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Regular colouring defect of a cubic graph and the conjectures of Fan-Raspaud and Fulkerson*

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Abstract

We introduce a new invariant of a cubic graph – its regular colouring defect – which is defined as the smallest number of edges left uncovered by any collection of three perfect matchings that have no edge in common. This invariant is a modification of colouring defect, an invariant introduced by Steffen in 2025, whose definition does not require the empty intersection condition. In this paper we discuss the relationship of this invariant to the well-known conjectures of Fulkerson (1971) and Fan and Raspaud (1994) and prove that colouring defect and regular colouring defect can be arbitrarily far apart.

Keywords: Cubic graph, perfect matching, colouring defect, Fulkerson Conjecture, Fan and Raspaud Conjecture.

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Regularni barvni defekt kubičnega grafa in domnevi Fan–Raspaudova ter Fulkersona*

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Povzetek

Vpeljemo novo invarianto kubičnega grafa – njegov regularni barvni defekt – ki je definiran kot najmanjše število povezav, ki jih ne pokriva nobena trojica popolnih prirejanj, ki nimajo skupne povezave. Ta invarianta je predelava barvnega defekta, invariante, ki jo je Steffen uvedel leta 2025 in pri kateri v definiciji ni zahtevan prazen presečni pogoj. V tem članku obravnavamo razmerje med tema invariantama in dobro znanima domnevama Fulkersona (1971) ter Fana in Raspauda (1994) ter dokažemo, da sta barvni defekt in regularni barvni defekt lahko poljubno daleč narazen.

Ključne besede: kubični graf, popolno prirejanje, barvni defekt, Fulkersonova domneva, domneva Fana in Raspauda.

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