

# Hamilton cycles in primitive graphs of order $2rs^*$

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## Abstract

After long term efforts, it was recently proved by Du, Kutnar and Marušič in 2021 that except for the Petersen graph, every connected vertex-transitive graph of order  $rs$  has a Hamilton cycle, where  $r$  and  $s$  are primes. A natural topic is to solve the hamiltonian problem for connected vertex-transitive graphs of  $2rs$ . This topic is quite nontrivial, as the problem is still unsolved even for that of  $r = 3$  and  $5$ . In this paper, it is shown that except for the Coxeter graph, every connected vertex-transitive graph of order  $2rs$  contains a Hamilton cycle, provided the automorphism group acts primitively on vertices.

*Keywords:* Vertex-transitive graph, Hamilton cycle, primitive group, automorphism group, orbital graph.

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# Hamiltonski cikli v primitivnih grafih reda $2rs^*$

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## Povzetek

Po dolgotrajnih prizadevanjih so Du, Kutnar in Marušič nedavno, leta 2021, dokazali, da ima vsak povezan točkovno tranzitiven graf reda  $rs$ , razen Petersenovega grafa, hamiltonski cikel, kjer sta  $r$  in  $s$  praštevilici. Ob tem se naravno zastavlja vprašanje reševanja Hamiltonovega problema za povezane točkovno tranzitivne grafe reda  $2rs$ . To vprašanje je precej zahtevno, saj problem še vedno ni rešen niti za  $r = 3$  in  $5$ . V tem članku pokažemo, da, razen za Coxeterjev graf, vsak povezan vozliščno-tranzitiven graf reda  $2rs$  vsebuje hamiltonski cikel, pri pogoju, da grupa avtomorfizmov na točkah grafa deluje primitivno.

*Ključne besede:* Točkovno tranzitiven graf, hamiltonski cikel, primitivna grupa, grupa avtomorfizmov, orbitalni graf.

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