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Dynamics of P11 and P33 resonances in quark models with chiral mesons*

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Abstract. We apply the coupled channel formalism for the K-matrix to the calculation of the P11 and P33 scattering amplitudes in the region the N(1440) and $\Delta(1600)$ resonances.

1 Introduction

In this work [1] we extend the coupled channel formalism for the K matrix derived in [2] using the static approximation to take into account the correct relativistic kinematics of the meson-baryon system. We apply this method that has been used in [2] to explain the peculiar behaviour of the scattering amplitudes in the energy range of the Roper resonance to the calculation of the $\Delta(1600)$ resonance, the Roper's counterpart in the P33 partial wave.

In quark models these two resonances are assumed to have a similar spatial structure, this similarity is however not reflected in the scattering amplitudes. While in the P11 partial wave the phase shift reaches 90 degrees around $W \sim 1500$ MeV the phase shift in the P33 case shows no sign of resonanant behaviour in the energy range of $W \sim 1600$ and above, which is a strong signal of the important role of inelastic channels. This is further supported by the unusual behaviour of the inelasticity which in the P11 case rapidly rises from zero to unity and remains close to this value in a broad energy region, while in the P33 case it rises rather slowly and reaches the unitarity limit only at much higher energies.

We show that these features can be explained by assuming that in this energy range the inelastic channels are dominated by the two and three-pion decays proceeding mainly through two channels: (i) in the $\pi\Delta$ channel the resonance first decays into the pion and the Δ isobar of invariant mass M, $M_N + m_\pi < M < W - m_\pi$, and (ii) the σ channel in which the resonance first decays into the σ-meson mimicking two pions in the relative s-state and either the nucleon (in the P11 case) or the Δ isobar (in the P33 case).

* Talk delivered by B. Golli

2 Coupled-channel K-matrix formalism

We consider a class of chiral quark models in which mesons (the pion and the sigma meson in our case) couple linearly to the quark core:

$$H_{\text{meson}} = \int dk \sum_{lmt} \left\{ \omega_k\, a^\dagger_{lmt}(k) a_{lmt}(k) + \left[V_{lmt}(k) a_{lmt}(k) + V^\dagger_{lmt}(k)\, a^\dagger_{lmt}(k) \right] \right\},$$

where $a^\dagger_{lmt}(k)$ is the creation operator for a meson with angular momentum l and the third components of spin m and isospin t. In the case of the pion, we include only $l = 1$ pions, and

$$V_{mt}(k) = -v(k) \sum_{i=1}^{3} \sigma^i_m \tau^i_t \tag{1}$$

is the general form of the pion source, with the quark operator, $v(k)$, depending on the model. It includes also the possibility that the quarks change their radial function which is specified by the reduced matrix elements $V_{BB'} = \langle B \| V(k) \| B' \rangle$, where B are the bare baryon states (e.g. the bare nucleon, Δ, Roper, ...) In the case of the σ mesons we assume only $l = 0$ mesons, coupled to the quark core with

$$\widetilde{V}^\mu(k) = G_\sigma \frac{k}{\sqrt{2\omega_{\mu k}}} w_\sigma(\mu)\,, \qquad w_\sigma(\mu)^2 \approx \frac{1}{\pi} \frac{\frac{1}{2}\Gamma_\sigma}{(\mu - m_\sigma)^2 + \frac{1}{4}\Gamma_\sigma^2}\,. \tag{2}$$

Here $\omega^2_{\mu k} = k^2 + \mu^2$ and $w_\sigma(\mu)$ is the mass distribution function modeling the resonant decay into two pions. In this work we take the values consistent with the recent analysis of Leutwyler [5], $m_\sigma = 450$ MeV and $\Gamma_\sigma = 550$ MeV. The strength parameter G_σ in (2) is a free parameter of the model.

Chew and Low [4] have shown that in such models it is possible to find the exact expression for the T matrix (and consequently for the K matrix) without explicitly specifying the form of asymptotic states. In the basis with good total angular momentum J and isospin T, in which the K and T matrices are diagonal, it is possible to express the K matrix for the elastic channel in the form:

$$K^{JT}_{\pi N \pi N}(k, W) = -\pi \mathcal{N}_N \langle \Psi^N_{JT}(W) \| V(k) \| \Psi_N \rangle\,,$$

where W is the invariant mass of the meson-baryon system. In the inelastic channels we find

$$K^{JT}_{\pi\Delta\pi N}(k, W, M) = -\pi \mathcal{N}_\Delta \langle \Psi^N_{JT}(W) \| V(k) \| \widetilde{\Psi}_\Delta(M) \rangle\,,$$
$$K^{JT}_{\pi\Delta\pi\Delta}(k, W, M', M) = -\pi \mathcal{N}_\Delta \langle \Psi^\Delta_{JT}(W, M) \| V(k) \| \widetilde{\Psi}_\Delta(M') \rangle\,,$$

where $\widetilde{\Psi}_\Delta(M)$ is the intermediate Δ state with invariant mass M normalized as $\langle \widetilde{\Psi}_\Delta(M') | \widetilde{\Psi}_\Delta(M) \rangle = \delta(M - M')$. The matrix elements of the K matrix involving the σN channel in the P11 case read

$$K^{\frac{1}{2}\frac{1}{2}}_{\sigma N}(k, W, \mu) = -\pi \mathcal{N}_{\sigma N} \langle \Psi^N_{\frac{1}{2}\frac{1}{2}}(W) | \widetilde{V}^\mu(k) | \Psi_N \rangle\,,$$
$$K^{\frac{1}{2}\frac{1}{2}}_{\sigma \Delta}(k, W, \mu, M) = -\pi \mathcal{N}_{\sigma N} \langle \Psi^\Delta_{\frac{1}{2}\frac{1}{2}}(W, M) | \widetilde{V}^\mu(k) | \Psi_N \rangle\,,$$
$$K^{\frac{1}{2}\frac{1}{2}}_{\sigma \sigma}(k, W, \mu, \mu') = -\pi \mathcal{N}_{\sigma N} \langle \Psi^\sigma_{\frac{1}{2}\frac{1}{2}}(W, \mu') | \widetilde{V}^\mu(k) | \Psi_N \rangle\,,$$

and those involving the $\sigma\Delta$ channel in the P33 case:

$$K^{\frac{3}{2}\frac{3}{2}}_{N\sigma}(k,W,M,\mu) = -\pi\mathcal{N}_{\sigma\Delta}\,\langle\Psi^{N}_{\frac{3}{2}\frac{3}{2}}(W)|\widetilde{V}^{\mu}(k)|\widetilde{\Psi}_{\Delta}(M)\rangle\,,$$
$$K^{\frac{3}{2}\frac{3}{2}}_{\Delta\sigma}(k,W,\mu,M,M') = -\pi\mathcal{N}_{\sigma\Delta}\,\langle\Psi^{\Delta}_{\frac{3}{2}\frac{3}{2}}(W,M')|\widetilde{V}^{\mu}(k)|\widetilde{\Psi}_{\Delta}(M)\rangle\,,$$
$$K^{\frac{3}{2}\frac{3}{2}}_{\sigma\sigma}(k,W,\mu,M,\mu',M') = -\pi\mathcal{N}_{\sigma\Delta}\,\langle\Psi^{\sigma}_{\frac{3}{2}\frac{3}{2}}(W,\mu',M')|\widetilde{V}^{\mu}(k)|\widetilde{\Psi}_{\Delta}(M)\rangle\,,$$

where $\mathcal{N}_B = \sqrt{\omega E_B/kW}$, $\mathcal{N}_{\sigma B} = \sqrt{\omega_\mu E_B/k_\mu W}$, ω is the energy of the scattering pion, $k = \sqrt{\omega^2 - m_\pi^2}$, and ω_μ is the energy of the scattering σ-meson of invariant mass μ, $k_\mu = \sqrt{\omega^2-\mu^2}$. Here Ψ^H_{JT} is the principal value state for which we use the following ansatz that takes into account the proper relativistic kinematics and replaces the similar expression in [2] derived in the static (no-recoil) approximation:

$$\begin{aligned}|\Psi^H_{JT}(W,m_H)\rangle = \mathcal{N}_H\Bigg\{&\sum_B c^H_B(W,m_H)|\Phi_B\rangle + [a^\dagger(k_H)|\widetilde{\Psi}_H\rangle]^{JT}\\
&+\int dk\,\frac{\chi^{NH}_{JT}(k,W,m_H)}{\omega_k+E_N(k)-W}\,[a^\dagger(k)|\Psi_N(k)\rangle]^{JT}\\
&+\int dM\int dk\,\frac{\chi^{\Delta H}_{JT}(k,W,M,m_H)}{\omega_k+E(k)-W}\,[a^\dagger(k)|\widetilde{\Psi}_\Delta(M)\rangle]^{JT}\\
&+\int d\mu\int dk\,\frac{\chi^{\sigma H}_{JT}(k,W,\mu,m_H)}{\omega_{\mu k}+E(k)-W}\,b^\dagger(k)|\widetilde{\Psi}_{JT}\rangle\Bigg\}\,. \end{aligned} \qquad (3)$$

Here H stands for either the πN, $\pi\Delta$, σN or the $\sigma\Delta$ channel, m_H is the invariant mass of the corresponding intermediate hadron in the inelastic channels, $E(k)$ is the energy of the recoiled baryon (nucleon or Δ). The first term consists of the sum over *bare* tree-quark states Φ_B, involving different excitations of the quark core, the next term corresponds to the free meson (pion or σ-meson) and the baryon (N or Δ) and defines the channel, the next two terms represent the pion cloud around the nucleon and the Δ isobar, respectively, and the last term the σ-meson cloud around the nucleon (for $JT = \frac{1}{2}\frac{1}{2}$) or the Δ (for $JT = \frac{3}{2}\frac{3}{2}$), here $b^\dagger$ is the creation operator for the σ-meson.

The on-shell meson amplitudes $\chi^{H'H}_{JT}$, describing the corresponding meson clouds around the nucleon and the Δ are proportional to corresponding matrix elements of the on-shell K matrix

$$K_{H'H} = \pi\mathcal{N}_{H'}\mathcal{N}_H\,\chi^{H'H}_{JT}(k_{H'},k_H)\,.$$

From the variational principle for the K matrix it is possible to derive the integral equation for the amplitudes which is equivalent to the Lippmann-Schwinger equation for the K matrix.

Using a simplified ansatz for the principal value states in which the terms involving the integrals are neglected amounts to taking only the non-homogeneous part of the corresponding Lippmann-Schwinger equation. Such an approximation is widely used in phenomenological analysis of scattering amplitudes and is known as the Born approximation for the K matrix.

The T matrix is calculated from the K matrix through the Heitler equation: $T = -K + iKT$.

3 Results for the scattering amplitudes in the Cloudy Bag Model

We illustrate the method by calculating scattering amplitudes for the P11 and the P33 partial waves. Though the expressions derived in the previous sections are general and can be applied to any model in which mesons linearly couple to the quark core, we choose here the Cloudy Bag Model, primarily because of its simplicity. In this model, the matrix element of the pion source (1) between the model 3-quark states can be written as

$$\langle \Phi_{B'} \| V(k) \| \Phi_B \rangle = r_q \, v(k) \, \langle J_{B'}, T_{B'} = J_{B'} \| \sum_{i=1}^{3} \sigma_m^i \tau_t^i \| J_B, T_B = J_B \rangle \,,$$

where

$$v(k) = \frac{1}{2f} \frac{k^2}{\sqrt{12\pi^2 \omega_k}} \frac{\omega_{MIT}}{\omega_{MIT} - 1} \frac{j_1(kR_{bag})}{kR_{bag}}$$

and

$$r_q = \begin{cases} 1 & \text{for} \quad B = B' = (1s)^3 \text{ configuration} \\ r_\omega = \left[\dfrac{\omega^1_{MIT}(\omega^0_{MIT}-1)}{\omega^0_{MIT}(\omega^1_{MIT}-1)} \right]^{1/2} = 0.457 & \text{for} \quad B = (1s)^3, B' = (1s)^2(2s)^1 \\ \frac{2}{3} + r_\omega^2 & \text{for} \quad B = B' = (1s)^2(2s)^1 \end{cases} .$$

In this work we use $R_{bag} = 0.9$ fm, $f = 76$ MeV yielding the correct value for the πNN coupling constant. Similar results are obtained for $0.85 \text{ fm} < R_{bag} < 1.0$ fm. In addition, the energies of the 3-quark states in different excited states are also taken as free parameters.

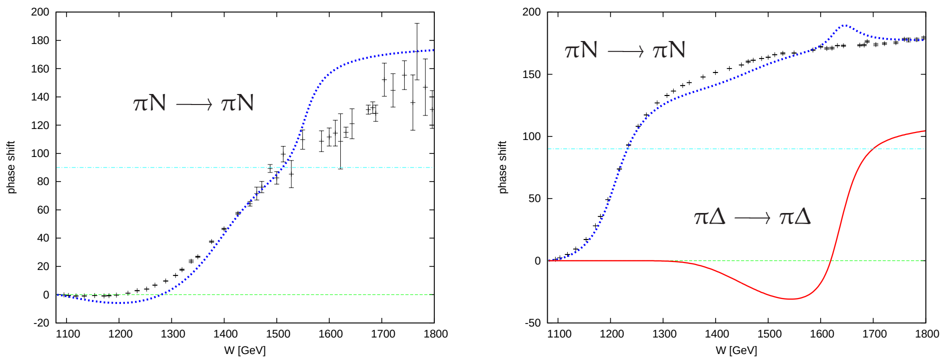

Fig. 1. The P11 (left panel) and P33 (right panel) phase shifts. The corresponding thin line in the P33 case represent the phase shift for the pion scattering on the Δ. The data points in this and subsequent figures are from the SAID $\pi N \to \pi N$ partial-wave analysis [6] The model parameters are $M_R = 1510$ MeV, $M_\Delta = 1232$ MeV, $M_{\Delta^*} = 1700$ MeV

The results in the simplest approximation, the Born approximation for the K matrix without background, are displayed in Figure 1. This approximation is equivalent to keeping only the the first term in the ansatz (3). By increasing the $\pi N\Delta$ interaction strength by 60 % and the πNRoper by 80 % with respect to the above model values, the widths of the N(1440) and $\Delta(1232)$ are reproduced. This simplified approach explains why the resonant behaviour of the phase shift is not observed for the $\Delta(1600)$ ($\equiv \Delta^*$) in the elastic channel: in this energy region, the matrix element $\pi\Delta\Delta^*$ becomes stronger than $\pi N\Delta^*$ in which case the resonance disappears in the $\pi N \to \pi N$ but appears is the non-observable $\pi\Delta \to \pi\Delta$ channel.

Including the background in the the Born approximation for the K matrix through the term $[a^\dagger(k_H)|\widetilde{\Psi}_H(M)\rangle]^{JT}$ in (3) we obtain almost perfect agreement of the calculated scattering amplitudes compared to the amplitudes extracted in the partial-wave analysis but still at the expense of considerably larger $\pi N\Delta$ and πNRoper interaction strength compared to those predicted by the quark model. This inconsistency is resolved when solving the Lippmann-Schwinger equation for the pion amplitudes; it turns out that for our particular choice of the bag radius we are able to reproduce the experimental scattering amplitudes starting from the bare values as predicted by the Cloudy Bag Model (see Fig. 2).

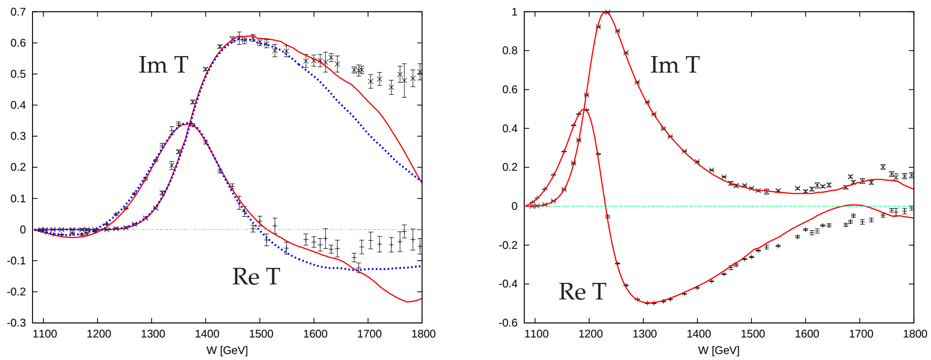

Fig. 2. The real and the imaginary parts of the T matrix for the P11 (left panel) and P33 (right panel) partial waves. The dashed/full curves in the left panel show the effect of omitting/including the $N(1710)$ state in the sum over B in (3)

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