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INTRODUCTION TO METEOROLOGY: SOLVED PROBLEMS

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Contents

Preface

This booklet is an English translation of the Slovenian language booklet *Rešene naloge iz osnov meteorologije* (ISBN: 978-961-6619-45-5). It is intended to supplement the *Introduction to Meteorology* course at the Faculty of Mathematics and Physics, University of Ljubljana. An integral part of this course is problem-solving exercises. The booklet is based on the problems initially collected by Prof. Zdravko Petkovšek, Prof. Jože Rakovec, Asst. Prof. Tomaž Vrhovec, Neva Pristov, MS, and Gregor Gregorič, PhD. The initial set of problems was a core that the authors later extended. Each problem in the booklet is presented with a solution.

The booklet is organized into sections related to the book *Osnove meteorologije za naravoslovce in tehnike* (ISBN: 978-961-6619-39-4)[1], written by Prof. Jože Rakovec and Asst. Prof. Tomaž Vrhovec.

The booklet is dedicated to our colleague Tomaž Vrhovec, who lost his life in an avalanche in the Julian Alps. The authors also want to thank Prof. Jože Rakovec for reviewing and providing useful advice and to Dr. Katarina Kosovelj and student Žiga Valentič for help in finding errors in the original Slovenian language version of the booklet. The authors also want to thank Veronika Hladnik, MS, for help with the English translation of the booklet.

The Authors

[1] freely available at `https://www.dlib.si/details/URN:NBN:SI:DOC-EXG6P2Z0`

1 Units

1.1 Convert the following units for pressure: [Solution]

1000 mbar = Pa,

600 mbar = hPa,

500 mbar = bar.

1.2 Convert the following units for temperature: [Solution]

12.5 °C = K,

290 K = °C,

−14 °C = K.

1.3 Convert from one set of units to another: [Solution]

3 days = seconds,

20000 h = years,

900 m^2 = km^2,

3 litres = m^3,

30 m/s = km/h,

100 km/h = m/s,

100 MW = W,

1400 W/m^2 = W/km^2,

0.005 K/s = °C/hour,

1.5 mbar/100 km = Pa/m.

100 MJ = kWh,

5 kWh = MJ.

2 Structure and layers of the atmosphere

2.1 Calculate the molar mass of the air, assuming that the mass fraction of oxygen in the atmosphere is 24% and nitrogen 76%.

Solution:

$$M_{O_2} = 32 \text{ kg/kmol}, \quad M_{N_2} = 28 \text{ kg/kmol}.$$

$$p = \frac{mR^*T}{VM} = p_{O_2} + p_{N_2} = \frac{m_{O_2}R^*T}{VM_{O_2}} + \frac{m_{N_2}R^*T}{VM_{N_2}}.$$

The mass fractions: $m_{O_2} = m \cdot 0.24, \quad m_{N_2} = m \cdot 0.76.$

From here we obtain: $\frac{1}{M} = \frac{0.24}{M_{O_2}} + \frac{0.76}{M_{N_2}}$,

$$M = 28.866 \text{ kg/kmol}.$$

2.2 What is the mass of the air in a classroom with dimensions 10 m × 10 m × 3 m, if the atmospheric pressure is 1013 mbar and the temperature 25 °C? [Solution]

2.3 What is the molar mass of moist air that has the partial water vapour pressure of 15 mbar and total atmospheric pressure (the sum of dry air and water vapour) of 1010 mbar? [Solution]

2.4 What is the density of dry air if the atmospheric pressure is 1000 mbar and the temperature 30 °C (−16 °C)? [Solution]

2.5 How much does the air density change if we descend from an altitude of 1500 metres, where the temperature is 20 °C and the atmospheric pressure is 855 mbar to an altitude of 450 m, where the pressure is 960 mbar and the temperature 30 °C? [Solution]

2.6 What is the mass of argon in the room in which the temperature is 15 °C, the atmospheric pressure 1000 mbar, and the room volume 100 m^3? The mass fraction of argon is 1.28%. [Solution]

2.7 Estimate the mass of the atmosphere of the Earth. [Solution]

3 Hydrostatics

3.1 At sea level the measured atmospheric pressure was 1005 mbar and the temperature was 15 °C. Calculate the pressure at the altitude of 4000 m for the following two cases:

a) atmosphere is isothermal (temperature does not change with height),

b) Temperature is linearly decreasing with height at a rate of 5 K/km.

Solution:
a) Because the atmosphere is isothermal, we can use the equation:

$$p = p_0 \cdot \mathrm{e}^{-\frac{g(z-z_0)}{RT}},$$

where p_0 is the atmospheric pressure on the sea level – 1005 mbar and z_0 the altitude of the sea level – 0 m.

$$p = 625 \text{ mbar}.$$

b) Because the temperature is linearly changing with height, we can use equation:

$$p = p_0 \cdot \left[1 + \left(\frac{\partial T}{\partial z}\right)\frac{z - z_0}{T_0}\right]^{-\frac{g}{R\left(\frac{\partial T}{\partial z}\right)}},$$

where for $\left(\frac{\partial T}{\partial z}\right)$ we insert -0.005 K/m, because the temperature is decreasing with height. p_0, z_0 and T_0 are atmospheric pressure, altitude (0 m) and temperature at the sea level.

$$p = 614 \text{ mbar}.$$

3.2 For the examples below, calculate the height at which the atmospheric pressure equals 10 mbar. For all examples, take the default assumption that the atmospheric pressure and the temperature at sea level are 1013 mbar and 273 K.

a) Homogeneous atmosphere (air density is constant with height).

b) Isothermal atmosphere.

c) Atmosphere in which the temperature linearly decreases with height at a rate of 6.5 K/km.

Solution: a)

$$\Delta z = \frac{\Delta p}{\rho g} = 7908 \text{ m},$$

b)

$$\Delta z = \frac{RT}{g} \ln \frac{p_0}{p} = 36884 \text{ m},$$

c)

$$\Delta z = \frac{T_0}{\left(\frac{\partial T}{\partial z}\right)}\left[\left(\frac{p_1}{p_0}\right)^{-\frac{R\left(\frac{\partial T}{\partial z}\right)}{g}} - 1\right] = 24548 \text{ m}.$$

3.3 Calculate the ratio between the atmospheric pressures at the following altitudes: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 km and the ground, if the temperature decreases with height linearly at a rate of -0.01 K/m and the temperature at the ground is 10 °C? [Solution]

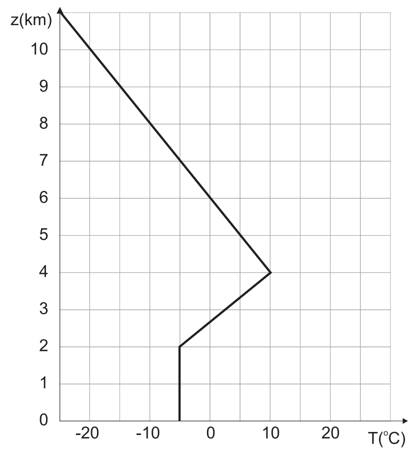
3.4 Calculate the height of 10 mbar level for the atmosphere with the temperature and atmospheric pressure at the ground at 15 °C and 1013 mbar. Between altitudes of 0 m and 100 m, the temperature increases by 2 K, between 100 m and 1000 m the temperature decreases by 7 K, above this height to the altitude of 3000 m the temperature is constant and from there upward the temperature decreases for 6.5 K/km up to the tropopause at the height of 12 km. From there, upward, the temperature is constant. [Solution]

3.5 At a height of 1500 m, the atmospheric pressure is 850 mbar. Calculate the pressure at the ground for the following two cases: [Solution]

a) The temperature from the ground upward is decreasing at 5 K/km and reaches -7 °C at the height of 1500 m,

b) Up to a height of 300 m above the ground lies cold air with the temperature of -1 °C. Above this height, the temperature rapidly (discontinuously) increases to 4 °C and then decreases with height by 8 K/km.

3.6 A meteorological balloon measured a vertical profile of temperature that is shown on the graph below. Independently, a meteorological station at an altitude of 2 km measures the temperature at -5 °C and the atmospheric pressure 850 mbar. [Solution]



a) What is the atmospheric pressure at sea level?

b) What is the atmospheric pressure at the height of 4 km? What is the air density at that height?

c) What is the atmospheric pressure at the height of 9 km?

d) At which altitude will the atmospheric pressure be 850 mbar, if conditions of standard atmosphere are assumed?

3.7 Determine the thickness of the air layer if the atmospheric pressure at the lower boundary is 779 mbar and at the upper boundary 545 mbar, if the layer is isothermal with the temperature 273 K. [Solution]

3.8 Calculate the height at which the atmospheric pressure is equal to 700 mbar. The atmospheric pressure at the ground is 1000 mbar, and the ground is located 207 m above the sea level. In the intermediate layer, the temperature decreases linearly with height with the gradient of 6.5 K/km, and the temperature at the ground is 22 °C. [Solution]

3.9 Calculate the height at which the atmospheric pressure is equal to 300 mbar if standard atmospheric conditions are assumed. [Solution]

3.10 The aeroplane uses an altimeter, which works based on atmospheric pressure measurements. The altimeter assumes conditions of ICAO standard atmosphere ($p_0 =$ 1013 mbar, $T_0 = 15$ °C, $\left(\frac{\partial T}{\partial z}\right) = -6.5$ K/km) and shows an altitude of 9000 m. What is the real altitude of the plane above the region if the atmospheric pressure at sea level is 980 mbar, the temperature 0 °C, and the temperature decreases with height by 8 K/km? [Solution]

3.11 When a mountaineer went from Aljažev dom in Vrata (the altitude of 1015 m), where the atmospheric pressure was 880 mbar, he set his altimeter and looked at the thermometer, which showed 15 °C. When he came to Kredarica, the temperature was 5 °C, and a meteorological observer told him that the temperature was essentially constant all day. His altimeter was showing 2500 m. By how much did the atmospheric pressure change on Kredarica, which is at the altitude of 2515 m? [Solution]

3.12 How thick and at which altitude is the air layer between 900 mbar and 800 mbar in the atmosphere? Atmospheric pressure and the temperature at the sea level are 1000 mbar and 10 °C, and from there upward, the temperature decreases by 5 K/1 km? [Solution]

3.13 A plane is circling above Portorož. The temperature and atmospheric pressure are 5 °C and 1020 mbar. A zero value of the altimeter, which assumes an ICAO standard atmosphere, is set to this pressure, and the altimeter shows that the plane is at the altitude of 3000 m. Because the bora wind is blowing, the air in the lower layer of the troposphere is well mixed so that the temperature decreases with the altitude by 9 K/km. What is the true altitude of the plane? [Solution]

3.14 During the night, the layer of air between 1000 mbar and 950 mbar losses 1 MJ/m^2 of energy due to emitted radiation. How much do the temperature and the thickness of the layer decrease during the night? [Solution]

3.15 The layer at the ground, between 1013 mbar and 950 mbar, absorbs 6 kWh of solar radiation energy on each square metre. What is the difference between the original and new thickness of this layer? [Solution]

3.16 What is the difference in height where the atmospheric pressure equals 925 mbar at Ljubljana and Koper? The ground level atmospheric pressure and temperature at Koper (which is at sea level) are 1009 mbar and 14 °C while at Ljubljana (300 m above sea level) the temperature is the same as at Koper while the atmospheric pressure is 975 mbar. Assume a temperature decrease with height of −6.5 K/km. [Solution]

3.17 What is the atmospheric pressure if the height of the column of mercury on the barometer is 732.5 mm and the temperature 15 °C? The density of mercury is 13.5 kg/l. [Solution]

3.18 How large is the error when performing the atmospheric pressure reduction at a meteorological measuring station that is 300 m above the sea level if the measured temperature is 288 K and the actual temperature between the station and sea level is higher by 1 K? [Solution]

3.19 In Slovenia, the atmospheric pressure reduction uses the temperature measured at the meteorological station. That kind of methodology leads to major errors for the stations at high altitudes. This is the reason that high-altitude stations use a different methodology for reduction. Thus, on Kredarica (altitude 2515 m), they determine the height of the 700 mbar isobar. What was the true measured atmospheric pressure on Kredarica if the temperature was 4 °C and the 700 mbar level was calculated to be at the height of 3000 m? [Solution]

3.20 How does the temperature change with height in a homogeneous atmosphere? [Solution]

3.21 During the winter, the formation of a cold-air lake in basins is a common phenomenon. The temperature at the ground is −5 °C, and in the lower 300 m of the atmosphere, an inversion exists with a vertical temperature increase of 0.001 K/m. Above this altitude, the vertical temperature gradient is the same as in the standard atmosphere. How much influence does the inversion have on the calculation of atmospheric pressure reduction? Assume that the bottom of the basin is 300 m above the sea level. [Solution]

3.22 What is the geopotential of 500 mbar level in the standard atmosphere? [Solution]

4 Basic laws

4.1 Calculate the size of the horizontal component of the Coriolis acceleration at different latitudes (10°, 30°, 50°, 80°) for the movement with speed 10 m/s.

Solution:
Horizontal component of the Coriolis acceleration is written as fv, where v is the wind speed and f the Coriolis parameter. $f = 2\Omega \sin\varphi$, where Ω is the angular velocity of the rotation of the Earth and φ is the latitude.

$$
\begin{aligned}
fv(\varphi = 10°) &= 2.52 \cdot 10^{-4}\ \mathrm{m/s^2},\\
fv(\varphi = 30°) &= 7.27 \cdot 10^{-4}\ \mathrm{m/s^2},\\
fv(\varphi = 50°) &= 1.11 \cdot 10^{-3}\ \mathrm{m/s^2},\\
fv(\varphi = 80°) &= 1.43 \cdot 10^{-3}\ \mathrm{m/s^2}.
\end{aligned}
$$

4.2 At what circling radius is the centrifugal force equal to 10% of the Coriolis force? Assume a latitude of 30° N and velocity 15 m/s. [Solution]

4.3 The air is moving horizontally towards the northeast at the speed of 25 m/s. To which direction does the horizontal component of the Coriolis force point, and how big is the force at 60° N? [Solution]

4.4 How much lighter does the mountaineer feel if he climbs on Kilimanjaro, which is 5800 m high and is located at 2° N? Otherwise, the climber lives at the altitude of 300 m at 50° N? [Solution]

4.5 How much less weight is displayed on a scale if a man is weighed on a train that is travelling through Slovenia at the speed of 120 m/s towards the west? And how much if the train is travelling at the same speed along the equator? [Solution]

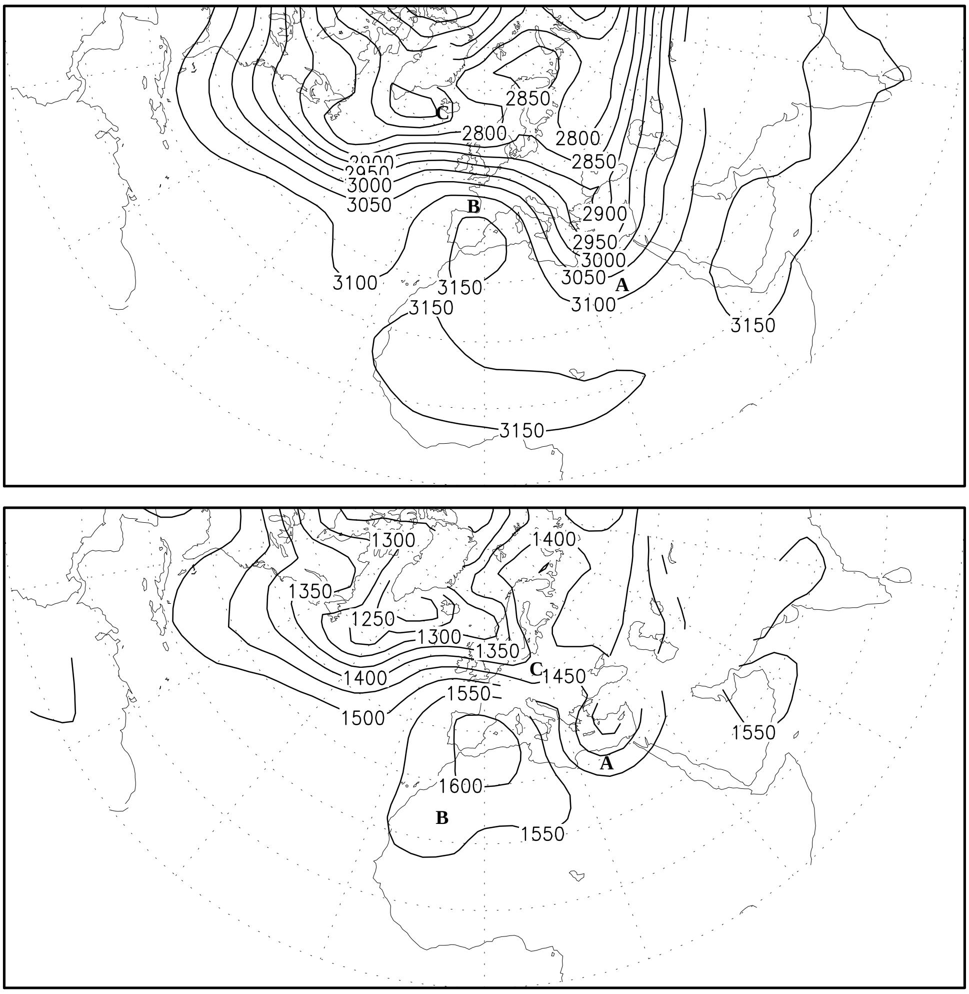
4.6 What is the ratio between the size of vertical and horizontal components of the pressure gradient force at the ground if the atmosphere is standard and the atmospheric pressure changes in the horizontal direction by 3 mbar at the distance of 180 km? [Solution]

4.7 What should be the volume of the hot air balloon, in which the temperature of the air is 50 °C, to raise itself and the basket with a combined weight of 300 kg? The temperature of the surrounding air is 10 °C and the atmospheric pressure 1020 mbar. [Solution]

4.8 The two maps below show the height of the 700 mbar and 850 mbar levels. For the three shown points (A, B and C), calculate:

a) compare the size of the specific horizontal pressure gradient force with the vertical pressure gradient force.

b) specific centrifugal force.



The height of the 700-mbar (top) 850-mbar (bottom) levels in metres.

Meridians are spaced apart by 20° and parallels by 10°. The temperatures are known in the three points:

p (mbar)	T_{A} (K)	T_{B} (K)	T_{C} (K)
700	273	272	255
850	278	290	262

Solution:
First, calculate the air density:

p (mbar)	ρ_A (kg/m^3)	ρ_B (kg/m^3)	ρ_C (kg/m^3)
700	0.9	0.8	0.96
850	1.06	1.02	1.13

Calculate the specific horizontal pressure gradient force using the distance between the neighbouring contour lines of the height of the constant atmospheric pressure:

$$-\frac{1}{\rho}\frac{\partial p}{\partial n} = -\frac{\partial \Phi}{\partial n}.$$

Calculate vertical component using the hydrostatic equation:

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = g.$$

Calculate specific centrifugal force indirectly via the gradient wind.

a) 700 mbar

point	R (km)	f (s^{-1})	V (m/s)	$-\frac{1}{\rho}\frac{\partial p}{\partial n}$ (m/s^2)	$-\frac{1}{\rho}\frac{\partial p}{\partial z}$ (m/s^2)	$\frac{V^2}{R}$
A	1000	$8.3 \cdot 10^{-5}$	12.8	$1.22 \cdot 10^{-3}$	9.81	$1.6 \cdot 10^{-4}$
B	-900	$1.0 \cdot 10^{-4}$	20.7	$1.63 \cdot 10^{-3}$	9.81	$4.8 \cdot 10^{-4}$
C	500	$1.3 \cdot 10^{-4}$	8.4	$1.22 \cdot 10^{-3}$	9.81	$1.4 \cdot 10^{-4}$

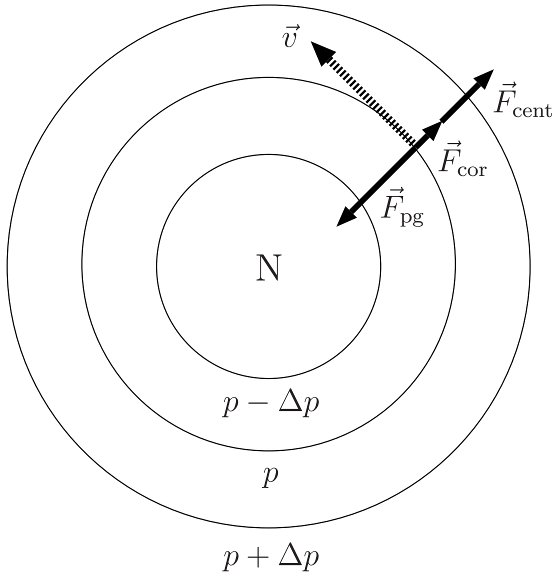
b) 850 mbar

point	R (km)	f (s^{-1})	V (m/s)	$-\frac{1}{\rho}\frac{\partial p}{\partial n}$ (m/s^2)	$-\frac{1}{\rho}\frac{\partial p}{\partial z}$ (m/s^2)	$\frac{V^2}{R}$
A	1000	$7.3 \cdot 10^{-5}$	14.1	$1.22 \cdot 10^{-3}$	9.81	$2.0 \cdot 10^{-4}$
B	-1000	$6.2 \cdot 10^{-5}$	6.9	$3.7 \cdot 10^{-4}$	9.81	$4.8 \cdot 10^{-5}$
C	500	$1.2 \cdot 10^{-4}$	7.5	$9.8 \cdot 10^{-4}$	9.81	$1.1 \cdot 10^{-4}$

5 Steady state horizontal winds

5.1 For a circular area of low pressure, draw the equilibrium of the forces and the velocities that are valid over the ocean (while neglecting the friction). Write the equation for determining the wind speed.

Solution:



From the equilibrium of the three forces, the following equation is obtained:

$$\vec{F}_{\mathrm{cent}} + \vec{F}_{\mathrm{cor}} + \vec{F}_{\mathrm{pg}} = 0,$$
$$\frac{v^2}{R} + fv - \frac{1}{\rho}\left|\nabla p\right| = 0.$$

v can be expressed via a solution to a quadratic equation as (only the positive solution is physically meaningful):

$$v = \frac{1}{2}\left(-fR + \sqrt{f^2R^2 + 4\,\frac{R}{\rho}\,\left|\nabla p\right|}\right) .$$

5.2 By what angle does the wind deviate away from the isobars in a circular area of low atmospheric pressure if the friction is not neglected? The friction coefficient is 0.00001 s^{-1}, the radial component of the pressure gradient is 2 mbar/100 km and the air density 1 kg/m^3. The cyclone is located at 45° N. What is the wind speed at a distance of 300 km from the centre?

Solution:
Due to friction, the wind in the cyclone deviates towards the centre. If we equalise the force components in the two directions, we obtain:

$$fv + \frac{v^2}{R} = \frac{1}{\rho}\left|\nabla p\right|\cos\beta,$$
$$kv = \frac{1}{\rho}\left|\nabla p\right|\sin\beta.$$

Assuming that β is small: $\cos\beta \approx 1$ and $\sin\beta \approx \beta$. The wind velocity can be expressed from the first equation, which is the same as for the example without friction. The velocity is 13.52 m/s. From the second equation, the deviation angle can be expressed:

$$\beta = \frac{kv\rho}{|\nabla p|} = 0.068 \text{ radian} = 4°.$$

5.3 In a cyclone, at what radius does the wind equal to 8 m/s, if the gradient of the atmospheric pressure in the radial direction is 1 mbar/100 km. The friction can be neglected, and the air density is 1 kg/m^3. Assume the cyclone is located at a latitude of 45°. [Solution]

5.4 Calculate the wind speed in the anticyclone at 45° N, if the gradient of the atmospheric pressure is 2 mbar/400 km and the radius of the curvature of isobars is 400 km. Assume air density of 0.7 kg/m^3. [Solution]

5.5 In the anticyclone, the air circulates at 45° N at radius of 1000 km. At the height at which the atmospheric pressure equals 500 mbar, the wind speed is 20 m/s. What is the size of the horizontal atmospheric pressure gradient that determines this movement? [Solution]

5.6 How strong is the geostrophic wind at 30° N if the size of the atmospheric pressure gradient is 2 mbar/100 km and the air density 0.5 kg/m^3?

Solution:
The geostrophic equilibrium applies when the pressure gradient force is balanced with the Coriolis force:

$$fv_{\text{g}} = \frac{1}{\rho}\left|\nabla p\right|,$$
$$v_{\text{g}} = \frac{1}{\rho f}\left|\nabla p\right| = 55 \text{ m/s}.$$

5.7 In the field of straight isobars at our latitudes (46° N, 15° E), the wind is blowing at the speed of 30 km/h and deviates by 15° from the direction of the isobars. Under the influence of which forces does the wind blow? What are the sizes of these forces? How large is the gradient of the atmospheric pressure? The air density is 0.5 kg/m^3. [Solution]

5.8 Calculate the wind speed in a cyclone at 70° N, if the atmospheric pressure gradient size is 4 mbar/450 km and the radius of the isobar curvature is 400 km. The air density is 0.7 kg/m^3. [Solution]

5.9 What is the ratio of the speeds between the gradient and the geostrophic wind in a cyclone located at 50° N if the atmospheric pressure gradient size is 1 mbar/100 km and the radius is 1500 km? The air density is 1 kg/m^3. [Solution]

5.10 On a weather map in a scale 1 : 50 000 000 at 60° N, the distance between the two nearby straight isobars is 1 cm. The atmospheric pressure interval between the isobars is 4 mbar. What atmospheric pressure gradient size and what geostrophic wind speed corresponds to this field? What is the wind speed at the same interval between the isobars at 50° N? The wind direction in this example is not relevant. The air density is 1 kg/m^3. [Solution]

5.11 In the field of the straight isobars, the wind is blowing at 60° N over an extensive plane. The atmospheric pressure gradient size is 2 mbar/150 km and the ground is uniformly rough, such that the coefficient of the linear friction is 10^{-4} s^{-1}. Draw the balance of the forces, specify the size of the specific forces, and calculate the wind speed. The air density is 1 kg/m^3. [Solution]

5.12 Due to friction, the wind deviates by 30° to the left of the straight isobars. The isobars are plotted every 5 mbar, and the distance between two nearby isobars is 200 km. The coefficient of the linear friction is 10^{-4} s^{-1}. What is the wind speed, and how large are the forces that hold the balance at that wind? The air density is 0.7 kg/m^3. [Solution]

5.13 What is the difference in atmospheric pressure between the point on the edge and in the centre of the tropical cyclone, if on the edge, 500 km from the centre, the wind is blowing at a speed of 200 km/h and the atmospheric pressure is decreasing towards the centre linearly? What are the pressure differences, if you assume that the frictional force exists that is proportional to the square of the speed? The coefficient of the square friction is 10^{-7} m^{-1}. The air density is in both cases 1 kg/m^3. [Solution]

5.14 What is the atmospheric pressure at the centre of the tropical cyclone, if on its edge, 400 km from the centre, the wind blows at the speed of 150 km/h and the atmospheric pressure is 970 mbar? The pressure field in a hurricane is parabolic, with the minimum in the centre. The air density is 1 kg/m^3. [Solution]

5.15 The undulating westerly wind is blowing at the height of 5500 m around the Earth. At 30° N, there is a trough, where the air circulates through the part of the circle with the radius of 1000 km and the wind blows at the speed of 20 m/s. What is the radius of the rotation at the ridge, at 60° N, if the speed is the same there? [Solution]

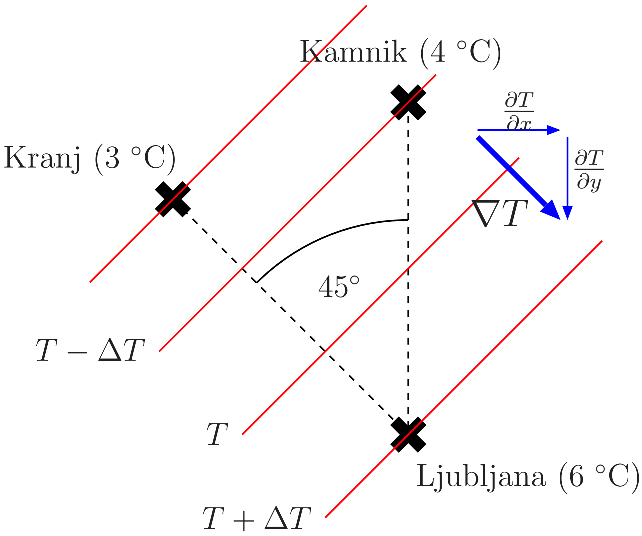
5.16 A tornado is rigidly rotating (the angular speed is independent of the radius). What is the atmospheric pressure field around the tornado centre if the air density is the same everywhere (horizontal and vertical homogeneity)? [Solution]

5.17 A tornado axis is tilted by 30 degrees from the vertical. What should be the speed of the rotation 100 m from the axis of the rotation if the atmospheric pressure is not changing with the height? [Solution]

6 Local, individual and advective changes

6.1 The temperature in Ljubljana is 6 °C, while in Kranj, which lies 20 km northwest of Ljubljana, the temperature is 3 °C. In Kamnik, which lies 20 km north of Ljubljana, the temperature is 4 °C. The atmosphere is calm, and the temperature field changes linearly everywhere (the temperature field can be written with the equation $T = T_0 + ax + by$). In which direction does the temperature gradient point, and what is its value? Sketch the temperature field with the isotherms.

Solution:



To calculate the temperature gradient, the constants T_0, a, and b need to be determined first. The three known temperatures can be inserted into the equation of the temperature field ($T = T_0 + ax + by$), which produces three linear equations for the three unknown variables.

If the origin of the coordinate system ($x = 0$, $y = 0$) is placed at Ljubljana and temperature ($T_{\mathrm{LJ}} = 6$ °C) inserted into the equation, the $T_0 = 6$ °C is obtained. If the data for Kamnik is inserted ($T_{\mathrm{KM}} = 4$ °C, $x = 0$, $y = 20$ km) the $b = -0.10$ K/km is obtained. Finally, from the data for Kranj ($T_{\mathrm{KR}} = 3$ °C, $x = -20$ km $\cos 45°$, $y = 20$ km $\sin 45°$) the $a = 0.11$ K/km is obtained.

The temperature gradient is

$$\nabla T = \left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}\right) = (a, b) = (0.11 \text{ K/km}, -0.1 \text{ K/km}).$$

6.2 The field of the atmospheric pressure can be described with the function

$$p = p_0 + a \cdot y,$$

where $p_0 = 1000$ mbar and $a = -2$ mbar/100 km. Calculate the atmospheric pressure gradient. Draw the pressure field using the isobars and draw the vectors of the gradient into this field. What is the pressure at a point that has the coordinates $x = 100$ km, $y = 200$ km? [Solution]

6.3 Over the ocean, the atmospheric pressure field can be described with the following a function:

$$p = p_0 + ay + bx^2,$$

where $p_0 = 1000$ mbar, $a = -2$ mbar/100 km and $b = 3 \cdot 10^{-4}$ mbar/km^2. Calculate the pressure gradient. Draw the pressure field using the isobars, and draw the vectors of the gradient on this field. What is the pressure at the point with the coordinates $x = 100$ km, $y = 100$ km? [Solution]

6.4 Over the ocean, the atmospheric pressure field can be described with the following a function:

$$p = p_0 + a(y - y_0) + b(x - x_0),$$

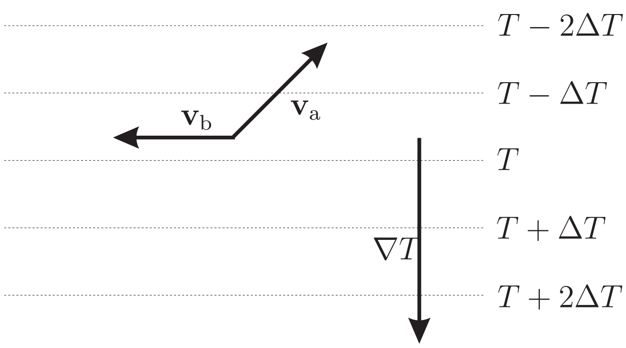
where $x_0 = 500$ km, $y_0 = 300$ km, $a = 1$ mbar/100 km, $b = 0.5$ mbar/100 km and $p_0 = 1010$ mbar. Draw the pressure field with the interval between 2 mbar isobars. Calculate the pressure gradient. Draw the vector of the pressure gradient on the atmospheric pressure field. Calculate the atmospheric pressure at the point with the coordinates $x = 300$ km, $y = -100$ km? [Solution]

6.5 Over Slovenia, the temperature decreases from south to north at a rate of 3 K/100 km. How much will the temperature change in three hours, if

a) there is a south-westerly wind with a speed of 10 m/s,

b) there is an easterly wind with a speed of 10 m/s,

Solution:
The temperature field, the temperature gradient (∇T) and the wind vectors ($\mathbf{v_a}$ and $\mathbf{v_b}$) are shown in the sketch below.



The temperature gradient always points in the direction of the maximum increase of temperature and, in this case, only has the component in the y direction. It can be written as $\nabla T = (0,\ -3\ \mathrm{K}/100\ \mathrm{km})$.

Similarly, the wind vector for case a can be determined:

$$\mathbf{v_a} = (10 \cdot \cos 45^\circ\ \mathrm{m/s},\ 10 \cdot \sin 45^\circ\ \mathrm{m/s}) = (5\sqrt{2}\ \mathrm{m/s},\ 5\sqrt{2}\ \mathrm{m/s}).$$

The equation that connects the local, individual, and advective changes can be used. In this case, the individual change ($\frac{dT}{dt}$) is equal to zero because the air is not heating or cooling.

$$\begin{aligned}\frac{\partial T}{\partial t} &= \frac{dT}{dt} - \mathbf{v_a}\nabla T = -\mathbf{v_a}\nabla T\\ &= -(5\sqrt{2}\ \mathrm{m/s},\ 5\sqrt{2}\ \mathrm{m/s}) \cdot (0,\ \ -3\ \mathrm{K}/100\ \mathrm{km})\\ &= -5\sqrt{2}\ \mathrm{m/s} \cdot 0 - 5\sqrt{2}\ \mathrm{m/s} \cdot (-3\ \mathrm{K}/100\ \mathrm{km})\\ &= 0.764\ \mathrm{K/h}.\end{aligned}$$

In three hours, the temperature will change by

$$\Delta T = \frac{\partial T}{\partial t}\Delta t = 0.764\ \mathrm{K/h} \cdot 3\ \mathrm{h} = 2.3\ \mathrm{K}$$

For the second case (the easterly wind), the wind vector can be composed in a similar way $\mathbf{v_b} = (-10, 0)$ m/s. Since the wind and the temperature gradient are perpendicular, the $\frac{\partial T}{\partial t}$ will be equal to zero, and the temperature will not change.

6.6 Over an area, the temperature is decreasing northwards at the rate of 1 K/100 km and the wind is blowing from a southwest direction at the speed of 10 m/s. How does the temperature change if there are no individual changes in temperature? How fast should the wind blow from the south so that the temperature will change at the same rate as before? [Solution]

6.7 Determine the temperature change rate on the meteorological station if there is cloudy weather, the south-westerly wind is blowing at the speed of 12 km/h, and the temperature in the atmosphere is decreasing from south to north: 100 km towards the south the temperature is 12 °C and 50 km towards the north the temperature is 6 °C. [Solution]

6.8 A ship is sailing straight from one island to a second island. The first island lies at 14° E and 38° N and the second at 15° E and 39° N. On the ship, it was recorded that the temperature increased for 1 K in three hours. The ship sails at the speed of 12 knots (6 m/s). What is the temperature at the second island if the temperature is 22 °C at the first island? The temperature between the islands is changing linearly, and the atmosphere is calm. [Solution]

6.9 Over Slovenia, the temperature increases from the northeast to the southwest at a rate of 5 K/100 km. The air is becoming warmer because of the received energy of the solar radiation, and its temperature increases by 3 K/h. What will be the temperature after 2 hours, if now the temperature is 15 °C and the southerly wind is blowing at a speed of 10 m/s? [Solution]

6.10 Over Europe, the temperature field can be described with the equation

$$T = T_0(ax^2 + bxy^2 + c).$$

The temperature was measured at three locations: $T(x = 20 \text{ km},\ y = 100 \text{ km}) = 10$ °C, $T(x = -200 \text{ km},\ y = 100 \text{ km}) = 5$ °C and $T(x = 100 \text{ km},\ y = -150 \text{ km}) = -7$ °C. What is the rate of temperature change at location $x = 100$ km, $y = 100$ km, if a north-westerly wind is blowing with speed of 15 m/s? [Solution]

6.11 An aeroplane is flying at the height of 12 km, and its path intersects a medium sized cyclone. The closest distance to the centre of the cyclone that the aeroplane reaches is 1500 km. In the cyclone's centre, the air is colder, while in the surrounding area, the air is warmer. When the aeroplane is 2000 km from the centre, it measures the temperature as being -55 °C; when it is closest to the centre, it measures -58 °C. What is the temperature gradient in the radial direction if we assume that the cyclone is circularly symmetric? [Solution]

7 Humidity

7.1 The relative humidity in the room is 70% and the temperature 18 °C. The atmospheric pressure is 1000 mbar. What are the values of a) the vapour pressure, b) the absolute humidity, c) the specific humidity, d) the mixing ratio and e) the dew point temperature?

Solution:

a) To determine the vapour pressure, the saturated vapour pressure first needs to be calculated (it is a function of the temperature only) using the Claussius-Clapeyron equation:

$$\begin{aligned} e_s(T) &= e_{s0}\, \mathrm{e}^{\frac{h_\mathrm{i}}{R_v}\left(\frac{1}{T_0}-\frac{1}{T}\right)} \\ &= 20.84 \text{ mbar}, \end{aligned}$$

where $e_{s0} = 6.1$ mbar and $T_0 = 273$ K. h_i and R_v are the latent heat of vaporisation and the meteorological gas constant for the water vapour (2.5 MJ/kg and 461 J/kg K).

Since relative humidity is $f = 70\%$, the vapour pressure is 70% of the saturated vapour pressure.

$$e = e_s \cdot f = 14.59 \text{ mbar}.$$

b) The absolute humidity is the density of the water vapour ρ_v for which the ideal gas equation can be used $e = \rho_v R_v T$

$$\rho_v = \frac{e}{R_v T} = 0.0108 \text{ kg/m}^3.$$

c) The specific humidity q is the mass concentration of the water vapour in the air

$$\begin{aligned} q &= \frac{m_v}{m} = \frac{\rho_v}{\rho} = \frac{e}{p}\frac{R}{R_v} \\ &= 9.08 \text{ g/kg}. \end{aligned}$$

d) The mixing ratio is defined as the ratio between the mass of the water vapour and the mass of dry air

$$\begin{aligned} r &= \frac{m_v}{m_z} = \frac{\rho_v}{\rho_z} = \frac{e}{p-e}\frac{R}{R_v} \\ &= 9.21 \text{ g/kg}. \end{aligned}$$

e) The dew point temperature T_d is defined as the temperature by which the current vapour pressure ($e = 14.59$ mbar) becomes saturated. Again, the Claussius-Clapeyron equation can be used so that T is expressed and for e_s the current vapour pressure ($e = 14.59$ mbar) is used.

$$\begin{aligned} T_d &= \left(\frac{1}{T_0} - \frac{R_v}{h_\text{i}} \ln \frac{e}{e_{s0}} \right)^{-1} \\ &= 285.5 \text{ K.} \end{aligned}$$

7.2 Determine the relative humidity, if the temperature is 5 °C and the dew point temperature is −5 °C. The atmospheric pressure is 1000 mbar. [Solution]

7.3 Calculate the dew point temperature under the following conditions: the temperature is 18 °C, the atmospheric pressure is 990 mbar, and the relative humidity is 65%. [Solution]

7.4 Determine the dew point temperature, if the relative humidity is 60%, the temperature −15 °C and the atmospheric pressure 1000 mbar? [Solution]

7.5 How much lighter is one cubic metre of moist air with 90% relative humidity from the air in which the relative humidity is only 10%? The temperature is in both cases 20 °C, and the atmospheric pressure is 1000 mbar. [Solution]

7.6 What is the mass of the water vapour in the classroom that is high 3 m, long 6 m and wide 5 m. The temperature is 22 °C, atmospheric pressure is 990 mbar and vapour pressure is 13 mbar? [Solution]

7.7 What is the air density at the ground if the atmospheric pressure is 1020 mbar and the temperature is 13 °C? What is its density if the relative humidity of the air is 70%? [Solution]

7.8 What is the specific humidity of the air if the vapour pressure is 5 mbar, the air density 1.1 kg/m^3 and the air temperature 10 °C? [Solution]

7.9 What is the saturated specific humidity at 1000 mbar and the temperatures 30 °C and −15 °C? [Solution]

7.10 How much does the relative humidity relatively change, if at the normal conditions: [Solution]

a) the air temperature changes for 3 K and the vapour pressure does not change?

b) the vapour pressure of the water vapour changes for 1% (0.1 mbar), and the temperature stays the same?

7.11 We are cooling the moist air, which has a temperature of 30 °C and vapour pressure of 10 mbar. At which temperature will saturation occur? How much of water is condensed, per unit of the air volume, when the air is cooled to the final temperature of −16 °C? [Solution]

7.12 The relative humidity of the air is 65% and the temperature is 15 °C. What will the new relative humidity be if the air would isobarically heat up due to receiving 5000 J of the solar energy per kilogram? The process takes place at the ground where the air has the density of 1 kg/m^3. [Solution]

7.13 In the evening the measured air temperature is 20 °C and the relative humidity is 80%. Over the night, the air temperature near the ground will decrease by 7 K. Will dew form during the night on the ground? [Solution]

7.14 The temperature and dew point temperature of the 20-m thick air layer at the ground are 20 °C and 10 °C. The ground is moist, and the water evaporates into the air. What is the mass of the evaporated water in every quadratic metre of the ground if you assume that the ground-level air becomes saturated and that there is no mixing with its surroundings? The atmospheric pressure is 1000 mbar. [Solution]

7.15 During the winter, we sometimes ventilate our apartment. Into the room in which the temperature is 20 °C and 50% relative humidity, we let in the outside air, which has the temperature −5 °C and 90% relative humidity. The cold air replaces half of the volume of the warm air in the room. The air is then mixed, and the mixture is again heated to 20 °C while the atmospheric pressure remains constant at 1000 mbar. What is the final relative humidity? [Solution]

7.16 How much of the water has to evaporate from the evaporator if we want to achieve a 70% humidity in the room, which is 4 m × 3 m × 2.5 m large and has a constant temperature of 25 °C and the initial relative humidity of 45%. Assume a constant atmospheric pressure of 1000 mbar. [Solution]

7.17 At sunset, the temperature is 15 °C and the relative humidity is 80%. On a clear night, the air is cooling by 1 K per hour. If the night lasts for ten hours, will there be dew and fog in the morning? [Solution]

7.18 Two air masses are uniformly mixed so that in every kilogram of air, there is exactly half a kilogram of air from one and half kilogram of air from the second air mass. Mixing takes place at the constant atmospheric pressure of 1000 mbar. The temperature of the warm air is 21 °C, and the temperature of the cool air is 5 °C. The warm air is saturated, and in the cool air, the relative humidity is 80%. Determine the temperature after the mixing. Did condensation occur, and if it did, how much water did condense? [Solution]

7.19 The air has the temperature 10 °C and the atmospheric pressure is 1000 mbar. How much of the water is condensed from each cubic metre of the saturated air if the air cools by 1 K? [Solution]

7.21 Calculate the saturated vapour pressure at −12 °C over water and over ice? [Solution]

7.22 What is the ratio of the relative humidity over the water and over the ice at a temperature of −5 °C? [Solution]

7.23 How is the relative humidity changing if the temperature at the ground is changing sinusoidally with an amplitude of 10 K, has the maximum at 14:00 and the minimum at 8:00 and the average temperature is 15 °C. The specific humidity and the atmospheric pressure are constant at 3 g/kg and 1020 mbar, respectively. What will be the relative humidity at 10:00? [Solution]

7.24 Calculate the relative humidity if the temperature of the dry bulb thermometer is 12 °C and the temperature of the wet bulb thermometer is 10 °C. Assume an atmospheric pressure of 960 mbar. [Solution]

7.25 Calculate the dew point temperature if you know the temperature of the dry bulb thermometer is 20 °C and the temperature of the wet bulb thermometer is 12 °C and the atmospheric pressure is 1020 mbar. [Solution]

7.26 Calculate the temperature of the wet bulb thermometer at the atmospheric pressure of 1000 mbar, when the temperature is 18 °C and the relative humidity is 65%. [Solution]

7.27 When the temperature is 10 °C, it starts raining with an intensity of 5 mm/h. At the beginning of the rain, a bridge has a temperature of −2 °C, so icing starts forming on it. The bridge is 20 cm thick. The specific heat capacity of the bridge is 800 J/kg K and the density 2500 kg/m^3. The drops have the same temperature as the air. [Solution]

a) How long should the rain fall before the bridge heats up by 1 °C?

b) How long should the rain fall before no more ice is on the bridge?

7.28 What is the mass of the precipitation that is intercepted by the rain gauge over a time of three hours? Assume the horizontal wind speed is 10 m/s, the falling speed of the raindrops is 18 m/s and that in each cubic metre of the air, there is 1 g of water in liquid state. The surface area of an ombrometer is 4 dm^2. What is the mass of the precipitation if, instead of raindrops, snowflakes are falling at the speed of 5 m/s? [Solution]

7.29 Calculate the virtual temperature of the air (T_v) at 30 °C, if the specific humidity is $20 \cdot 10^{-3}$. The virtual temperature is the temperature that dry air would have, if its density were the same as for moist air. [Solution]

8 Adiabatic changes

8.1 At which height will a cloud base form if the air is rising from the ground upwards? At the ground, the air temperature and dew point temperature are 15 °C and 11.6 °C?

Solution:
The rising air is cooling at 10 K/km ($\Gamma_a = 10$ K/km). Γ_a is, by definition, always positive, although the temperature of the rising air decreases with height. When using Γ_a, one needs to be careful to prevent errors due to incorrect usage of a positive or negative sign.

Besides the temperature, the dew point temperature decreases in the rising air. The decrease is approximately equal to $\frac{1}{6}\Gamma_a$.

The cloud base is at a height where the temperature of the rising air is equal to the dew point temperature of this air.

$$T - \Gamma_a \cdot z_B = T_d - \frac{1}{6}\Gamma_a \cdot z_B$$
$$z_B = 0.4 \text{ km}$$

8.2 Part of the air, which is close to the ground, is overheated by 10 K above the ambient temperature and is rising in the standard atmosphere. What is the hydrostatically unbalanced part of the specific buoyancy force 500 m above the ground?

Solution:
The air temperature at the ground in the standard atmosphere is $T_{00} = 288.15$ K and is decreasing with height at a rate of 6.5 K/km ($\frac{\partial T}{\partial z} = -0.0065$ K/m).

The temperatures of the rising and surrounding air at 500 m above the ground are:

$$T_{ok} = T_0 + \left(\frac{\partial T}{\partial z}\right)\Delta z = 284.9 \text{ K},$$
$$T = T_0 - \Gamma_a \Delta z = 293.15 \text{ K}.$$

The hydrostatically unbalanced part of the specific buoyancy force is:

$$\frac{dw}{dt} = g\frac{T - T_{ok}}{T_{ok}} = 0.28 \text{ m/s}^2.$$

8.3 The air, which has the temperature of 15 °C and the specific humidity 1.1 g/kg at the ground, is rising to the height of 6000 m because of the unbalanced buoyancy. What will the temperature of air at this height be and at which height will the air become saturated? The altitude of the ground is 0 m and the atmospheric pressure at the ground is 1000 mbar; the moist adiabatic lapse rate is 7 K/km. [Solution]

8.4 From Radovljica, one can see that the base of the orographic cloud is at the height of 1700 metres on the slope of mountain Stol. A light southerly wind is blowing, and air from Radovljica is climbing along the slope of Stol. What is the relative humidity in Radovljica if the air temperature there is 15 °C? The altitude of Radovljica is 450 m. [Solution]

8.5 The wind is blowing at the speed of 8 m/s along an extensive slope, that has a tilt of 4 °C. Dry air travels from the bottom to the top of the hill in 26 minutes. How much does the temperature change on the top of the slope after the wind starts to blow, if the horizontal temperature gradient is negligible and the vertical component of the gradient is 5 K/1000 m? Evaluate the temperature change, if the air is saturated. Take into account the approximate value of the moist adiabatic lapse rate $\Gamma_s = 6$ K/km. [Solution]

8.6 The wind is blowing from the valley up to the hill. In the valley, the temperature is 15 °C and the relative humidity is 80%. What is the relative humidity 500 m above the bottom of the valley? [Solution]

8.7 A part of the air with temperature 17 °C and absolute humidity 7 g/m^3 is rising adiabatically. How much will the absolute humidity decrease if the air rises by 1000 m without exchanging the heat or the humidity with the surroundings? [Solution]

8.8 How much does the relative humidity of the air change if the air rises by 500 m at the constant specific humidity of 5 g/kg? The temperature at the ground is 20 °C, and the atmospheric pressure is 1000 mbar. Calculate the height of the condensation level. [Solution]

8.9 The air is raised adiabatically from 1000 mbar and 28 °C, to 850 mbar, where it becomes saturated. What is the relative humidity of air at the ground? At 200 metres above the level of condensation, the temperature decreases by 1 K and the atmospheric pressure by 18 mbar. How many grams of the water is condensed per unit of the air mass if the air rises by an additional 200 m? [Solution]

8.10 Calculate the height of the free convection of the unsaturated air if the vertical temperature gradient is 6.5 K/km and the air near the ground warms up by 5 °C. [Solution]

8.11 In the morning, the atmosphere above the airport is stable, with a temperature decrease of 3 K per 1000 m. The temperature at the ground was 10 °C, and the dew point temperature was 2 °C. During the day, the air near the ground becomes warmer, and at 11:00, the temperature is 18 °C. To which height did the convection extend? Did cumulus clouds form? If they did, where was their base? [Solution]

8.12 Dry air is rising from 1000 mbar to 700 mbar without mixing or exchanging the heat with its surroundings. At the beginning, it has a temperature of 10 °C. What is the initial density of the air? What are the final temperature and the density of the air? [Solution]

8.13 The wind blows towards a 800-m high ridge, and the air, which has a temperature of 15 °C and 70% relative humidity, has to forcibly lift along the slope of the ridge. Does an orographic cloud form at the ridge? At which height will the cloud base be in case of 90% relative humidity? [Solution]

8.14 In the morning, over the sea, the air temperature is 20 °C with a 3 K/km decrease to the height of 2 km. From there, upward, a strong inversion exists, and the temperature is increasing by 2 K/km. To which height will the convection extend if the air at the ground warms up to 27 °C during the day? Do the clouds appear if the relative humidity at the ground is 60% in the morning? [Solution]

8.15 The air with the initial temperature 294 K and specific humidity 10 g/kg is rising at the hill from 1000 mbar to 700 mbar. What is the dew point temperature at 1000 mbar? At which atmospheric pressure is the lifted condensation level? [Solution]

8.16 The air with a temperature of 18 °C and a relative humidity of 85% is flowing over the Alps. On the south side, it rises from the level of the Adriatic Sea to the top of the ridge (3000 m), and on the north side, it sinks to Bavaria (700 m) without precipitation fall out. What are the temperature and the relative humidity north of the Alps? Assume a moist adiabatic lapse rate of $\Gamma_s \sim 7$ K/km. [Solution]

8.17 Over the mountains blows the Foehn wind, which has the temperature of 38 °C and the mixing ratio of 4 g/kg at 1000 mbar. Can this be the same air as on the windward side of the mountain, where the temperature is 21.5 °C and the mixing ratio 10 g/kg at 1000 mbar? [Solution]

8.18 The air at 20 °C and mixing ratio 8 g/kg is raised from 1000 mbar along the hill to 700 mbar. What was the dew point temperature before the rise? What is the temperature on the other side of the hill at 900 mbar, if 80% of the mass of condensed water falls from the cloud and the precipitation water does not evaporate into the air? [Solution]

8.19 The air flows over the Alps (altitude 3000 m). On the southern side at the bottom, the air has a temperature of 18 °C and a relative humidity of 85%. On the northern side at the bottom, the southerly Foehn is blowing with the temperature of 25 °C and the relative humidity of 36%. How much precipitation falls when crossing the Alps? [Solution]

8.20 In the lower troposphere, during peaceful dry weather, at night, we measure at the ground the temperature of 16 °C. The temperature decreases with height by 7 K on km to the height of 3 km and by 5 K on km to the height of 5 km. During the day, the air at the ground warms up to 28 °C. To which height does the air mix due to convection? [Solution]

8.21 What is the frequency of vertical oscillation of vertically displaced unsaturated air in a stable atmosphere ($\partial T/\partial z = -5$ K/km, the temperature is 15 °C)? What happens, if $\partial T/\partial z = -11$ K/km? [Solution]

8.22 The saturated air with the temperature of 20 °C and the atmospheric pressure of 1000 mbar blows at the speed of 10 m/s towards a slope with the tilt of 20°. What is the maximum possible intensity of the precipitation that falls from the rising air if the cloud top is at the pressure 500 mbar? Assume moist adiabatic lapse rate of $\Gamma_s = \beta(p)\Gamma_a$: [Solution]

atmospheric pressure (mbar)	β
1000	0.38
850	0.45
700	0.50
500	0.62

9 Emagrams

9.3 The meteorological balloon measured the following temperatures at different heights:

atmospheric pressure	temperature	dew point temperature
1000 mbar	20 °C	10 °C
950 mbar	20 °C	10 °C
600 mbar	−14 °C	−40 °C
300 mbar	−45 °C	−60 °C
above 300 mbar	−45 °C	no data

a) Draw the vertical temperature profile on the Skew-T Log-P emagram (between the measurements, you can draw the straight lines).

b) Draw the vertical profile of the dew point temperature on the emagram.

c) What are the temperature and the dew point temperature at the height, where the atmospheric pressure equals 450 mbar?

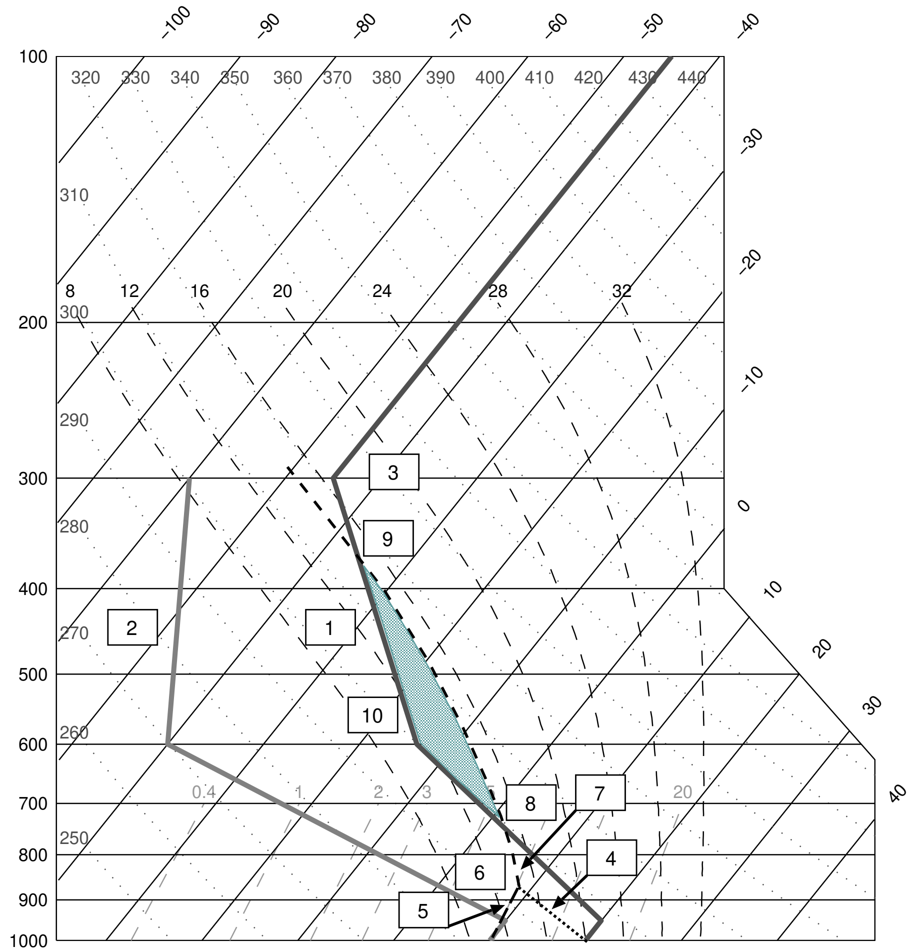
d) Determine the lower limit of the tropopause.

e) At which height will cloudiness appear if the air is forced up along the slope? Draw the vertical profile of the temperature during the rising on the emagram.

f) To which height would the air have to be additionally raised so that the free convection will occur? Where will the cloud top be in this case? (Mark on the emagram.)

g) Mark the CAPE (Convective Available Potential Energy).

Solution:



a),b) On a Skew-T Log-P emagram the atmospheric pressure is on the ordinate axis (marked on the left side and goes from 1000 mbar to 100 mbar). The abscissa axis shows the temperature and is rotated by 45° in the clockwise direction (solid black lines inclined to the right). The temperature values are labelled at the right and top sides and go from 40 °C to −100 °C. The vertical profiles of the temperatures with the height (atmospheric pressure) that are given in the example are shown with thick lines. The right line represents the temperature, and the left is the dew point temperature.

c) You have to read the temperature from the graph (points 1 and 2). The temperatures are approximately −27 °C (air) and −48 °C (dew point).

d) The tropopause is the isothermal layer, which is located approximately 10 km above the troposphere. To accurately determine its lower limit, you have to determine the height where the temperature becomes constant from the graph. In our case, that is at point 3 (300 mbar). At this point, the vertical profile of the air temperature becomes parallel to the inclined temperature axis.

e) If an air parcel is rising, its temperature and the dew point temperature are changing. The temperature of the rising air with the height decreases parallel with the dotted lines that are inclined to the left. When the air rises from the ground, the temperature parallelly follows these lines (marked with 4). At the same time, the dew point temperature is decreasing. It is decreasing in parallel with the shorter grey dashed lines inclined to the right (marked with 5). When the air temperature and the dew point temperature become equal, the air becomes saturated (the relative humidity becomes 100%). This is labelled with 6. At that height, the cloud droplets will start to form and cloudiness appears. This height is also known as the rising condensation level.

f) From the rising condensation level, the air is rising parallel to the black dashed curve that is inclined to the left (labelled with 7). The dew point temperature is equal to the air temperature (the relative humidity is 100%). The rising air is still cooling, but somewhat more slowly than before the rising condensation level. If, at some point, the temperature of the rising air becomes higher than the temperature of the surrounding air at the current height, the unbalanced buoyancy force starts to point upwards, and free convection occurs. In our case, this happens at the atmospheric pressure around 750 mbar (labelled with 8). From here on, the air will, due to the buoyancy, raise itself (it does not need to forcibly raise itself along the hill) until it becomes colder from the surroundings. It will become colder than the surroundings at the atmospheric pressure around 360 mbar (labelled with 9). In this case, the cloud will extend from the rising condensation level (point 6) to the upper level of the free convection (point 9).

g) Convective Available Potential Energy (CAPE) is defined as the surface area between the curves of the temperature of the rising and the surrounding air, where the first is warmer than the second (greyness labelled with 10). It is difficult to evaluate the CAPE from the emagram qualitatively, but it is generally true that the larger the CAPE surface area is, the more intense the convection will be (if the convection does occur – do not forget that the air first has to be forcibly raised at the slope of the hill to the height of 750 mbar. If there is no forced rising, the convection will not occur).

9.4 The meteorological balloon measured the following temperatures at different heights: [Solution]

atmospheric pressure	temperature	dew point temperature
1000 mbar	34 °C	23 °C
950 mbar	30 °C	22 °C
900 mbar	25 °C	20 °C
850 mbar	22 °C	17 °C
800 mbar	21 °C	13 °C
750 mbar	18 °C	5 °C
700 mbar	12 °C	−1 °C
650 mbar	6 °C	−10 °C
600 mbar	2 °C	−12 °C
550 mbar	−3 °C	−21 °C
500 mbar	−7 °C	−25 °C
450 mbar	−12 °C	−30 °C
400 mbar	−17 °C	−36 °C
350 mbar	−25 °C	−39 °C
300 mbar	−32 °C	−42 °C
250 mbar	−42 °C	−51 °C
200 mbar	−52 °C	−60 °C
150 mbar	−51 °C	no data
100 mbar	−46 °C	no data

a) Draw the vertical temperatures profile on the blank Skew-T Log-P emagram.

b) What is the relative humidity of the surrounding air at the ground and at 700 mbar?

c) Determine the lower limit of the tropopause.

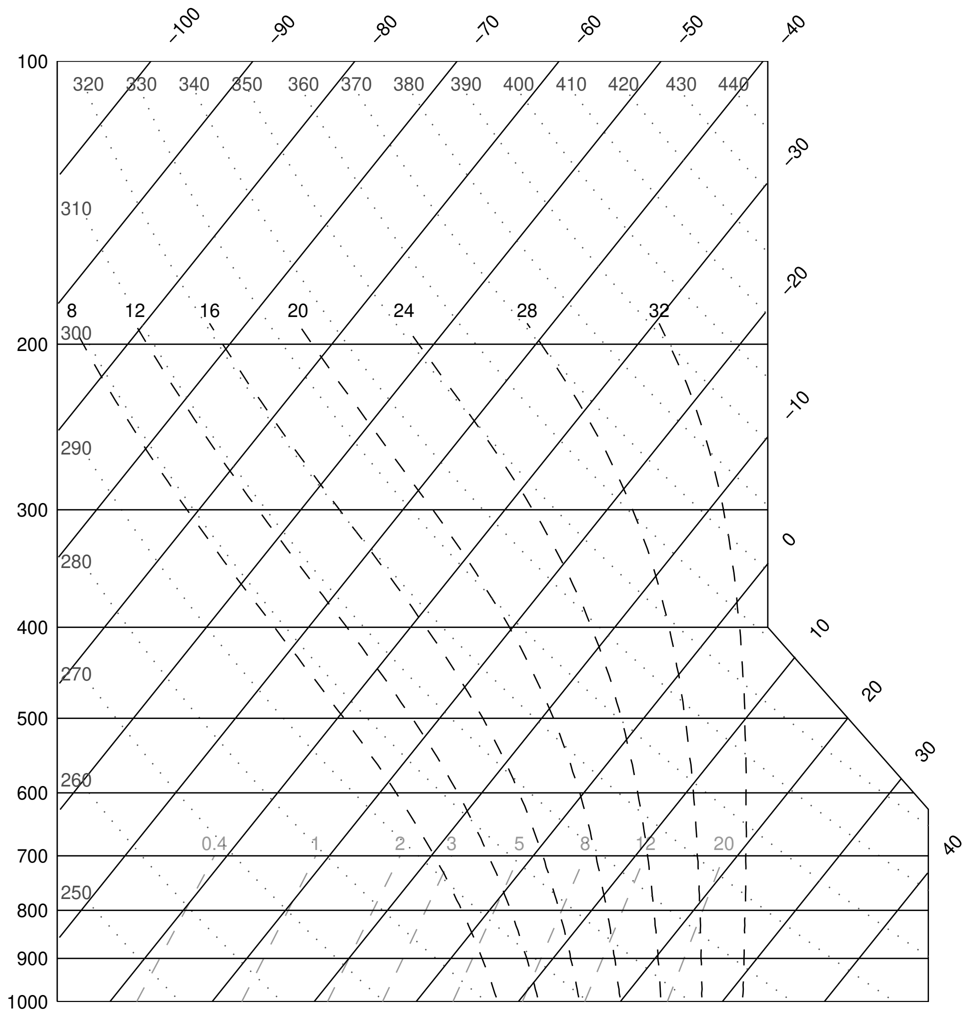
d) Did any cloud layers exist at the time of the measurement?

e) Determine the lifted condensation level.

f) Determine the height of the cloud base.

g) Determine the level of free convection. h) In the case of free convection, to what height will the cloud reach?

i) How much would the air at the ground have to warm up so that free convection with condensation would occur?


–100
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16
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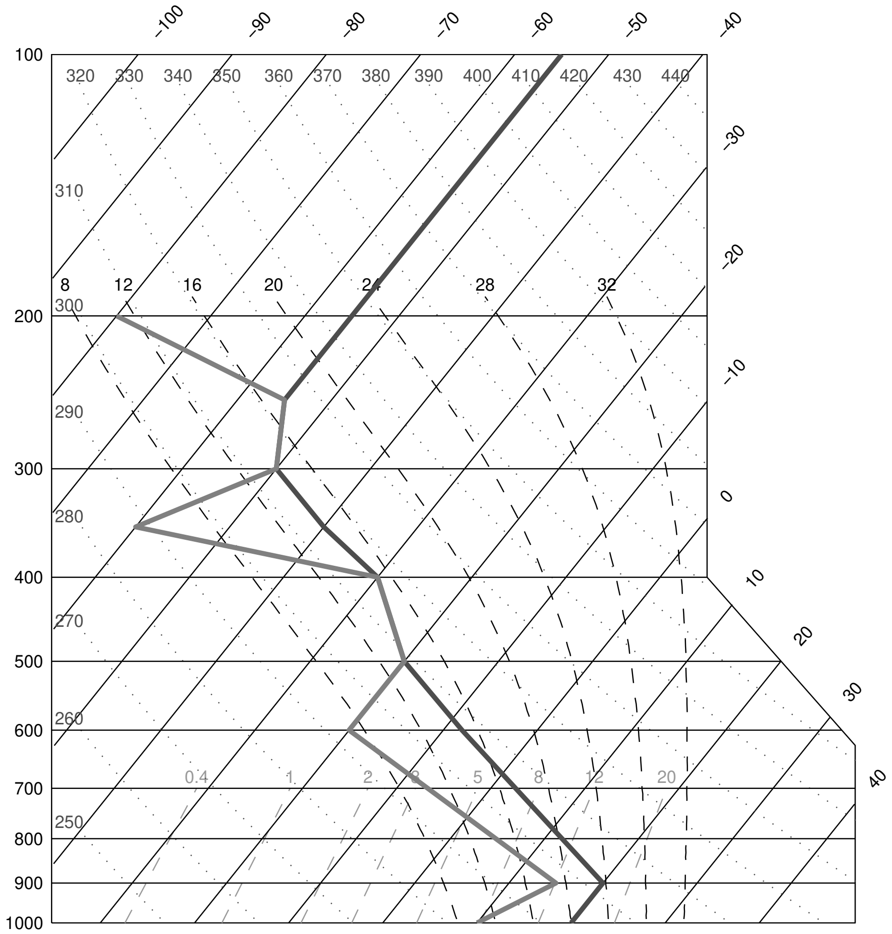
20

9.5 Using the provided Skew-T Log-P emagram determine the following: [Solution]



a) For how many degrees does the temperature decrease in the layer between 800 mbar and 500 mbar?

b) Mark on the emagram where the cloud layers were located at the time of the measurement.

c) On the emagram, mark the lower limit of the tropopause.

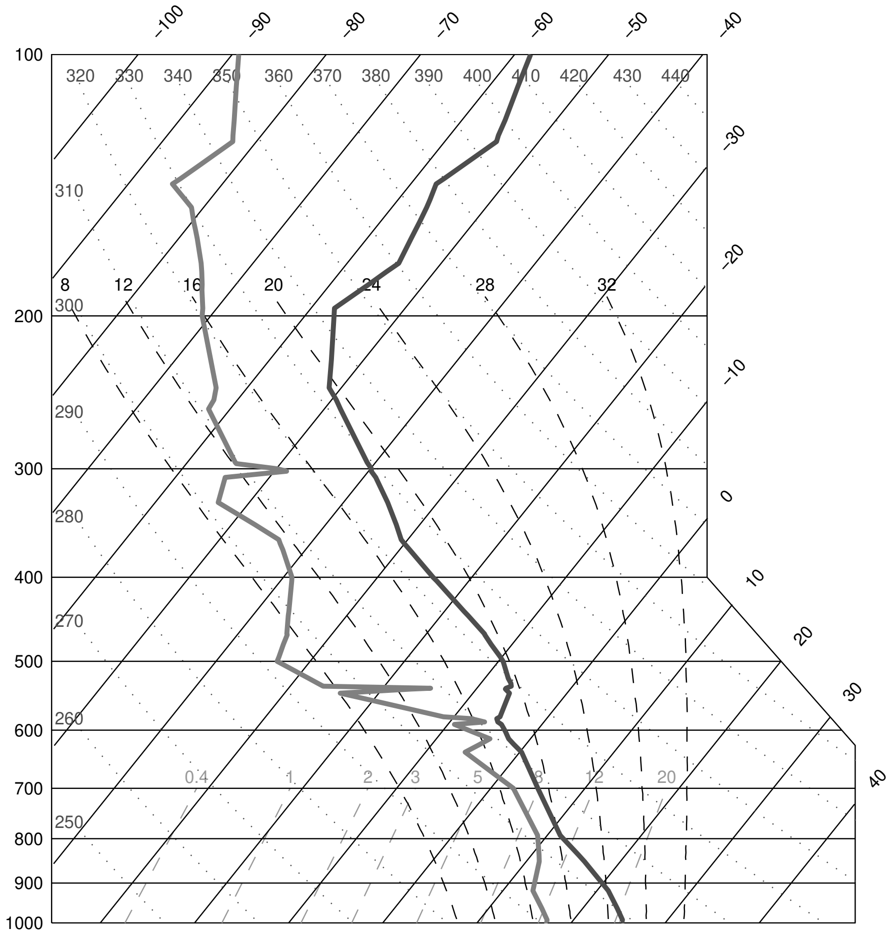
d) On the emagram, mark and evaluate the lifted condensation level.

e) On the emagram mark to which height the air would additionally need to be raised so that the free convection would occur?

f) For how many degrees should the air at the ground warm up, so that the free convection with the condensation will occur?

g) On the emagram mark, to which height would the cloud extend in this case.

9.6 Using the provided Skew-T Log-P emagram determine the following: [Solution]



a) What is the temperature at the height of 700 mbar?

b) Did any cloud layers exist at the time of the measurement?

c) On the emagram, mark and determine the height of the lifted condensation level.

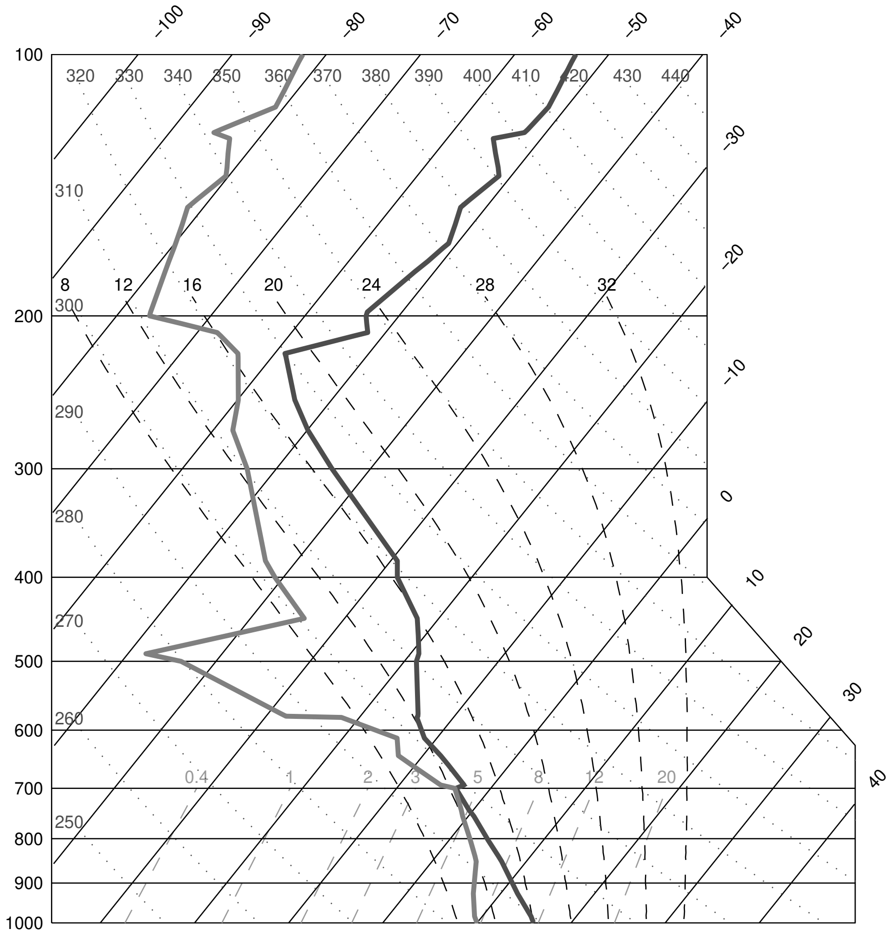
d) The air is forcibly rising at the slope of the hill that has the top at 850 mbar. Will the free convection occur?

e) To which height will the cloud extend in this case?

f) By how many degrees should the air at the ground warm up, so that the free convection with the condensation will occur?

g) To which height will the cloud extend in this case?

9.7 Using the provided Skew-T Log-P emagram, determine the following: [Solution]



a) What is the temperature at the height of 450 mbar?

b) Did any cloud layers exist at the time of the measurement?

c) On the emagram, mark and determine the height of the lifted condensation level.

d) On the emagram, mark to which height the air would additionally need to be raised so that the free convection would occur.

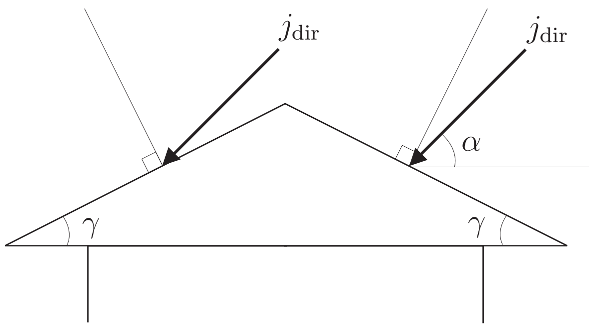
e) At least for how many degrees should the air at the ground warm up so that the free convection that will take place to the height of 200 mbar will occur?

f) To which height will the cloud extend in this case?

10 Radiation

10.1 Calculate the ratio between the incoming power of solar radiation for sunny and shady parts of a roof with a tilt of 30°. The ridge of the roof is in the east-west direction, the sun is positioned in the south at the elevation of 45° above the horizon. The flux density of the incoming direct solar radiation is 800 W/m^2, and the flux density of the diffuse radiation is 44 W/m^2.

Solution:



The sunny and the shady sides are receiving power:

$$P_{\text{sunny}} = j_{\text{dir}} \cdot \cos\left(\frac{\pi}{2} - (\alpha + \gamma)\right) \cdot S + j_{\text{dif}} \cdot S$$
$$P_{\text{shady}} = j_{\text{dir}} \cdot \cos\left(\frac{\pi}{2} - (\alpha - \gamma)\right) \cdot S + j_{\text{dif}} \cdot S$$

The ratio of the energy fluxes is: $P_{\text{sunny}}/P_{\text{shady}} = 4.4$.

10.2 The temperature of all walls in a prison cell is 10 °C. In the cell is a naked prisoner, and the temperature of the human skin is 30 °C. What is the net radiative energy loss by the prisoner if you assume that the surface area of the human body is 1 m^2 and the walls and the human body radiate as a black body? [Solution]

10.3 Fog has risen from the ground to a height of a few tens of metres. The emissivity of the fog is 0.8 and temperature 282 K. What is the power of the incoming radiation that is received by a flat surface on the ground? The area of the surface is 1 m^2. No IR radiation is absorbed between the fog and the ground. [Solution]

10.4 The estimate of the average evaporation of the water for the whole Earth is 2.7 mm/day. This corresponds to the average precipitation. What portion of the incoming average solar radiation the Earth receives is needed for the evaporation of the water? Assume the average albedo of the Earth $a = 0.35$. [Solution]

10.5 How many days would the sun have to irradiate the horizontal plane on the Earth's surface that 1200 mm of water would evaporate from it? Assume the average length of the day is 12 h and the height of the Sun at noon is 60° above the horizon. Assume a linear function for the sun's elevation during the day. [Solution]

10.7 A nuclear power station uses a nearby lake for cooling and is dumping waste heat with the power of 500 MW. The lake has a surface area of 100 km^2 and an average depth of 10 m. Assume that the waste heat is evenly distributed over the entire lake. [Solution]

a) How fast would the lake warm up if it did not emit the energy to the surrounding area?

b) If the lake has a natural temperature of 10 °C, how will its temperature change if the surplus of the heat is emitted only via radiation?

c) How much evaporation should occur from the lake in order to carry away all the additional heat from the power plant so that the temperature of the lake does not change?

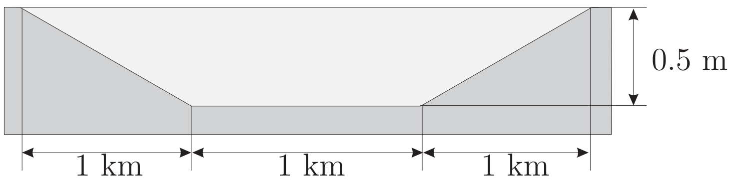
10.8 A meadow with the surface area of 1.2 ha, absorbs 18000 MJ of the energy from solar radiation and evaporates 1000 kg of water in one hour. If the emissivity of the meadow ground is 0.95, what is the temperature of the ground on the meadow? [Solution]

10.9 In Greenland, a circular lake with the radius of 1 km and the depth of 0.5 m exists. On a polar day, the Sun is at elevation 25° above the horizon all the time, and the flux density of direct solar radiation is 800 W/m^2. 80% of the incoming solar radiation is absorbed in the water. All the absorbed energy is used only for the evaporation of the water. [Solution]

a) How long would it take the lake to dry out if the situation stays unchanged and there is no precipitation?

b) How many days would pass before the lake would dry out if it has a constant river inflow with the capacity of 500 m^3/h?

10.10 In Greenland, a circular lake with a cross-sectional profile, as shown in the figure below, exists. On a polar day, the Sun is at elevation 25° above the horizon all the time, and the flux density of direct solar radiation is 800 W/m^2; 80% of the incoming solar radiation is absorbed in the water. All the absorbed energy is used only for the evaporation of the water. [Solution]

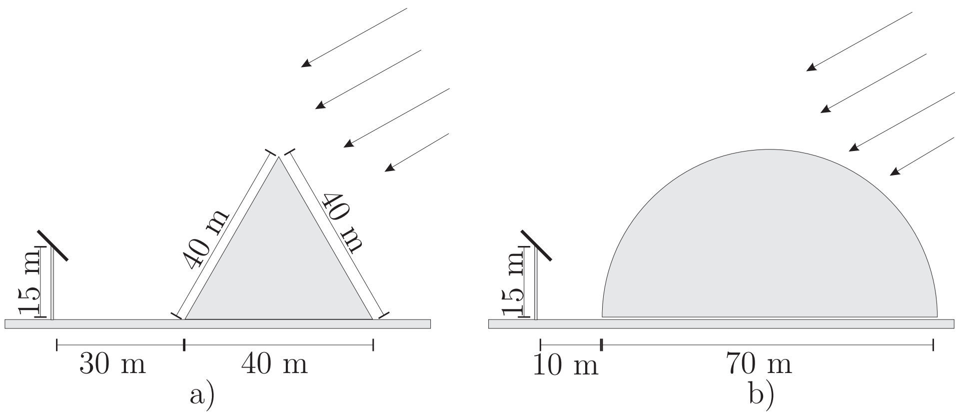


a) How long would it take for the lake to dry out?

b) What would be the equilibrium depth of the lake if the lake had a constant river inflow with the capacity of 1000 m^3/h?

c) How long would it take for the lake to dry out if the lake would have constant river inflow with the capacity of 200 m^3/h?

10.11 On a 15-m high pillar, there is a solar collector with a surface area of 1 m^2. The collector is tilted at 45° with respect to the horizon. Between the collector and the Sun is a triangular or a half-circular hill (see figure). What is the power of the incoming solar radiation that falls on the collector in the morning when the Sun shines on it? Assume that the flux density of direct solar radiation is 800 W/m^2. [Solution]



10.12 During the day, the temperature of the soil surface changes sinusoidally with the amplitude of 10 K, with a maximum at 14:00 and a minimum at 8:00. The average temperature of the soil surface is 15 °C. What is the temperature amplitude at the depth of 15 cm in the soil? Assume thermal conductivity of soil $\lambda = 100$ W/m^2, soil density 2000 kg/m^3 and specific heat capacity of soil of 2000 J/kg K? [Solution]

10.13 The albedo of Earth together with the atmosphere for shortwave radiation is 0.3, and the absorptivity of the atmosphere is 0.1 for the shortwave and 0.7 for the longwave radiation. What are the equilibrium temperatures of the ground and the atmosphere if the solar constant j_0 is known? [Solution]

10.14 Assume that some area on the Earth is isolated from its surroundings so that the energy fluxes between this area and the surroundings can be neglected. This area is irradiated by the Sun; near the ground, the flux density of solar radiation is 600 W/m^2. The albedo of the soil is 0.36. The atmosphere consists of three layers, which do not mix with each other. We neglect the transfer of energy with the conduction between the nearby layers. The transmissivity of the three layers is 0.70, and the reflectivity is 0. What will the ground temperature be if the layers and the ground are in the radiative balance? Assume, that the layers and the ground affect only the neighbouring layers. [Solution]

10.15 How much energy of solar radiation is received by a black horizontally oriented plate of the radiometer, with surface 6 cm^2 at the time between 11:30 and 12:30 on the local solar time, when the Sun is at 60° above the horizon? What is its equilibrium temperature at noon? What is the total daily received energy if the Sun rises at 5:45 and the elevation angle of the Sun changes linearly with time from 0° to 60° at noon and then back to 0°? Assume that the solar radiation flux density near the ground is 800 W/m^2. [Solution]

10.16 A thermometer is placed in direct sunlight and shows the temperature of 39 °C. The true air temperature is 25 °C, while the emissivity of air is 0.7 and the thermometer radiates as a black body. What is the albedo of a thermometer bulb for solar radiation if it is exposed to a solar radiation flux density of 1000 W/m^2? Assume the bulb is spherical in shape. [Solution]

11 Fronts

11.1 What is the tilt of the cold front if the difference in the temperature between the air masses is 3 K, a south-westerly wind is blowing before the front at the speed of 13 m/s, and after the front, a north-westerly wind is blowing at the speed of 10 m/s? The front is oriented in the north-south direction. The warm air temperature is 16.5 °C. The front is located at 45° N.

Solution:
We use the equation for the tilt of the front:

$$\tan\alpha = \frac{f\overline{T}}{g}\left(\frac{v_\mathrm{T} - v_\mathrm{H}}{T_\mathrm{H} - T_\mathrm{T}}\right),$$

where v_T and v_H are the tangential components of the wind, which blows along the front. $\tan\alpha$ is defined as positive in the cold front and negative in the warm front. In the example, we first need to determine the tangential components of the wind, that are $v_\mathrm{T} = -13\ \mathrm{m/s} \cdot \sin 45°$ and $v_\mathrm{H} = 10\ \mathrm{m/s} \cdot \sin 45°$.

$$\tan\alpha = 0.0164.$$

11.2 How fast will the cold front move from Koper towards central Slovenia if in the warm air the wind blows from the azimuthal direction 210° at the speed of 25 m/s and in the cold air the atmospheric pressure increases towards the southwest by 1.3 mbar/100 km? The air density is 1 kg/m^3. [Solution]

11.3 The front is located in the northern hemisphere in the east-west direction and is vertically tilted by 1°. In the warm air, the temperature is 6 °C and in the cold air, −2 °C. In the cold air, the wind blows from the northeast at the speed of 40 m/s. Determine the wind blowing in the warm air. [Solution]

11.4 What is the orientation of the front and how fast is it moving if in cold air northerly wind at the speed of 10 m/s is blowing while in warm air north-westerly wind at the speed of 7.1 m/s is blowing? [Solution]

11.5 At the cold front with a tilt of 1/200 and orientation in the SW-NE direction, the cold air is advancing at the speed of 50 km/h. The cold air is 5 K colder than the warm air. In the warm air, where the temperature is 15 °C, a westerly wind blows. What are the wind's speed and direction in the cold air? [Solution]

11.7 An isothermal lake of cold air ($T_1 = 5$ °C) lies in the basin where the weather is calm. Above the inversion that separates the lower air mass from the upper air mass, the air is warmer ($T_2 = 15$ °C) and westerly geostrophic wind at the speed of 15 m/s is blowing. At the side of the basin, where the inversion level is the lowest, the inversion is 100 m high. At which height is the inversion on the opposite side of the basin, which is 40 km away. In which direction is the opposite side? [Solution]

11.8 What is the slope of the upper limit of the calm lake of the cold air, in which the temperature is −8 °C if above it wind speed is 10 m/s and temperature 5 °C? [Solution]

11.9 Determine the tilt of a stationary front, which separates the two air masses at the latitude of 45°. Characteristics of the air masses: the atmospheric pressure at the ground is 1000 mbar, the temperature in the cold air is 263 K, and in the warm air 273 K. The horizontal gradient of the atmospheric pressure, perpendicular to the front in the cold air, is 1 hpa/100 km. In the warm air, the geostrophic wind blows at the speed of 15 m/s. How much would the tilt change if the warm air would stop moving? [Solution]

11.10 How far ahead of the warm front at the ground is the warm air at the height of 8 km (usually, we see it via cirrus clouds)? Assume the incoming air is 10 K warmer than the cold air, that its temperature is 5 °C and that at the front, the wind turns by 45° and strengthens by 5 m/s? How far away on the horizon can this cloudiness be seen if we stand on the hill 1000 metres high? Before the front, the wind is blowing at the speed of 10 m/s perpendicular to the direction of the front movement. [Solution]

12 Solutions

Solutions: Units

1.1: 100000 Pa, 600 mbar, 0.5 bar.

1.2: 285.5 K, 17 °C, 259 K.

1.3: 259200 s, 2.3 years, 0.0009 km^2, 0.003 m^3, 108 km/h, 27.7 m/s, 10^8 W, $1.4{\cdot}10^9$ W/km^2, 18 °C/hour, 0.0015 Pa/m, 27.8 kWh, 18 MJ.

Solutions: Structure and atmospheric layers

2.2: 355 kg.

2.3:

$$M = \frac{\Sigma m_i}{\Sigma n_i} = \frac{\Sigma m_i}{\Sigma \frac{m_i}{M_i}} = \frac{\Sigma \frac{V}{R^* T} p_i M_i}{\Sigma \frac{V}{R^* T} p_i} = \frac{\Sigma p_i M_i}{\Sigma p_i} = 28.8 \text{ g/mol.}$$

2.4: a) 1.1 kg/m^3, b) 1.4 kg/m^3.

2.5: Air density increases from 1.02 kg/m^3 to 1.10 kg/m^3.

2.6: From the ideal gas equation for air, we calculate the mass of air in the room to be 121 kg. Considering the mass fraction of argon, we obtain the mass of argon to be 1.6 kg.

2.7: The average atmospheric pressure at sea level is 1013 mbar. The radius of the Earth is 6370 km.

$$p = \frac{F}{S} = \frac{mg}{4\pi R_z^2},$$
$$m = 5.3 \cdot 10^{18} \text{ kg.}$$

Solutions: Hydrostatics

3.3:

z	1 km	2 km	3 km	4 km	5 km	6 km	7 km	8 km	9 km	10 km
$p(z)/p_0$	0.88	0.78	0.68	0.59	0.51	0.44	0.38	0.32	0.27	0.22

3.4: The problem is solved progressively from the lower layer upwards.

height (m)	atmospheric pressure (mbar)	temperature (K)
0	1013	288
100	1001.1	290
1000	899.2	283
3000	706.2	283
12000	208.0	224.5
31964	10	224.5

3.5: a) 1028.8 mbar, b) 1026.2 mbar.

3.6: a) 1098.0 mbar, b) 663.2 mbar, 0.82 kg/m^3 c) 352.4 mbar, d) 1454 m.

3.7: 2853 m.

3.8: 3187 m.

3.9: 9153 m.

3.10: Using the parameters of standard atmosphere, the atmospheric pressure at the height of the plane can be calculated (306.8 mbar). Then, the actual data can be used to calculate the real height of the plane (8119 m).

3.11: We can calculate the pressure on Kredarica in two ways: first with $\left(\frac{\partial T}{\partial z}\right)$ for the standard atmosphere, where for the height of Kredarica we use the value shown by the altimeter, and second with the use of true temperature and height data from the temperature profile. The pressure increases for 1.5 mbar.

3.12: The layer is located between 866 m and 1818 m.

3.13: 2863 m.

3.14:

$$dQ = mc_p dT = \rho S h c_p dT,$$

$$\Delta z = z_1 - z_0 = \frac{R}{g}\overline{T}\int \mathrm{d}\ln p,$$

$$d\Delta z = \frac{R}{g}\ln\frac{p_0}{p_1}\,d\overline{T} = \frac{R}{g}\ln\frac{p_0}{p_1}\frac{1}{\rho h c_p}\frac{dQ}{S} = \frac{R}{\Delta p}\ln\frac{p_0}{p_1}\frac{1}{c_p}\frac{dQ}{S} = 2.9\text{ m}.$$

3.15:

$$d\Delta z = \frac{R}{\Delta p} \ln \frac{p_0}{p_1} \frac{1}{c_p} \frac{dQ}{S} = 63 \text{ m}.$$

3.16: 16 m.

3.17:

$$p = \rho_{Hg} \cdot g \cdot h = 970{,}1 \text{ mbar}.$$

3.18: By calculating the atmospheric pressure at the sea level it is assumed that the atmosphere is isothermal with the temperature measured at the station:

$$\frac{\Delta p}{p_0} = \mathrm{e}^{\frac{g\Delta z}{RT}} - \mathrm{e}^{\frac{g\Delta z}{R(T+1\text{ K})}} = 1.2 \cdot 10^{-4}.$$

3.19: 743.4 mbar.

3.20: In the case of homogeneous atmosphere, the density is constant and $dp = \rho R dT$. We can use the hydrostatic equation and use it to express the change of the temperature with the height:

$$\frac{\partial p}{\partial z} = -\rho g,$$
$$\frac{\partial T}{\partial z} = -\frac{g}{R} = -0.03 \text{ K/m}.$$

3.21: When there is no inversion in the layer above the ground, the temperature at the ground is −2.8 °C. The ratio of the calculated values of the atmospheric pressure is

$$\frac{p_0}{p_0'} = \mathrm{e}^{\frac{gh}{R}\left(\frac{1}{T}-\frac{1}{T}\right)} = 1.0003.$$

3.22: First, we calculate the height of the 500-mbar layer, by assuming the standard atmosphere: $p_0 = 1013$ mbar, $T_0 = 288$ K, $\left(\frac{\partial T}{\partial z}\right) = -6.5$ K/km.

$$z = \frac{T_0}{\left(\frac{\partial T}{\partial z}\right)} \left[\left(\frac{p_1}{p_0}\right)^{-\frac{R\left(\frac{\partial T}{\partial z}\right)}{g}} - 1 \right] = 5567 \text{ m}.$$

When calculating the geopotential, we assume that g is not changing with height

$$\Phi = \int_0^H g \,\mathrm{d}z = 54612 \text{ m}^2/\text{s}^2.$$

Solutions: Basic laws

4.2:

$$\omega^2 R = 0.1 f u,$$
$$R = 20.7 \text{ km}.$$

4.3: It points towards the southeast. The force is $3.1 \cdot 10^{-3}$ m/s^2.

4.4: One must consider the difference in acceleration of gravity, which results from the change in gravitational force acceleration with altitude ($g(z) = g_0 \left(\frac{r_0}{r_0+z}\right)^2$, where g_0 is the gravitational force acceleration at sea level and r_0 is the Earth's radius and z is the altitude) and the change in the radial component of the centrifugal force acceleration ($f_{cent} = \omega^2 (r_0 + z) \cos\varphi$, where ω is the angular velocity of the Earth's rotation and φ is the latitude). The acceleration of gravity on Kilimanjaro is 9.76 m/s^2, and at the other location, it is 9.79 m/s^2. A mountaineer feels approximately 0.3% lighter.

4.5: The difference is due to the radial component of the Coriolis acceleration, which points in the opposite direction as the gravitational acceleration. In Slovenia ($\varphi = 45°$), this difference is 0.0123 m/s^2 and on the equator 0.017 m/s^2. For a human with the mass of 75 kg, this means 0.92 kg or 1.3 kg less.

4.6:

$$\frac{|-\rho g|}{|\frac{\partial p}{\partial n}|} = 7240.$$

4.7: The balloon will not be moving in the vertical direction if the sum of the forces that act upon it in the vertical direction is equal to zero. The gravitational force and the buoyancy force are balanced:

$$0 = -m_b g - m_k g + V_b \rho_{ok} g = \left(\frac{\rho_{ok}}{\rho_{zr}} - 1\right) - \frac{m_k}{\rho_{zr} V_b}.$$
$$V_b = \frac{R_s m_k}{p(T_{ok}^{-1} - T_{zr}^{-1})} = 1928 \text{ m}^3.$$

Solutions: Steady state horizontal winds

5.3: 320 km.

5.4: 8.9 m/s.

5.5: First, we have to calculate the air density for standard atmosphere at 500 mbar (0.7 kg/m^3). The horizontal pressure gradient is 1.15 mbar/100 km.

5.7: It blows under the influence of the Coriolis force ($8.7 \cdot 10^{-4}$ m/s^2), the friction force ($2.3 \cdot 10^{-4}$ m/s^2) and the pressure gradient force ($9.0 \cdot 10^{-4}$ m/s^2). $k = \tan\beta\, f = 10^{-5}$ s^{-1}. $|\nabla p| = 0.45$ mbar/100 km.

5.8: 8.1 m/s.

5.9: $\frac{v_{\text{grad}}}{v_{\text{geo}}} = 0.95$.

5.10: a) $|\nabla p| = 4$ mbar/500 km. $v_g = 6.4$ m/s. b) $v_g = 7.2$ m/s.

5.11: $v_g = 8.3$ m/s, $F_{\text{cor}} = 1.0 \cdot 10^{-3}$ m/s^2, $F_{\text{fr}} = 0.83 \cdot 10^{-3}$ m/s^2, $F_{\text{gra}} = 1.3 \cdot 10^{-3}$ m/s^2.

5.12: $v_g = 17.9$ m/s, $F_{\text{cor}} = 3.1 \cdot 10^{-3}$ m/s^2, $F_{\text{fr}} = 1.7 \cdot 10^{-3}$ m/s^2, $F_{\text{gra}} = 3.6 \cdot 10^{-3}$ m/s^2.

5.13: a) We assume that in a tropical cyclone, the centrifugal and the pressure gradient force are equal (we neglect the Coriolis force). $\Delta p = 30.86$ mbar. b) $\Delta p = 30.90$ mbar.

5.14: The parabolic pressure field can be expressed as $p(r) = p_0 + kr^2$, where r is the distance from the center of the cyclone, where the pressure is equal to p_0. The coefficient k is obtained from the balance of forces on the periphery of the cyclone, where $\frac{\partial p}{\partial n} = \frac{\partial p(r)}{\partial r} = 2kr$, resulting in $k = 5.43 \cdot 10^{-9}$ Pa/m^2, through which the pressure at the center is determined to be $p_0 = 961$ mbar.

5.15: 602 km.

5.16: Assuming that the centrifugal force and the gradient force are balanced in the tornado, we obtain $p(r) = p_0 + \frac{\rho\omega^2 r^2}{2}$.

5.17: The pressure is preserved, if we move from the starting point up along the axis that is tilted for 30 degrees:

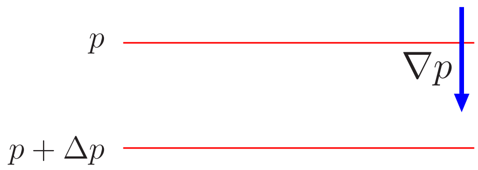
$$\Delta p = \left(\frac{\partial p}{\partial r}\right) R - \rho g \Delta z = 0,$$

$$\left(\frac{\partial p}{\partial r}\right) = \frac{\rho g \Delta z}{R} = \frac{\rho g}{\tan\alpha},$$

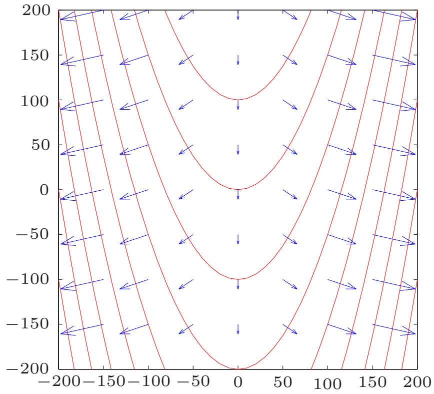
$$v = \sqrt{\frac{R}{\rho}\left(\frac{\partial p}{\partial r}\right)} = \sqrt{\frac{Rg}{\tan\alpha}} = 41.2 \text{ m/s}.$$

Solutions: Local, individual and advective changes

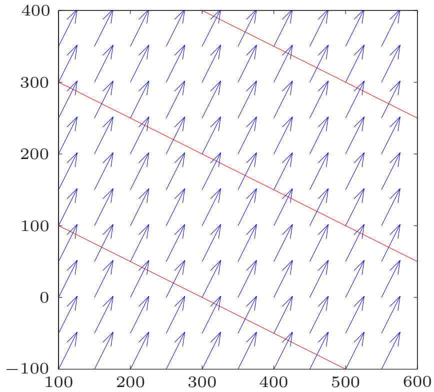
6.2: a) $\nabla p = (0, a)$, b) 996 mbar.



6.3: a) $\nabla p = (2bx, a)$, b) 1001 mbar.



6.4: a) $\nabla p = (b, a)$, b) 1005 mbar.



6.6: a) 0.25 °C/h b) 7.1 m/s.

6.7: 0.34 °C/h.

6.8: 24.4 °C.

6.9: 23.6 °C.

6.10: 1.2 °C/h.

6.11: $\frac{\partial T}{\partial r} = \frac{3\ \mathrm{K}}{500\ \mathrm{km}}$.

Solutions: Humidity

7.2:

$$f = e/e_s = 0.48.$$

7.3: 284.4 K.

7.4: 255.3 K (if we take into account the latent heat of vaporisation of water instead of the latent heat of sublimation).

7.5:

$$e(f = 90\%) = 21.3\ \mathrm{mbar}, \quad e(f = 10\%) = 2.4\ \mathrm{mbar},$$
$$\Delta m = m(f = 90\%) - m(f = 10\%) = -8.5\ \mathrm{g}.$$

7.6:

$$m_v = \rho_v \cdot V = \frac{eV}{R_v T} = 0.86\ \mathrm{kg}.$$

7.7: a) 1.242 kg/m^3, b) 1.238 kg/m^3.

7.8: One can first calculate the atmospheric pressure and then the specific humidity, which is $3.5 \cdot 10^{-3}$.

7.9: a) $e_s(T = 30\ °\mathrm{C}) = 43.6$ mbar, $q_s = 0.027$, b) $e_s(T = -15\ °\mathrm{C}) = 1.9$ mbar, $q_s = 0.0012$ (if we take into account the latent heat of vaporisation of water instead of the latent heat of sublimation).

7.10:

$$f = \frac{e}{e_s},$$
$$\frac{df}{f} = \frac{de}{e} - \frac{de_s}{e_s} = \frac{de}{e} - \frac{h_\mathrm{i}}{R_v}\frac{dT}{T^2}.$$

a) Increasing (decreasing) the temperature for 3 K causes the relative decrease (increase) of the relative humidity by 0.22.

b) Increasing (decreasing) the vapour pressure for 1% causes the relative increase (decrease) of the relative humidity by 0.01.

7.11: a) 279.9 K.

b) In case of saturation, the absolute humidity is $\rho(T_d) = \frac{e_s(T_d)}{R_v T_d} = 7.7$ g/m^3. The final absolute humidity is $\rho_2 = \frac{e_s(T_2)}{R_v T_2} = 1.5$ g/m^3. The difference of condensed water per unit volume is 6.2 g (if we take into account the latent heat of vaporisation of water instead of the latent heat of sublimation).

7.12: We assume that the water vapour does not affect the energy balance. The received energy is spent for two things: expansion of the air ($p\Delta V$) and heating of the air ($mc_v\Delta T$). Using the gas equation and a little rearrangement, we find the equation $A = m(c_v + R)\Delta T = mc_p\Delta T$. We obtain the temperature change $\Delta T = 5$ K from this equation. We calculate the initial and the new saturated vapour pressure to get the relative humidity. The new relative humidity is 47%.

7.13: The actual partial pressure of water vapor is $e = 18.9$ mbar, from which we find that the dew point temperature is 16.5°C. Since the morning temperature will be lower than the dew point temperature, dew will form.

7.14: The amount of the evaporated water per square metre of the soil is proportional to the difference between the actual and the saturated vapour pressure in the layer of the air at the ground:

$$m/S = \frac{\Delta e \cdot h}{R_v T} = 0.17 \text{ kg/m}^2.$$

7.15: In the warm air (T_2), the actual vapour pressure is 11.8 mbar and in the cold air (T_1) 3.8 mbar. From here on, we calculate the absolute humidity of the mixture:

$$\rho(\text{mixture}) = \frac{1}{2}\left(\frac{e_1}{R_v T_1} + \frac{e_2}{R_v T_2}\right),$$

then the partial pressure of the water vapour of the mixture:

$$e(\text{mixture}) = \rho(\text{mixture}) R_v T_2,$$

and finally the new relative humidity:

$$f = 0.34.$$

7.16:

$$m = \frac{\Delta e V}{R_v T} = 0.176 \text{ kg}.$$

7.17: At sunset, the partial pressure of water vapor is $e = 13.7$ mbar, from which we find that the dew point temperature is 11.6 °C. Both dew and fog will form since the morning temperature will be lower than the dew point temperature.

7.18: First, we determine the specific humidity of the mixture, which is 0.0100. If the saturation is not reached, the final temperature will be the average temperature between 21 °C and 5 °C (because the air masses are mixing in the same proportions), which is 13 °C. The air will become saturated because the saturated vapour pressure at this temperature is lower than the vapour pressure of the mixture. We can write the energy equation

$$mc_{\mathrm{pz}}(T - T_{\mathrm{H}}) = m_a h_{\mathrm{i}} + mc_{\mathrm{pz}}(T_{\mathrm{T}} - T),$$

where the indexes H and T are the temperature of the warm and the cold air and m_a the mass of the condensed water. We can write this as the difference in the masses of the water vapour before and after the condensation

$$m_a = m_{v1} - m_{v2} = mq - mq_s(T) = mq - \frac{mR}{pR_v} e_s(T),$$

where q is the specific humidity of the mixture. From there, we obtain

$$c_{\mathrm{pz}}(T - T_{\mathrm{H}}) = qh_{\mathrm{i}} - \frac{Rh_{\mathrm{i}}}{pR_v} e_s(T) + c_{\mathrm{pz}}(T_{\mathrm{T}} - T).$$

We need to solve the equation numerically because we cannot analytically express the temperature from this equation ($e_s(T)$ appears in the exponent). The result is 286.4 K. From here, we determine that $m_a = 0.35$ g of the water condensed per kilogram of the air.

7.19: The saturated vapour pressures are 12.3 mbar and 11.5 mbar. From every cubic metre, 0.59 g of water is eliminated.

7.21: Above the water, the saturated vapour pressure is 2.4 mbar. Above the ice, we use the same Claussius-Clapeyron equation in which the specific latent heat of evaporation h_{i} is replaced with the specific heat of sublimation h_{s}. The result is 2.2 mbar.

7.22:

$$\frac{f_{\mathrm{water}}}{f_{\mathrm{ice}}} = \frac{e_{s,\mathrm{ice}}}{e_{s,\mathrm{water}}} = 1.05.$$

7.23: We want to write the temperature profile with the sinusoidal function of time (t). Therefore

$$T(t) = T_A + T' \cdot \sin[a \cdot t + b],$$

where $T_A = 288$ K and $T' = 10$ K. We have to determine the constants a and b so that the temperature at 14:00 will be maximal and at 8:00 minimal. This is completed when the argument in the sinus is at 14:00, equal to $\pi/2$, and at 8:00 to $-\pi/2$. Therefore, for a, b we obtain the system of the two equations, which yield $a = \frac{\pi}{6}$ h^{-1} and $b = -\frac{11\pi}{6}$.

The relative humidity is dependent on the temperature:

$$f = \frac{e}{e_s(T)} = \frac{qpR_v}{R}\frac{1}{e_s(T)} = \frac{qpR_v}{Re_{s0}}\,\mathrm{e}^{-\frac{h_\mathrm{i}}{R_v}\left(\frac{1}{T_0}-\frac{1}{T}\right)}$$
$$= \frac{qpR_v}{Re_{s0}}\,\mathrm{e}^{-\frac{h_\mathrm{i}}{R_v}\left(\frac{1}{T_0}-\frac{1}{T_A+T'\cdot\sin[a\cdot t+b]}\right)}.$$

At 10:00, the relative humidity will be 40%.

7.24: The saturated vapour pressures at the temperature of the dry and the wet bulb thermometers are $e_s(T) = 14.08$ mbar and $e_s(T_m) = 12.13$ mbar. We calculate the vapour pressure with the psychometric equation:

$$e = e_s(T_m) - \frac{c_\mathrm{pz}pR_v}{h_\mathrm{i}R}(T - T_m) = 11.1 \text{ mbar}.$$

The relative humidity is 79%.

7.25: From the psychometric equation, we calculate the partial pressure of the water vapour 8.8 mbar. The dew point temperature is 278.1 K.

7.26: With the use of the psychometric equation, we arrive at the following equation:

$$f \cdot e_s(T) = e_s(T_m) - \frac{c_\mathrm{pz}pR_v}{h_\mathrm{i}R}(T - T_m).$$

The equation is implicit because we cannot express T_m analytically. The numerical solution is $T_m = 14$ °C.

7.27: a) We assume that all the droplets that fall on the bridge immediately cool down and freeze to the bridge's temperature. The energy released during the cooling and freezing of the water droplets, as well as the cooling of the frozen droplets, is used to heat the bridge. The energy for heating the bridge can be written as $m_M c_M \Delta T' = \rho_M S d c_M \Delta T'$. Here, S and d are the surface area and thickness of the bridge, c_M is the specific heat capacity of the bridge, and $\Delta T'$ is 1 K.

The energy released by cooling the droplets to the freezing point can similarly be written as $\rho_a S R R t c_{pa} \Delta T''$, where RR is the precipitation rate (5 mm/h), t is the time, c_a is the specific heat capacity of water, and $\Delta T''$ is 10 K. Similarly, for cooling the frozen droplets by one degree, we use the specific heat capacity for ice c_{pl} instead of c_{pa}. The energy released by freezing the droplets is $\rho_a S R R t h_t$, where ρ_a is the density of water and h_t is the specific latent heat of fusion for water. The result is approximately 0.2 h.

b) The energy for heating the bridge is written in the same way as before, but for the droplets, we only consider the heat released during cooling to the freezing point (we do not need to consider the heat released during freezing and further cooling of the droplets below the freezing point, as the same amount of heat will be used later to warm them back up to the freezing point and melt them). The result is 3.8 h.

7.28: The total rainfall accumulated in the rain gauge is independent of the horizontal wind (if the rain gauge is placed horizontally). It is dependent only on the vertical velocity of the droplets. In three hours, the mass is:

$$m = \rho_k \Delta t w S_0 = 7.8 \text{ kg}.$$

If the snowflakes are falling, the accumulated mass is 2.2 kg.

7.29:

$$T_v = T(1 + \frac{R_v - R}{R} q) = 306.7 \text{ K}.$$

Solutions: Adiabatic changes

8.3: We calculate the dew point temperature at the ground. Starting from the equation for the specific humidity $q = \frac{e}{p}\frac{R}{R_v}$, we calculate the vapour pressure $e = 176.7$ Pa, which determines the dew point temperature $T_d = -16$ °C. The air first rises from the ground along the dry adiabat, until the air becomes saturated at the height of $z_B = 3.72$ km. Then it rises along the moist adiabat to the final height of 6 km, where it has the temperature $T = -38.2$ °C.

8.4: From the height of the cloud base, we calculate the dew point temperature of the air at the ground: $T_d = 4.6$ °C. The relative humidity is the ratio between the saturated pressure of the water vapour at the dew point temperature ($e_s(4.6\ °\text{C}) = 848.1$ Pa) and the saturated pressure of the water vapour at the temperature 15 °C, ($e_s(15\ °\text{C}) = 1704.1$ Pa); therefore, $f = 50\%$.

8.5: The height of the slope is $h = vt \cdot \sin\alpha = 870$ m. a) $\Delta T = -\Gamma_a h - \left(\frac{\partial T}{\partial z}\right) h = -4.4$ K. b) $\Delta T = -\Gamma_s h - \left(\frac{\partial T}{\partial z}\right) h = 0.9$ K.

8.6: The dew point temperature at the ground is $T_d = 11.6$ °C. The height of the cloud base is $h = 0.4$ km, and above, the air is saturated. The relative humidity at 500 m is therefore 100%.

8.7: When rising, the air temperature decreases to $T_2 = 7$ °C. The partial pressure of the water vapour before the rising is $e_1 = \rho_{v1} R_1 T_1 = 935.8$ Pa; at the height of 1000 m, the partial pressure of water vapour and the absolute humidity are:

$$e_2 = e_1 \left(\frac{T_2}{T_1}\right)^{\frac{c_p}{R}} = 827.7 \text{ Pa}$$
$$\rho_{v2} = \frac{e_2}{R_v T_2} = 6.4 \text{ g/m}^3$$

8.8: From the specific humidity, we calculate the partial pressure of the water vapour at the ground $e_1 = 803$ Pa and then the relative humidity of the air at the ground $f_1 = 33.9\%$. At the height of 500 m, the rising air has the temperature of 15 °C, the partial pressure of the water vapour during the rising is changing depending on the temperature:

$$e_2 = e_1 \left(\frac{T_2}{T_1}\right)^{\frac{c_p}{R}} = 756 \text{ Pa}$$

The relative humidity at the height of 500 m is $f_2 = e_2/e_{s2} = 44.0\%$. The condensation level is at the height of 1.94 km.

8.9: a) We calculate the air temperature (T_2) and the saturated vapour pressure (e_2) at that temperature. Then we calculate the partial pressure of the water vapour at the height of 1000 mbar and the final humidity f_1 at the ground:

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{R}{c_p}} = 287.3 \text{ K}$$
$$e_2 = e_s(T_2) = 1628.75 \text{ Pa}$$
$$e_1 = e_2 \left(\frac{T_1}{T_2}\right)^{\frac{c_p}{R}} = 1917.0 \text{ Pa}$$
$$f_1 = \frac{e_1}{e_s(T_1)} = 50.7\%$$

b) The changes are small and happening fast; we can use the differential form of the energy equation:

$$mc_p\Delta T - V\Delta p + h_\text{i}\Delta m_v = 0$$
$$c_p\Delta T - \frac{\Delta p}{\rho} + h_\text{i}\Delta q = 0$$

The quantity of the condensed water vapour that is expressed as the change of the specific humidity:

$$\Delta q = \frac{\Delta p}{\rho h_\text{i}} - \frac{c_p}{h_\text{i}}\Delta T = 0.3 \text{ g/kg}$$

8.10: $z = 1.4$ km

8.11: The height, at which the cumulus clouds will occur (if the convection extends to this height) is $z = 2$ km. In reality, the free convection extends only to the height of 1.1 km. Therefore, clouds did not form.

8.12: $\rho_1 = 1.2$ kg/m^3, $T_2 = 255.6$ K, $\rho = 0.95$ kg/m^3.

8.13: a) The dew point temperature at the ground is $T_d = 9.6$ °C. The cloud base is at the height of $z_B = 648$ m. Yes, the hill has orographic clouds. b) $T_d = 13.4$ °C, $z_B = 192$ m.

8.14: a) The air is mixed to the height of 1 km. b) No, cloudiness did not appear.

8.15: a) $T_d = 14.1$ °C.

b) The condensation level is at the height of $z = 846$ m, where the air temperature is $T = T_0 - \Gamma_a\Delta z = 12.5$ °C, and the atmospheric pressure is

$$p_2 = p_1 \cdot \left(\frac{T_2}{T_1}\right)^{\frac{c_p}{R}} = 903 \text{ mbar}$$

8.16: The dew point temperature at the ground is $T_d = 15.45$ °C. The condensation occurs at the height of $z_1 = 300$ m above the sea level. $z = 700$ m above the sea the relative humidity is 100% and the air temperature $T = T_0 - \Gamma_a z_1 - \Gamma_s(z - z_1) = 12.2$ °C.

8.17: Yes. Solve with the help of the rotated $(T, \ln p)$ diagram.

8.18: a) $e = 1274.8$ Pa, $T_d = 10.6$ °C.

b) Solve with the help with the rotated $(T, \ln p)$ diagram. $T = 18$ °C.

8.19: Calculate the partial pressure of the water vapour on both sides of the hill. We assume that the atmospheric pressure is 1000 mbar and then calculate the specific humidity on both sides. From the air, $\Delta q = 4$ g/kg of precipitation is removed.

8.20: The air mixes to the height of 3.6 km. We can solve the example by calculation or with the help of a (T, z) diagram.

8.21:

$$\omega = \sqrt{g\frac{\Gamma_a + \left(\frac{\partial T}{\partial z}\right)}{T_{ok}}} = 0.01\ \mathrm{s}^{-1}$$

In a non-stable atmosphere, it does not come to the fluctuations, but the air displaced from the equilibrium position keeps on accelerating.

8.22: We assume that the vertical speed of the wind $w = v \cdot \sin\alpha = 3.4$ m/s is not changing with the height. The maximum amount of the precipitation is equal to the total amount of the condensed water vapour:

$$RR = \int_{\mathrm{base}}^{\mathrm{top}} \frac{c_p}{h_\mathrm{i}}(\Gamma_a - \Gamma_s(z))\, w(z)\rho(z)\,\mathrm{d}z = \frac{c_p}{gh_\mathrm{i}} \int_{1000\ \mathrm{mbar}}^{500\ \mathrm{mbar}} (\Gamma_a - \Gamma_s(p))\, w(p)\,\mathrm{d}p$$
$$\doteq \frac{wc_p}{gh_\mathrm{i}} \sum_i (\Gamma_a - \Gamma_s(p))\Delta p_i \doteq 347.5\ \mathrm{mm}$$

Solutions: Emagrams

9.4: The solution is not drawn, b) the example is solved similarly as the examples from the caption about the humidity: 52% (1000 mbar) and 40% (700 mbar), c) 200 mbar, d) Nowhere. The dew point temperature is nowhere equal to the air temperature e) around 890 mbar, f) around 890 mbar, g) around 770 mbar, h) around 170 mbar. i) The ground warms up during the day when the Sun shines on them. At the same time as the ground warms up, the air at the ground also warms up, while the air temperature higher up does not change. This warming at the ground can cause the air at the ground to become warmer than the surroundings (the convection begins). In this case, forced rising is not needed. Meanwhile, the amount of water vapour in the air near the ground stays constant (assuming there is no evaporation from the ground). That means that the dew point temperature near the ground does not change. The problem seeks to determine how much the air on the ground will need to warm up. It needs to be warmer than the surrounding air when it reaches the lifting condensation level. This is determined by following the curve of the dew point temperature to the height at which it crosses the temperature of the surrounding air (around the height of 750 mbar). The lifted condensation level has to be at this height (or higher). From this height, we simply follow the dry adaiabat line downwards to the ground. Where the line crosses the ground, we can read the temperature (around 43 °C). The air at the ground should thus have warmed up by 9 °C (from 34 °C to 43 °C).

9.5: a) decreases from 12 °C (800 mbar) to −20 °C (500 mbar), therefore the temperature decreases by approximately 32 °C, b) Two cloudy layers: 500–400 mbar and 300–250 mbar, c) 250 mbar, d) around 850 mbar, e) around 610 mbar, f) by approximately 12 °C (to approximately 32 °C), g) to the height around 240 mbar.

9.6: a) around 5 °C (700 mbar) and −40 °C (500 mbar), b) There are no cloudy layers, c) 900 mbar, d) No. For free convection, the hill needs to reach 800 mbar, e) to the top of the hill (850 mbar), f) for approximately 6 °C (to approximately 31 °C), g) to the height around 200 mbar.

9.7: a) around −23 °C, b) One cloudy layer is around 700 mbar, c) 930 mbar, d) the free convection almost does not occur. There is just a small region of free convection below the height of 700 mbar, e) The example is a bit reversed from the previous cases, but the logic is similar. From the temperature of the surrounding air at the height of 200 mbar, we descend along the moist adiabat line to where it intersects the line, which describes the change of the dew point temperature. At this height (around 580 mbar), the lifted condensation level is located. From this height, you follow the dry adiabat line downward to the ground. In our case, the line intersects the ground around the temperature of 50 °C. The air at the ground has to warm up by 35 °C (from 15 °C to 50 °C), f) the cloud will extend from the lifted condensation level (around 580 mbar) to the height of 200 mbar.

Solutions: Radiation

10.2: $P = \varepsilon\sigma S(T^4_{\text{wals}} - T^4_{\text{skin}}) = 114$ J/s.

10.3: $P = \varepsilon\sigma T^4 S = 286.9$ W.

10.4: The energy that the Earth receives from the Sun in one day is: $E_s = (1-a)j_0 t \cdot \pi R_z^2 \cdot (1-a)$. The energy that is consumed by the evaporation: $E_i = mh_i = RR \cdot t\rho_v h_i \cdot 4\pi R_z^2$, where $RR = 2,7$ mm/day. The energy portion, that is used for the evaporation: $E_i/E_s = 0,34$.

10.5: We assume a linear profile of the Sun's elevation during the day and obtain a sinusoidal changing of the flux density. The energy the area receives in one day is:

$$E_d = \int_0^{t_0} j_0 S \sin\left(\frac{\pi}{3}\frac{t}{t_0}\right) \mathrm{d}t + \int_{t_0}^{2t_0} j_0 S \sin\left(\frac{2\pi}{3} - \frac{\pi}{3}\frac{t}{t_0}\right) \mathrm{d}t = j_0 S \frac{3t_0}{\pi},$$

where t_0 is 6 h. For the evaporation of $h = 1200$ mm of precipitation the energy of $E_i = qm_v = qSh\rho_v$ is needed. The number of days that are needed for the evaporation of all water is: $N = E_i/E_d = 106$.

10.7: a) All received power is used for heating the lake: $P_e t = mc_v \Delta T$. We can calculate that $\frac{\Delta T}{t} = \frac{P_e}{mc_v} = 0.01$ K/day.

b) When the new equilibrium is established, the power from the power plant is equal to the power by radiation: $\varepsilon T_{\text{new}}^4 S = \varepsilon T^4 S + P_e$. The new equilibrium temperature:

$$T_{\text{new}} = \sqrt[4]{T^4 + \frac{P_e}{\varepsilon \sigma S}} = 284 \text{ K}.$$

The lake warms up by 1 K.

c) In this case, the heat required for evaporation must equal the waste heat from the power plant, and we get the evaporation rate (change in the lake's water height over time) as $\frac{dh}{dt} = \frac{P_e}{h_i \rho_a S} = 0.17$ mm/day

10.8: From the equation for the energy equilibrium: $E = m_v q_i + \varepsilon \sigma T^4 S t$, we calculate the temperature $T = 12.7$ °C.

10.9: a) 53.5 days, b) 90.5 days.

10.10: a) Considering that the absorbed solar radiation energy is used solely for water evaporation, the rate of change of the water column height due to evaporation can be expressed as $\frac{dz_i}{dt} = k_i = \frac{j_t \sin 25° \beta}{\rho_a h_i}$, where j_t is the solar radiation energy flux density, β is the fraction of radiation absorbed, ρ_a is the density of water, and h_i is the specific latent heat of water evaporation. We find $k_i = 1.08 \cdot 10^{-7}$ m/s, which means that a water column 0.5 m high would evaporate in 53.5 days.

b) In equilibrium, the volume of water evaporating will equal the volume of water flowing into the lake. Given that the evaporation rate k_i is constant, the total volume of evaporating water depends only on the current lake surface area S, which can be expressed as $\frac{dV_i}{dt} = k_i S$. In equilibrium, evaporation must equal the volumetric inflow of 1000 m^3/h, from which we obtain $S = 2.57 \cdot 10^6$ m^2. Due to the shape of the lake's basin, its surface area depends on its depth h and can be expressed as $S = \pi \left(\frac{r_0}{2} + \frac{h}{h_0} r_0 \right)^2$, where $r_0 = 1000$ m and $h_0 = 0.5$ m. From this, we find that the previously calculated lake surface area corresponds to a depth of $h = 0.20$ m.

c) The change in the volume of the lake can be expressed as the difference between the volumetric inflow ϕ and evaporation: $\frac{dV}{dt} = \phi - k_i S(h)$. Considering $dV = S(h)dh$, we obtain the differential equation $\frac{dh}{dt} = \frac{\phi}{S(h)} - k_i$ in which h and t are the independent variables. By inserting the expression for $S(h)$ into the equation and solving it through integration, we can express the time required for the entire lake to evaporate, which gives us a result of 71.8 days.

10.11: a) 733 W, b) 769 W.

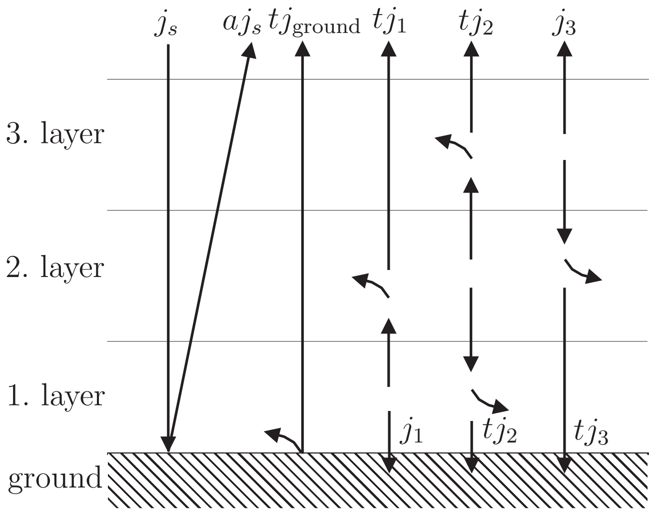
10.12: We solve the diffusion equation:

$$\frac{\lambda}{\rho c_p}\frac{\partial^2 T}{\partial t^2} = \frac{\partial T}{\partial t}$$

with the initial condition $T = T_0 + (T_{max} - T_0)\sin((t-t_0)\frac{\pi}{12\text{ h}})$. We get the equation, that describes the changing of the temperature with the time at the different heights: $T = T_0 + (T_{max} - T_0)\,\mathrm{e}^{-k_1 z}\sin((t-t_0)\frac{\pi}{12\text{ h}}k_1 z)$, where $k_1 = \pi/(2D\cdot 12\text{ h})$ where $D = \lambda/\rho c_p$. At a depth of 15 cm, the amplitude of the temperature fluctuation is 8 K.

10.13: $T_{\text{ground}} = 276$ K.

10.14: For the ground and for every atmosphere layer the sum of the energy fluxes is 0.



ground:	$j_{\text{ground}} = (1-a)j_s + j_1 + tj_2 + tj_3$
1. layer:	$2j_1 = (1-t)j_{\text{ground}} + (1-t)j_2$
2. layer:	$2j_2 = (1-t)j_1 + (1-t)j_3$
3. layer:	$2j_3 = (1-t)j_2$

The ground temperature is 300 K.

10.15: a) We assume that the elevation of the sun between 11:30 and 12:30 is not changing.

$$Q = j_g S_0 \sin \varphi_0 t = 1497 \text{ J}$$

b) We calculate the equilibrium temperature at noon using the radiation balance for the black body, where we assume that the pad radiates only upwards:

$$T = \sqrt[4]{\frac{j_0 \sin \varphi_0}{\sigma}} = 332.5 \text{ K}.$$

c) The change in the angle of the sun's elevation over time from sunrise ($t = 0$) to noon ($t = t_0$) can be described by the equation:

$$\varphi(t) = \varphi_0 \frac{t}{t_0}$$

Since the panel will receive the same amount of energy in the afternoon as in the morning, the total energy received can be obtained as:

$$Q_s = 2 \int_0^{t_0} P(t) \mathrm{d}t = 2 \int_0^{t_0} j_g S_0 \sin(\varphi(t)) \, \mathrm{d}t = \frac{2 j_g S_0 t_0 (1 - \cos \varphi_0)}{\varphi_0} = 10313 \text{ J}$$

10.16: We assume that the thermometer bulb is spherical. The received power of the solar radiation and the radiation of the surrounding air is equal to the radiated power of the bulb:

$$(1 - a_{\text{ther}}) j_g S_0 + \varepsilon_{\text{air}} \sigma T_{\text{air}}^4 4 S_0 = \varepsilon_{\text{ther}} \sigma T_{\text{ther}}^4 4 S_0,$$

where S_0 is the surface area of the spherical bulb. From this, we obtain

$$a_{\text{ther}} = 1 - \frac{4\sigma(\varepsilon_{\text{ther}} T_{\text{ther}}^4 - \varepsilon_{\text{air}} T_{\text{air}}^4)}{j_g} = 0.1$$

Solutions: Fronts

11.2: The movement speed of the front is defined by the wind component, which is perpendicular to the front. This component has to be the same in the warm and cold sectors. If β is the angle between the front and the wind vector in the warm sector, the equation: $v_t \sin \beta = v_h \sin(\pi/4 + \pi/6 - \beta)$ has to be true. We calculate the angle $\beta = 29.2°$. The speed of the front is then $u = v_t \sin \beta = 12.5$ m/s.

11.3: From the tilt of the front, we calculate the wind shear at the front and the wind in the warm sector. The latter blows at the speed of 35 m/s from the direction of 129.2°.

11.4: The front orientation is SW-NE and travels at the speed of 7.1 m/s.

11.5: Speed: 14.9 m/s, direction: 294.1°.

11.7: We use the equations that are also valid for the front. The tilt of the upper limit of the cold-air lake is $\tan\alpha = 1/231$. The inversion level on the other side of the basin is 273 m, which is at the southern part of the basin.

11.8: $\tan\alpha = 1/456$.

11.9: The tilt of the front is $\tan\alpha = 1/159$. After the winds in the warm air calm down, the tilt of the surface will be $\tan\alpha = 1/484$.

11.10: The tilt of the front is 1/326. The cirrus cloud appears approximately 2600 km before the front. From the 1000-m high hill, we can see the cirrus clouds that at approximately 400 km away (we take into account that the radius of the Earth is $R_z = 6378$ km).

Appendix

A list of used symbols and codes

c_{pa}	specific heat of water at constant pressure, $c_{\mathrm{pa}} = 4181$ J/kg K
c_{pl}	specific heat of ice at constant pressure, $c_{\mathrm{pl}} = 2114$ J/kg K
c_{pv}	specific heat of water vapour at constant pressure, $c_{\mathrm{pv}} = 1847$ J/kg K
c_{pz}	specific heat of dry air at constant pressure, $c_{\mathrm{pz}} = 1004$ J/kg K
g_0	standard value of gravitational acceleration, $g_0 = 9.81$ m/s^2
h_{i}	specific latent heat of water evaporation, $h_{\mathrm{i}} = 2.50$ MJ/kg
h_{s}	specific latent heat of water sublimation, $h_{\mathrm{s}} = 2.83$ MJ/kg
h_{t}	specific heat of fusion for water, $h_{\mathrm{t}} = 0.33$ MJ/kg
j_0	solar constant, energy flux density of solar radiation at the top of the Earth's atmosphere, $j_0 \approx 1400$ W/m^2
R^*	specific gas constant, $R^* = 8317$ J/kmol K
R	specific gas constant for air, $R = 287$ J/kg K
R_v	specific gas constant for water vapour, $R_v = 461.5$ J/kg K
Γ_a	(negative) individual change of temperature at adiabatic displacement of unsaturated air in vertical direction, $\Gamma_a = -\frac{\mathrm{d}T}{\mathrm{d}z} = 10$ K/km
Γ_s	(negative) individual change of temperature at adiabatic displacement of saturated air in vertical direction, $\Gamma_s = -\frac{\mathrm{d}T}{\mathrm{d}z}$

Table of the saturated vapour pressure

Table of the saturated vapour pressure over water in Pa. The following equation was used (Rogers & Yau: *A Short Course in Cloud Physics*, Pergamon Press, page 16):

$$e_s(T) = 611.2\ \mathrm{Pa} \cdot \exp\left(\frac{17.67 \cdot T}{T + 243.5}\right),$$

where the temperature T is in °C. Example: $e_s(-3.2\ °\mathrm{C}) = 483.05$ Pa, $e_s(3.2\ °\mathrm{C}) = 768.64$ Pa.

T [°C]	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
−20	125.74	124.66	123.59	122.53	121.47	120.43	119.39	118.36	117.34	116.32
−19	137.00	135.83	134.68	133.53	132.39	131.26	130.14	129.03	127.92	126.83
−18	149.15	147.89	146.65	145.41	144.18	142.96	141.75	140.55	139.36	138.17
−17	162.26	160.90	159.56	158.22	156.90	155.58	154.28	152.98	151.69	150.42
−16	176.39	174.93	173.48	172.04	170.61	169.19	167.79	166.39	165.00	163.63
−15	191.61	190.04	188.48	186.93	185.39	183.86	182.34	180.84	179.34	177.86
−14	208.00	206.30	204.62	202.95	201.30	199.65	198.02	196.40	194.79	193.20
−13	225.62	223.80	221.99	220.20	218.42	216.65	214.89	213.15	211.42	209.70
−12	244.57	242.61	240.67	238.74	236.82	234.92	233.03	231.16	229.30	227.45
−11	264.92	262.82	260.73	258.66	256.60	254.56	252.53	250.52	248.52	246.54
−10	286.77	284.51	282.28	280.05	277.84	275.65	273.47	271.31	269.17	267.04
−9	310.21	307.79	305.39	303.01	300.64	298.29	295.95	293.63	291.33	289.04
−8	335.35	332.76	330.18	327.62	325.08	322.56	320.06	317.57	315.10	312.65
−7	362.28	359.50	356.75	354.01	351.29	348.58	345.90	343.23	340.59	337.96
−6	391.12	388.15	385.20	382.26	379.35	376.46	373.58	370.73	367.89	365.08
−5	421.99	418.81	415.65	412.51	409.39	406.30	403.22	400.17	397.13	394.12
−4	455.01	451.60	448.22	444.87	441.53	438.22	434.93	431.66	428.42	425.19
−3	490.30	486.66	483.05	479.46	475.90	472.36	468.84	465.35	461.88	458.43
−2	528.00	524.11	520.26	516.42	512.62	508.84	505.08	501.35	497.64	493.95
−1	568.25	564.10	559.99	555.90	551.83	547.79	543.78	539.80	535.84	531.90
−0	611.20	606.78	602.39	598.02	593.69	589.38	585.10	580.84	576.62	572.42
0	611.20	615.65	620.13	624.64	629.17	633.74	638.33	642.96	647.61	652.29
1	657.01	661.75	666.52	671.33	676.16	681.03	685.93	690.86	695.82	700.81
2	705.83	710.89	715.97	721.09	726.24	731.43	736.64	741.89	747.18	752.49
3	757.84	763.23	768.64	774.09	779.58	785.10	790.65	796.24	801.87	807.52
4	813.22	818.95	824.71	830.52	836.35	842.23	848.14	854.08	860.07	866.09
5	872.15	878.24	884.38	890.55	896.76	903.00	909.29	915.61	921.98	928.38
6	934.82	941.30	947.82	954.38	960.98	967.62	974.31	981.03	987.79	994.60
7	1001.44	1008.33	1015.26	1022.23	1029.24	1036.30	1043.40	1050.54	1057.73	1064.95
8	1072.23	1079.54	1086.90	1094.30	1101.75	1109.25	1116.78	1124.37	1132.00	1139.67
9	1147.39	1155.16	1162.97	1170.83	1178.74	1186.69	1194.69	1202.74	1210.83	1218.98
10	1227.17	1235.41	1243.70	1252.04	1260.43	1268.86	1277.35	1285.89	1294.48	1303.11
11	1311.80	1320.54	1329.33	1338.18	1347.07	1356.02	1365.02	1374.07	1383.17	1392.33
12	1401.54	1410.80	1420.12	1429.49	1438.92	1448.40	1457.94	1467.53	1477.18	1486.88
13	1496.64	1506.46	1516.33	1526.26	1536.25	1546.29	1556.39	1566.55	1576.77	1587.05
14	1597.39	1607.78	1618.23	1628.75	1639.32	1649.96	1660.65	1671.41	1682.23	1693.11
15	1704.05	1715.05	1726.12	1737.25	1748.44	1759.69	1771.01	1782.40	1793.84	1805.35
16	1816.93	1828.57	1840.28	1852.05	1863.89	1875.80	1887.77	1899.81	1911.92	1924.09
17	1936.34	1948.65	1961.03	1973.48	1986.00	1998.58	2011.24	2023.97	2036.77	2049.64
18	2062.58	2075.60	2088.68	2101.84	2115.07	2128.38	2141.75	2155.21	2168.73	2182.33
19	2196.01	2209.76	2223.58	2237.48	2251.46	2265.51	2279.65	2293.85	2308.14	2322.50
20	2336.95	2351.47	2366.07	2380.75	2395.51	2410.35	2425.27	2440.27	2455.35	2470.52
21	2485.76	2501.09	2516.51	2532.00	2547.58	2563.24	2578.99	2594.82	2610.74	2626.74
22	2642.83	2659.00	2675.26	2691.61	2708.05	2724.57	2741.19	2757.89	2774.68	2791.56
23	2808.53	2825.59	2842.74	2859.98	2877.31	2894.74	2912.25	2929.86	2947.57	2965.36
24	2983.25	3001.24	3019.32	3037.50	3055.77	3074.13	3092.60	3111.16	3129.82	3148.57
25	3167.43	3186.38	3205.44	3224.59	3243.84	3263.19	3282.65	3302.20	3321.86	3341.62
26	3361.48	3381.45	3401.51	3421.69	3441.96	3462.35	3482.84	3503.43	3524.13	3544.94
27	3565.85	3586.87	3608.00	3629.24	3650.59	3672.05	3693.62	3715.30	3737.09	3758.99
28	3781.00	3803.13	3825.37	3847.72	3870.19	3892.77	3915.47	3938.28	3961.21	3984.25
29	4007.41	4030.69	4054.09	4077.61	4101.24	4124.99	4148.87	4172.86	4196.98	4221.22
30	4245.58	4270.06	4294.66	4319.39	4344.24	4369.22	4394.33	4419.55	4444.91	4470.39

Empty Skew-T Log-P emagram