A MATHEMATICAL MODEL FOR THE STATIONARY PROCESS OF ROLLING OF TUBES ON A CONTINUOUS MILL

MATEMATIČNI MODEL PROCESA KONTINUIRNEGA VALJANJA CEVI

Yu. G. Gulyayev[1], Ye. I. Shyfrin[2], Ilija Mamuzić[3]

[1]National Metallurgical Academie of Ukraine, Dnipropetrovsk, Ukraine
[2]Tube MetallurgicCompany, Russia
[3]University of Zagreb, Faculty of Metallurgy Sisak, Croatia
mamuzic@simet.hr

A mathematical model has been developed for the calculation of process parameters in continuous lengthwise plugless tube rolling. Examples of concrete calculations of rolling parameters, their comparison with experimental data and the results obtained with the application of other calculation procedures are given.

Key words: tubes, plugless rolling, mathematical model

Razvit je bil matematični model za izračun parametrov procesa neprekinjenega valjanja cevi brez notranjega trna. Dani so konkretni primeri izračunov parametrov valjanja, rezultati pa so primerjani z eksperimentalnimi podatki in z izračuni po drugih postopkih.

Ključne besede: cevi, valjanje brez trna, matematični model

1 INTRODUCTION

The prospects of enhancement of the production efficiency at numerous tube rolling units are closely linked with the possibility of a reliable prediction of the forming parameters at the final stage of plastic deformation in the plugless tube reducing or sizing processes. In this connection, the problem of development of a universal mathematical model applicable in studying the process of lengthwise plugless rolling in the tube rolling mills equipped with the roll drives of different types is of a high interest.

2 STATE OF THE ISSUE AND THE AIM OF INVESTIGATION

The analysis of the relevant references shows that the problem of determination of kinematical, deformational and power-and-force parameters of the continuous plugless lengthwise tube rolling process was solved up to now by *consecutive* analysis of forming in each individual stand. Solutions based on integration of the deformational parameters in all N stands of the continuous mill into a common system of equations are proposed, also [1,2]. In the development of mathematical models of the continuous rolling process, e.g. in [1,2] two assumptions were made.

Firstly, the mean angle of the neutral section θ_{ni} (**Figure 1**) is defined for the condition of coincidence of the roll and the mother tube speeds within the section of the deformation zone *exit* in the i-th stand, though it would be logical to choose some section between the entry and exit of the deformation zone.

Secondly, for the determination of the effective roll diameter D_{ki}, the approximate formula is used: $D_{ki} = D_{ui} - D_i \cos\theta_{ni}$ (where D_{ui}, D_i – are the ideal roll diameter and the mean tube diameter after rolling in the i-th stand respectively) that introduces an error because in reality $D_{ki} = D_{ui} - 2r_{\theta i} (\theta_{ni}) \cos\theta_{ni}$ (where D_{ui} is the ideal roll diameter; $r_{\theta i} (\theta_{ni})$ is the pass radius at $\theta = \theta_{ni}$, see **Figure 2**).
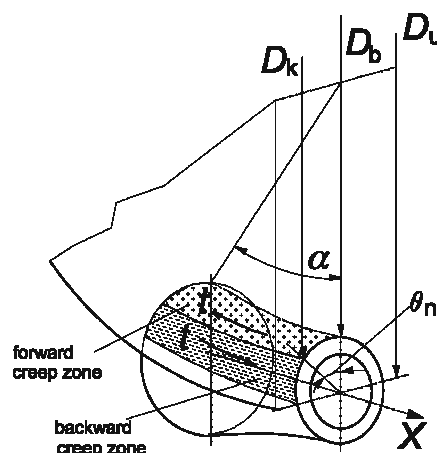

Figure 1: Scheme for the determination of the value of the effective diameter D_k
Slika 1: Shema za določitev efektivnega premera D_k

In accord with the model consisting of $2N$ equations proposed[1], the effective roll diameter D_{ki} cannot be greater than the ideal roll diameter D_{ui} and smaller than the roll diameter at the swell D_{bi}. This distorts the real picture of the rolling process kinematics, namely, when the rolls slip on the mother tube surface two conditions are met: $D_{ki} < D_{bi}$ or $D_{ki} > D_{ui}$. The model proposed [2] is free from this shortcoming but it is a system of $3N$ equations that when being solved at $N > 16$ is connected with considerable difficulties because of the great number of unknowns to be determined.

This work is aimed at the verification and simplification of the mathematical models proposed [1,2] and to the assessment of the verification results on the basis of comparison of calculated and experimental data and it is, for this reason, of scientific and practical interest.

3 PROBLEM STATEMENT

The following values have to be calculated:
– angular roll rotation velocity n_{Bi} in each i-th mill stand (for the mill with individual roll drives);
– angular velocities of rotation of the main (N_Γ) and auxiliary (N_B) motors (for the mill with differential-group roll drives);
– ideal roll diameters D_{ui} (for the mill with group roll drives).

These values ensure that tubes of required size ($D_t \cdot S_t$, mm) are rolled from the mother tube of given size (($D_0 \times S_0$) mm) at a specified rolling speed V_0/(m/s) in the first stand of the multiple-stand mill.

Initial data for the calculation are as follows:
– the total diameter and wall reduction (or just diameter reduction), i.e. initial mother tube dimensions $D_0 \times S_0$ (or just D_0) and final tube dimensions $D_t \times S_t$;
– the distribution of partial mother tube diameter reductions m_i (%) among the mill stands of total number of N;
– the value of external friction f_i;
– the mother tube rolling speed V_0/(m/s) in the first mill stand (the problem can also be stated for V_0 as the value to be determined);
– the gear ratios $\eta_{\Gamma i}$, η_{Bi} from the motors to the rolls in the lines of the main and auxiliary drives (for the mills with differential-group roll drives);
– the absence of backward pull in the first mill stand ($Z_{31} = 0$) and of front pull in the last mill stand ($Z_{nN} = 0$);
– the number of rolls N_b forming passes in the mill stands.

4 PHYSICAL MODEL OF THE PROCESS

No mother tube forming occurs in interstand spaces and the wall thickness S_j at the exit from the stand of ordinal number $j = i - 1$ is equal to the wall thickness S_{0i}

at the entry to the stand of ordinal number i. The deformation resistance K_{fj} of the mother tube material at the exit from the stand of ordinal number j is equal to the deformation resistance K_{f0i} of the mother tube material at the entry to the stand of ordinal number i. It follows that the coefficient of front plastic pull Z_{nj} for the stand of ordinal number j is equal to coefficient of backward plastic push Z_{3i} for the stand of ordinal number i. The area F_{ki} of the contact surface of the mother tube with one roll in the stand of ordinal number i is equal to the area of a rectangle with sides

$$L_i = \sqrt{\frac{\beta_i \cdot D_j \cdot \varepsilon_i \cdot (D_{ui} - D_i)}{2 \sin \beta_i}} \qquad (1)$$

with

$$B_i = \beta_i \cdot D_j \qquad (2)$$

where D_j and D_i are the mean mother tube diameters at the entry to and at the exit from the deformation zone in the stand of ordinal number i;

$$\varepsilon_i / \% = \frac{m_i}{100}; \quad \beta_i = \frac{\pi}{N_{bi}}$$

The area F_i^+ of the zone of forward creep at the surface of contact between one roll and the mother tube in the deformation zone of the stand of ordinal number i is defined as the surface of a rectangle with sides

$$L_i^+ = L_i \qquad (3)$$

and

$$L_i^+ = \theta_{ni} D_j \qquad (4)$$

where θ_{ni} is the neutral section angle characterizing the position of the neutral line differentiating the zone of forward creep and the zone of backward creep on the surface of contact between the mother tube and the roll in the deformation zone (**Figure 1**).

In a real process, the magnitude of angle θ_{ni} is a function of the angle α characterizing the position of a concrete diametrical section of the deformation zone relative to the diametrical section of the mother tube exit from the reduction zone. In accord with the assumption[4], the magnitude of angle θ_{ni} is assumed to be equal to some quantity averaged over the contact surface length. It will be regarded that θ_{ni} is the value of the neutral angle in the "neutral" diametrical section of the deformation zone where the extension is equal to the mean extension in the i-th stand. The axial velocity V_{Mn} of metal and axial component of the roll surface velocity V_{Bn} in the "neutral" diametrical section are given with

$$V_{Mni} = V_0 \, \mu_{\Sigma i}^{cp} \qquad (5)$$

$$V_{Bni} = \frac{\pi n_{Bi} A_{ti}^{cp} D_{\theta i}}{\xi} \qquad (6)$$

where $\mu_{\Sigma i}^{cp} = \frac{2 S_0 (D_0 - S_0)}{S_j (D_j - S_j) + S_i (D_i - S_i)}$ is the total elongation from the mill entry to the "neutral" diametrical section of the i-th stand;
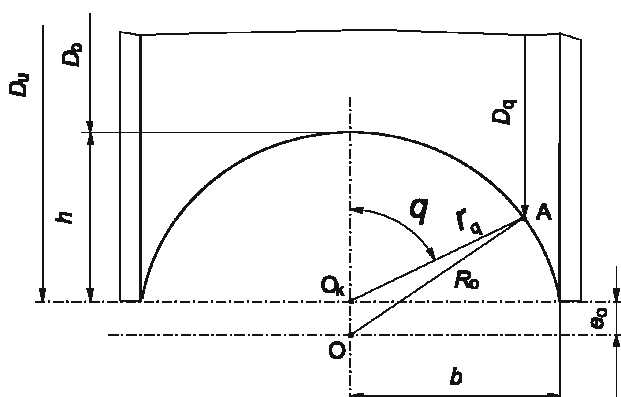
Figure 2: Scheme for the determination of the value of the variable across the pass perimeter of the roll diameter D_θ
Slika 2: Shema za določitev spremembe premera valjev D_θ na obodu vtika

A_{ti}^{cp} is the mean value of the guiding cosine of the contact friction stresses [1];

$D_{\theta i} = D_{ui} - 2r_{\theta i} \cos\theta$ is the varying of the roll pass diameter across the pass perimeter (**Figure 2**);

$$r_{\theta i} = \overline{O_k}\,\overline{A} = R_{oi}\left[\sqrt{1-\left(\frac{e_{oi}}{R_{oi}}\right)^2 (1-\cos^2\theta)} - \frac{e_{oi}\cos\theta}{r_{oi}}\right]$$ is

the varying across the pass perimeter value of the pass diameter;

$$R_{oi} = \overline{OA} = \frac{h_i(\lambda_{oi}^2 + 1 - 2\lambda_{ki}\sin\psi)}{2(1-\lambda_{ki}\sin\psi)}$$ is the pass generatrix

radius;

$$e_{oi} = \overline{O_k}\,\overline{A} = \frac{h_i(\lambda_{oi}^2 - 1)}{2(1-\lambda_{ki}\sin\psi)}$$ is the pass generatrix eccen-

tricity;

$\lambda_{ki} = \dfrac{b_i}{h_i}$ is the pass ovality;

$\psi = \dfrac{(N_b - 2)}{2N_b}$ is the pass shape index;

b_i, h_i are the pass width and the pass height correspondingly;

$\xi = 6\cdot10^4$ is coefficient of quantity dimension reduction $(s\cdot mm\cdot\dfrac{r}{min}\cdot m)$

The angle θ_{ni} is defined as root of the transcendental equation

$$V_{Mni} - V_{Bni} = 0 \qquad (7)$$

Taking in account that in a physical sense $0 \le \theta_{ni} \le \beta_i$, the condition for the determination of the neutral angle assumes the following form (in symbols of Math-CAD programming language)

$$\theta_{ni} = \begin{vmatrix} \arccos Q_i \text{ if } \cos\beta_i \le Q_i \le 1 \\ 0 \text{ if } 1 < Q_i \\ \beta_i \text{ if } \cos\beta_i > Q_i \end{vmatrix} \qquad (8)$$

The quantity Q_i in (8) is defined as the root of equation

$$\frac{2S_0V_0(D_0-S_0)}{S_j(D_j-S_j)+S_i(D_i-S_i)} - \frac{\pi n_{Bi}A_{ti}^{cp}}{\xi} \cdot$$

$$\cdot\left\{ D_{ui} - 2Q_iR_{oi}\left[\sqrt{1-\left(\frac{e_{oi}}{R_{oi}}\right)^2(1-Q_i^2)} - \frac{e_{oi}Q_i}{R_{oi}}\right]\right\} = 0 \qquad (9)$$

Note that as distinct from the conditions used[1,2], the condition (8) reflects the relation of the neutral angle θ_{ni} value with the roll design parameters (R_{0i}, e_{0i}, λ_{ki}, b_i, h_i).

Taking into consideration relationships (5) and (6), the effective roll diameter D_{ki} can be defined by the following equation:

$$D_{ki} = \frac{2S_0V_0(D_0-S_0)\xi}{\pi\left[S_j(D_j-S_j)+S_i(D_i-S_i)\right]n_{Bi}A_{ti}^{cp}} \qquad (10)$$

For $D_{bi} < D_{ki} < D_{ui}$, on the contact surface of each roll appears a forward creep zone with the area equal of $F_i^+ = \theta_{ni}D_jL_i$ in accord with (3), (4) and the backward creep zone with the area equal to $F_i^- = (\beta_i - \theta_{ni})D_jL_i$. For $D_{bi} > D_{ki}$, the backward creep zone extends over the entire contact surface area and the "forward roll slippage" takes place on the metal surface. If $D_{ui} < D_{ki}$, the forward creep zone extends over the entire contact surface area and "backward roll slippage" takes place on the metal surface.

The magnitude of neutral angle θ_{ni} must meet the condition of force equilibrium of the metal volume in the geometrical deformation zone of the i-th stand [1] that can be expressed as:

$$\theta_{ni} = \beta_i \cdot \begin{vmatrix} X_i \text{ if } 0 \le X_i \le 1 \\ 0 \text{ if } 0 > X_i \\ 1 \text{ if } 1 < X_i \end{vmatrix} \qquad (11)$$

where

$$X_i = \frac{F_i^+}{F_{ki}} = \frac{1}{2}\cdot\left\{1 - \frac{A_{pi}^{cp}}{f_i A_{ti}^{cp}} + \frac{Z_{ni}S_i(D_i-S_i)-Z_{nj}S_j(D_j-S_j)}{f_i A_{ti}^{cp}\left(2L_i+\sqrt{S_jD_j}\right)n_{ti}S_i\frac{D_j}{D_i}\left[1-(Z_{cp})\right]}\right\}$$

is the coefficient of forward creep in the i-th mill stand calculated for the equilibrium of forces in the volume of the metal in the geometrical deformation zone of the i-th stand;

A_{ti}^{cp} is the mean, over the contact surface, value of the guiding cosine for normal contact stresses;

f_i is coefficient of external friction;

$n_{ti} = 1 + 0.36 f_i$ is coefficient accounting for the effect of the contact friction stresses upon normal contact stresses[3];

Z_π is coefficient of forward plastic pull;

$(Z_{cp})_i = \dfrac{Z_{ni}}{3} + \dfrac{2Z_{nj}}{3}$ is the mean value of the plastic pull coefficient in the i-th stand.

Table 1: Parameters of rolling a (57 × 11.6) mm tube from a (117 × 14.8) mm mother tube
Tabela 1: Parametri valjanja cevi (57 × 11.6) mm iz cevi (117 × 14.8) mm

i	D_i/mm	m_i/%	λ_{ki}	S_i/mm			Z_{ni}		θ_{ni}/°		P_i/kN		M_i/(kN·m)	
				E	A	B	A	B	A	B	A	B	A	B
1	115.25	1.50	1.037	14.77	14.86	14.85	0.327	0.361	53.7	56.8	93	92	-14.5	-16.6
2	112.59	2.30	1.024	14.82	14.82	14.81	0.568	0.606	51.3	53.2	74	69	-10.0	-10.6
3	109.67	2.60	1.032	15.55	14.68	14.64	0.721	0.756	46.9	48.2	49	44	-5.3	-5.2
4	106.49	2.90	1.028	14.35	14.47	14.40	0.729	0.766	19.9	19.1	37	32	3.0	2.7
5	102.97	3.30	1.038	14.07	14.22	14.13	0.730	0.761	17.7	14.0	38	32	3.8	3.9
6	99.47	3.40	1.034	13.75	13.97	13.86	0.729	0.755	17.00	14.4	37	33	3.8	3.9
7	96.09	3.40	1.036	13.50	13.73	13.60	0.724	0.742	16.6	13.5	37	33	3.8	4.2
8	92.82	3.40	1.035	13.28	13.49	13.35	0.715	0.743	16.3	17.2	37	33	3.9	3.4
9	89.57	3.50	1.037	13.00	13.26	13.10	0.705	0.721	16.7	14.1	37	34	4.0	4.2
10	86.44	3.50	1.036	12.70	13.03	12.87	0.693	0.711	17.2	15.4	38	35	3.9	4.0
11	83.41	3.50	1.037	12.49	12.82	12.65	0.679	0.693	17.5	16.1	38	36	3.9	4.0
12	80.49	3.50	1.036	12.41	12.62	12.44	0.665	0.676	18.2	17.3	39	37	3.8	3.9
13	77.68	3.50	1.036	12.31	12.42	12.25	0.650	0.659	18.8	18.2	40	38	3.7	3.7
14	74.96	3.50	1.036	12.18	12.24	12.06	0.635	0.642	19.4	18.9	41	39	3.7	3.7
15	72.33	3.50	1.036	11.95	12.07	11.89	0.618	0.623	19.9	19.4	42	41	3.6	3.6
16	69.80	3.50	1.036	11.78	11.91	11.74	0.595	0.597	19.7	19.2	43	42	3.8	3.8
17	67.36	3.50	1.036	11.69	11.77	11.59	0.568	0.564	20.1	19.6	45	44	3.8	3.9
18	65.00	3.50	1.036	11.58	11.64	11.47	0.532	0.518	20.1	19.2	47	47	4.0	4.4
19	62.73	3.50	1.036	11.53	11.53	11.39	0.459	0.414	18.2	16.1	52	54	5.3	6.3
20	60.53	3.50	1.036	11.40	11.49	11.41	0.267	0.078	12.6	6.5	63	72	9.2	13.8
21	59.02	2.50	1.020	11.50	11.53	11.44	0.036	-0.140	14.3	17.4	71	87	9.4	9.3
22	57.96	1.80	1.018	11.58	11.58	11.58	0.001	-0.105	25.7	29.0	74	85	3.0	1.3
23	57.60	0.61	1.000	11.60	16.00	11.60	0	0	28.4	34.3	51	54	1.0	-1.6

NOTES:
A = calculation by the procedure proposed in [1]
B = calculation by the procedure proposed in this work
E = experimental data

5 MATHEMATICAL MODEL

Equate right parts of equations (8) and (11) and use the equation of relation between the change of the mean wall thickness and force conditions of the mother tube deformation in each i-th mill stand [4,5] to obtain the mathematical model of the continuous mother tube rolling process in N stands of the mill as a system of $2N$ equations:

$$\theta_{ni} = \begin{vmatrix} \arccos Q_i & \text{if } \cos\beta_i \le Q_i \le 1 \\ 0 & \text{if } 1 < Q_i \\ \beta_i & \text{if } \cos\beta_i > Q_i \end{vmatrix} =$$

$$= \beta_i \cdot \begin{vmatrix} X_i & \text{if } 0 \le X_i \le 1 \\ 0 & \text{if } 0 > X_i \\ 1 & \text{if } 1 < X_i \end{vmatrix} \qquad (12)$$

$$S_i - S_j \left\{ 1 + \varphi_i \cdot \frac{2(Z_{cp})_i(T_i - 1) + (1 - 2T_i)}{(Z_{cp})_i(1 - T_i) - (2 - T_i)} * \right.$$

$$\left. + \frac{1}{2}\left[\varphi_i \cdot \frac{2(Z_{cp})_i(T_i - 1) + (1 - 2T_i)}{(Z_{cp})_i(1 - T_i) - (2 - T_i)} \right]^2 \right\} \qquad (13)$$

where

$$\varphi_i = \ln\frac{D_i - S_i}{D_j - S_j}; \ T_i = \left(\frac{S_j}{D_j} + \frac{S_i}{D_i} \right)^K;$$

$$(Z_{cp})_i = \frac{1}{2}(Z_{nj} + Z_{ni});$$

$K = 1.57$ for $N_b = 2$; $K = 1.20$ for $N_b = 3$; $i = 1, 2, \ldots , N-1, N$

Distinct from the known solution[2], the mathematical model includes $2N$ and not $3N$ equations that simplifies the search of solution and makes it possible to analyze the rolling process in stretch-reducing mills with $N \le 25$ stands.

Depending on the type of the mill drive, the problem of determination of the rolling parameters with the use of the system of equations (12)-(13) can be formulated in different ways. For the mills with *individual* drives, it is necessary to determine $2N$ values of n_{Bi} (where $i = 1, 2, \ldots , N$) and S_i (where $i = 0, 1, 2, \ldots , N-1$) for the specified values of S_t, V_0 and Z_{ni} (where $i = 1, 2, \ldots , N-1$). For the mills with *differential-group* drives, it is necessary to find $2(N-1)$ values of the quantities S_i and Z_{ni} (where $i = 1, 2, \ldots , N-1$) and the values of N_Γ and N_B for the specified values of S_0, S_t, V_0. For the mills with *individual* drives, it is necessary to determine $N-1$ value

of the quantities D_{ui} (where $i = 2, 3, \ldots, N$), N values of the quantities S_i ($i = 0, 1, 2, \ldots, N–1$) and the angular roll velocity n_B that is constant for all stands at the specified values of D_{u1}, V_0 and Z_{ni} (where $i = 1, 2, \ldots, N–1$).

Solve the system of equations (12)-(13) using (8) or (11) to find values of neutral angles θ_{ni} and determine the values of the effective diameters in correspondence with expression (10).

6 RESULTS OF MODEL CALCULATIOINS

The model has been successfully used in the calculation of tube rolling parameters for mills with the roll drives of individual, group and differential-group types.

As an example, let us consider the results of the calculation of the nature of change in the mean wall thickness S_i, rolling pressure P_i, rolling moments acting in the stand M_i and the values of Z_{ni}, θ_{ni} in rolling a $D_t \cdot S_t = (57 \times 11.6)$ mm tube from a $D_0 \cdot S_0 = (117 \times 14.8)$ mm mother tube in 23 stands of the tube rolling unit "30-102" reducing mill with differential-group roll drives ($N_B = 3$, $V_0 = 0.7$ m/s with the mother tube material: Grade 45 steel). In this case, the mathematical model (12)-(13) is a system of 46 equations with 46 unknowns: 22 values of S_i and Z_{ni} each and the values of N_Γ as well as N_B. The rolling parameters, results of calculation by the procedure given[1], by the proposed model and experimental data are given in **Table 1**. For the calculation of P_i and M_i values, the procedure[3] was used.

The processing of data in **Table 1** shows that when the procedure[1] was used, the standard deviation

$$\Delta = \sqrt{\frac{\sum_{i=1}^{N-1} (S_i^C - S_i^A)^2}{N-1}}$$

of calculated values of the wall thickness S_i^C from the actual values of this parameter S_i^A was $\Delta = 0.150$ mm. When the present mathematical model was used, the value of Δ was of 0.085 mm and for 1.76 times smaller. Hence, the rolling parameter calculation accuracy is improved when the proposed procedure is used.

7 CONCLUSION

The mathematical model of the lengthwise continuous plugless tube rolling process has been developed and successfully tested. It improves the accuracy of calculation of the process parameters in comparison with the earlier developed procedure.

ACKNOWLEDGEMENT

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