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# 15 Degrees of Freedom of Massless Boson and Fermion Fields in Any Even Dimension

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**Abstract.** This is a discussion on degrees of freedom of massless fermion and boson fields, if they are free or weakly interacting. We generalize the gauge fields of the *spin-charge-family* to the gauge fields of all possible products of $\gamma^a$'s and of all possible products of $\tilde{\gamma}^a$'s, the first taking care in the *spin-charge-family* theory of the spins and charges ($S^{ab}\ \omega_{abc}$) of fermions, the second ($\tilde{S}^{ab}\ \tilde{\omega}_{abc}$) taking care of families.

**Povzetek.** Avtorja diskutirata v prispevku prostostne stopnje brezmasnih prostih ali šibko sklopljenih fermionskih in ustreznih bozonskih polj, v primeru, da dovolita, da so bozonska polja umeritvena polja vseh produktov Cliffordovih operatorjev $\gamma^a$ in umeritvena polja vseh operatorjev $\tilde{\gamma}^a$. Produkti dveh Cliffordovih operatorjev $\gamma^a$ določajo v teoriji *spina-nabojev-družin* naboje ene družine kvarkov in leptonov, produkti dveh Cliffordovih operatorjev $\tilde{\gamma}^a$ pa družine kvarkov in leptonov.

## 15.1 Introduction

The purpose of this contribution to the Discussion section of this Proceedings to the Bled 2015 workshop is to hopefully better understand: : **a.** Why is the simple starting action of the *spin-charge-family* theory doing so well in manifesting the observed properties of the fermion and boson fields? **b.** Under which condition would more general action lead to the starting action of Eq. (15.1)? **c.** What would more general action, if leading to the same low energy physics, mean for the history of our Universe? **d.** Could the fermionization procedure of boson fields or the bosonization procedure of fermion fields, discussed in this Proceedings for any dimension $d$ (by the authors of this contribution, while one of them, H.B.F.N. [5], has succeeded with another author to do the fermionization for $d = (1+1)$), help to find the answers to the questions under **a. b. c.**?

In the *spin-charge-family* theory of one of us (N.S.M.B.) [1–4], which offers the possibility to explain all the assumptions of the *standard model*, with the appearance of families, the scalar higgs and the Yukawa couplings included, as well as the matter-antimatter asymmetry in our universe and the appearance of the dark matter, a very simple starting action for massless fermions and bosons in $d =$

$(1+13)$ is assumed. In this action

$$\mathcal{A} = \int d^dx\, E\, \frac{1}{2}\, (\bar{\psi}\, \gamma^a p_{0a} \psi) + h.c. + \int d^dx\, E\, (\alpha\, R + \tilde{\alpha}\, \tilde{R})\,, \tag{15.1}$$

where $p_{0a} = f^{\alpha}{}_a p_{0\alpha} + \frac{1}{2E}\{p_\alpha, E f^{\alpha}{}_a\}_-$, $p_{0\alpha} = p_\alpha - \frac{1}{2} S^{ab}\omega_{ab\alpha} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\alpha}$, $R = \frac{1}{2}\{f^{\alpha[a} f^{\beta b]}\ (\omega_{ab\alpha,\beta} - \omega_{ca\alpha}\, \omega^c{}_{b\beta})\} + h.c.$, $\tilde{R} = \frac{1}{2}\{f^{\alpha[a} f^{\beta b]}\ (\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha}\, \tilde{\omega}^c{}_{b\beta})\} + h.c.$, the two kinds of the Clifford algebra objects, $\gamma^a$ and $\tilde{\gamma}^a$,

$$\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab} = \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+\,, \tag{15.2}$$

which anticommute, $\{\gamma^a, \tilde{\gamma}^b\}_+ = 0$ and determine one of them spins and charges of spinors, another determines families. Here [1] $f^{\alpha[a} f^{\beta b]} = f^{\alpha a} f^{\beta b} - f^{\alpha b} f^{\beta a}$. There are correspondingly for spinors two kinds of the infinitesimal generators of the groups - $S^{ab}$ for $SO(13,1)$ and $\tilde{S}^{ab}$ for $\widetilde{SO}(13,1)$. The generators $S^{ab} = \frac{i}{4}(\gamma^a\, \gamma^b - \gamma^b\, \gamma^a)$, $\tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a\, \tilde{\gamma}^b - \tilde{\gamma}^b\, \tilde{\gamma}^a)$, determine in the theory the spin and charges of fermions, $S^{ab}$, and the family quantum numbers, $\tilde{S}^{ab}$.

The curvature $R$ and $\tilde{R}$ determine dynamics of gauge fields.

The infinitesimal generators of the Lorentz transformations for bosons operate as follows $\mathcal{S}^{ab}\, A^{d\dots e\dots g} = i\, (\eta^{ae}\, A^{d\dots b\dots g} - \eta^{be}\, A^{d\dots a\dots g})$.

We discuss in what follows properties of free massless fermion fields, Sect. 15.1.1, of free massless boson fields and suggest the interaction among fermions and bosons, which fulfill the Aratyn-Nielsen theorem [5], but is in general not gauge invariance.

### 15.1.1 Properties of general fermion fields

Let us make a choice of one kind of the Clifford algebra objects, let say $\gamma^a$'s, and express correspondingly the linear vector space of fermions as follows

$$\bar{\Psi}(\gamma) = \psi + \sum_{k=1}^{d} \psi_{a_1 a_2 \dots a_k}\, \gamma^{a_1}\gamma^{a_2}\dots\gamma^{a_k}\,, \quad a_i \le a_{i+1}\,. \tag{15.3}$$

We could as well make a choice of $\tilde{\gamma}^a$'s instead of $\gamma^a$'s. We define that operation of $\gamma^a$ and $\tilde{\gamma}^a$ on such a vector space is understood as the *left* and the *right* multiplication, respectively, of any Clifford algebra object. Let $f(\gamma)$ be one of the (orthogonal) fermion states in the Hilbert space. The *left* and the *right* multiplication

[1] $f^{\alpha}{}_a$ are inverted vielbeins to $e^a{}_\alpha$ with the properties $e^a{}_\alpha f^\alpha{}_b = \delta^a{}_b$, $e^a{}_\alpha f^\beta{}_a = \delta^\beta_\alpha$, $E = \det(e^a{}_\alpha)$. Latin indices $a, b, .., m, n, .., s, t, ..$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, .., \mu, \nu, ..\sigma, \tau, ..$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a, b, c, ..$ and $\alpha, \beta, \gamma, ..$ ), from the middle of both the alphabets the observed dimensions $0, 1, 2, 3$ ($m, n, ..$ and $\mu, \nu, ..$), indexes from the bottom of the alphabets indicate the compactified dimensions ($s, t, ..$ and $\sigma, \tau, ..$). We assume the signature $\eta^{ab} = diag\{1, -1, -1, \cdots, -1\}$.

can be understood as follows

$$\gamma^a\, f(\gamma)\, |\psi_0\ >: = (\, a_0\, \gamma^a \ + a_{a_1}\, \gamma^a\, \gamma^{a_1} + \\ a_{a_1 a_2}\, \gamma^a\, \gamma^{a_1} \gamma^{a_2} + a_{a_1 \cdots a_d}\, \gamma^a\, \gamma^{a_1} \cdots \gamma^{a_d}\, )\, |\psi_0\ >\,, \\ \tilde{\gamma}^a\, f(\gamma)\, |\psi_0\ >: = (\, i\, a_0 \gamma^a - i\, a_{a_1} \gamma^{a_1}\, \gamma^a + i\, a_{a_1 a_2} \gamma^{a_1} \gamma^{a_2}\, \gamma^a + \cdots + \\ i\, (-1)^d\, a_{a_1 \cdots a_d} \gamma^{a_1} \cdots \gamma^{a_d}\, \gamma^a\, )\, |\psi_0\ >\,, \quad (15.4)$$

where $|\psi_0\ >$ is a vacuum state.

Eq. (15.3) represents $2^d$ internal degrees of freedom, that is $2^d$ basic states. Let us arrange the basis to be orthogonal in a way that operators $S^{ab}$ transform $2^{\frac{d}{2}-1}$ members of these basic states among themselves. They represent one family. The operators $\tilde{S}^{ab}$ transform each family member into the same family member of one of $2^{\frac{d}{2}-1}$ families.

There are obviously four such groups of $2^{\frac{d}{2}-1}$ families with $2^{\frac{d}{2}-1}$ family members ($2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1} \times 2^2 = 2^d$). These four groups differ in the eigenvalues of the two operator of handedness, $\Gamma^{(1+(d-1))}$ and $\widetilde{\Gamma}^{(1+(d-1))}$,

$$\Gamma^{(1+(d-1))} = (-i)^{\frac{d-2}{2}}\, \gamma^{a_1} \gamma^{a_2} \ldots \gamma^a_d\,, \\ \widetilde{\Gamma}^{(1+(d-1))} = (-i)^{\frac{d-2}{2}}\, \tilde{\gamma}^{a_1} \tilde{\gamma}^{a_2} \ldots \tilde{\gamma}^a_d\,, \\ a_k < a_{k+1}\,. \quad (15.5)$$

The eigenvalues of $[(\Gamma^{(1+(d-1))}, \widetilde{\Gamma}^{(1+(d-1))}]$ are $= [(+,+),(-,+),(+,-),(-,-)]$. Each of the groups can be extracted from the basis due to requirement

$$\text{A.}\ \ (1 - \widetilde{\Gamma}^{(1+(d-1))})\, (1 - \Gamma^{(1+(d-1))})\, \bar{\Psi} = 0\,, \\ \text{B.}\ \ (1 - \widetilde{\Gamma}^{(1+(d-1))})\, (1 + \Gamma^{(1+(d-1))})\, \bar{\Psi} = 0\,, \\ \text{C.}\ \ (1 + \widetilde{\Gamma}^{(1+(d-1))})\, (1 - \Gamma^{(1+(d-1))})\, \bar{\Psi} = 0\,, \\ \text{D.}\ \ (1 + \widetilde{\Gamma}^{(1+(d-1))})\, (1 + \Gamma^{(1+(d-1))})\, \bar{\Psi} = 0\,. \quad (15.6)$$

In ($d = 4n$)-dimensional spaces the first and the last condition share the space of spinors determined by an even number of $\gamma^a$'s in each product, Eq. (15.3), while the second and the third share the rest half of the spinor space determined by an odd number of $\gamma^a$'s in each product. In ($d = 2(2n + 1)$)-dimensional spaces is opposite: The first and the last condition determine spinor space of and odd number of $\gamma^a$'s in each product, while the second and the third require an even number of $\gamma^a$'s in each product.

Let us denote these four groups of states, defined in Eqs. (15.3,15.6) with the values of $[(\Gamma^{(1+(d-1))}, \widetilde{\Gamma}^{(1+(d-1))}] = [(+,+),(-,+),(+,-),(-,-)]$, by ($\bar{\Psi}_{++}$, $\bar{\Psi}_{-+}$, $\bar{\Psi}_{+-}$, $\bar{\Psi}_{--}$), respectively.

States of each group can be chosen to fulfill the Weyl dynamical equation for free massless spinors

$$\gamma^0\, \gamma^a p_a \bar{\Psi}_{ij} = 0\,, \\ (i,j) \in \{(+,+),(-,+),(+,-),(-,-)\}\,. \quad (15.7)$$

In the *spin-charge-family* theory one family contains, if analyzed with respect to the spin and charges of the *standard model*: the left handed weak charged quarks and the leptons - electrons and neutrinos - and the right handed weak chargeless quarks and leptons, with by the *standard model* assumed hyper charges, as well as the right handed weak charged quarks and leptons and left handed weak chargeless quarks and leptons. The break of the starting symmetry than leads to two groups of four families, which gain masses at the electroweak break. All the rest families ($2^{\frac{14}{2}-1}$ $-8$) gain masses interacting with the scalar fields.

These $2^d$ orthogonal basic states can be reached from any one of them by applying on such a state the products of operators: a constant, $\gamma^{a_1}, \tilde{\gamma}^{a_1}$, and products of $\gamma^{a_i}$ and products of $\tilde{\gamma}^{b_1}$.

Let us see on the case of $d = 2$, how do these four groups of families and family members distinguish among themselves.

We shall check also conditions under which these fermion states fulfill the Weyl equation, (Eq. (15.7)), for free (massless) fermions.

**Properties of four groups of fermion states defined in Eq. (15.6)** To better understand the meaning of the four groups (Eq. (15.6)) of families and family members let start with the simplest case: $d = (1 + 1)$ - dimensional spaces.

- **o d=(1+1) case.**

The requirement A. of Eq. (15.6) ($(1 - \tilde{\Gamma}^{(1+1)})\,(1 - \Gamma^{(1+1)})\,\bar{\Psi}_{\bar{\Psi}} = 0$, $\bar{\Psi}_{++} = \psi + \gamma^0\psi_0 + \gamma^1\psi_1 + \gamma^0\gamma^1\,\psi_{01}$) leads to $\psi_0 + \psi_1 = 0$, or consequently $\bar{\Psi}_{++} = \psi_{++}\,(\gamma^0 - \gamma^1)$. This state fulfills the Weyl equation provided that $(p_0 - p_1)\,\psi_{++} = 0$.

The requirement B. of Eq. (15.6) ($(1 - \tilde{\Gamma}^{(1+1)})\,(1 + \Gamma^{(1+1)})\,\bar{\Psi} = 0$) leads to $\psi + \psi_{01} = 0$, or consequently $\bar{\Psi}_{+-} = \psi_{+-}\,(1 - \gamma^0\gamma^1)$. This state fulfills the Weyl equation provided that $(p_0 + p_1)\,\psi_{+-} = 0$.

The requirement C. of Eq. (15.6) ($(1 + \tilde{\Gamma}^{(1+1)})\,(1 - \Gamma^{(1+1)})\,\bar{\Psi} = 0$) leads to $\psi - \psi_{01} = 0$, or consequently $\bar{\Psi}_{-+} = \psi_{-+}\,(1 + \gamma^0\gamma^1)$. This state fulfills the Weyl equation provided that $(p_0 - p_1)\,\psi_{-+} = 0$.

The requirement D. of Eq. (15.6) ($(1 - \tilde{\Gamma}^{(1+1)})\,(1 - \Gamma^{(1+1)})\,\bar{\Psi} = 0$) leads to $\psi_0 - \psi_1 = 0$, or consequently $\bar{\Psi}_{--} = \psi_{--}\,(\gamma^0 + \gamma^1)$. This state fulfills the Weyl equation provided that $(p_0 + p_1)\,\psi_{--} = 0$.

Making a choice of $p_1$ showing in the positive direction, the first and the third choice correspond to the positive energy solution, while the second and the fourth choice correspond to the negative energy solution of the Weyl equation (15.7).

Each of the four groups of states contains $2^{\frac{d}{2}-1} = 1$ state and $2^{\frac{d}{2}-1} = 1$ familiy. The operators $(1, \gamma^0\gamma^1, \tilde{\gamma}^0\tilde{\gamma}^1)$ are diagonal, the operators $(\gamma^0, \gamma^1, \tilde{\gamma}^0, \tilde{\gamma}^1)$ are off diagonal. Let us present the matrices for, let say, $\gamma^0$, $\tilde{\gamma}^0$ and $\gamma^0\tilde{\gamma}^0$ for the basic states, arranged as follows $\stackrel{01}{(+i)} = \frac{1}{2}(\gamma^0 - \gamma^1)$ (the case A.), $\stackrel{01}{(-i)} = \frac{1}{2}(\gamma^0 + \gamma^1)$ (the case D.), $\stackrel{01}{[+i]} = \frac{1}{2}(1 + \gamma^0\gamma^1)$ (the case C.), $\stackrel{01}{[-i]} = \frac{1}{2}(1 - \gamma^0\gamma^1)$ (the case B.). Let us notice that $\Gamma^{(1+1)}\,(\stackrel{01}{(+i)}, \stackrel{01}{(-i)}, \stackrel{01}{[+i]}, \stackrel{01}{[-i]}) = (\stackrel{01}{(+i)}, -\stackrel{01}{(-i)}, \stackrel{01}{[+i]}, -\stackrel{01}{[-i]})$, while $\widetilde{\Gamma}^{(1+1)}\,(\stackrel{01}{(+i)}, \stackrel{01}{(-i)}, \stackrel{01}{[+i]}, \stackrel{01}{[-i]}) = (\stackrel{01}{(+i)}, -\stackrel{01}{(-i)}, -\stackrel{01}{[+i]}, \stackrel{01}{[-i]})$. One finds the matrix representation for $\gamma^0$ and $\tilde{\gamma}^0$ and $\gamma^0\tilde{\gamma}^0$

$$\gamma^0 = \begin{pmatrix} 0&0&0&1 \\ 0&0&1&0 \\ 0&1&0&0 \\ 1&0&0&0 \end{pmatrix}, \tilde{\gamma}^0 = \begin{pmatrix} 0&0&i&0 \\ 0&0&0&i \\ -i&0&0&0 \\ 0&-i&0&0 \end{pmatrix}, \gamma^0\tilde{\gamma}^0 = \begin{pmatrix} 0&-i&0&0 \\ -i&0&0&0 \\ 0&0&0&i \\ 0&0&i&0 \end{pmatrix}. \tag{15.8}$$

While $\gamma^0$ causes the transformations among states, which have the opposite handedness $\Gamma^{(1+1)}$, while they have the same handedness $\tilde{\Gamma}^{(1+1)}$, transforms $\tilde{\gamma}^0$ among states of opposite handedness $\tilde{\Gamma}^{(1+1)}$, leaving handedness $\Gamma^{(1+1)}$ unchanged. The operator $\gamma^0\tilde{\gamma}^0$ causes transformations among the states, which differ in both handedness. Interaction of the type $S^{ab}\omega_{abc}$ and $\tilde{S}^{ab}\tilde{\omega}_{abc}$, appearing in the action Eq.(15.1) do not cause in this $d = (1+1)$ case transformations among the basic states $(\overset{01}{(+i)}, \overset{01}{(-i)}, \overset{01}{[+i]}, \overset{01}{[-i]})$.

- **d=(13+1) case.**

In the case of $d = (13+1)$-dimensional space the operators $S^{ab}$ transform all the members of one family among themselves. Table IV of Ref. [4] represents one family representation analyzed with respect to the *standard model* gauge and spinor groups. The $2^{d/2-1} = 64$ members represent quarks and leptons, left and right handed, with spin up and down and with the hyper charges as required by the *standard model*. There are also the anti-members, reachable from members not only by $S^{ab}$ but also by $\mathcal{C_N}\mathcal{P_N}$ [7].

The operators $\tilde{S}^{ab}$ transform each family member of a particular family into another family, keeping the family member quantum numbers unchanged.

There are four groups of such families, having

$$(\Gamma^{(13+1)}, \tilde{\Gamma}^{(13+1)}) = ((+,+), (-,-), (+,-), (-,+)),$$

respectively. As seen in the simple case of $d = (1+1)$ all four groups could be reachable from the starting one only by the operators $\gamma^a$, $\tilde{\gamma}^a$ and $\gamma^a\tilde{\gamma}^b$.

We have some experience with the toy model in $d = (5+1)$, Refs. [8–10], that when breaking symmetries not only that only spinors of one handedness remain masless, but also most of families can get heavy masses.

After the break of $SO(13,1)$ to $SO(7,1) \times SO(6)$ (and correspondingly also of $\widetilde{SO}(13,1)$) $S^{st}$, $s \in (0,\ldots,8), t \in (9,\ldots,14)$ (and correspondingly also of $\tilde{S}^{st}$, $s \in (0,\ldots,8), t \in (9,\ldots,14)$) are no longer applicable. Anti spinors (spinors with quantum numbers of the second part, numerated by 33 up to 64, of Table IV in Ref. [4]) are after the break reachable only by $\mathcal{C_N}$ $\mathcal{P_N}$ [7].

The break of $SO(6)$ to $SU(3) \times U(1)$ disables transformations from quarks to leptons.

When breaking symmetries, like from $SO(13,1)$ to $SO(7,1) \times SO(6)$, the break must be done in a way that only spinors of one handedness remain massless in order that the break leads to observed (almost massless) fermions and that most of families get masses of the energy of the break [8–10]. Our studies so far support the assumption that only the families with $\tilde{\Gamma}^{(7+1)} = 1$ and $\tilde{\Gamma}^{(6)} = -1$ remain massless.

Correspondingly only eight families ($2^{(7+1)/2-1}$) remain massless.

At the further break of $SO(7,1) \times SU(3) \times U(1)$ to $SO(3,1) \times SU(2) \times SU(3) \times U(1)$ all the eight families of quarks and leptons remain massless due to the fact that left handed and right handed quarks and leptons have different charges and are correspondingly mass protected.

### 15.1.2 Properties of general boson fields

We have discussed so far only fermion fields. The *spin-charge-family* theory action, Eq (15.1), introduces the vielbeins and the two kinds of the spin-connection fields, with which the fermions interact. These are the gauge fields of the two kinds of charges, which take care of the family members quantum numbers ($S^{ab}$) and of the family quantum numbers ($\tilde{S}^{ab}$).

The Lagrange density (15.1) of each kind of the spin connection fields is linear in the curvature. This action seems to be the simplest action of the Kaluza-Klein kinds of theories, in which fermions carry the family and the family members quantum numbers, while the gravitational field - the vielbeins and the two kinds of the spin connection fields take care of the interaction among fermions. Vielbeins and spin connections are the only boson fields in the theory. They manifest at the low energy regime all the phenomenologically needed vector and scalar bosons.

Let us define boson fields, which in the case of $d = (1+1)$, $d = (13+1)$, or any $d$, transform the $2^d$ fermion states among themselves? The fields $S^{ab}\omega_{abc}$ and $\tilde{S}^{ab}\tilde{\omega}_{abc}$ can, namely, cause transitions only among fermions with the same Clifford character: The Clifford even (odd) fermion states are transformed into the Clifford even (odd) fermion states, as we have seen in subsection 15.1.1.

Let us assume for this purpose that there exist to each of products

$$\gamma^{a_1}\gamma^{a_2}\ldots\gamma^{a_k},$$

the number of products of $\gamma^a$'s running from zero to $d$, the corresponding gauge fields: $\omega_{a_1 a_2 \ldots a_k}$. There are obviously $2^d$ such gauge fields. These gauge fields, carrying $k$ vector indexes $a_1 \ldots a_k$, transform a fermion state

$$\underline{\Psi}_{ij}, (i,j) = [(+,+), (-,+), (+,-), (-,-)]$$

belonging to one of the four groups (with the eigenvalues of ($\Gamma^{(d)}$,$\tilde{\Gamma}^{(d)}$= $(i,j)$, respectively), discussed in subsection 15.1.1, into another state, belonging to the same or to one of the rest free groups: If starting with the state of either the A. or B. groups, these bosons transform such a state to one of the states belonging to either the group A. (if the number of $a_j$ is even) or to the group B. (if the number of $a_j$ is odd). If we start from the group C. or D., then the transformed state remains within these two groups.

Correspondingly we define to each of products $\tilde{\gamma}^{a_1}\ \tilde{\gamma}^{a_2}\ldots\tilde{\gamma}^{a_k}$, again the number of products of $\tilde{\gamma}^a$'s running from zero to $d$, the corresponding gauge fields $\tilde{\omega}_{a_1 a_2 \ldots a_k}$, which again transform the state $\underline{\Psi}_{ij}$, belonging to one of the four groups, discussed in subsection 15.1.1, into another state, belonging to the same (if the number of $a_k$ is even), or to one of the rest free groups (if the number of $a_k$ is odd). In this case the transformations go from A. to C., or from B. to D..

All the states of one group of fermions are reachable from the starting state under the application of $\omega_{abc}$ and $\tilde{\omega}_{abc}$. The operators $S^{ab}$ and $\tilde{S}^{ab}$ keep the handedness $\Gamma^{(d)}$ and $\tilde{\Gamma}^{(d)}$, respectively, unchanged. (Let us remind the reader that all the $2^{(13+1)/2-1}$ states of one family (Table IV of Ref. [4]) are reachable by $S^{ab}\omega_{abc}$ and all the $2^{(7+1)/2-1}$ families (Table V of Ref. [4]) are reachable by $\tilde{S}^{ab}\tilde{\omega}_{abc}$).

The by the products of $\tilde{\gamma}^a$'s transformed state $\tilde{\Psi}$ differs in general from the one transformed by the product of $\gamma^a$'s according to the definition in Eq. (15.2).

Let us assume that all the boson fields obey the equations of motion

$$\begin{aligned} \partial^a\partial_a \omega_{a_1 a_2 \dots a_k} &= 0\,, \\ \partial^a\partial_a \tilde{\omega}_{a_1 a_2 \dots a_k} &= 0\,. \end{aligned} \tag{15.9}$$

For the boson fields, which are the gauge fields of the products of $\tilde{\gamma}^{a_1}\,\tilde{\gamma}^{a_2}\dots\tilde{\gamma}^{a_k}$ or of $\gamma^{a_1}\gamma^{a_2}\dots\gamma^{a_k}$ Eq. (15.9), this can only be true in the weak fields limit.

Let us see the action of this boson fields on fermion basic states in the case of $d=(1+1)$. The boson fields bring to fermions the quantum numbers, which they carry. We can calculate these quantum numbers by taking into account Eq. (16) in Ref. [4]

$$\mathcal{S}^{ab}\,A^{d\dots e\dots g} = i\,(\eta^{ae}\,A^{d\dots b\dots g} - \eta^{be}\,A^{d\dots a\dots g})\,, \tag{15.10}$$

or we can simply calculate the action of the operators, the gauge fields of which are boson fields.

$$\begin{aligned} 1\ \ (1,\,\gamma^0,\gamma^1,\,\gamma^0\gamma^1) &= (1,\gamma^0,\gamma^1,\gamma^0\gamma^1)\,, \\ \tilde{1}\ \ (1,\gamma^0,\gamma^1,\gamma^0\gamma^1) &= (1,\gamma^0,\gamma^1,\gamma^0\gamma^1)\,, \\ \gamma^0\ \ (1,\gamma^0,\gamma^1,\gamma^0\gamma^1) &= (\gamma^0,1,\gamma^0\gamma^1,\gamma^1)\,, \\ \tilde{\gamma}^0\ \ (1,\gamma^0,\gamma^1,\gamma^0\gamma^1) &= i\,(\gamma^0,-1,\gamma^0\gamma^1,-\gamma^1)\,, \\ \gamma^1\ \ (1,\gamma^0,\gamma^1,\gamma^0\gamma^1) &= (\gamma^1,-\gamma^0\gamma^1,-1,\gamma^0)\,, \\ \tilde{\gamma}^1\ \ (1,\gamma^0,\gamma^1,\gamma^0\gamma^1) &= i\,(\gamma^1,-\gamma^0\gamma^1,1,-\gamma^0)\,, \\ \gamma^0\gamma^1\,(1,\gamma^0,\gamma^1,\gamma^0\gamma^1) &= (\gamma^0\gamma^1,-\gamma^1,-\gamma^0,1) \\ \tilde{\gamma}^0\tilde{\gamma^1}\,(1,\gamma^0,\gamma^1,\gamma^0\gamma^1) &= (i)^2\,(\gamma^0\gamma^1,\gamma^1,\gamma^0,1)\,, \end{aligned} \tag{15.11}$$

*It is obvious that the two kinds of fields influence states in a different way, except the two constants, which leave states untouched.*

One can conclude that there are correspondingly $2\times 2^d-1$ independent real boson fields (only one of the two constants has the meaning), and there are also, as we have learned in Subsec. 15.1.1 $2^d$ complex fermion fields, which means $2\times 2^d$ real fermion fields in any dimension. This supports the Aratyn-Nielsen theorem [5].

**o** Comments on **d=(1+1) case**.

Let us make a choice of $2^{\frac{d}{2}-1}$ fermion states, which is for $d=2$ only one state, say $\overset{01}{(+i)}$. It is the complex field and accordingly with two degrees of freedom.

One can make then (any) one choice of the boson field, let say $\omega_{01}$, which is the gauge field of the "charge" $\Gamma^{(1+1)}$. This is in agreement with the Aratyn-Nielsen theorem.

All the (complex) Clifford odd fermion states, $(\stackrel{01}{(+i)}, \stackrel{01}{(-i)}$, need three of the independent boson fields, let say $(\gamma^1\omega_1, \gamma^0\gamma^1\omega_{01}, \tilde{\gamma}^1\tilde{\omega}_1)$, to be in agreement with the Aratyn-Nielsen theorem.

**Bosons in interaction with fermions** If we expect gauge boson fileds to appear in the covariant derivative of fermions, as we are used to require, then all the gauge fields must curry the space index, like it is the case of the covariant derivative for fermions, presented in Eq. (15.1): $p_{0a} = p_a - \frac{1}{2}S^{bc}\omega_{bca} - \frac{1}{2}\tilde{S}^{bc}\tilde{\omega}_{bca}$.

Let us generalize this covariant momentum by replacing $\frac{1}{2}S^{a_1 a_2}\omega_{a_1 a_2 a} + \frac{1}{2}\tilde{S}^{a_1 a_2}\tilde{\omega}_{a_1 a_2 a}$ by

$$\begin{aligned} p_{0a} = p_a - & \\ & \{\omega_a + \gamma^{a_1}\omega_{a_1 a} + \gamma^{a_1}\gamma^{a_2}\omega_{a_1 a_2 a} + \cdots + \gamma^{a_1}\gamma^{a_2}\ldots\gamma^{a_d}\omega_{a_1 a_2 \ldots a_d\, a} \\ & + \tilde{\gamma}^{a_1}\tilde{\omega}_{a_1 a} + \tilde{\gamma}^{a_1}\tilde{\gamma}^{a_2}\tilde{\omega}_{a_1 a_2 a} + \cdots + \tilde{\gamma}^{a_1}\tilde{\gamma}^{a_2}\ldots\tilde{\gamma}^{a_d}\tilde{\omega}_{a_1 a_2 \ldots a_d\, a}\}\,. \end{aligned} \tag{15.12}$$

We assumed that all the $\gamma^a$'s in products appear in the ascending order. Correspondingly is $\frac{1}{2}S^{a_1 a_2}\omega_{a_1 a_2 a}$ replaced by $\frac{i}{2}\,\gamma^{a_1}\gamma^{a_2}\omega_{a_1 a_2 a}$, the factor $\frac{i}{2}$ appears due to $S^{a_1 a_2} = \frac{i}{2}\gamma^{a_1}\gamma^{a_2}$, $a_2 > a_1$.

This theory would neither be gauge invariant nor do the corresponding gauge fields fulfill the equations of motion, Eq. (15.9), except in the weak limit if the gauge fields appear as the background fields. The degrees of freedom of bosons and fermions no longer fulfill the Aratyn-Nielsen theorem, unless we again allow either only Clifford even or Clifford odd fermion states and only one of the two fields with the space index zero, let say $\omega_0$ among the boson fields is allowed. And yet we have in addition nonphysical degrees of freedom due to gauge invariance for almost free massless fields in the weak limit, which should be possibly removed.

If nature has ever started with the boson fields as presented above, most of these fields do not manifest in $d = (3+1)$.

## 15.2 Conclusions

We have started the fermionization of boson fields (or bosonization of fermion fields) in any $d$ (the reader can find the corresponding contribution in this proceedings )to understand better why, if at all, the nature has started in higher dimensions with the simple action as assumed in the *spin-charge-family* theory, offering in the low energy regime explanation for all observed degrees of freedom of fermion and boson fields, with the families of fermions included. This theory is a kind of the Kaluza-Klein theories with two kinds of the spin connection fields. We also hope that the fermionizasion can help to see which role can the same number of degrees of freedom of fermions and bosons play in the explanation, why the cosmological constant is so small.

This contribution is a small step towards understanding better the open problems of the elementary particle physics and cosmology. We discussed for any d-dimensional space the degrees of freedom for free massless fermions and the degrees of freedom for free massless bosons, which are the gauge fields of all possible products of both kinds of the Clifford algebra objects, either of $\gamma^a$ or of $\tilde{\gamma}^a$.

Although we have not yet learned enough to be able to answer any of the four questions, presented in the introduction (**a.** Why is the simple starting action of the *spin-charge-family* theory doing so well in manifesting the observed properties of the fermion and boson fields? **b.** Under which condition can more general action lead to the starting action of Eq. (15.1)? **c.** What would more general action, if leading to the same low energy physics, mean for the history of our Universe? **d.** Could the fermionization procedure of boson fields or the bosonization procedure of fermion fields, discussed in this Proceedings for any dimension d (by the authors of this contribution, while one of them, H.B.F.N. [5], has succeeded with another author to do the fermionization for $d = (1+1)$), help to find the answers to the questions under **a. b. c.**?), yet we have started to understand better the topic.

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