

The Sierpiński product of graphs*

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***Dedicated to Professor Wilfried Imrich on
the occasion of his 80th birthday.***

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Abstract

In this paper we introduce a product-like operation that generalizes the construction of the generalized Sierpiński graphs. Let G, H be graphs and let $f: V(G) \rightarrow V(H)$ be a function. Then the *Sierpiński product of graphs G and H with respect to f* , denoted by $G \otimes_f H$, is defined as the graph on the vertex set $V(G) \times V(H)$, consisting of $|V(G)|$

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copies of H ; for every edge $\{g, g'\}$ of G there is an edge between copies gH and $g'H$ of form $\{(g, f(g')), (g', f(g))\}$.

Some basic properties of the Sierpiński product are presented. In particular, we show that the graph $G \otimes_f H$ is connected if and only if both graphs G and H are connected and we present some conditions that G, H must fulfill for $G \otimes_f H$ to be planar. As for symmetry properties, we show which automorphisms of G and H extend to automorphisms of $G \otimes_f H$. In several cases we can also describe the whole automorphism group of the graph $G \otimes_f H$.

Finally, we show how to extend the Sierpiński product to multiple factors in a natural way. By applying this operation n times to the same graph we obtain an alternative approach to the well-known n -th generalized Sierpiński graph.

Keywords: Sierpiński graphs, graph products, connectivity, planarity, symmetry.

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Sierpińskijev produkt grafov*

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Povzetek

V tem prispevku vpeljemo operacijo, podobno produktu, ki posplošuje konstrukcijo posplošenih grafov Sierpińskega. Naj bosta G, H grafa in naj bo $f: V(G) \rightarrow V(H)$ funkcija. *Sierpińskijev produkt grafov* G in H glede na f , oznaka $G \otimes_f H$, je definiran kot graf na množici vozlišč $V(G) \times V(H)$, sestavljen iz $|V(G)|$ kopij H ; za vsako povezavo $\{g, g'\}$ grafa G obstaja povezava med kopijama gH in $g'H$ oblike $\{(g, f(g')), (g', f(g))\}$.

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Predstavljene so nekatere osnovne lastnosti Sierpińskijevega produkta. Pokažemo, da je graf $G \otimes_f H$ povezan, če in samo če sta oba grafa G in H povezana, in predstavimo pogoje, ki jih morata izpolnjevati G in H , da je graf $G \otimes_f H$ ravninski. V zvezi s simetrijskimi lastnostmi pokažemo, kateri avtomorfizmi grafov G in H se dajo razširiti na avtomorfizme $G \otimes_f H$. V več primerih lahko opišemo tudi celotno grupo avtomorfizmov grafa $G \otimes_f H$.

Na koncu pokažemo, kako Sierpińskijev produkt naravno razširimo na več faktorjev. Uporaba te operacije n -krat na istem grafu predstavlja alternativni pristop k dobro znanemu n -temu posplošenemu grafu Sierpińskega.

Ključne besede: Grafi Sierpińskega, produkti grafov, povezanost, ravninskost, simetrija.

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