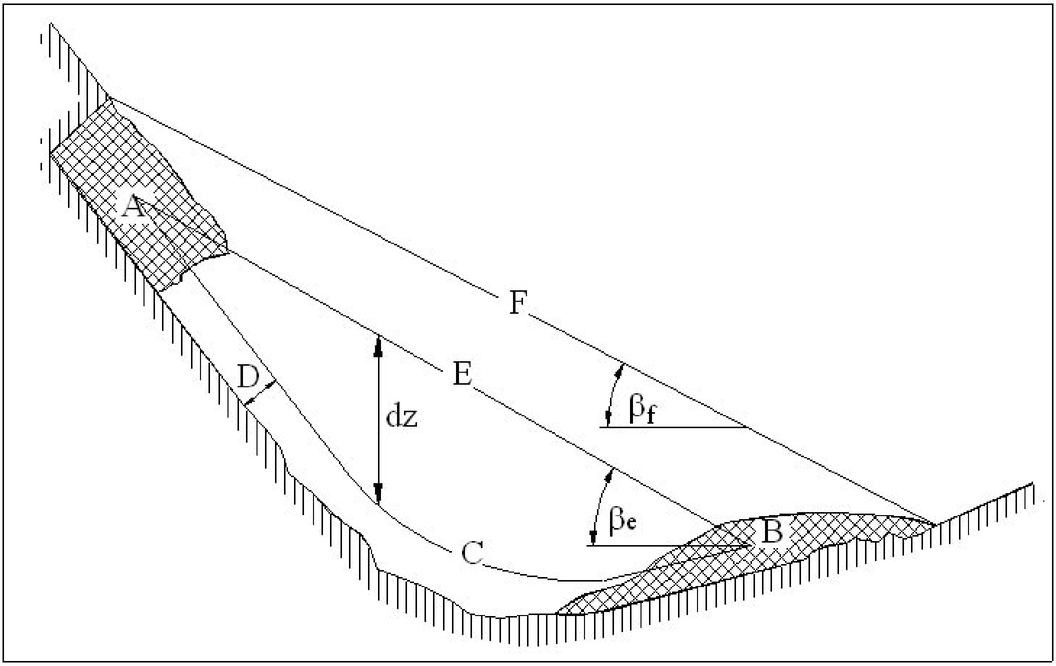

Fig. 1. Determining of rockfall mass velocity using the energy method. Points A and B represent centres of gravity of rockfall mass before and after the release, respectively, line C is flowpath of the centre of gravity, line E is energy line, and line F is average gradient or travel angle of the rockfall mass.

Slika 1. Določitev hitrosti podorne mase po energijski metodi. Točki A in B predstavljata težišči podorne mase pred in po premiku mase, C pot težišča, E energijsko črto in F povprečen naklon oziroma kot gibanja podorne mase.

ent of friction and thus also the calculation of the rock fall mass is correct.

In the next section a detailed description of ways of motion of rock mass along slopes with relevant mathematical representation of motion equations will be discussed, which are the basis for state-of-the-art mathematical simulation models of rockfalls.

Free fall

Free fall occurs when the slope (below the potential release area) is steeper than 76°, however, the boundary values given in the literature differ, also giving the value of 70° as the boundary value for free fall (Ritchie, 1963). The characteristic of free fall is motion in the air, without any contact with the ground. It can occur in the rolling or sliding phases, where a great change in the slope occurs, or upon impact with the ground. During the free fall, two types of motion occur. The first one involves the translation of the rock centre, which is analytically described with the quadratic equation, and the other involves rotation around the centre. Translation and rotation carry special significance because rocks are rarely round. Due to the rotation in the air, the rock rebounds upon impact into different directions as compared to its previous direction. Velocity is also affected by air friction, however air friction does not have significant effects on the rock motion (Bozzolo & Pamini, 1986). The next factor to influence the falling rocks and their trajectories is their mutual collision. However, the analysis of these effects is rather difficult (Azzoni et al., 1991).

During rock fall, gravitation is more significant than friction. If the Coulomb friction is presupposed as a working hypothesis for the start of motion, then acceleration a and gravity g are as follows:

$$\frac{a}{g} = \sin\beta - \mu \cdot \cos\beta \, , \qquad (1)$$

where β is slope angle and μ is coefficient of friction. Falling occurs at the relationship $a/g > 0.6$ (Figure 2) or with a slope angle greater than 45°–50° (Table 1).

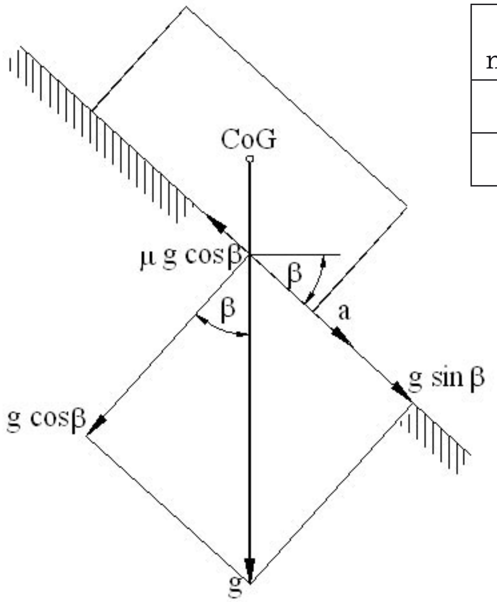

Fig. 2. Free falling should develop at the ratio $a / g > 0.6$ (CoG – centre of gravity).

Slika 2. Padanje naj bi se pojavilo pri razmerju $a / g > 0{,}6$ (CoG – težišče)

Until a falling rock has no contact with the ground, the forces are reduced to gravitation and aerodynamic effects only and they are unproblematic.

Equations describing motion are:

$$\frac{dv_x}{dt} = a_x = -v_x^2 \frac{c\partial_1 A}{2\partial_2 V} \rightarrow v_x = \int a_x dt \rightarrow x = \int v_x dt \quad (2)$$

$$\frac{dv_y}{dt} = a_y = \pm v_y^2 \frac{c\partial_1 A}{2\partial_2 V} - g \rightarrow v_y = \int a_y dt \rightarrow y = \int v_y dt$$

where a is the acceleration component, c is coefficient of friction, A is cross section perpendicular to the velocity vector, V is volume, g is gravitation, ∂_1 is air density and ∂_2 is rock density; x and y are the co-ordinates.

For practical calculations and to simplify the problem the following presuppositions can be assumed:

- Compressibility effects in the air can be disregarded if the fall velocity is less than 100 m/s.
- The effects of wind for large rock blocks (diameter of 1 m or more) can be disregarded, except in thunderstorms.
- Drag coefficient c depends upon the shape ($c \approx 0.5$ for spherical bodies).
- The aerodynamic lift can be disregarded, since it occurs under rare conditions.

slope angle – naklon pobočja β	45°	50°	55°	60°
a/g (μ = 0.2)	0.57	0.64	0.70	0.77
a/g (μ = 0.4)	0.42	0.51	0.59	0.67

Table 1. Free falling at slope angle β – a comparison between rock acceleration a and gravity acceleration g (Erismann & Abele, 2001).

Tabela 1. Padanje po pobočju z naklonom β – primerjava pospeška a s pospeškom prostega pada g (Erismann & Abele, 2001).

- In the case of a fast enough rotation the Magnus effect can be observed. Because of the boundary layer of the air around the spinning surface, an aerodynamic force perpendicular to both the vectors of velocity and spin is created. The result is a negative lift and the reach of the bounce is reduced.
- For the falling velocity of about 100 m/s vacuum trajectories can be predicted. This holds true for motion of large single rock blocks as well as for coherent mass motion and disintegrated mass that is not too "loose".
- In the loose disintegrated mass, large particles rebound farther than smaller ones. The result is the tendency for a two-dimensional deposition. Large rocks travel further and are deposited at the top of the debris.

If we disregard the air friction, the equations are simplified (Figure 3).

Acceleration:

$$a_x(t) = 0 \quad a_y(t) = -g \quad (3)$$

Velocity initial values in time t_0:

$$v_x(t_0) = v_{0x} \quad v_y(t_0) = v_{0y} \quad (4)$$

Co-ordinates of the initial position of the centre of gravity:

$$x(t_0) = x_A \quad y(t_0) = y_A + h_0 \quad (5)$$

By integration of acceleration in time:

$$x(t) = v_{0x}(t - t_0) + x_A \quad y(t) = -\frac{1}{2}g(t - t_0)^2 + v_{0y}(t - t_0) + (y_A + h_0) \quad (6)$$

equations of parabola are obtained:

$$y(t) = -\frac{1}{2}g(t - t_0)^2 + v_{0y}(t - t_0) + (y_A + h_0) \quad (7)$$

$$y(t) = -\frac{1}{2}g\frac{(x(t) - x_A)^2}{v_{0x}^2} + \frac{v_{0y}}{v_{0x}}(x(t) - x_A) + (y_A + h_0) \quad (8)$$

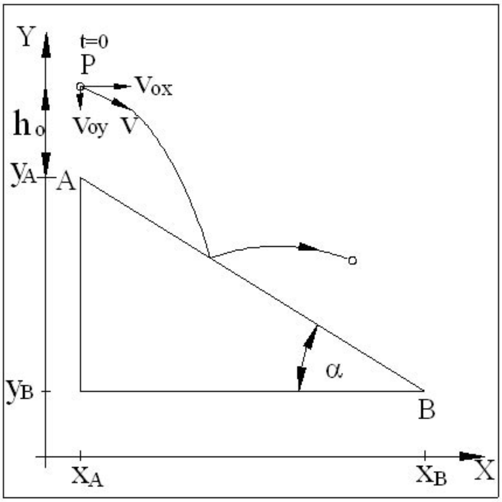

Fig. 3. Definition of a free fall for a body with lumped mass (Azzoni et al., 1995).

Slika 3. Definicija prostega pada za telo z maso, skoncentrirano v točki (Azzoni et al., 1995).

The energy equation can be written as follows:

$$v = \sqrt{2 \cdot g \cdot h} \tag{9}$$

$$E_{kin} = \frac{m \cdot v^2}{2} \tag{10}$$

Primary falling or free fall occurs, when the rock fall motion is undisturbed. Each disturbance in falling causes the motion to become secondary. Such disturbances can last a longer period of time. A rock fall that is completely undisturbed (free fall) is much more rare than falling, where rebounds from the not completely vertical wall occur. Because of the rebounds, the direction, length and shape of trajectory depend on many factors, which make it highly complex. The motion trajectory and the run-out zone depend on:

- slope (height, angle, orientation, shape);
- falling stones (size, shape, strength);
- characteristics of the bedrock (rebound coefficient, vegetation, gravel, terraces);
- angle of rock hitting the ground;
- deformations of rock and ground.

The factors that increase rock rebounds are hard bedrock, lack of vegetation, slopes with high rebound factors and slopes with the so-called "ski jumps", which cause the rocks to rebound far away from the foot of the hill and redirect the motion from the vertical fall into the horizontal direction. Upon impact, the rocks from very hard bedrock do not fall apart and on clean surfaces without gravel they rebound strongly.

Free fall, bouncing and rolling are strongly related. A falling rock will sooner or later start rolling. In turn, fast rolling of a body of irregular shape on uneven surface necessary entails impact and rebounds.

Bouncing

When the air trajectory (parabola) intersects with the slope (straight line), ground hit, rebound and bouncing occur. Upon first contact with the slope, rocks have a tendency to break. Regardless whether they crush or not, with the block size of 0.3 m^3 the energy loss is between 75 % and 85 %. Similar observations were made with other rockfalls for blocks of the size of 1 to 10 m^3 (Descoeudres & Zimmermann, 1987). Internal forces of reaction between two bodies during impact are much bigger than the active external forces (such as weight). A precise determination of internal forces is highly significant, however, it is hard to obtain. Experimental analyses of impacts show that the way of motion after the impact strongly depends on the block shape, slope geometry and quantity of dissipated energy that depends on the geomechanical characteristics of the block, slope and impact angle. Field observations have shown (Bozzolo et al., 1988) that due to the impacts of falling rocks, the rocks that were initially in place then started to move. For this probably a minimum slope angle is required, being the same as the dynamic coefficient of friction (arctg μ_r, where μ_r is the coefficient of friction). The rocks that are hit rotate upon impact and start sliding and then rolling.

The impact of the rock with the ground is a complex event to describe mathematically. The energy loss of the rock depends on the plastic deformations of the rock and the ground. Mostly, the models do not take into account breaking and crushing of rocks (Bozzolo, 1987), although this often occurs when the rock is brittle and the ground hard. In the mathematical models that consider the rock as lumped mass, the velocity after the contact can be determined based on the principle of preservation of the angular momentum in short time steps before and after the impact (with certain simplifications, such as elliptical shape of the block, impact in one point only, rotation around the point after impact), with the help of the coefficient of restitution ε, usually defined as the relationship of velocity before and after the impact:

$$\varepsilon = \frac{v}{v_0} . \tag{11}$$

In most models the normal component of velocity v_y is considered and it is assumed that the tangential velocity component is preserved. Thus the coefficient of restitution can be written as (Bozzolo, 1987):

$$\varepsilon = \left| \frac{v_y}{v_{0y}} \right| . \tag{12}$$

With models that consider the rock as a rigid body, two parameters are needed for the calculation of the change of momentum and the new angular velocity: coefficient of friction μ and coefficient of losses ε, written as (Bozzolo, 1987):

$$\varepsilon = \frac{\int_{t_1}^{t_2} F_y(t)dt}{\int_0^{t_1} F_y(t)dt} , \tag{13}$$

where F_y is the reaction force perpendicular to the impact surface, $t = 0$, t_1 and t_2 are the time at the beginning of the impact, at maximum ground pressure, and the time at the end of the impact, respectively. The equation is valid only at the central impact.

In the CADMA model (Azzoni et al., 1991), where the rock is taken as a rigid body, the energy-based coefficient of restitution is based on the principle of preservation of angle momentum in the time interval before and after the impact (Figure 4):

$$I \cdot \omega_0 + v_{0x} \cdot d_y - v_{oy} \cdot d_x = I \cdot \omega + v_x \cdot d_y - v_y \cdot d_x , \tag{14}$$

$$v_x = \omega_z d_y \quad v_y = -\omega_z d_x \tag{15}$$

$$d_y = Y_G - Y_P$$

$$d_x = X_G - X_P$$

where I is the moment of inertia around the centre, ω_0 and ω are angular velocities before and after the impact, v_{0x}, v_{oy}, v_x, v_y are velocity components before and after the impact and d_x and d_y are co-ordinates of the centre of gravity of the rock.

If equation (15) is inserted into the right hand-side of equation (14), the following equation is obtained:

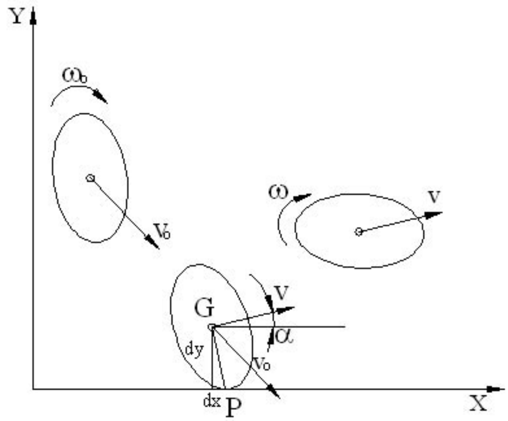

Fig. 4. Rock block before and after the impact (Azzoni et al., 1995).

Slika 4. Blok pred in po trku (Azzoni et al., 1995).

$$\omega = \frac{I \cdot \omega_0 + v_{0x} \cdot d_y - v_{0y} \cdot d_x}{I + d_x^2 + d_y^2} \tag{16}$$

The velocity component after the impact is obtained by inserting the value of ω, calculated in equation (16), into equation (15). The total kinetic energy per unit of mass after the impact can be described as:

$$K = \frac{1}{2}\left(I \cdot \omega^2 + v_x^2 + v_y^2\right) = \frac{1}{2} \cdot \omega^2 \cdot \left(I + d_x^2 + d_y^2\right) = \frac{1}{2} \cdot \omega^2 \cdot \left(I + r^2\right) \tag{17}$$

Now it is possible to describe the coefficient of restitution as:

$$\varepsilon = \frac{K}{K_0} = \frac{\omega^2}{2K_0}\left(I + r^2\right) \quad 0 \le \varepsilon \le 1 \tag{18}$$

$$r^2 = (d_x^2 + d_y^2) \tag{19}$$

where K_0 is the total kinetic energy during the contact, I is angular momentum around the mass centre, ω_0 and ω are angular velocities before and after the contact (Azzoni et al., 1991).

The relationships between energy losses and other variables are not exactly determined. In most cases, the effects of plastic deformations of the ground and the geometric configuration of the contact are taken into account by the so-called »contact functions«, describing the rock kinematics (velocity) or dynamics (energy) before and after contact. These functions are expressed as the coefficient of restitution and the coefficient of friction.

In bouncing, the trajectory is a parabola. The falling phase (motion along the parabola) is followed by a contact with the ground.

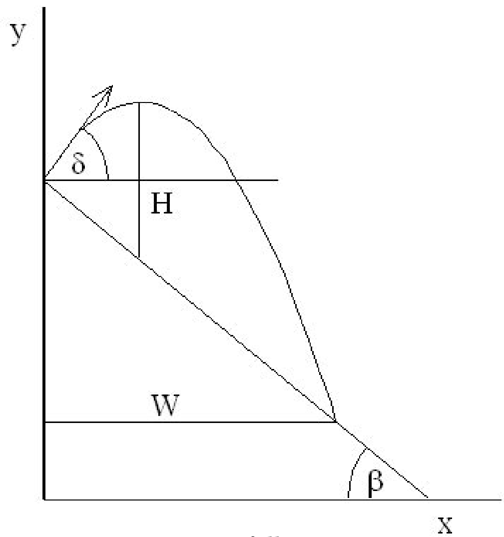

Fig. 5. Free-fall trajectory.
Slika 5. Trajektorija padanja.

This may be followed by another bounce, or change of the way of motion: sliding or rolling may occur. The way of motion after the contact with the ground depends on the slope and the block size. Some authors claim that bouncing is the prevailing type of motion at the slope angle in the interval between 45° and 63° (John & Spang, 1979). Observations have shown that large blocks hardly bounce at all, but rather roll. After a large bounce they usually fall apart. Smaller blocks, on the other hand, can bounce for a long time. The parabolic trajectory of fall (Figure 5) can be written in the following way:

$$x(t) = v_0 \cdot t \cdot \cos\delta \quad y(t) = v_0 \cdot t \cdot \sin\delta - \frac{g \cdot t^2}{2} \tag{20}$$

We obtain the equation of parabola:

$$y = x \cdot \tan\delta - \frac{x^2 \cdot g}{2 \cdot v_0^2 \cdot \cos^2\delta} \tag{21}$$

Height of bounce:

$$H = \frac{v_0^2 \cdot \sin\delta}{2}\left(\frac{\sin\delta}{g} + \cos\delta \cdot \tan\beta\right) \tag{22}$$

Length of bounce:

$$W = \frac{2 \cdot v_0^2}{g}\left(\sin\delta \cdot \cos\delta + \tan\delta \cdot \cos^2\delta\right) \tag{23}$$

Velocity:

$$v_x = v_0 \cos\delta = konst. \quad v_y = v_0 \cdot \sin\delta - g \cdot t \tag{24}$$

Between two bounces the total energy and the rotation energy remain constant. Based on this it can be assumed that with smaller height the potential energy decreases, and due to the increase of velocity (due to gravitation) translational energy increases. Each bounce ends, as free fall, with a contact with the ground or an obstacle. Plastic deformations of the block and the ground and the potential breakage of the block into smaller parts cause loss of energy. The loss is higher when:

- the surface roughness is higher as compared with the block size;
- the upper layer of the slope can be plastically deformed;
- the impact angle is steeper (Ritchie, 1963).

During contact with an obstacle, damage or total destruction (of a tree, house) may occur or the energy is transferred (to e.g. a still rock).

The velocity before contact (index *i*):

$$v_{xi}(t) = \frac{x(t)}{t} = \frac{v_{0x}(t-t_0) + x_A}{t} = v_{0x} \tag{25}$$

$$v_{yi}(t) = \frac{y(t)}{t} = -\frac{1}{2} g \frac{(t-t_0)^2}{t} + v_{0y}\frac{(t-t_0)}{t} + \frac{(y_A + h_0)}{t}$$

The velocity can be split into a component perpendicular to the ground, and the tangential component:

$$v_{in} = v_y(t)\cos\alpha - v_x(t)\sin\alpha \quad v_{it} = v_y(t)\sin\alpha - v_x(t)\cos\alpha \tag{26}$$

Velocity after contact:

$$v_{rn} = R_n \cdot v_{in} \quad v_{rt} = R_t \cdot v_{it} \tag{27}$$

If after rebound, the velocity is transformed into horizontal and vertical components, we obtain:

$$\begin{aligned} v_{xr}(t) &= v_{rn}(t) sin\,\alpha + v_{rt}\, cos\,\alpha \\ v_{yr}(t) &= v_{rt}(t) sin\,\alpha - v_{rt}\, cos\,\alpha \end{aligned} \tag{28}$$

The slope characteristics that determine surface material elasticity and thus the normal coefficient of restitution R_n, are a common characteristic of slope material and vegetation cover. Slope characteristics that determine the tangential coefficient of restitution R_t, are surface roughness, vegetation (mostly trees and bushes) and the number of trees on the slope that function as an obstacle (standing or fallen trees). The next factor determining the tangential coefficient of restitution is the radius of the falling rock, where for bigger rocks the effective slope roughness is smaller than for smaller rocks (Dorren et al., 2004).

The tangential coefficient of restitution has a value between 0.90 and 0.92 for very hard surfaces and bedrock and decreases with an increase of vegetation (Table 2). On fans that are covered with some vegetation, the coefficient is between 0.80 and 0.87. On softer ground covered with soil and loose rock the coefficient is between 0.78 and 0.80. The forest has a major effect on the decrease of the tangential coefficient of restitution: the normal coefficient of restitution is between 0.28 and 0.30 for soft surfaces, between 0.33 and 0.37 for slopes with large rocks, and 0.37 to 0.53 for smooth, hard surfaces (some give the value of up to 0.75); the coefficient of friction is between 0.2 and 0.35 for soft surfaces and 0.75 to 0.80 for rock slopes.

Until now, the bounce was treated in this paper as a one-time event, which is initiated in a steep slope and ends with a landing. Often, the bounce will not end with a landing, but with a second bounce. The reason lies in the complexity of the landing: irregular shapes, a body, possibly in rotation, hitting the non-vegetated (bare) ground, scree or non-consolidated soil. That is why the laboratory-determined coefficients have such a wide range, making it hard to determine the exact value.

In order to determine the initial conditions when the rebound occurs again, two parameters must be considered. The first parameter is the coefficient of elastic restitution:

$$J = \frac{v_p}{-u_p}, \tag{29}$$

where u_p is the velocity component perpendicular to the slope in landing, and v_p is the velocity component at the start of rebound. The other parameter is coefficient of friction μ of a non-rotating body in the process of rebounding. If we suppose the validity of the Coulomb's rule during the impact, the loss of velocity is obtained from the loss in momentum, which is a product of μ with the change of momentum perpendicular to the surface. Written in terms of velocity:

$$v_L = u_L - \mu(v_p - u_p), \tag{30}$$

where u_L and v_L are the longitudinal velocity components before and after the impact.

The following equation represents the space of time between two consecutive impacts:

$$\Delta t = -u_p \frac{J+1}{g \cdot \cos\beta} = -u_p \cdot \mu \frac{J+1}{g \cdot \sin\beta} = -2u_p \frac{J}{g \cdot \cos\beta}. \tag{31}$$

Interestingly, it can be assumed from $\mu = \tan\beta$ that motion through the air is much more energy rational than the motion on the ground, that is, within the validity of the Coulomb's Law.

Rolling

If the average slope angle is reduced, bouncing changes into rolling with or without bouncing. In rolling the rock is almost constantly in contact with the surface (Hungr & Evans, 1988). During motion in the form of rolling and bouncing, the rock is in fast rotation and only the rock areas with the biggest radius maintain contact with the ground. This is why the centre of gravity moves almost in a straight line, which is the most economical way of motion in relation to energy loss. The combination of rolling and short bounces is one of the most economical ways of motion.

Rolling is the prevailing way of motion:

- with long trajectories on a moderate slope (Evans & Hungr, 1993). The slope can reach up to 45°;
- in fans where the previous deposited blocks were smaller than the arriving rockfall blocks.

Rolling occurs when the vertical projection of the centre of gravity is outside the polygon defined by the contact surface. These conditions are identical to those describing the topple of mass. Rolling is in fact a repeated toppling, where geometry plays an important role.

A right polygonal prism will start rolling at slope angle β higher than 180/n and coefficient of friction $\mu > \tan\beta$ (Table 3). Smaller friction may lead to sliding.

If we take a look at a single body, the following four conditions define the way of motion:

- The possibility of rolling depends to a high degree on the body shape. The ability to roll is asymptotically increased by the approximation of the round cross-section, with the centre of gravity (mass centre) in its geometric centre.
- Once the body starts rolling, the rolling continues even under the conditions that would not allow rolling to start (unfavourable slope angle and coefficient of friction).
- Even with completely plane ground, non-circular rolling bodies start bouncing at a critical velocity.
- Next to the body shape, the critical ve-

Land cover pokrovnost	R_t	R_n	μ
Cliff faces – strma stena (60°–90°)	0.95	0.45	0.25
Steep bare slope – strmo golo pobočje (40°–60°)	0.90	0.40	0.45
Scree slope – gruščnato pobočje (30°–40°)	0.88	0.32	0.60
Bare slope – golo pobočje (0°–30°)	0.87	0.35	0.50
Meadow – travnik	0.87	0.30	0.55
Alpine shrubs – alpsko grmovje	0.85	0.30	0.60
Bushes – grmovje	0.83	0.30	0.65
Forest (200 trees/ha) – gozd (200 dreves/ha)	Up to/do 0.85 Average/srednji 0.67	0.28	1.00
Forest (300 trees/ha) – gozd (300 dreves/ha)	Up to/do 0.85 Average/srednji 0.57	0.28	1.50
Forest (500 trees/ha) – gozd (500 dreves/ha)	Up to/do 0.85 Average/srednji 0.38	0.28	2.00
Forest (700 trees/ha) – gozd (700 dreves/ha)	Up to/do 0.85 Average/srednji 0.27	0.28	2.20

Table 2. The tangential R_t and the normal coefficient of restitution R_n and the coefficient of friction μ for the different land-cover types (after Dorren & Seijmonsbergen, 2003).
Tabela 2. Tangencialni R_t in normalni R_n koeficient odboja ter koeficient trenja μ za različne pokrovnosti tal (povzeto po Dorren & Seijmonsbergen, 2003).

n – number of sides – št. stranic prizme	4	6	8	10	12	14	16
β (°) – minimum slope angle required for the start of rolling – minimalni naklon pobočja, da se prične drsenje	45.0	30.0	22.5	18.0	15.0	12.9	11.2
$\mu = \tan \beta$ (-) – minimum coefficient of friction required for the start of rolling at slope angle β – minimalni koeficient trenja, da se prične drsenje pri naklonu β	1.00	0.58	0.41	0.32	0.27	0.23	0.20
e/L (%) – e = deviation of centre of gravity with respect to a straight course – razdalja težišča od ravne črte, ki povezuje potovanje težišča; L = side length of polygon – dolžina stranice prizme (if – če $n \rightarrow \infty$: sphere – krogla and – in $e = 0$)	20.7	13.4	9.9	7.9	6.6	5.6	4.9

Table 3. Conditions for rolling of a regular polygonal prism (Erismann & Abele, 2001).
Tabela 3. Pogoji kotaljenja pravilne poligonalne prizme (Erismann & Abele, 2001).

locity depends on its size: for geometrically similar bodies it is proportional to the square root of the linear dimensions.

The process of rolling becomes a more complex one, when there are more bodies involved. Field configuration also affects motion. When surfaces are very uneven high acceleration of the body occurs as well as rotating motion. In this way, the rebound may cause rotation.

To define the velocity of rolling we must first have a look at the energy equation:

$$E_{kin} = E_{trans} + E_{rot} = \frac{m \cdot v^2}{2} + \frac{I \cdot \omega^2}{2} \quad (32)$$

where m is the weight of the released mass [kg], v is the velocity [m/s], I is the moment of inertia and ω is the angular velocity [s^{-1}].

$$v = \omega \cdot r \quad (33)$$

$$I = k^2 \cdot m \quad (34)$$

$$E_{kin} = \frac{m \cdot v^2}{2} \left(1 + \frac{k^2}{r^2}\right) \quad (35)$$

Spherical blocks show a minor loss of energy and have the greatest runout. For this »worst case scenario« the equations can be written as follows (Figure 6):

$$k^2 = \frac{2}{5} r^2 \quad (36)$$

$$E_{kin} = \frac{7 \cdot m \cdot v^2}{10} \quad (37)$$

The energy in point A at the start of motion is given as:

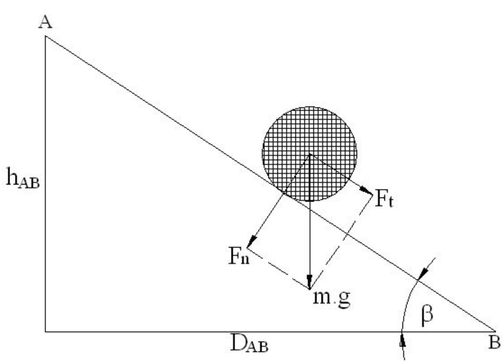

Fig. 6. Rolling spherical rock on a slope with gradient β (F_t translational force, F_n normal force).

Slika 6. Kotaleča okrogla skala na pobočju z naklonom β (F_t translacijska sila, F_n normalna sila).

$$E_{pot} + E_{kin} = m \cdot g \cdot h_{AB} + \frac{7 \cdot m \cdot v_A^2}{10} \quad (38)$$

The energy in point B at the end of the slope is given as:

$$E_{kin} + E_{tr} = \frac{7 \cdot m \cdot v_B^2}{10} + \mu_r \cdot m \cdot g \cdot \cos\beta_{AB} \cdot S_{AB} \quad (39)$$

After equalisation of both equations we obtain the velocity in the point B:

$$v_B = \sqrt{v_A^2 + \frac{10}{7} \cdot g \cdot \left(h_{AB} - \mu_r \cdot D_{AB}\right)} \quad (40)$$

If we compare the translational force in the point B in sliding and rolling (at $\mu_r = \mu_g = 0$), it becomes clear that the translational force in rolling is smaller by 15 % than the one in sliding.

F_{tr} friction force [N]
E_{tr} energy of friction [J]
F_n normal force [N]
E_{pot} potential energy [J]
E_{kin} kinetic energy [J]
v_A velocity in point A [m/s]
v_B velocity in point B [m/s]
m weight of released mass [kg]
g earth gravity [m/s^2]
μ_r friction coefficient in rolling [–]
h_{AB} difference in height between A and B [m]
D_{AB} horizontal distance between A and B [m]
S_{AB} slope distance between A and B [m]

The friction coefficient in rolling μ_r corresponds in motion of a spherical rock to the tangent of slope angle and thus to the tangent of friction angle of the rockfall material (Scheidegger, 1975). Size of μ_r largely depends on the following parameters (Bozzolo, 1987; Chau et al., 2002):

- Shape of a block: spherical blocks indicate smaller friction in rolling than sharp-edged bodies, which also stop sooner. Flat, and rarely also prism-shaped blocks, often travel great distances if they move in similar way to a bicycle (Azzoni et al., 1991).
- Relative roughness (relationship between block size and roughness height affecting the rolling block): the impact between a rolling block and talus scree does not significantly change velocity, if the block is sufficiently larger than the talus scree. When, however, the block is of similar dimensions as the talus scree, the impact bears significant effect on the block velocity.

- Mechanic slope characteristics: energy loss in rolling depends on whether elastic and/or plastic slope deformations occur.
- Slope angle.

The dynamic friction angle can be expressed as (Kirkby & Statham, 1975; Statham, 1976; after Meißl, 1998):

$$\tan\phi_{\mu d} = \tan\phi_0 + k\frac{d}{2R}, \tag{41}$$

where ϕ_0 is the friction angle (°), k is the shape coefficient (between 0.17 and 0.26), d is the median diameter of rocks on the slope (m) and R is the diameter of the falling block (m).

Energy losses in rolling are mainly smaller than in sliding and falling. Rolling blocks usually reach the longest runout. However, since the rolling blocks are not perfect spheres and the surface is not perfectly even, rolling rarely occurs. Usually, there is a combination of rolling and bouncing. This is probably the most complex way of motion of rock mass and it represents the greatest danger from the dynamic point of view. This way of motion enables the block to collect a lot of energy and the resulting trajectory of motion is extremely hard to define.

Sliding and runout

Sliding is the next mechanism of motion to be discussed, which occurs only in initial and final phases of motion. If the slope gradient increases, the sliding rock starts falling, rolling or bouncing (Bozzolo, 1987). If along sliding pathway the slope gradient does not change, the motion, due to energy loss, usually stops. The equations describing sliding are firstly used at the start of motion when the body has potential and kinetic energy (as also shown on Figure 6 for rolling):

$$E_{pot} + E_{kin} = m\cdot g\cdot h_{AB} + \frac{m\cdot v_A^2}{2} \tag{42}$$

After Coulomb's Law the friction force is written as:

$$F_{tr} = \mu_g \cdot F_n = \mu_g \cdot m\cdot g\cdot\cos\beta_{AB} \tag{43}$$

and energy as:

$$E_{tr} = F_{tr}\cdot S_{AB} = \mu_g\cdot m\cdot g\cdot\cos\beta_{AB}\cdot S_{AB} \tag{44}$$

When the body reaches point B, it has the following energy:

$$E_{kin} + E_{tr} = \frac{m\cdot v_B^2}{2} + \mu_g\cdot m\cdot g\cdot\cos\beta_{AB}\cdot S_{AB} \tag{45}$$

After the law of conservation of energy, the energy in point A and point B can be equalized, and we obtain the velocity in point B:

$$v_B = \sqrt{v_A^2 + 2\cdot g\cdot\left(h_{AB} - \mu_g\cdot D_{AB}\right)} \tag{46}$$

If the following condition is fulfilled:

$$v_A^2 \le 2\cdot g\cdot\left(\mu_g\cdot D_{AB} - h_{AB}\right), \tag{47}$$

the body stops. The horizontal distance can be written as:

$$D_{AB} = \frac{v_A^2}{2g\left(\mu_g - tg\beta_{AB}\right)} \tag{48}$$

F_{tr} friction force [N]
F_n normal force [N]
E_{pot} potential energy [J]
E_{kin} kinetic energy [J]
E_{tr} energy of friction [J]
v_A velocity in point A [m/s]
v_B velocity in point B [m/s]
m weight of released mass [kg]
μ_g friction coefficient [–]
h_{AB} difference in height between A and B [m]
D_{AB} horizontal distance between A and B [m]
S_{AB} slope distance between A and B [m]
β_{AB} slope gradient between A and B [rad]

During sliding the body is in constant contact with the ground. The sliding occurs only if the friction coefficient is smaller than the tangent of the slope angle. In order for a rock to travel the distance from point A to point B, the condition must be met that the velocity component runs parallel to the following segment:

$$\left|\frac{v_x}{v_y}\right| \ge \mu_g \tag{49}$$

A sliding body can in a specific segment:

- stop by itself;
- slide to the end of the segment;
- or the sliding changes into rolling.

In order for a body to stop, the kinetic energy must equal 0. For the transition from the sliding phase into the rolling phase, the kinetic energy must be larger than the potential energy $mg\Delta h$, where Δh is the vertical drop of the centre of gravity of the body: $E_{kin} \ge mg\Delta h$.

Two blocks sliding next to each other must overcome twice as much friction than one block only. In this particular example, it can be maintained, in relation to the Coulomb's Law, that the total friction remains the same if two blocks move so that one is on top of the other, instead of being next to each other. In order to assume that this postulate is true, there must exist similar conditions related to friction. The basic idea of explanation for technical materials is that no surface can be perfectly even. More pressure first means more and larger removed fragments and not higher stresses. The factor of proportionality μ can be determined as a function of shear stress and hardness (Bowden & Tabor, 1964). Deformational changes of the material microstructure close to its surface occur, resulting in the instability of the material to the local shear forces (Spang, 1987). This causes the particles of the deformed material to move, which results in the production of crushed or ground ultrafine material. When compression stresses exceed the resistance of the material, crushing locally destroys rock asperities and crushed fragments are partly pushed aside. The geometries of the two surfaces in contact correspond better to each other.

The Coulomb's rule provides useful basis for a quantitative approach to sliding (Bowden & Tabor, 1964; Rigney et al., 1984):

- sliding between two blocks usually occurs at the contact surface composed of an abundance of fragments, which constantly adapt to the crushing (and relative transport) and compression under almost critical pressure;
- reasonably accurate calculations of energy losses with the mechanisms mentioned above are not possible, since we do not have the knowledge on the relevant parameters;
- energy loss in crushing is independent of velocity;
- energy losses in accelerated crushing are proportional to the velocity squared.

At the initial stage, the rockfall mass has potential energy that changes into:

- kinetic energy;
- internal energy (energy due to friction);
- energy for internal crushing of mass.

Between the moving rockfall mass and slope surface, energy loss occurs due to:

- friction;
- plastic deformations of the contact zone;
- non-plastic components during the falling stage.

The rock stops when the kinetic energy equals zero. This occurs during constant loss of energy or due to the total transfer of energy to the obstacle (Spang, 1987).

In comparison to other types of motion, in free fall occurs a total transfer of potential energy into kinetic energy and thus to high velocities. In long fall trajectories, however, the efffect of air friction cannot be neglected. As a rule, the kinetic energy changes from rolling to bouncing and sliding (Figure 7).

Lubrication

Lubrication is a technical (tribological) term describing the reduction of frictional resistance by a third medium (called lubricant) between two separate surfaces in relative motion to each other. The lubricant can be a liquid (e.g. water or oil), a solid matter (e.g. graphite) or a gas (e.g. air). It may be present due to its own coherence (oil) or the hydraulic pressure. Hypothetic possibilities of lubrication in rock motion are as versatile as in its technical uses. The four major groups of mechanisms depending on the type of the third medium (water, snow, ice, mud, clay, dust etc.) can be divided into four classic governing elements that were defined already by Empedocles (after Erismann & Abele, 2001): »water« (for fluids); »air« (for gases); »fire« (for heat); and »earth« (for solid matter).

The key word that is used most often in relation to motion of a rock mass down-slope is »water«. Up until now, water has always been considered as a destructive force. The significant characteristic of water is its low viscosity. Low viscosity means that it will leave almost unhindered relative motion of the involved bodies, however, it will flow away very quickly under overload, if it is not present in sufficient quantity. When water should take the role of a lubricant in rockslides, one must understand under what conditions can water take on the overload during periods of time long enough to contribute to the efective coefficient of friction.

The volume of potentially available liquid (pore water or fluid mud) is limited by the pore volume between the large particles of the fill. In the first assumption, if particles may be taken as equal in size and spherical, the pore volume is around 26 % of the total

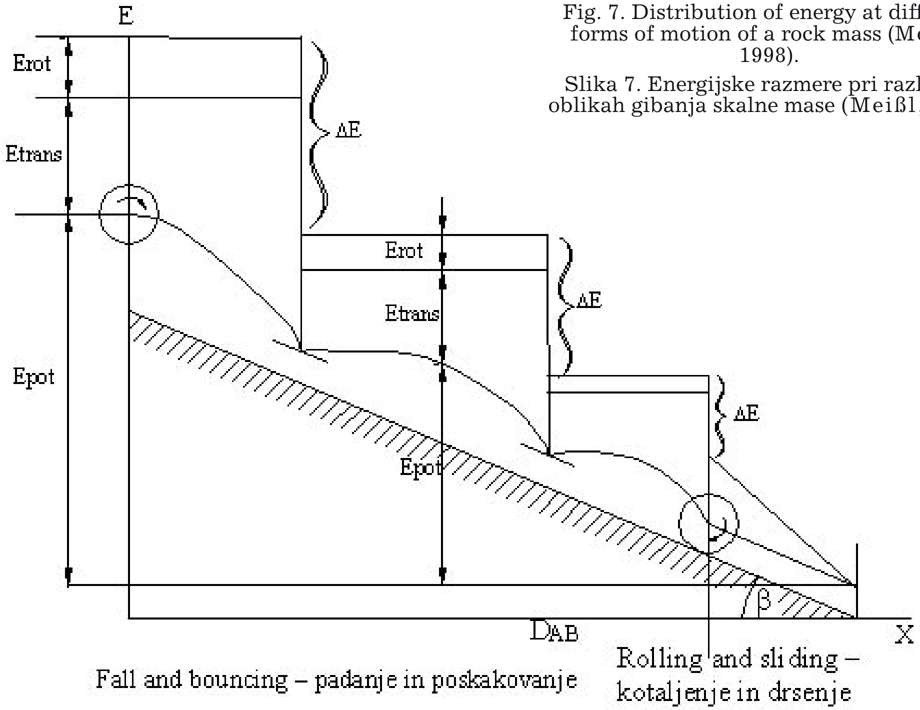

Fig. 7. Distribution of energy at different forms of motion of a rock mass (Meißl, 1998).

Slika 7. Energijske razmere pri različnih oblikah gibanja skalne mase (Meißl, 1998).

volume of the fill. In reality, the pore volume is some percent of the total volume of the fill (the lubricant layer can be several metres thick). There is a question of the actual availability of the lubricant under the compression of the sliding mass. In the sliding mass, the filling material moves downwards, which is the result of elastic deformation, disintegration, redistribution of particles or a combination of these mechanisms.

Fluidization

The main difference between lubrication and fluidization is in the location of the governing mechanism achieving a reduction of resistance. In the case of lubrication, the mechanism is concentrated close to the boundary between the moving mass and the ground. In fluidization the mechanism is active in a much larger part, normally in the entire thickness of the moving mass. The geomorphological consequences of these differences are clear. In motion of a fluidized mass, relative displacements will occur in its entire volume. Lubrication, on the other hand, causes only moderate relative displacements between the parts of the disintegrated mass, however, the mass retains its »shape«. This is important in order to exclude the suspicion that fluidization of rockslides often occurs during their motion downslope. In some cases, fluidization can be considered as multi-layered lubrication, with layers parallel to each other and to the ground. Most commonly in rockslides, the fluidization occurs with water.

Let us imagine a multi-layered mass with interchanging layers of water and rock (Figure 8). The thickness of water layer is e_w, the thickness of rock layer e_r and the respective densities are ∂_w and ∂_r. First, let us assume the negligible rugosity of impermeable rock layers (roughness height is small as compared to the thickness of water layer). In turn, the layer thickness e_w and e_r is small compared to the entire thickness of mass H, so that the sliding mass can be described as a Newtonian fluid with averaged density:

$$\partial = \frac{\partial_r e_r + \partial_w e_w}{e_r + e_w} \tag{50}$$

and averaged viscosity:

$$E = E_w \frac{e_r + e_w}{e_w}, \tag{51}$$

where $E_w = 0.00134 \mathrm{kgm^{-1}s^{-1}}$ is the viscosity of water. This highly simplifies the quantitative treatment.

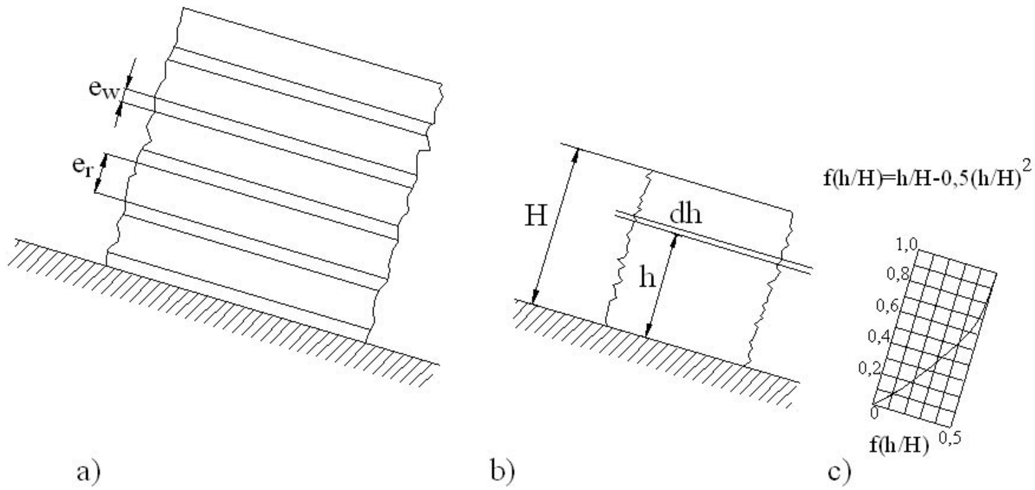

Fig. 8. Fluidisation by water (Erismann & Abele, 2001):
a) Simplified model of a water saturated layer, where e_r is the thickness of the impermeable rock layer and e_w is the thickness of the water layer, respectively;
b) Homogenous viscous layer of depth *H* (see also eqs. (52) and (53));
c) Velocity distribution valid for all mass having a constant viscosity.

Slika 8. Utekočinjenje z vodo (Erismann & Abele, 2001):
a) Poenostavljen model z vodo saturirane plasti, kjer je e_r debelina plasti neprepustne kamnine in e_w debelina plasti vode;
b) Homogena viskozna plast debeline *H* (glej tudi enačbi (52) in (53));
c) Porazdelitev hitrosti, ki velja za vse mase, ki imajo konstantno viskoznost.

For motion at constant velocity, the shear stress at the level *h* can be expressed by the gravitational acceleration *g* and viscosity *E*:

$$(H-h)g\partial \sin\beta = s = E\frac{du}{dh} \qquad (52)$$

β is the slope angle and *u* is the velocity at level *h*. The differential equation can be solved and we obtain:

$$u = f\left(\frac{h}{H}\right)gH^2\frac{\partial}{E}\sin\beta\,, \qquad (53)$$

where f(*h*/*H*)=*h*/*H*–0.5(*h*/*H*)^2 is the dimensionless parameter. The maximum velocity at the surface of the siding mass is obtained at f(*h*/*H*)=1/2.

If we consider a mass with a finite breadth *B* (this is not necessarily the total width of the mass, since the mass may have longitudinal cracks), the water may escape from the mass. The water layers thus narrow down, and the resistance increases. The following equation describes the laminar flow of the escaping water:

$$\frac{de_w/dt}{e_w} = \frac{2}{3}\left(\frac{e_w}{B}\right)^2\frac{p}{E_w}\ . \qquad (54)$$

On the left hand-side of equation (54) is the velocity that causes the water layer to narrow down (expressed as the relative loss in layer thickness per time unit), and on the right hand-side of equation (54) there is a term related to the geometry and the relationship between the driving force (pressure *p*) and the braking force (viscosity E_w).

When fluidization is suspected, the following checklist of five points is used to check this possibility:

- general behaviour of the disintegrated mass saturated with water; as long as flow is laminar and the mass is capable of retaining water, the behavior is like that of a viscous liquid (equation (53)).
- In such cases it is necessary that the velocity increases from the bottom to the top of the mass. The main consequence of the mass with a finite length is the reverse order of elements and fast loss of layer thickness.
- A strong effect of size is given by the thickness *H* squared (equation (53)).
- Based on equations (54) and (55) the relative loss of water (and increase of effective viscosity) is slower with larger mass than smaller mass.

- The absolute extension of transition of width e_w exerts the critical influence on the duration of functional life.

Conclusions

A detailed description of motion of rock mass downslope is mathematically demanding, and the knowledge of the motion mechanisms is essential. These mostly depend on slope angle and slope characteristics as well as rock characteristics in the release area, thereby also the characteristics and volume of rock mass in motion. Only good knowledge of the terrain can enable the selection of a proper mathematical description of the motion, which must be based on simplified premises. A potential field investigation of the motion of rock mass is time-consuming, difficult and costly. Most often, empirically acquired values of coefficients from field observations of rockfall and rockslide motion or measurements of their deposits are built into mathematical models.

The paper shows the prevailing ways of rock mass motion and for each way a corresponding mathematical description is given. The review is intended for a better understanding of kinematics and dynamics of these dangerous phenomena and should serve as a decision-support tool when deciding, whether in a particular case of establishing hazard areas the simplified empirical models of rockfall runout should be used (review given in Petje et al., 2005a), or whether it is necessary to use physically more accurate models with several parameters discussed in this paper, which are used in computer simulations of rock mass motion (review given in Petje et al., 2005b).

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