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S and D-wave resonances in chiral quark models: coupled channel approach

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Abstract. We apply a coupled channel formalism incorporating quasi-bound quark-model states to calculate the S11 and D13 scattering amplitudes. The meson-baryon verteces for πN, ηN, $\pi\Delta$, ρN and $K\Lambda$ channels are determined in the Cloudy Bag Model. Using the same values for the model parameters as in the case of the P11 and P33 amplitudes the elastic as well as most of the inelastic amplitudes are reasonably well reproduced.

1 Introduction

This work is a continuation of a joint project on the description of baryon resonances performed by the Coimbra group (Manuel Fiolhais, Luis Alvarez Ruso, Pedro Alberto) and the Ljubljana group (Simon Širca and B. G.)

We have developed a general method to incorporate excited baryons represented as quasi-bound quark-model states into a coupled channel formalism using the K-matrix approach [1]. In our method, the meson-baryon and the photon-baryon verteces are therefore determined by the underlying quark model rather than fitted to the experimental data as is the case in phenomenological approaches. The method can be applied to meson scattering as well as to electro and weak-production of mesons.

In the previous work we have investigated the P33 and P11 amplitude dominated by the low lying positive parity resonances $\Delta(1232)$, $\Delta(1600)$ and $N(1440)$ [1,2]. We have found a good agreement between the model prediction and experiment for the scattering as well as the electro-production amplitudes. We have shown that the pion and the σ-meson considerably contribute in particular to the scattering amplitudes in the energy region just above the two pion threshold and to the electro-excitation amplitudes in the region of low Q^2 transfer. In the present work we investigate the extension of the approach to low lying negative parity resonances. This implies the inclusion of new channels involving the s-wave and the d-wave pions, the η and the ρ mesons, and the $K\,\Lambda$ channel.

In the next section we give a short review of the method and in the following sections we discuss in more detail scattering in the S11 and D13 partial waves.

2 A short overview of the K-matrix approach

We consider a class of chiral quark models in which mesons couple linearly to the quark core:

$$H_{\text{meson}} = \int dk \sum_{lmt} \left\{ \omega_k\, a^\dagger_{lmt}(k) a_{lmt}(k) + \left[V_{lmt}(k) a_{lmt}(k) + V^\dagger_{lmt}(k)\, a^\dagger_{lmt}(k) \right] \right\}, \tag{1}$$

where $a^\dagger_{lmt}(k)$ is the creation operator for a meson with angular momentum l its third components m and isospin t (absent in the case of s-waves and isoscalar mesons). Here $V_{lmt}(k)$ is a general form of the meson source involving the quark operators and is model dependent. In the following section we give a few examples for $V_{lmt}(k)$ in the Cloudy Bag Model.

We have shown [1] that in such models the elements of the K matrix in the basis with good total angular momentum J and isospin T take the form:

$$K^{JT}_{M'B'\,MB} = -\pi \mathcal{N}_{M'B'} \langle \Psi^{MB}_{JT} \| V_{M'}(k) \| \widetilde{\Psi}_{B'} \rangle \,, \qquad \mathcal{N}_{MB} = \sqrt{\frac{\omega_M E_B}{k_M W}}\,, \tag{2}$$

where ω_M and k_M are the energy and momentum of the incoming (outgoing) meson, E_B is the baryon energy and W is the invariant energy of the meson-baryon system. In addition, the channels are specified by the relative angular momentum of the meson-baryon system and parity. Here $|\Psi^{MB}\rangle$ is the principal value state and assumes the form:

$$|\Psi^{MB}_{JT}\rangle = \mathcal{N}_{MB} \left\{ [a^\dagger(k_M)|\widetilde{\Psi}_B\rangle]^{JT} + \sum_{\mathcal{R}} c^{MB}_{\mathcal{R}} |\Phi_{\mathcal{R}}\rangle \right.$$
$$\left. + \sum_{M'B'} \int \frac{dk\, \chi^{M'B'\,MB}(k, k_M)}{\omega_k + E_{B'}(k) - W} [a^\dagger(k)|\widetilde{\Psi}_{B'}\rangle]^{JT} \right\}. \tag{3}$$

The first term represents the free meson (π, η, ρ, K, ...) and the baryon (N, Δ, Λ, ...) and defines the channel, the next term is the sum over *bare* tree-quark states $\Phi_{\mathcal{R}}$ involving different excitations of the quark core, the third term introduces meson clouds around different isobars, $E(k)$ is the energy of the recoiled baryon. In our approach we assume the commonly used picture in which the two pion decay proceeds either through an unstable meson (ρ-meson, σ-meson, ...) or through a baryon resonance ($\Delta(1232)$, $N^*(1440)$ ...). In such a case the state Ψ^{MB} depends on the invariant mass of the subsystem (either $\pi\pi$ or πN) and the sum over $M'B'$ in (3) implies also integration over the invariant mass. The state $\widetilde{\Psi}_B$ is the asymptotic state of the incoming (outgoing) baryon; in the case it corresponds to an unstable baryon it depends on the invariant mass of the πN subsystem, M_B, and is normalized as $\langle \widetilde{\Psi}_B(M'_B)|\widetilde{\Psi}_B(M_B)\rangle = \delta(M'_B - M_B)$, where M_B is the invariant mass M_B of the Nπ subsystem. The meson amplitudes $\chi^{M'B'\,MB}(k, k_M)$ are proportional to the (half) off-shell matrix elements of the K-matrix

$$K_{M'B'\,MB}(k, k_M) = \pi \mathcal{N}_{M'B'} \mathcal{N}_{MB}\, \chi^{M'B'\,MB}(k, k_M) \tag{4}$$

and obey a Lippmann-Schwinger type of equation:

$$\chi^{M'B'\,MB}(k,k_M) = -\sum_{\mathcal{R}} c_{\mathcal{R}}^{MB}\, V_{B'\mathcal{R}}^{M'}(k) + \mathcal{K}^{M'B'\,MB}(k,k_M) + \sum_{M''B''}\int dk'\, \frac{\mathcal{K}^{M'B'\,M''B''}(k,k')\chi^{M''B''\,MB}(k',k_M)}{\omega_k' + E_{B''}(k') - W}\,, \quad (5)$$

where

$$\mathcal{K}^{M'B'\,MB}(k,k') = \sum_{B''} f_{BB'}^{B''}\, \frac{\widetilde{\mathcal{V}}_{B''B'}^{M'}(k')\, \widetilde{\mathcal{V}}_{B''B}^{M}(k)}{\omega_k + \omega_k' + E_{B''}(\bar{k}) - W}\,, \quad (6)$$

$$f_{AB}^{C} = \sqrt{(2J_A+1)(2J_B+1)(2T_A+1)(2T_B+1)}\, W(1J_AJ_B1;J_C,J)W(1T_AT_B1;T_C,T)\,.$$

The coefficients $c_{\mathcal{R}}^{MB}$ obey the equation

$$(W - M_{\mathcal{R}}^{(0)})c_{\mathcal{R}}^{MB} = V_{B\mathcal{R}}^{M}(k_M) + \sum_{M'B'}\int dk\, \frac{\chi^{M'B'\,MB}(k,k_M)V_{B'\mathcal{R}}^{M'}(k)}{\omega_k + E_{B'}(k) - W}\,. \quad (7)$$

Here $V_{B\mathcal{R}}^{M}(k)$ are the matrix elements of the quark-meson interaction between the baryon state B and the bare 3-quark state $\Phi_{\mathcal{R}}$, and $M_{\mathcal{R}}^{(0)}$ is the energy of the bare state. Solving the coupled system of equations (5) and (7) using a separable approximation [1] for the kernels (6), the resulting amplitudes take the form

$$\chi^{M'B'\,MB}(k,k_M) = -\sum_{\mathcal{R}} \tilde{c}_{\mathcal{R}}^{MB}\, \widetilde{\mathcal{V}}_{B'\mathcal{R}}^{M'}(k) + \mathcal{D}^{M'B'\,MB}(k,k_M)\,, \quad (8)$$

where the first term represents the contribution of various resonances while $\mathcal{D}^{M'B'\,MB}(k)$ originates in the non-resonant background processes. Here

$$\tilde{c}_{\mathcal{R}}^{MB} = \frac{\widetilde{\mathcal{V}}_{B\mathcal{R}}^{M}}{Z_{\mathcal{R}}(W)(W - M_{\mathcal{R}})}\,, \quad (9)$$

$\widetilde{\mathcal{V}}_{B\mathcal{R}}^{M}$ is the dressed matrix element of the quark-meson interaction between the resonant state and the baryon state in the channel MB, and $Z_{\mathcal{R}}$ is the wave-function normalization. The physical resonant state $\mathcal{R}$ is a superposition of the dressed states built around the bare 3-quark states $\Phi_{\mathcal{R}'}$. The T matrix is finally obtained by solving the Heitler's equation

$$T = K + iTK\,. \quad (10)$$

In this work we concentrate on the negative parity partial waves S11 and D13; in both cases the amplitudes are dominated by two rather closely lying resonances, either the N(1535) S_{11} and N(1650) S_{11}, or the N(1520) D_{13} and N(1700) D_{13}. We have performed the calculation of the scattering amplitudes in the same model, the Cloudy Bag Model (CBM), with the same choice of model parameters as in the case of positive parity resonances. For the bag radius we use $R = 0.83$ fm and $f_\pi = 76$ MeV for the parameter determining the interaction strength, while the energies of the bare states are taken as free parameters.

3 The meson coupling to negative parity states in the CBM

We have used the bag model description for the resonances assuming that one of the three quarks is excited from the 1s state to the $1p_j$ state with the total angular momentum j either 1/2 or 3/2. The relevant quark bispinors in the $j\, m_j$ basis are

$$\psi_s(\mathbf{r}) = \frac{N_s}{\sqrt{4\pi}\, j_0(\omega_s)} \begin{pmatrix} -i\, j_0(\omega_s r/R) \\ \boldsymbol{\sigma}\cdot\hat{\mathbf{r}}\, j_1(\omega_s r/R) \end{pmatrix} \chi_{m_j}\,,$$

$$\psi_{p_{1/2}}(\mathbf{r}) = \frac{N_{p_{1/2}}}{\sqrt{4\pi}\, j_0(\omega_{p_{1/2}})} \begin{pmatrix} i\, j_1(\omega_{p_{1/2}} r/R)\boldsymbol{\sigma}\cdot\hat{\mathbf{r}} \\ j_0(\omega_{p_{1/2}} r/R) \end{pmatrix} \chi_{m_j}\,,$$

$$\psi_{p_{3/2}}(\mathbf{r}) = \frac{N_{p_{3/2}}}{\sqrt{6\pi}\, j_1(\omega_{p_{3/2}})} \begin{pmatrix} -i\, j_1(\omega_{p_{3/2}} r/R) \\ \boldsymbol{\sigma}\cdot\hat{\mathbf{r}}\, j_2(\omega_{p_{3/2}} r/R) \end{pmatrix} \sum_{m_s m} \chi_{m_s}\hat{r}_m C^{\frac{3}{2}m_j}_{\frac{1}{2}m_s 1m}\,.$$

Here χ_m is the spinor for spin $\frac{1}{2}$, R is the bag radius, $\omega_s = 2.043$, $\omega_{p_{1/2}} = 3.811$, $\omega_{p_{3/2}} = 3.204$, and

$$N_s^2 = \frac{\omega_s}{2R^2(\omega_s - 1)}\,,\quad N_{p_{1/2}}^2 = \frac{\omega_{p_{1/2}}}{2R^2(\omega_{p_{1/2}} + 1)}\,,\quad N_{p_{3/2}}^2 = \frac{9\omega_{p_{3/2}}}{4R^2(\omega_{p_{3/2}} - 2)}\,.$$

The wave function of the negative parity states in the j–j coupling scheme are taken from [5].

For the *quark pion coupling* we use the usual CBM form yielding

$$V^{\pi}_{l=0,t}(k) = \frac{1}{2f_\pi}\sqrt{\frac{\omega_{p_{1/2}}\omega_s}{(\omega_{p_{1/2}}+1)(\omega_s-1)}}\,\frac{1}{2\pi}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_0(kR)}{kR}\sum_{i=1}^{3}\tau_t(i)\,\mathcal{P}_{sp}(i)\,,$$

$$V^{\pi}_{1mt}(k) = \frac{1}{2f_\pi}\,\frac{\omega_s}{(\omega_s-1)}\,\frac{1}{2\pi}\,\frac{1}{\sqrt{3}}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_1(kR)}{kR}\sum_{i=1}^{3}\tau_t(i) \times\left(\sigma_m(i) + r_{p_{1/2}} S^{[\frac{1}{2}]}_{1m}(i) + r_{p_{3/2}} S^{[\frac{3}{2}]}_{1m}(i)\right)\,,$$

$$V^{\pi}_{2mt}(k) = \frac{1}{2f_\pi}\sqrt{\frac{\omega_{p_{3/2}}\omega_s}{(\omega_{p_{3/2}}-2)(\omega_s-1)}}\,\frac{\sqrt{2}}{2\pi}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_2(kR)}{kR}\sum_{i=1}^{3}\tau_t(i)\,\Sigma^{[\frac{1}{2}\frac{3}{2}]}_{2m}(i)\,.$$

Here

$$\mathcal{P}_{sp} = \sum_{m_j} |s m_j\rangle\langle p_{1/2} m_j|\,,\quad S^{[\frac{1}{2}]}_{1m} = \sqrt{3}\sum_{m_j m_j'} C^{\frac{1}{2}m_j}_{\frac{1}{2}m_j' 1m} |p_{1/2} m_j\rangle\langle p_{1/2} m_j'|\,,$$

$$\Sigma^{[\frac{1}{2}\frac{3}{2}]}_{2m} = \sum_{m_s m_j} C^{\frac{1}{2}m_s}_{\frac{3}{2}m_j 2m} |s m_s\rangle\langle p_{3/2} m_j|\,,\quad S^{[\frac{3}{2}]}_{1m} = \frac{\sqrt{15}}{2}\sum_{m_j m_j'} C^{\frac{3}{2}m_j}_{\frac{3}{2}m_j' 1m} |p_{3/2} m_j\rangle\langle p_{3/2} m_j'|\,,$$

and

$$r_{p_{1/2}} = \frac{\omega_{p_{1/2}}(\omega_s - 1)}{\omega_s(\omega_{p_{1/2}} + 1)}\,,\qquad r_{p_{3/2}} = \frac{2\omega_{p_{3/2}}(\omega_s - 1)}{5\omega_s(\omega_{p_{3/2}} - 2)}\,.$$

The ρ *meson coupling* to quarks is similar to the EM coupling. For the coupling of negative parity states to ρN, the dominant contribution is expected to arise from the transverse ρ-mesons with the total $J = 1$ and the orbital angular momentum of the ρN system equal to either 0 or 2. In the spirit of the CBM model we assume that the ρ meson couples to the quarks only on the bag surface [6]. Assuming further a pure vector coupling $\gamma^\mu\rho_\mu$ we find (note that m in (1) and below refers to the total angular momentum rather than to the orbital one):

$$V^\rho_{l=0mt}(k) = \frac{1}{2f_\rho}\sqrt{\frac{\omega_s}{(\omega_s-1)}}\,\frac{1}{2\pi}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_0(kR)}{kR}\sum_i \tau_t(i)$$
$$\times\left(\frac{\sqrt{8}}{3}\sqrt{\frac{\omega_{p_{1/2}}}{\omega_{p_{1/2}}+1}}\,\Sigma^{[\frac12]}_{1m} + 3\sqrt{\frac{\omega_{p_{3/2}}}{\omega_{p_{3/2}}-2}}\,\Sigma^{[\frac12\frac32]}_{1m}(i)\right),$$
$$V^\rho_{l=2mt}(k) = \frac{1}{2f_\rho}\sqrt{\frac{\omega_{p_{3/2}}\omega_s}{(\omega_{p_{3/2}}-2)(\omega_s-1)}}\,\frac{1}{2\pi}\,\frac{1}{3}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_2(kR)}{kR}\sum_{i=1}^{3}\tau_t(i)\Sigma^{[\frac12\frac32]}_{1m}(i)\,.$$

Here

$$\Sigma^{[\frac12]}_{1m} = \sum_{m_s m_j} C^{\frac12 m_s}_{\frac12 m_j 1m}|s m_s\rangle\langle p_{1/2} m_j|\,,\qquad \Sigma^{[\frac12\frac32]}_{1m} = \sum_{m_s m_j} C^{\frac12 m_s}_{\frac32 m_j 1m}|s m_s\rangle\langle p_{3/2} m_j|\,,$$

and f_ρ is the ρ-meson decay constant with the experimental value 208 MeV. For the coupling of the ρ meson to the nucleon we obtain a similar expression as for the coupling of the p-wave pions, with f_π replaced by f_ρ, yielding $g_{\pi NN}/g_{\rho NN} = f_\rho/f_\pi$ which is experimentally well fulfilled. The choice of the above form with $f_\rho \approx 200$ MeV is therefore not insensible.

For the *s-wave* η and K *mesons* we assume the SU(3) symmetry yielding

$$V^\eta(k) = \frac{1}{2f_\pi}\sqrt{\frac{\omega_{p_{1/2}}\omega_s}{(\omega_{p_{1/2}}+1)(\omega_s-1)}}\,\frac{1}{2\pi}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_0(kR)}{kR}\sum_{i=1}^{3}\lambda_8(i)\,\mathcal{P}_{sp}(i)\,,$$
$$V^K_t(k) = \frac{1}{2f_\pi}\sqrt{\frac{\omega_{p_{1/2}}\omega_s}{(\omega_{p_{1/2}}+1)(\omega_s-1)}}\,\frac{1}{2\pi}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_0(kR)}{kR}\sum_{i=1}^{3}(V_t(i)+U_t(i))\,\mathcal{P}_{sp}(i)\,,$$

with $t = \pm\frac12$, and $V_{\pm t} = (\lambda_4 \pm i\lambda_5)/\sqrt{2}$ and $U_{\pm t} = (\lambda_6 \pm i\lambda_7)/\sqrt{2}$.

The peculiar oscillating shape of the CBM form factor has little influence in the case of the p and d-wave pions but leads to the unphysical behaviour of the s-wave scattering amplitude since it crosses zero already at $W \sim 1950$ MeV. We have cured this problem by replacing $j_0(kR)$ by an exponential tail for $k > 1.6/R$ in such a way as not to alter the value of the self energy integral.

4 S11 resonances

For the S11 partial wave we have included the πN, πΔ, πN(1440), ρN and KΛ channels and the N(1535) and N(1650) resonances. We have used the quark-model wave-functions for the negative-parity states using the j–j coupling scheme [5]:

$$\Phi_{\mathcal{R}} = c^{\mathcal{R}}_A|(1s)^2(1p_{3/2})^1\rangle + c^{\mathcal{R}}_P|(1s)^2(1p_{1/2})^1\rangle_1 + c^{\mathcal{R}}_{P'}|(1s)^2(1p_{1/2})^1\rangle_2\,,$$

where the mixing coefficients $c_A^{\mathcal{R}}$, $c_P^{\mathcal{R}}$, and $c_{P'}^{\mathcal{R}}$ can be expressed in terms of the mixing angle ϑ_s between the spin-1/2 and spin-3/2 3-quark configurations. The mixing is a consequence of the gluon and the meson interaction; since the quark-gluon interaction is not included in the model, the mixing angle due to gluons is taken as a free parameter independent of W. In the energy region of the N(1535) and N(1650) resonances we obtain the best results using $\vartheta_s = -34°$ in agreement with the phenomenological analysis [7].

Resonance	Γ_{tot} [MeV]	Γ_i/Γ_{tot}					
		πN	ηN	$\pi\Delta$	KΛ	ρ_1N	πN(1440)
N(1535)	165	0.29	0.69	0.01	-	0.01	0
PDG	125 – 175	0.35 – 0.55	0.53	0.01	-	0.02	0
N(1650)	188	0.59	0	0.19	0.13	0.04	0.04
N(1650)*mod*	156	0.72	0	0.08	0.10	0.05	0.04
PDG	150 – 180	0.60 – 0.95	0.02	0.02	0.03	0.01	0.03

Table 1. The total and the partial widths for the N(1535) and the N(1650) resonance at the K-matrix pole (1545 MeV and 1695 MeV, respectively) using the unmodified and the modified (*mod*) quark-model values for the quark-meson couplings. The PDG values are from [3].

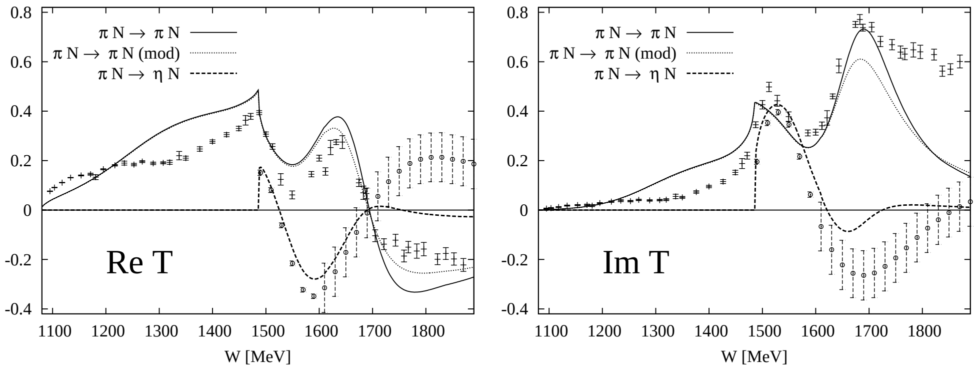

Fig. 1. The real and the imaginary part of the scattering amplitudes for the S11 wave. Dotted lines are for the elastic channel with unmodified quark-model verteces, full lines are those with the modified values, dashed lines correspond to the ηN channel. The data points for the elastic channel are from the SAID $\pi N \to \pi N$ partial-wave analysis [4], those for the inelastic one are taken from [9].

In the vicinity of the lower resonance, just above the η meson threshold, the elastic and inelastic amplitudes are dominated by the *s*-wave ηN channel. In the energy region of the upper resonance, additional channels open or become more important. We have considered the following additional channels: the $\pi\Delta(1232)$ channel with $l = 2$, the $K\Lambda(1116)$ channel with $l = 0$, two channels involving the ρ meson with $l = 0$ (ρ_1N) and $l = 2$ (ρ_3N), and the $\pi N^*(1440)$ channel with $l = 0$.

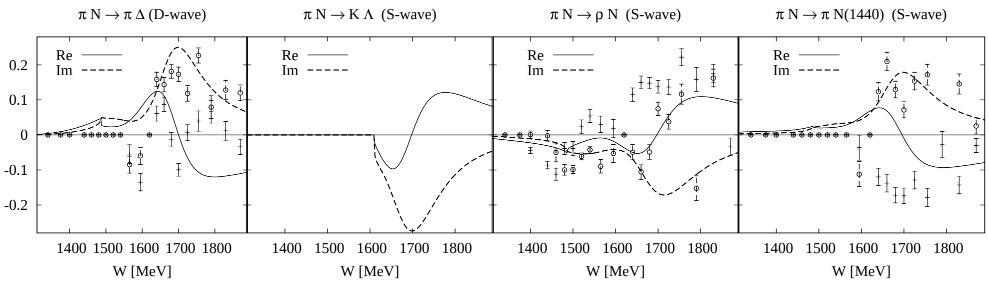

Fig. 2. The real and the imaginary part of the inelastic amplitudes for S11 partial wave. The experimental points are from [8].

Using the quark-model values for the quark-meson coupling as introduced in the previous section we obtain a good agreement between the model prediction and the experimental analysis for the lower resonance; for the upper resonance the agreement is worse. Though the extraction of the experimental data is less reliable and considerably differs between different authors, it clearly indicates that the strengths of the $\pi\Delta$ (d-wave) vertex is overestimated in our model; the same is probably true also for the $K\Lambda$ (s-wave) channel (Table 1). Multiplying the strength of the $\pi\Delta$ and the $K\Lambda$ vertex by 0.6 and 0.8, respectively, yields a better agreement in particular for the imaginary part of the T matrix (Figs. 1 and 2).

5 D13 resonances

In the D13 partial wave the model yields a consistent picture for the upper resonance but fails to reproduce the behaviour of the scattering amplitudes for the lower resonance. In the latter case, the quark-model values for the d-wave πN vertex and the s-wave $\pi\Delta$ vertex are of comparable strength and relatively weak. Dressing the verteces and introducing the mixing of the two (bare) resonances considerably enhances the verteces. However, the enhancement is stronger in the case of the $\pi\Delta$ channel and, as a consequence, the resonance disappears in the elastic channel. A reasonable agreement is obtained if the quark-model strength of the $\pi\Delta$ is multiplied by 0.3 (Table 2 and Fig. 3).

Resonance	Γ_{tot} [MeV]	Γ_i/Γ_{tot}			
		πN	$\pi\Delta$ (s-wave)	$\pi\Delta$ (d-wave)	ρN (s-wave)
N(1520)	64	0.56	0.40	0.00	0.03
PDG	100 – 125	0.55 – 0.65	0.15	0.11	0.09
N(1700)	55	0.02	0.10	0.70	0.18
PDG	50 – 150	0.05 – 0.15	0.11	0.79	0.07

Table 2. The total and the partial widths for the N(1520) and the N(1700) resonance at the K-matrix pole (1515 MeV and 1700 MeV, respectively) using the unmodified quark-model values for the quark-meson couplings except for the s-wave $\pi\Delta$ coupling which is taken with only 30 % of the model strength. The PDG values are from [3].

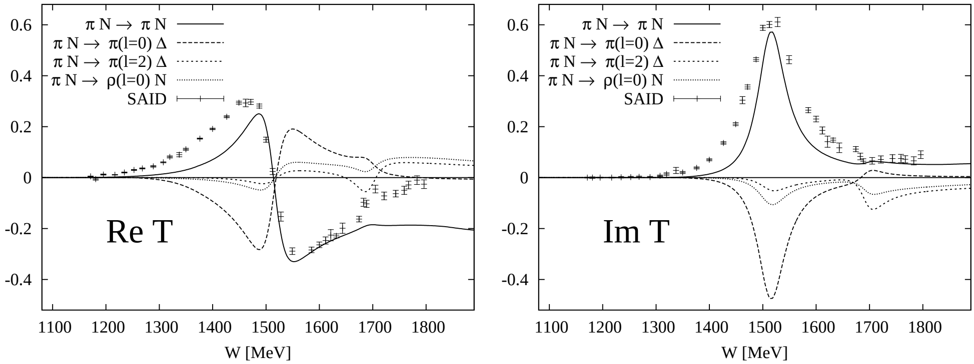

Fig. 3. The real and the imaginary part of the elastic and the dominant inelastic scattering amplitudes for the D13 wave. The data points for the elastic channel are from [4]

For the upper resonance (N(1700)), the model predicts the dominance of the d-wave $\pi\Delta$ channel in agreement with the phenomenological analysis.

6 Concluding remarks

The model reasonably well reproduces the behaviour of the amplitude in the S11 partial wave. The bare quark values for the meson verteces are generally too weak but are considerably enhanced through the meson cloud effects and the mixing of the bare quark resonances. At higher energies around and above $\mathrm{N}(1650)$, the model amplitudes are too small; here the contribution of higher resonances not included in our model becomes important as suggested by the phenomenological analysis [9]. The background contribution is in our model generated by the u-channel processes. We have not considered the Weinberg-Tomazawa term which would enhances the behaviour of the amplitudes near the pion threshold.

The situation is less favourable for the D13 partial wave. The model fails to reproduce a rather intriguing behaviour of the s and d-wave $\pi\Delta$ amplitudes in the energy region of the N(1520) resonance. Here the phenomenological analysis suggests that they are comparable in strength which is difficult to explain in a model calculation where at relatively low pion momenta the $l = 0$-wave coupling is strongly favoured over the $l = 2$ one.

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