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IZBOLJŠAVA OBRAZCEV ZA RAČUN HIDRAVLIČNIH TRENJSKIH IZGUB ZA TOK POD TLAKOM V CEVEH KROŽNEGA PREREZA
IMPROVEMENT OF THE HYDRAULIC FRICTION LOSSES EQUATIONS FOR FLOW UNDER PRESSURE IN CIRCULAR PIPES

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Po poskusih Nikuradzeja in konstrukciji Moodyjevega diagrama je izraz za hidravlične izgube v cevovodih dobil svojo dokončno obliko v Colebrook-Whiteovi formuli. Treba se je zavedati, da je ta formula sestavljena le za območje razvite turbulence, a se uporablja tudi za prehodno in laminarno območje. Opravljene primerjave med karakterističnimi izrazi koeficienta trenja ter Nikuradzejevimi meritvami potrjujejo potrebo po reviziji formule pri nižjih Reynoldsovih številih (Re < 4000). Revizija izraza za hidravlične izgube v cevovodih je še toliko bolj potrebna, ko imamo opravka z nestalnim tokom. V prispevku je prikazana izdelava novega izraza za določitev koeficienta trenja, ki združuje teoretično ozadje in empiriko. S tem izrazom določeni koeficienti trenja se skoraj popolnoma prilegajo izmerjenim in vsekakor bistveno bolje kot koeficienti, določeni s klasičnimi (že znanimi) enačbami. Omenjeni izraz postavlja temelje potrebnim poskusom, ki bodo omogočili nadaljnji razvoj enačbe za nestacionarne razmere toka.
Ključne besede: *hidravlične izgube, koeficient trenja, relativna hrapavost, optimizacija parametrov, cevovod, stalni tok, Colebrook-White, Nikuradze, Moody*

Following the experiments of Nikuradse and construction of the Moody diagram the term for hydraulic losses in pipes got its final form in the Colebrook-White formula. Despite the fact that the formula was developed only for the fully turbulent flow it is used also for the transition and laminar flow. Comparisons between the characteristic formulas for the friction factor and Nikuradse's experimental results show the need to revise this formula for low Reynolds numbers (Re < 4000). Indeed, the revision is even more necessary when one deals with unsteady flow. In this paper, a construction of a new formula for the friction factor, which successfully combines theoretical background and empirical knowledge, is shown. This formula gives better fit to the measured friction factor than the results of classical (known) formulas. The formula sets the basis for further experiments, which will improve the friction factor formula in unsteady flow conditions.
Key words: *hydraulic losses, friction factor, relative roughness, parameter optimization, pipes, steady flow, Colebrook-White, Nikuradse, Moody*

1. UVOD

Eden od odprtih problemov hidravlike je določitev ustreznejšega izraza za opis hidravličnih trenjskih izgub v cevovodih. Analitično je mogoče izpeljati izraza trenjske funkcije za laminarni tok in za popolnoma razvito turbulenco. Za prehodno območje pa to še ni bilo storjeno. Pač pa sta bila oba izraza verjetno mehanično združena v enega

1. INTRODUCTION

One of the unsolved problems in hydraulics is the definition of a more suitable term for determination of hydraulic friction losses in pipelines. Analytically, it is possible to determine the terms of the friction function for laminar flow and for a fully developed turbulent flow. For the transition zone, this has not been done yet. However, the two formulas

(Colebrook-White, 1939), in to precej »ponesrečeno«, saj bi namesto vsote pod logaritmom morali na nek način sešteti dva logaritma. Zaradi narave logaritmov pa je rezultat pravzaprav kar »posrečen«. Tak inženirski »zmazek« se še danes uporablja kot najboljši izraz za določanje hidravličnih izgub oz. koeficienta trenja λ v ceveh pri stalnem toku (Kompare, 1996). Naj pojasnimo, da je koeficient trenja λ dejansko funkcija in ne koeficient (konstanta), kot se je udomačil izraz v hidrotehnični stroki. Zaradi konsistentnosti izrazoslovja bomo ta izraz uporabljali tudi v nadaljevanju.

J. Nikuradze je leta 1933 objavil dosežene rezultate svojih raziskav, ki so bile izvedene leta 1924, o koeficientu trenja λ v ceveh, kjer nastopa enakomerna hrapavost. Eksperimenti, ki so bili izvedeni na krožnih ceveh z enakomerno, umetno prirejeno hrapavostjo za tok pod tlakom, naj bi omogočili določitev zveze med koeficientom trenja λ in Reynoldsovim številom Re. Meritve so potekale na območju Reynoldsovega števila $500 \le Re \le 10^6$ za šest različnih relativnih hrapavosti. Pri tem je relativna hrapavost ε/D definirana s srednjo višino hrapavosti ε, v odvisnosti od premera D (Petrešin, 1987, cit. po Nikuradze, 1933). Na sliki 1 so prikazani rezultati koeficienta trenja (f = λ) v odvisnosti od Reynoldsovega števila (N_R = Re). Nikuradze je enakomerno hrapavost ustvaril z lepljenjem mešanice laka in zrn peska na ostenje cevi konstantnega prereza. Velikost zrn peska, s premerom ϕ = 0,8 mm, je bila zato lahko vzeta kot velikost absolutne hrapavosti ε.

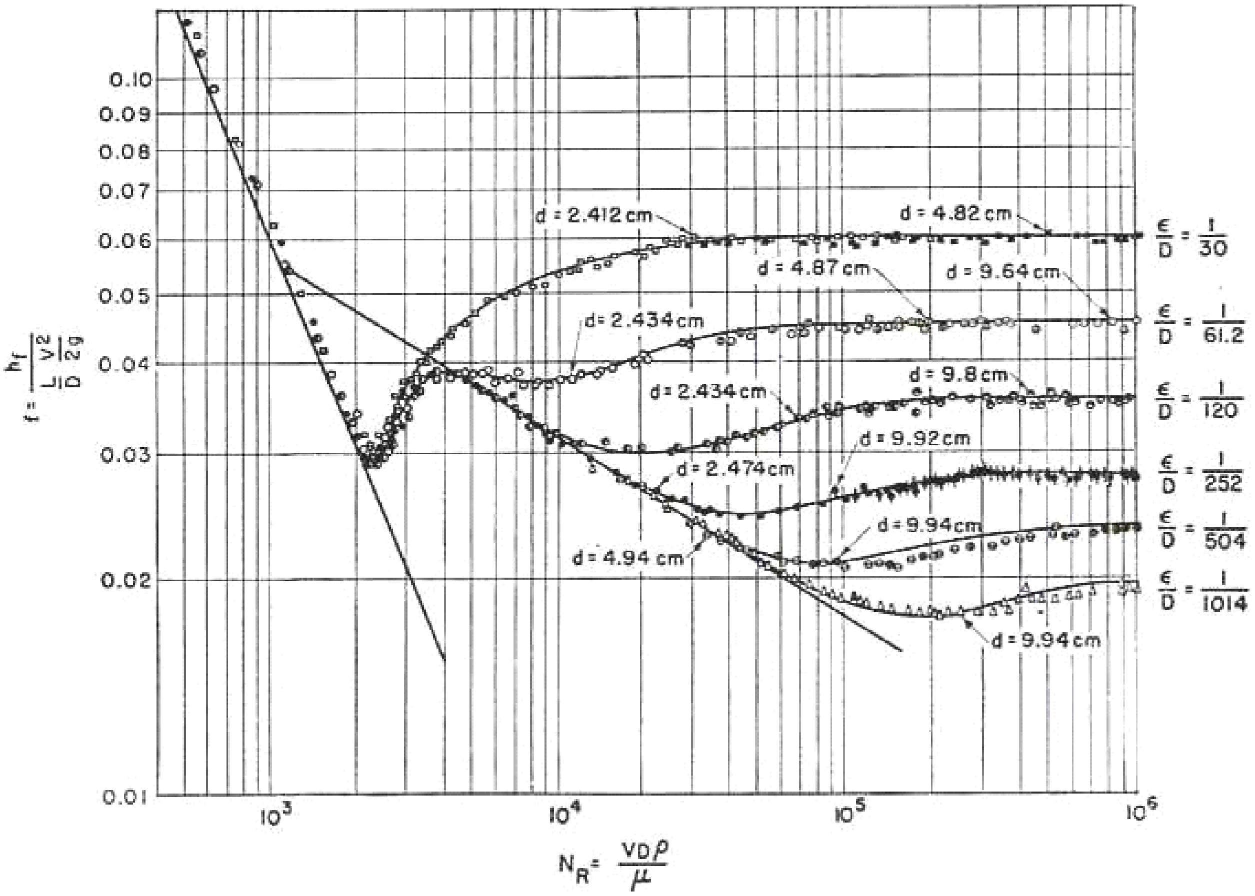
C.F. Colebrook in C.M. White sta pri svojih raziskavah upoštevala rezultate svojih predhodnikov. Delno sta jih tudi sama testirala in praktično upoštevala kot svoja izhodišča. Na podlagi eksperimentov, ki sta jih izvajala, sta Colebrook in White leta 1939 podala enačbo, ki velja tudi za prehodno območje, in sicer za vrednosti Reynoldsovih števil $4 \cdot 10^3 < Re < 10^8$ (Petrešin, 1987, cit. po Colebrooke, White, 1939):

were probably mechanically combined into one term (Colebrooke-White, 1939), which was done in a pretty "unsuccessful" way. Instead of the logarithm of the sum of the arguments a sum of logarithms should be used. Because of the nature of a logarithmic function the results have in fact proven "successful". Such an engineering "scrawl" is still being used as the best term to determine the hydraulic friction loss, respectively the friction factor λ in the pipelines at steady flow conditions (Kompare, 1996). Let us state that the friction factor λ is actually a function and not a constant, as it is usually taken in hydraulics. Because of the consistency in terminology this term will be used through this paper.

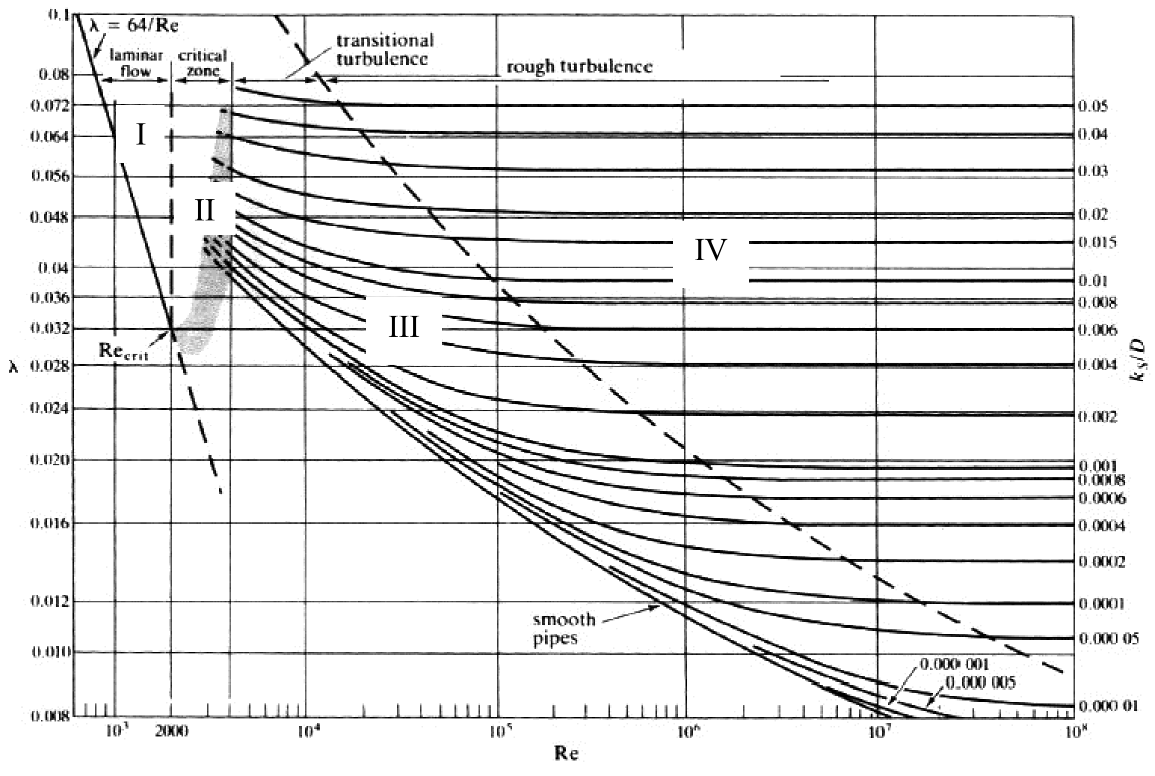
In the 1933 J. Nikuradse published the results of his research about the friction factor λ in pipes with uniform roughness. Experiments, which were carried out on circular pipes with artificially made uniform roughness for flow under pressure, should have made it posible to define the relation between the fricton factor λ and the Reynolds number Re. Mesurements were made in a range of Reynolds number of $500 \le Re \le 10^6$ for six different relative roughnesses. Relative roughness ε/D was defined with the medium height of roughness ε dependent on pipe diameter D (Petrešin, 1987, cit. Nikuradse, 1932). In Figure 1 the results of the friction factor in dependency of the Reynolds number are shown. Nikuradse created the uniform roughness by sticking grain of sand on the pipes wall of a constant cross section. The height of a grain of sand, with a diameter of ϕ = 0.8 mm, defined the height of absolute roughness ε.

In their research, C. F. Colebrook and C.M. White considered the results of their predecessors. They partly tested them and practically took them in consideration as their starting point. Based on their experiments, Colebrook and White published in 1939 the equation which is also valid in the transitional zone of flow but limited only to the values of the Reynolds number between $4 \cdot 10^3 < Re < 10^8$ (Petrešin, 1987, cit. Colebrook, White, 1939):

$$\frac{1}{\sqrt{\lambda}} = -2{,}0 \cdot \log\left(\frac{k}{3{,}7D} + \frac{2{,}51}{\mathrm{Re}\sqrt{\lambda}}\right) \tag{1}$$



Slika 1. Nikuradzejevi eksperimentalni rezultati za cevovod z umetno hrapavostjo (Streeter, 1951).
Figure 1. Nikuradse's experimental results for pipelines with uniform roughness (Streeter, 1951).



Slika 2. Moodyjev diagram (Streeter, 1951).
Figure 2. Moody's diagram (Streeter, 1951).

Za praktično uporabo Colebrook-Whiteove enačbe in hitrejšo oceno vrednosti koeficienta trenja je Lewis F. Moody leta 1944 izdelal diagram, ki je prikazan na sliki 2. Diagram je znan kot Moodyjev diagram.

Če zanemarimo čisto empirične in le delno konceptualne enačbe (npr. Manning v Evropi, Hazen-Williams v Ameriki), je od konceptualnih enačb še vedno najbolj v uporabi Colebrook-Whiteova enačba ter Moodyjev diagram, kjer ne smemo spregledati omejitev na intervalu Reynoldsovih števil $4000 \leq Re \leq 10^8$, ki jih sicer lahko dobimo v objavljeni literaturi, a so napačne. Zgornja meja omejitve ni problematična, težave nastopajo z omejitvijo na spodnji meji (Re = 4000). Kot se vidi iz diagrama na sliki 3, kjer je prikazana primerjava Colebrook-Whiteove enačbe z Nikuradzejevimi meritvami, je vrednost spodnje meje bistveno višja in se tako območje uporabe (vseh) enačb zoži zgolj na zelo visoka Reynoldsova števila. Celebrook-Whiteovemu izrazu lahko pripišemo tudi nekoliko daljšo pot do končnega rezultata, predvsem zaradi postopka iteracij, ki ga ta izraz zahteva za rešitev.

Poleg do sedaj omenjenih avtorjev so se z enačbami, ki opisujejo trenje v toku pod tlakom v prehodnem režimu, ukvarjali še številni drugi avtorji (Barr, Haaland, Swamee …). Barr je na primer nadomestil Colebrook-Whiteovo enačbo s svojim izrazom z natančnostjo ± 1 %. Območje veljave enačbe je podobno enačbi Colebrook-Whitea, in sicer za vrednosti Reynoldsovega števila $4 \cdot 10^3 < Re < 10^8$ (Barr, 1975). P. K. Swamee in A. K. Jain sta podala svojo enačbo, ki je omejena na Reynoldsova števila vrednosti $5 \cdot 10^3 < Re < 10^8$ ter relativno hrapavost $10^{-6} < \varepsilon/D < 10^{-2}$ (Swamee, 1976). Enačbe ostalih avtorjev in primerjave med njimi so obravnavane v diplomski nalogi z naslovom Izboljšava obrazcev za račun hidravličnih izgub v stalnem enakomernem toku (Uršič, 2003).

Pri hidravličnem preračunu vodovodnega sistema, še posebej pri programih, ki upoštevajo Colebrook-Whiteov izraz za koeficient trenja, se ta redno uporablja tudi

For practical use of the Colebrook-White equation and also for faster estimation of the value of the friction factor, Lewis F. Moody constructed in 1944 a diagram, which is shown on Figure 2. This diagram is known as the Moody Diagram.

If empirical and partly conceptual terms (Manning in Europe, Hazen-Williams in America) are disregarded, the most used conceptual terms are still the Colebrook-White equation and Moody's diagram. In both cases restrictions for the equation/diagram for the use between the Reynolds numbers $4000 \leq Re \leq 10^8$ can be found in the literature. But these bounds are wrong. The upper bound is not questionable, the problem is the reduction on the lower boundary condition (Re = 4000). As it can be seen from the diagram on Figure 3, where the comparisons between the Colebrook-White formula and Nikuradse’s results are shown, the value of the lower boundary condition is essentially higher. As a result the range of the use of (all) the equation/diagram is restricted just to the high values of Reynolds numbers. The Colebrook-White formula has also a longer way in getting the result, mainly because of the iteration procedure.

Beside the mentioned authors, who worked on the friction factor term for the transition flow, several others investigated the topic (Barr, Haaland, Swamee …). For example, Barr has replaced the Colebrook-White equation with his own and achieved an accuracy of ± 1%. The validity region of the equation is similar to that of the Colebrook-White equation and it is supposed to be used only for the values of the Reynolds number between $4 \cdot 10^3 < Re < 10^8$ (Barr, 1975). P. K. Swamee and A. K. Jain have given their own equation, which is limited to the values of the Reynolds numbers between $5 \cdot 10^3 < Re < 10^8$ and the relative roughness of $10^{-6} < \varepsilon/D < 10^{-2}$ (Swamee, 1976). The equations of other authors and the comparison between them are discussed in the graduation thesis entitled Improvement of the hydraulic losses terms in steady flow conditions (Uršič, 2003).

Regularly, when calculating the hydraulic losses in the pipelines, especially when dealing with computer programs, which use the zunaj definiranega območja. Do tega prihaja še posebej na območju vodovodnega omrežja, kjer so prisotne tehnološko pogojene majhne hitrosti pretokov. Poleg tega imajo vse funkcije, ki imajo logaritmično obliko, singularno točko, ko je vrednost argumenta logaritma enaka 1. To velja za Reynoldsova števila $5 < Re < 10$ (argument algoritma je v Colebrook-Whiteovi enačbi odvisen tudi od relativne hrapavosti, zato se pojavi singularna točka na intervalu in ne za samo za določeno vrednost Reynoldsovega števila). Če primerjamo vrednosti koeficienta trenja, dobljene po Colebrook-Whiteu ter ostalih avtorjih, in vrednosti s hiperbolično obliko $\lambda = 64/Re$, vidimo, da so dobljene vrednosti koeficienta trenja na intervalu $100 \leq Re \leq 2100$ nerealne (slika 3). Manjše oziroma večje vrednosti koeficienta trenja povzročajo nerealne vrednosti pretokov.

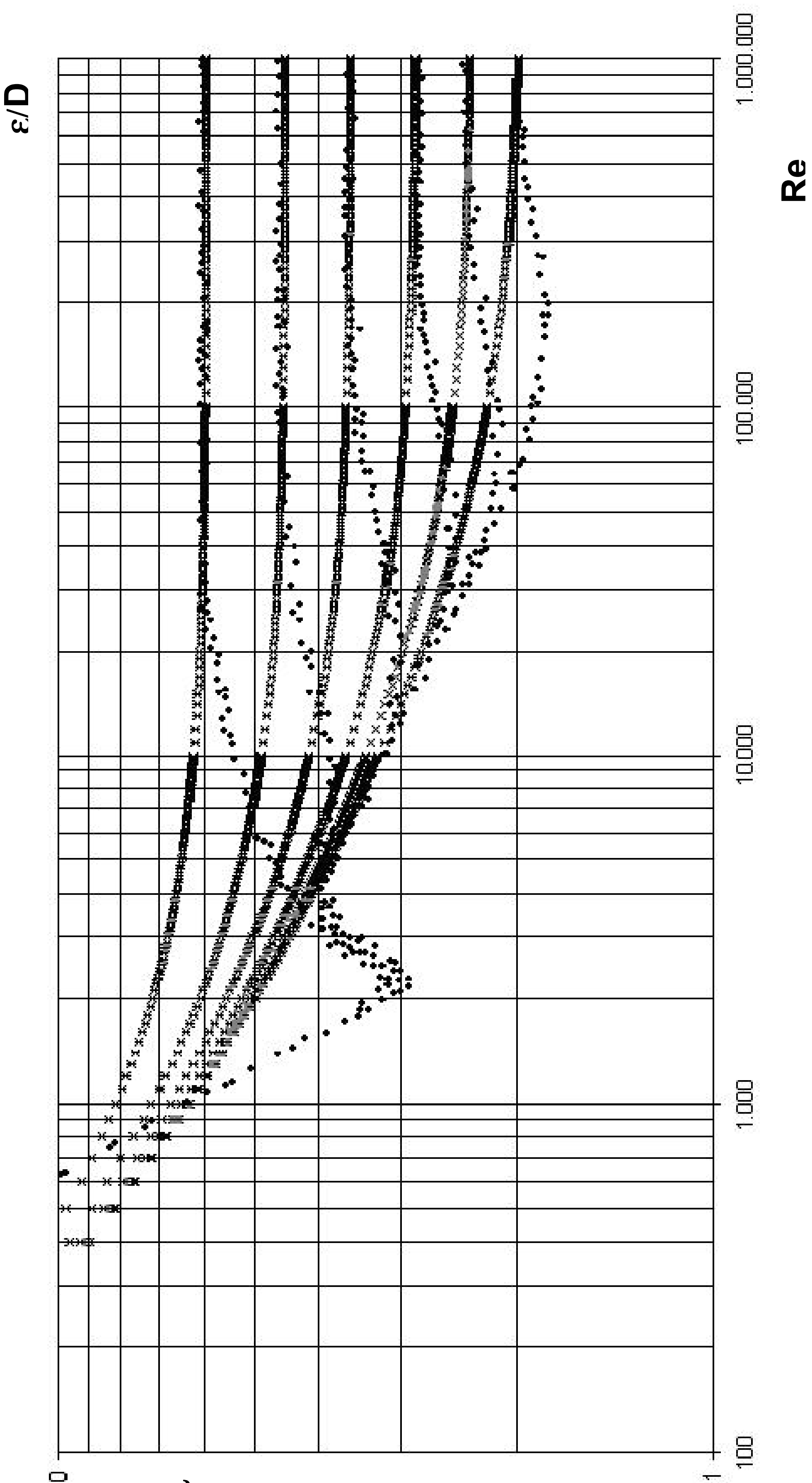
Če temu dodamo še posebne lastnosti vodovodnega omrežja, ki te razlike samo povečujejo, se lahko tudi vprašamo, ali je koeficient trenja sploh določen z zadostno natančnostjo? Vodovodno omrežje v pogojih obratovanja ponuja drugačne, zahtevnejše pogoje, kot jih lahko zasledimo pri krajšem cevovodu v preizkusnih pogojih laboratorija (Petrešin, 1987).

Poleg vsega zgoraj naštetega je treba poudariti, da je tako določen koeficient izgub veljaven le za stalni tok (saj so bili vsi poskusi izvedeni v režimu stalnega enakomernega toka) Pri nestalnem toku se tokovna slika namreč stalno spreminja, kar bi se moralo odraziti kot povečane izgube, ki pa niti v omenjenem izrazu Colebrook-White in niti v katerem drugem, ki so bili objavljeni, niso upoštevane. Obstajajo samo nekatere indikacije eksperimentatorjev, ki potrjujejo, da je koeficient trenja λ za nestalni tok dejansko večji od tistega pri stalnem toku (Kompare, 1996). Bergant *et al.* (2001) so objavili primerjavo modelov, ki se uporabljajo za določanje koeficienta trenja v nestalnih pogojih. Omenjenih je sicer več modelov, podrobneje pa so bili raziskani trije: pogojno-stacionarni, Zielkejev in Brunonejev model.

Colebrook-White formula for evaluation of the friction factor, the use of the Colebrook-White equation may be used outside the defined range of Reynolds numbers. Because of the low flow speed, conditioned by the nature of the system, the inappropriate use of the Colebrook-White equation happens especially when dealing with municipal water supply networks. Beside this, all functions that have a logarithmic form have a singular point when the value of the argument of the logarithmic function equals 1. Such point is between the Reynolds numbers $5 < Re < 10$ (in the Colebrook-White formula the argument of the logarithm also depends on the relative roughness, thus the singular point occurs between an interval and not only for one value of the Reynolds number). When comparing the values of the friction factor, obtained with the use of the Colebrook-White formula or other authors, with the values gained with the hyperbolic formula $\lambda = 64/Re$, it can be seen that the values of the friction factor in the range of Reynolds numbers $100 \leq Re \leq 2100$ are unreal (Figure 3). Lower or respectively higher values of the friction factor cause unreal values of flows.

If special properties of water supply networks are added, which amplify the differences, then a serious question may be raised: Is the friction factor defined with a sufficient accuracy? Operational networks have different, more demanding conditions than the pipelines in an experimental laboratory (Petrešin, 1987).

Beside the above listed problems it has to be emphasised that the friction factor, defined in this way, is valid only in steady flow conditions (since all the experiments were carried out in steady flow conditions). When dealing with unsteady flow conditions, the flow pattern continuously changes and this should reflect as an increased friction loss. Neither the Colebrook-White nor any other known published formulas takes these facts into consideration. There are only indications of experimentalists confirming that the value of the friction factor λ in unsteady flow conditions is greater than that in steady flow conditions (Kompare, 1996). Bergant *et al.* (2001) have published a comparison of different models used for evaluating the friction factor in unsteady flow conditions. Among several models mentioned three of them were investigated in detail. These are the Zielke, Brunone and quasi-steady models.



Slika 3. Diagram Colebrook-Whiteove enačbe (x) v primerjavi z Nikuradzejevimi meritvami (•).
Figure 3. Comparison of the Colebrook-White equation (x) and Nikuradse's measurements (•).

Rezultati, pridobljeni s temi modeli, so bili primerjani z laboratorijskimi meritvami v primerih vodnega udara v laminarnem režimu toka oziroma v primeru turbulentnega toka z nizkimi vrednostimi Reynoldsovih števil.

Glavni namen naše analize je bil preveriti veljavnost Colebrook-Whiteovega izraza (in tudi ostalih razpoložljivih podobnih izrazov) z eksperimentalnim izhodiščem na Nikuradzejevih meritvah ter dobiti ustreznejši obrazec za določevanje koeficienta trenja v stacionarnih pogojih.

2. PODATKOVNA BAZA

V vsej literaturi, ki smo jo imeli na voljo, na žalost nismo nikjer zasledili izvornih Nikuradzejevih podatkov o izmerjenih koeficientih trenja. Na voljo smo imeli le diagram opravljenih meritev, ki smo ga nato z ustreznim programskim orodjem obdelali in digitalizirali. Podatki vsebujejo relacije med koeficienti trenja, pripadajočimi Reynoldsovimi števili ter relativnimi hrapavostmi.

Natančnost podatkovne baze je težko določiti, saj izhaja digitalizirani diagram iz knjige Fluid Mechanics (Streeter, 1951), ki seveda ni izviren Nikuradzejev diagram oziroma ne predstavlja izvirnika meritev. Sam postopek digitalizacije pa bi lahko ocenil kot zelo natančen, saj so bile koordinate vsake na diagramu izrisane točke določene na ± 0,001 mm natančno. Iz tega lahko zaključimo, da natančnost enačbe koeficienta trenja »bistveno« omejuje le pomanjkanje izvornih podatkov in ne sam postopek digitalizacije.

3. IZRAZ KOEFICIENTA TRENJA, DOLOČEN S POMOČJO PREKLOPNE FUNKCIJE

Na eksperimentalno dobljenem diagramu, t. i. Nikuradzejevi harfi, se lahko jasno ločijo štiri območja, kjer je obnašanje koeficienta (funkcije!) trenja karakteristično drugačno od ostalih delov. Moody je to na svojem diagramu tudi jasno poudaril (glej sliko 2). Več različnih obnašanj pomeni, da preproste funkcijske odvisnosti oziroma opisa koeficienta trenja λ verjetno ne bo mogoče poiskati. Neustrezen

Numerical results obtained with these three models were compared with the results of laboratory measurements for water hammer cases with laminar and low Reynolds numbers in case of turbulent flows.

The main goal of this analysis was to check the validity of the Colebrook-White term (and also other similar published terms) by comparing it with the experimentally basic Nikuradse's measurements and to possibly coin a new term for evaluation of the friction factor in steady flow conditions.

2. DATA BASE

In the literature available we could not find Nikuradse's source data about the measured friction factor. The needed data were gained from digitalized Nikuradse's diagram. The data contain the relations between the friction factor, corresponding Reynolds numbers and relative roughness.

It is difficult to determine the accuracy of the database, since the digitalized diagram has been taken from the publication Fluid Mechanics (Streeter, 1951), which does not represent the originally published Nikuradse's diagram, neither does it represent Nikuradse's source data. The procedure of digitalization can be assessed as highly accurate since the co-ordinates of the points plotted on the diagram were determined at 0.001 mm accuracy. It can be resumed that the accuracy of the friction factor equation is "mainly" limited by the lack of source data and not by the digitalization procedure.

3. TERM OF THE FRICTION FACTOR DETERMINED WITH THE USE OF THE SWITCHING FUNCTION

On Nikuradse's experimental diagram, the so called Nikuradse's harp, four zones of different friction factor (function!) behaviour can be found. Notably, these zones were previously emphasized by Moody on his diagram (Figure 2). Several different zones of behaviour mean that a simple functional dependence, respectively a description of the friction factor λ, cannot be found. This fact is confirmed by the unsuitable construct in the konstrukt v obliki Colebrook-Whiteove enačbe to dejstvo potrjuje. Zato smo se odločili, da poskušamo poiskati funkcijsko odvisnost za vsako karakteristično območje posebej. Da bi se izognili vplivu posamičnih opisov zunaj njihovega območja veljave, smo morali uvesti preklopne funkcije, ki vklapljajo in izklapljajo posamičen opis, ko je to potrebno. Stopenjske Dirac-delta funkcije ne pridejo v poštev, saj so prehodi v našem primeru gladki in zvezni. Lahko bi uporabili linearne (ramp) funkcije, ki bi zagotovile zveznost, ne pa tudi dovolj gladkosti. Gladkost in zveznost najbolje zagotavljajo večkrat odvedljive funkcije. Kot možno rešitev smo zato iskali izpeljanke izraza e^x.

3.1 DOLOČITEV PREKLOPNE FUNKCIJE

Funkcija, ki bi se lahko uporabila kot preklopna funkcija, mora ustrezati tem lastnostim:

- $\lim_{x\to\infty} f(x) = 1$,
- $\lim_{x\to-\infty} f(x) = 0$,
- za zagotovitev zveznosti se mora prehod iz $f(x_A) = 0$ do $f(x_B) = 1$ »zgoditi« na ustreznem intervalu, tj. z ustreznim začetkom in koncem preklopa,
- za zagotovitev gladkosti mora imeti iskana funkcija ustrezno število odvodov (npr. vsaj 2).

Med različnimi funkcijami, ki ustrezajo tem lastnostim, je tudi funkcija:

form of the Colebrook-White equation. This was the main reason why we decided to find a suitable functional dependence for the friction factor in each characteristic zone separately. To avoid the use of the terms outside their validity zone, we introduced the switching functions that enable or disable single terms when necessary. Dirac-delta functions could not be taken into consideration because in our case the transitions between the zones are smooth and continuous. Linear (ramp) functions that would assure continuous transition but not smoothness could be used. But both, smoothness and the continuous transition are assured only by the multiple times differentiable functions. As possible candidates the e^x term and its derivatives were chosen.

3.1 DEFINITION OF THE SWITCHING FUNCTION

A function that could be used as a switching function has to have the following properties:

- $\lim_{x\to\infty} f(x) = 1$,
- $\lim_{x\to-\infty} f(x) = 0$,
- to assure a continuous transition, the passage from $f(x_A) = 0$ to $f(x_B) = 1$ must happen in the right interval, i.e. with a proper start and end of the switch,
- to assure smoothness, the sought function must have an adequate number of derivatives, at least 2.

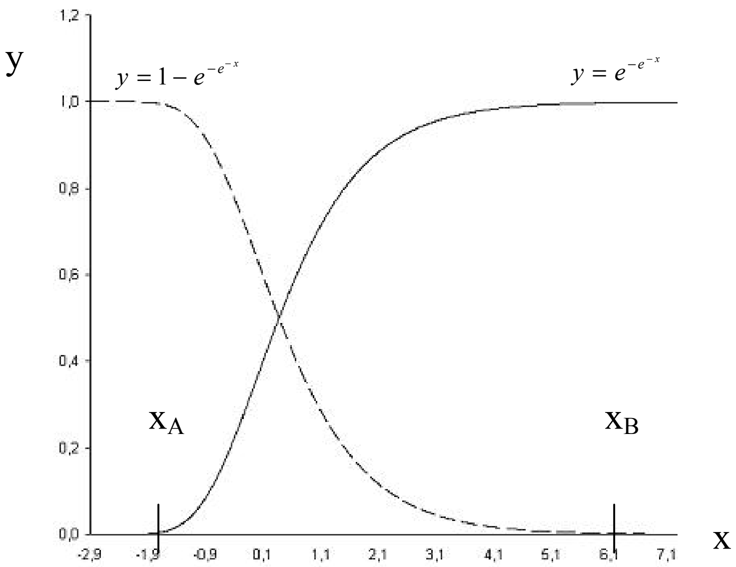
One of the functions that correspond to this properties is:

$$y = e^{-e^{-x}} \tag{2}$$

oziroma njena nasprotna funkcija:

or its opposite function:

$$y = 1 - e^{-e^{-x}} \tag{3}$$



Slika 4. Diagram preklopne funkcije $y = e^{-e^{-x}}$ ter $y = 1 - e^{-e^{-x}}$.

Figure 4. Diagram of the switching function $y = e^{-e^{-x}}$ *and* $y = 1 - e^{-e^{-x}}$.

Iz diagrama na sliki 4 so razvidne vse ključne lastnosti preklopne funkcije. Funkcija ima vse do vrednosti argumenta x_A = - 1,9 vrednost 0. Nato doseže v zelo ozkem intervalu x_B = 6,1 vrednost y = 1 in se pri tej vrednosti ustali.

Ustrezen interval preklopa med vrednostjo y = 0 in y = 1 smo določili tako, da smo argument eksponenta x izrazili kot funkcijo Reynoldsovega števila Re.

Enačba ima naslednjo obliko:

All crucial properties of the function are shown on diagram on Figure 4. Until the value of the argument x_A = -1.9 the value of the switching function is 0. Afterwards the value of y = 1 is reached in a narrow interval at the value of x_B = 6.1.

A suitable interval of the switch between the values of y = 0 and y = 1 was determined by expressing the argument of the exponential function x with the functional dependency of Reynolds number Re.

The equation has the following form:

$$x = b \cdot \mathrm{Re} + c \tag{4}$$

3.2 ENAČBA KOEFICIENTA TRENJA, SESTAVLJENA IZ TREH ČLENOV IN TREH PREKLOPNIH FUNKCIJ

S slik 1 in 2 je razvidno, da ima koeficient trenja štiri funkcijska območja oz. jasno definirana tri območja (I., III. in IV.) ter nejasno II. kritično območje. Prvo, tj. laminarno območje, je podano z linearno funkcijo v log-log diagramu, ki jo lahko izrazimo z enačbo:

3.2 THE EQUATION OF THE FRICTION FACTOR DEFINED WITH THREE TERMS AND THREE SWITCHING FUNCTIONS

As it can be seen from Figures 1 and 2, the friction factor has four different functional zones or three clearly defined zones (I, III and IV), and a dim critical zone II. The first, i.e. the laminar zone, is characterized by a linear function on a log-log diagram, which can be expressed by the following equation:

$$\lambda = \frac{64}{\mathrm{Re}}. \tag{5}$$

Četrto območje je območje popolnoma razvite turbulence, kjer je koeficient trenja odvisen samo od relativne hrapavosti. Prandtl in Karman (Petrešin, 1987) sta to odvisnost zapisala z enačbo:

The fourth zone is the zone of a fully developed turbulence, where the friction factor depends only on the relative roughness. Prandtl and Karman (Petrešin, 1987) give the following equation to describe this dependency:

$$\lambda = \frac{0{,}25}{\left(\log\left(\frac{\varepsilon}{3{,}71D}\right)\right)^2} \tag{6}$$

Tretje območje je prehodno območje, kjer se turbulenca šele razvija in je koeficient trenja odvisen tako od Re kot od ε/D. Ne moremo ga preprosto opisati oz. izpeljati analitičnega izraza kot za obe skrajni območji. Pač pa na Moodyjevem diagramu oz. na Nikuradzejevi harfi opazimo, da se, če potujemo po diagramu z desne proti levi, tj. od visokih Re proti nižjim, začne λ počasi dvigati. Ovojnica tega dviganja je črta, ki opisuje potek koeficinta trenja v hidravlično gladkem režimu toka in jo je Blasius (Petrešin, 1987) izrazil kot:

The third zone is a transitional zone where the turbulence is not yet fully developed and the friction factor depends on both the Re and ε/D. It cannot be easily described or analytically deduced as for zones I and IV. On Moody's or Nikuradse's diagrams it can be seen that if we move across the diagram from right to left, i.e. from high to low Reynolds numbers, the friction factor slowly rises. The envelope of this rise is a line that was expressed by Blasius (Petrešin, 1987) in the following way:

$$\lambda = \frac{0{,}3164}{\sqrt[4]{\mathrm{Re}}} \tag{7}$$

Drugo območje oziroma kritično območje se nahaja nekje med vrednostmi Reynoldsovih števil 2100 ≤ Re ≤ 5000 in ga lahko opazimo na Nikuradzejevem diagramu (slika 1) kot prehod med laminarnim (I.) in prehodnim (III.) režimom toka. Na tem območju je koeficient trenja odvisen tako od Reynoldsovega števila kot od relativne hrapavosti.

Naša ideja je bila, da za prehodno območje, tj. ovojnici hidravlično gladkih cevi, k izrazu za hidravlično hrapave cevi (mehko, z uporabo preklopnih funkcij) prištejemo še Blasiusov člen. Potem pogledamo, kako lahko s preklopnimi funkcijami opišemo prehod od gladkih cevi v turbulentnem režimu (cona III) v laminaren režim (cona I). Načeloma bi za opis štirih območij potrebovali štiri karakteristične funkcije. Mi smo predpostavili, da II., tj. kritično območje med laminarnim in turbulentnim območjem, nima svoje karakteristične opisne funkcije, pač pa je zgolj rezultat preklapljanja med laminarnim in

The second zone, the critical zone, can be found between Reynolds numbers 2100 ≤ Re ≤ 5000. It can be seen on Nikuradse's diagram (Figure 1) as a passage between the laminar (I) and transitional (III) zones. In this zone of flow and in the transitional zone the friction factor depends on both the Reynolds number and the relative roughness.

Our idea was to add the Blasius term in the transitional zone for the hydraulically smooth pipes to the term for the hydraulically rough zone (in a "soft" way with the use of the switching function). Then the transition between the hydraulically smooth pipes, in the turbulent zone (zone III), and the laminar zone (zone I) with the use of the switching function should be found. Fundamentally, to describe the four characteristic zones, four different functions should be used. Our supposition was that zone II, the transitional zone between the laminar and turbulent zones, is not defined by its own characteristic function but is merely a result of the switching between the laminar and turbulent regimes of flow. This is the turbulentnim režimom. Zato smo zapisali posamezne enačbe za tri režime toka (I., III. in IV. območje) in jih zlepili s tremi preklopnimi funkcijami. Dve funkciji namreč nista dovolj, ker je treba v razviti turbulenci v coni IV Blasiusov člen za III. cono izklopiti, prav tako, kot ga je potrebno izklopiti v II., kritičnem območju. Za obe skrajni območji pa je očitno dovolj ena sama preklopna funkcija.

Po tej logiki enačbo koeficienta trenja sestavljajo trije členi. Prvi člen enačbe pokriva laminarno območje režima toka, drugi člen sestavlja enačba za prehodno območje, tretji člen pa enačba za hidravlično hrapavo območje. Preprost seštevek vseh treh členov enačbe po pričakovanjih ni prinesel želenih rezultatov, zato smo uporabili preklopne funkcije, ki so omogočile optimalno prilagajanje Nikuradzejevim meritvam. Enačba se tako glasi:

reason why only three different equations for three regimes of flow (zones I, II and IV) were used. These equations were bound together with the use of three switching functions. The use of only two switching functions proved inadequate because the Blasius term in zone III has to be turned off in zone IV and also in the second, critical zone. For both extreme zones the use of only one switching function is enough.

Following this logic the friction factor equation is composed of three terms. The first term of the equation covers the laminar zone of the flow, the second term is an equation that covers the hydraulically smooth zone and the third term covers the hydraulically rough zone. As expected, a simple addition of these three terms did not bring the desired results, and thus the switching functions, which enabled an optimal adaptation to Nikuradse's measurements, were used. The equation has the following form:

$$\lambda = \frac{a}{\mathrm{Re}} \cdot (1 - y_1) + \frac{b}{\mathrm{Re}^{\beta}} \cdot (y_1 - y_3) + \frac{c}{\log^2\left(\dfrac{\varepsilon}{k_\varepsilon \cdot D}\right)} \cdot y_2 . \qquad (8)$$

Členi y_1 , y_2 ter y_3 nastopajo v enačbi (8) kot preklopne funkcije z naslednjo obliko:

y_1, y_2 and y_3 terms in the equation (8) are switching functions with the following form:

$$\begin{aligned} y_1 &= e^{-e^{-(\gamma_1 \cdot \mathrm{Re} + \delta_1)}} \\ y_2 &= e^{-e^{-(\gamma_2 \cdot \mathrm{Re} + \delta_2)}} \\ y_3 &= e^{-e^{-(\gamma_3 \cdot \mathrm{Re} + \delta_3)}} \end{aligned} \qquad (9)$$

Optimizirano vrednost parametrov (a, b, c … δ_3), ki nastopajo v enačbah (8) in (9), smo poiskali s programom SCILAB. Z downhill-simplex metodo optimizacije smo želeli optimizirati vse parametre istočasno, kar pa se je pokazalo za pretežko nalogo zaradi preveliko stopenj svobode in narave optimizacijske metode, da prehitro zapade v lokalne optimume. Zato smo zmanjšali število prostostnih stopenj tako, da smo iskali funkcije za vsako vejo Nikuradzejeve harfe posebej (Uršič, 2003). Optimizacija je bila tokrat uspešna, saj smo za vsako vejo relativne hrapavosti posebej dobili optimalne vrednosti parametrov. Tako smo dobili 6 različnih množic parametrov, ki so bili (implicitno)

Optimized values of the parameters (a, b, c … δ_3) that appear in the equations (8) and (9) were gained with the use of the SCILAB program. With the downhill-simplex method all the parameters were optimized simultaneously but this turned out to be quite a difficult task, because of too many degrees of freedom and the nature of the optimization method, which too quickly gets stuck in local optimums. The number of degrees of freedom was reduced by searching the optimal functions for each of the branches of the relative roughness on Nikuradse's diagram separately (Uršič, 2003). This time the optimization process ended in a successful way giving the optimized value of the parameters for each branch of the relative roughness. So, six different datasets of parameters were found, which were odvisni od relativne hrapavosti. Vrednosti parametrov so podane v preglednicah 1 in 2, diagram enačbe (8) pa na sliki 5.

(implicitly) dependent on the relative roughness. The values of the parameters are shown in Tables 1 and 2, meanwhile the diagram of the equation (8) is shown on Figure 5.

Preglednica 1. Optimizirani parametri enačbe (8) v odvisnosti od relativne hrapavosti.
Table 1. Optimized parameters in equation (8) as a function of the relative roughness.

ε/D	a	b	β	c	k_ε
0,000986	63,020918	0,103271	0,120128	0,221963	2,663000
0,001980	62,882743	0,047485	0,031636	0,223410	2,663219
0,003970	62,768000	0,053358	0,040424	0,225584	2,663470
0,008330	62,845016	0,071954	0,075679	0,224490	2,663428
0,016300	62,834293	0,027864	- 0,042818	0,223714	2,663349
0,033300	62,837410	0,006933	- 0,212001	0,218528	2,663367

Preglednica 2. Optimizirani parametri enačbe (9) v odvisnosti od relativne hrapavosti.
Table 2. Optimized parameters in equation (9) as a function of the relative roughness.

	y_1		y_2		y_3	
ε/D	γ_1	δ_1	γ_2	δ_2	γ_3	δ_3
0,000986	0,003968	- 8,643438	0,000069	- 1,353603	0,000068	- 1,490421
0,001980	0,003554	- 8,644670	0,000138	- 1,284863	0,000138	- 1,436383
0,003970	0,003381	- 8,646930	0,000014	1,306298	0,000092	0,253699
0,008330	0,003536	- 8,647870	0,000070	- 1,296255	0,000089	- 1,259867
0,016300	0,003526	- 8,647230	0,000140	- 1,413005	0,000165	- 1,373546
0,033300	0,003540	- 8,643022	0,000161	- 1,326516	0,000187	- 1,551611

Seveda je tako optimizirana enačba (8) v praksi neuporabna, saj je dejansko šest enačb, ki implicitno upoštevajo relativno hrapavost, in ne le ena splošna enačba, ki bi bila definirana za vse vrednosti relativne hrapavosti. Treba je bilo ugotoviti, kateri parametri so odvisni od relativne hrapavosti, ter določiti funkcijsko povezavo med temi ter relativno hrapavostjo.

Po opravljeni analizi parametrov iz zgornjih preglednic se je izkazalo, da imajo največji vpliv na potek enačbe (8) parametri, ki nastopajo v preklopnih fukcijah y_2 ter y_3. V naslednjem koraku je bilo treba določiti funkcijsko odvisnost parametrov, ki nastopajo v preklopnih funkcijah y_2 in y_3, od relativne hrapavosti. Splošna oblika enačbe koeficienta trenja tako ostaja nespremenjena (enačba 8),

Certainly co optimized equation (8) is practically useless since we deal with 6 equations, which implicitly consider the relative roughness, and not only with one generalized equation that would be defined for all values of the relative roughness. Thus it was necessary to determine, which of the parameters is dependent on the relative roughness and to define this functional dependency.

After the analysis of the parameters from the tables given above, it was shown that the most influential parameters on the course of the equation (8) were the parameters, which appear in the switching functions y_2 and y_3. In the next step in was necessary to define the functional dependency of the parameters, which appear in the switching functions y_2

spremeni se le funkcijska odvisnot argumentov eksponentov v preklopnih funkcijah y_2 ter y_3.

Enačba preklopne funkcije y_1 po opravljeni analizi ostaja nespremenjena:

and y_3, from the relative roughness. A generalized form of the friction factor equation remains unchanged (equation 8). The changes are applied to the functional dependency of the arguments of exponential functions in the switching functions y_2 and y_3.

After the analysis the equation of the switching function y_1 remains unchanged:

$$y_1 = e^{-e^{-(\gamma\cdot \mathrm{Re}+\delta)}} \tag{10}$$

Enačbi preklopnih funkcij y_2 ter y_3, v odvisnosti od relativne hrapavosti in Reynoldsovega števila, sta:

The equations of the switching functions y_2 and y_3 in dependency from the relative roughness and Reynolds number are:

$$\begin{aligned} y_2 &= e^{-e^{-\left(\left(\Psi_2\cdot\frac{\varepsilon}{D}+\Omega_2\right)\cdot \mathrm{Re}+\left(\psi_2\cdot\frac{\varepsilon}{D}+\omega_2\right)\right)}} \\ y_3 &= e^{-e^{-\left(\left(\Psi_3\cdot\frac{\varepsilon}{D}+\Omega_3\right)\cdot \mathrm{Re}+\left(\psi_3\cdot\frac{\varepsilon}{D}+\omega_3\right)\right)}} \end{aligned} \tag{11}$$

Optimizacijski postopek smo vnovič izvedli s programom SCILAB (Uršič, 2003) in metodo downhill-simplex ter dobili naslednje rezultate, prikazane v preglednicah 3, 4 in 5. Diagram novo predlagane (generalizirane) enačbe trenja (8) z optimiziranimi parametri iz preglednic 3, 4 in 5 je prikazan na sliki 6.

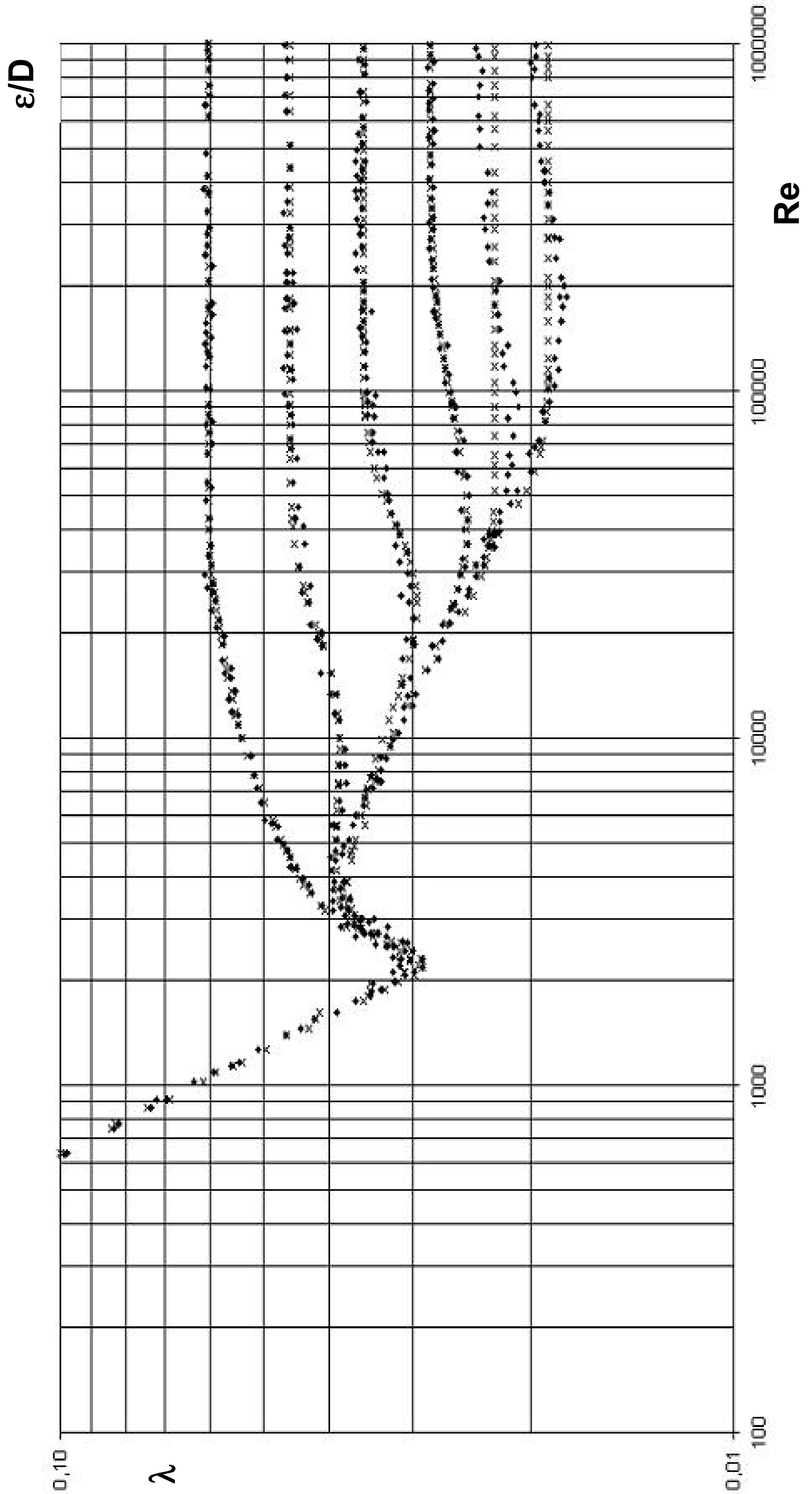
The optimization procedure was once again performed with the SCILAB program (Uršič, 2003) and the downhill-simplex method. The results are shown in Tables 3, 4 and 5. The diagram of the newly proposed (generalized) friction equation (8) with the optimized parameters from Tables 3, 4 and 5 is shown on Figure 6.

Preglednica 3. Optimizirani parametri za novo (generalizirano) enačbo koeficienta trenja (8).
Table 3. Optimized parameters for the new (generalized) friction factor equation (8).

območje – Area	laminarno – *Laminar*	prehodno – *Transitional*		turbulentno – *Turbulent*	
parameter – *Parameter*	a	b	β	c	k_ε
Vrednost – *Value*	67,7880110	0,2989496	0,2414664	0,2445573	3,4366602

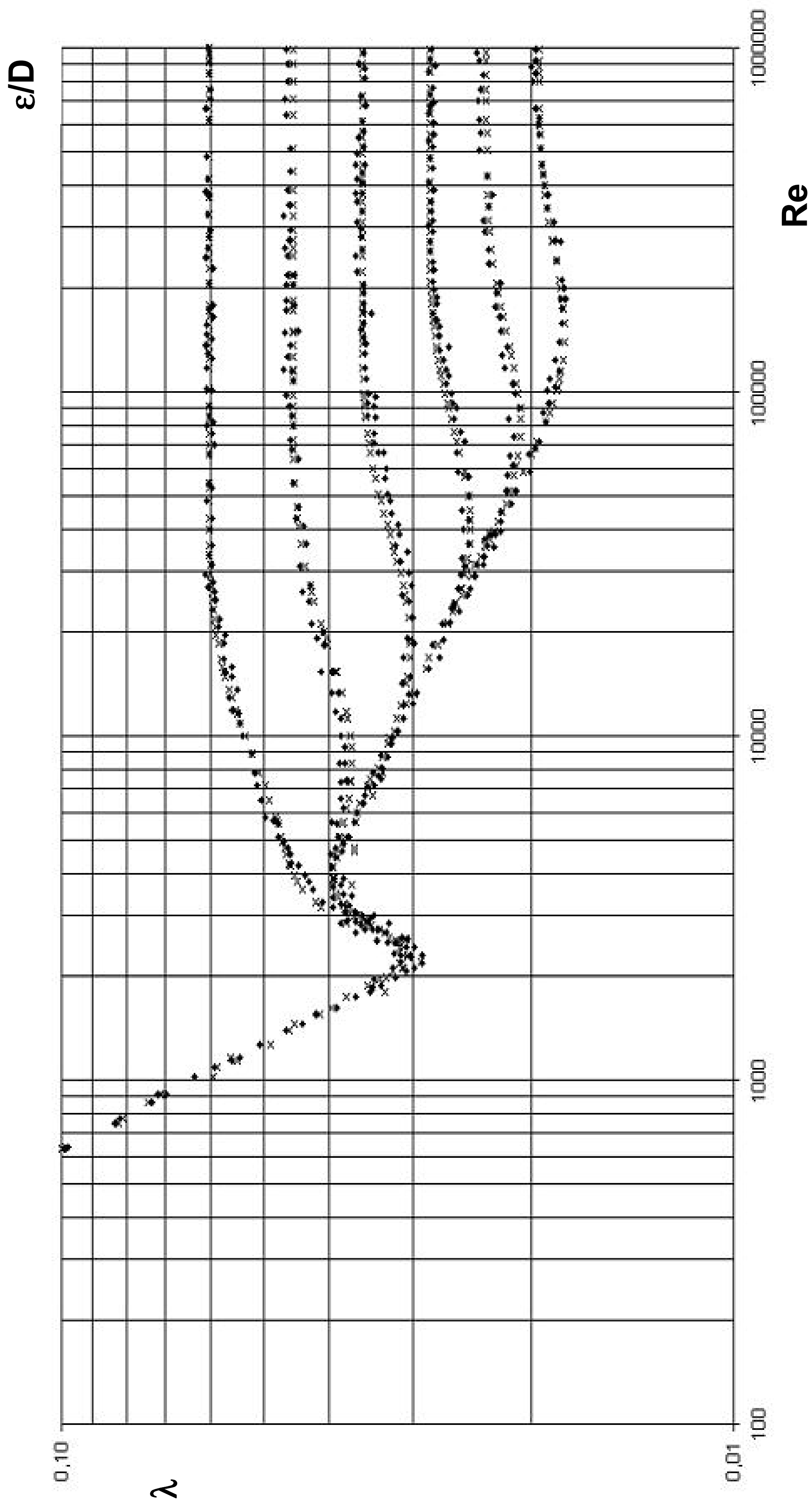
Preglednica 4. Optimizirani parametri enačbe preklopne funkcije y_1 (10).
Table 4. Optimized parameters for the switching function y_1 (10).

funkcija – *Function*	y_1	
parameter – *Parameter*	γ	δ
vrednost – *Value*	0,0024655	- 6,3820544



Slika 5. Diagram nove enačbe koeficienta trenja (8), optimizirane za posamezne relativne hrapavosti (x) v primerjavi z Nikuradzejevimi meritvami (•).

Figure 5. Comparison of the new friction factor equation (8) optimized for different relative roughnesses (x) and Nikuradse's measurements(•).



Slika 6. Diagram nove (generalizirane) enačbe koeficienta trenja (8) (x) v primerjavi z Nikuradzejevimi meritvami (•).
Figure 6. Comparison of the new (generalized) friction factor equation (8) (x) and Nikuradse's measurements(•).

Preglednica 5. Optimizirani parametri enačbe preklopnih funkcij y_2 ter y_3 (11).
Table 5. Optimized parameters for the switching functions y_2 and y_3 (11).

funkcija – *Function*	y_2			
parameter – *Parameter*	Ψ_2	Ω_2	ψ_2	ω_2
vrednost – *Value*	0,0048188	0,0000036	- 7,4288529	0,6901159
funkcija – *Function*	y_3			
parameter – *Parameter*	Ψ_3	Ω_3	ψ_3	ω_3
vrednost – *Value*	0,0158366	0,0000041	19,9028630	- 0,1301545

4. RAZPRAVA

Iz končnega predloga za izraz koeficienta (funkcije!) trenja lahko ugotovimo, da so bili prvotno zastavljeni cilji in želje po izgradnji enačbe, ki bi se popolnoma prilagajala Nikuradzejevim meritvam, doseženi. Ugotovljena je bila struktura odsekoma spremenljive enačbe in določene so bile vrednosti parametrov ter funkcijska povezava med relativno hrapavostjo in parametri enačbe. Enačba (8) se na vseh območjih režima toka skoraj popolnoma prilega Nikuradzejevim meritvam, kar dokazuje tudi korelacijski koeficient, ki znaša r = 0,998. Na diagramu na sliki 6 se lepo vidijo vse naštete ugotovitve.

Enačba koeficienta trenja je podana v eksplicitni obliki, kar je velika pridobitev glede na implicitno obliko Colebrook-Whiteove enačbe. To pride do veljave še posebej pri hidravličnem preračunu vodovodnega sistema, zaradi enostavnejše rešitve enačbe (8) brez iteracij.

Eksplicitni izraz koeficienta trenja (8) velja za vrednosti Reynoldsvih števil $0 \leq Re \leq 10^8$. To je omogočila uporaba preklopne funkcije, ki poleg tega, da izboljša natančnost rezultatov za vse vrednosti Reynoldsovih števil, tudi izniči singularne točke, ki bi jih enačbe tipa Colebrook-White (če se uporabljajo zunaj predpisanih omejitev) sicer imele na intervalu Reynoldsovih števil $5 \leq Re \leq 10$ zaradi narave logaritemske funkcije.

4. DISCUSSION

On the basis of our final proposal for the friction factor (function!) equation, it can be ascertained that the primary goals and desires to construct an equation, which would be fully adapted to the Nikuradze's measurements, have been achieved. The right form of a piecewise changing equation was ascertained, the values of the parameters were defined correctly and the functional dependence between the relative roughness and the parameters was found. In each of the four zones the equation (8) fits very well with the Nikuradse's measurements, what shows the correlation coefficient of r = 0.998. The diagram on Figure 6 clearly supports these statements.

The friction factor equation is given in an explicit form. This is a great achievement regarding the implicit form of the Colebrook-White equation, which is mainly used for the calculation of the hydraulic losses in water supply networks, because of an easy solution of the equation (8) without iterations.

The explicit friction term (8) is valid for values of the Reynolds numbers $0 \leq Re \leq 10^8$. This was made possible by the use of the switching function, which improves the accuracy of the results for all Reynolds numbers and avoids singular points, which a logarithmic function of Colebrook-White type would have (if used out of range of the directed limitations) in the range of the Reynolds numbers $5 \leq Re \leq 10$.

5. ZAKLJUČKI

Na podlagi digitaliziranih Nikuradzejevih meritev smo določili enačbo koeficienta (funkcije!) trenja (8) v eksplicitni obliki, ki je definirana za vrednosti Reynoldsovega števila $0 \leq Re \leq 10^8$ in vse vrednosti relativne hrapavosti.

Nova enačba (8) se tako rekoč popolnoma prilega vsem šestim krivuljam Nikuradzejevih meritev in je bistveno bol natančna od Colebrook-Whiteove oziroma vseh do sedaj znanih objavljenih enačb (Uršič, 2003).

Za celotno območje Reynoldsovih števil ter šest vrednosti relativne hrapavosti je dosežen zelo visok koeficient korelacije, ki znaša r = 0,998 ter nizka standardna napaka $\sigma = 9{,}625 \cdot 10^{-8}$. Nova enačba (8) nima nobene omejitve uporabe, saj je definirana za vsa Reynoldsovega števila in, kar lahko iz visoke vrednosti korelacijskega koeficienta ekstrapoliramo, tudi za vse vrednosti relativne hrapavosti. Tako lahko zaključimo in predlagamo, da se nova enačba (8) začne uporabljati za preračun hidravličnih izgub v vodovodnih omrežjih oziroma v zaprtih cevovodih.

5. CONCLUSIONS

On the bases of the digitalized Nikuradse's measurements we determined the friction factor (function!) equation (8) in an explicit form, which is defined for the values of the Reynolds numbers between $0 \leq Re \leq 10^8$ and all values of the relative roughnesses.

The new equation (8) has practically a full fit to all six curves of Nikuradse's measurements and it is essentially more accurate than the Colebrook-White or any other published equation by now (Uršič, 2003).

For the entire range of the values of the Reynolds numbers and relative roughness a high correlation factor, r = 0,998 and a low standard deviation $\sigma = 9{,}625 \cdot 10^{-8}$ is achieved. The new equation (8) has no restrictions in terms of its use since it is defined for all the values of the Reynolds numbers and relative roughnesses. We can thus conclude and propose a new equation (8) for the calculation of the hydraulic losses in water supply networks and respectively in all pipelines.

6. SIMBOLI

ε	… Absolutna velikost hrap
D	… Premer cevi
ε/D	… Relativna hrapavost
μ	… Viskoznost vode
v	… Hitrost vode v cevi
Re	… Reynoldsovo število
λ	… Koeficient trenja
y_1, y_2, y_3	… Preklopne funkcije
$a,b,c, ...\delta_3$	… Parametri nove enačbe koeficienta trenja

6. SYMBOLS

ε	… Absolute roughness height
D	… Diameter of a pipe
ε/D	… Relative roughness
μ	… Viscosity of water
v	… Water velocity in a pipe
Re	… Reynolds number
λ	… Friction factor
y_1, y_2, y_3	… Switching functions
$a,b,c…,\delta_3$	… Parameters of the new friction factor equation

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