BLED WORKSHOPS
IN PHYSICS
VOL. 8, NO. 1
p. 17

Proceedings of the Mini-Workshop
Hadron Structure and Lattice QCD
Bled, Slovenia, July 9-16, 2007

Does nucleon parity doubling imply $U_A(1)$ symmetry restoration?⋆

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Abstract. We examine the role of $U_A(1)$ symmetry and its breaking/restoration in two complete chiral multiplets consisting of the nucleon and the Roper and their two "chiral mirror" odd-parity resonances. We base our work on the recent classification of the chiral $SU_L(2) \times SU_R(2)$ transformation properties of the two (Ioffe) independent local tri-quark nucleon interpolating fields in QCD [1].

1 Introduction

Over the past five years there has been considerable activity on the question if the chiral $U_A(1)$ symmetry restoration is in any way related to the (purported) parity doubling in the nucleon spectrum [2,3]. In the previous additions to the literature [2], following an old and to a large extent formal example by Ben Lee [4], it was assumed that the nucleons admitted only certain specific linear non-Abelian chiral transformation properties - no assumptions were made about the Abelian ones, however.

Rather than guess at the chiral properties of the nucleon, we use the results of our study [1] of the $SU_L(2) \times SU_R(2)$ and $U_A(1)$ (the non-Abelian and the Abelian chiral symmetries, respectively) transformations of the over-complete set of (five) three-quark non-derivative (local) nucleon interpolating fields. We showed that the two independent nucleon fields form two different irreducible $U_A(1)$ representations: one with the axial baryon number minus one (the Abelian "mirror" field), and another with three (the Abelian triply "naive" nucleon in the parlance of Ref. [5]).

For odd-parity nucleons, on the other hand, the inclusion of at least one space-time derivative is natural. Once we allow for a derivative to exist in the interpolating field, we find two nucleon fields with chiral properties opposite to the non-derivative ones, e.g. the non-Abelian chiral properties of the derivative fields are "mirror" compared to the "naive" non-derivative ones. Thus, altogether we have four independent nucleon fields constructed from three quarks with or

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without one derivative. They can be classified as being non-Abelian “naive” or “mirror” and similarly for the Abelian chiral transformation properties.

As an illustrative example, we identify these four specific nucleon fields with the four lowest-lying nucleon resonances: the nucleon-Roper even-parity pair and the $N^*(1535)$, $N^*(1650)$ pair of odd-parity resonances, and construct an effective Lagrangian with the $U_A(1)$ and $SU_L(2) \times SU_R(2)$ symmetries. We show that, after spontaneous symmetry breakdown to $SU(2)_V$, the mass splitting induced by this effective interaction can reproduce all four nucleon's masses *even without explicit* $U_A(1)$ *symmetry breaking*. This is an explicit counter-example to the statement in the literature that the parity doubling in the nucleon spectrum is related to the restoration of the $U_A(1)$ symmetry.

Our method applies equally well to any, and not just the low-lying, $U_A(1)$ chiral quartet, i.e., pair of nucleon parity doublets. Of course, this result is subject to the assumption of three-quark nature of the corresponding nucleon states.

2 Three-quark nucleon interpolating fields

We start by summarizing the transformation properties of various quark trilinear forms with quantum numbers of the nucleon as shown in Ref. [1]. It turns out that every nucleon, i.e., spin- and isospin 1/2 field, besides having same non-Abelian transformation properties, comes in two varieties: one with “mirror” and another with “triple-naive” Abelian chiral properties. This allows us to address the old (Ioffe) problem of duplication/ambiguity of nucleon fields: For $J^P = \frac{1}{2}^+$ nucleons there is only one non-Abelian representation allowed, the $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$, but with the two afore-mentioned Abelian chiral properties, thus lending physical distinction to Ioffe's two nucleon fields: the nucleon ground state, the two odd-parity resonances and the Roper are the four mutually orthogonal admixtures of the Abelian “mirror”- (so called Ioffe current), the Abelian “triple naive”- and their non-Abelian mirror fields.

Table 1. The Abelian axial charges (+ sign indicates “naive”, - sign “mirror” transformation properties) and the non-Abelian chiral multiplets of $J^P = \frac{1}{2}^+$ nucleon interpolating fields in the Lorentz group representation $D(\frac{1}{2}, 0)$ without derivatives. In the last column we show the Fierz identical fields, see [1].

	$U_A(1)$	$SU_A(2)$	$SU_V(2) \times SU_A(2)$	Fierz identical
$N_1 - N_2$	-1	$+1$	$(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$	$N_3,\ N_4$
$N_1 + N_2$	$+3$	$+1$	$(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$	N_5

We can construct nucleon fields with “opposite” chiral transformations to those shown above by replacing γ_μ with $i\partial_\mu$: for example we may use the following two nucleon interpolating fields involving three quarks and one derivative

$$N_1^{'-} = \epsilon_{abc} i\partial_\mu(\tilde{q}_a q_b)\gamma^\mu\gamma^5 q_c, \tag{1}$$

$$N_2^{'-} = \epsilon_{abc} i\partial_\mu(\tilde{q}_a \gamma^5 q_b)\gamma^\mu q_c. \tag{2}$$

They are odd-parity, spin 1/2 and isospin 1/2 fields, i.e. they describe (some) nucleon resonances. A prime in the superscript implies that the fields contain a derivative, and we show below that therefore they have opposite, i.e., "mirror" non-Abelian chiral transformation properties to those of the corresponding non-derivative fields.

Taking the symmetric and antisymmetric linear combinations of two nucleon fields $N^{'-}_{1,2}$ as the new canonical fields

$$N^{'-}_{m} = \frac{1}{\sqrt{2}}(N^{'-}_{1} + N^{'-}_{2}) \tag{3}$$

$$N^{'-}_{n} = \frac{1}{\sqrt{2}}(N^{'-}_{1} - N^{'-}_{2}), \tag{4}$$

their Abelian chiral transformation properties read

$$\delta_5 N^{'-}_{m} = -3ia\gamma_5 N^{'-}_{m} \tag{5}$$

$$\delta_5 N^{'-}_{n} = \ \ ia\gamma_5 N^{'-}_{n}, \tag{6}$$

whereas the non-Abelian ones remain "mirror"

$$\delta_5 N^{'-}_{m,n} = -i\gamma_5 \boldsymbol{\tau}\cdot\mathbf{a} N^{'-}_{m,n}. \tag{7}$$

In summary, we have explicitly constructed four independent nucleon fields: two fields with "naive" and two fields with "mirror" Abelian and non-Abelian chiral transformation properties. In the present paper, we identify these fields with the nucleon ground state N(940) and its resonances N(1440), N(1535) and N(1650). We summarize the properties of the four fields in Table.2. With these fields we can construct the "naive-mirror" interactions.

Table 2. The axial charges of the nucleon fields.

Interpolating fields	$U_A(1)$	$SU_A(2)$	Assigned states
N_m	-1	$+1$	N(940)
N_n	$+3$	$+1$	N(1440)
N'_n	$+1$	-1	N(1650)
N'_m	-3	-1	N(1535)

3 The $U_A(1)$ symmetry in baryons

The $U_A(1)$ symmetry's explicit breaking due to the triangle anomaly and topologically non-trivial configurations in QCD has only a few firmly established observable consequences, all of which are in the flavor-singlet spin-less meson sector, see Ref. [11] and references therein, with lots of recent speculation about its role in the baryon sector ("parity doubling"), especially with regard to its alleged/purported "restoration high up in the hadron spectrum" Ref. [2]. This scenario has effectively been disproven in the meson case in Refs. [2].

The baryon case is much more difficult to handle, due to, *inter alia*, a fundamental lack of knowledge of the baryon chiral transformation properties. In the baryon sector, the empirically observed parity doubling has been quantitatively analyzed by Jaffe et. al. [3], who proposed that the physics behind that might be the (explicitly broken) $U_A(1)$ symmetry. In the absence of direct lattice measurements the best one can do is resort to chiral models.

Lee, DeTar, Kunihiro, Jido, Oka and others [4,5] have developed a Lagrangian formalism based on one pair of "naive" and "mirror" opposite-parity nucleon fields. They did not consider the $U_A(1)$ symmetry, however. Christos [8] has shown that there are two independent cubic interactions for each parity doublet that preserve both $U_A(1)$ and $SU(2)_L \times SU(2)_R$ symmetry. However Christos did not include Abelian chiral mirror fields, so he obtained vanishing off-diagonal $\pi N N^*$ couplings. Our strategy was first to construct the $SU_L(2) \times SU_R(2)$ chiral invariant interaction(s) for two pairs of nucleon ($N^+_{m,n}$ and $N^{'-}_{m,n}$) fields; and then to include the $U_A(1)$ symmetry [12]. We have classified these terms according to the power of the meson fields. We found that besides the linear (in meson fields) interactions there are also quadratic and cubic ones. The form of these interactions is uniquely dictated by the $U_A(1)$ symmetry; higher-order terms may appear only as products of these three lower-order ones. That allows altogether six interactions: four diagonal ones in the two doublets and two "inter-doublet" ones. Furthermore, we included all quadratic terms allowed by the non-Abelian "mirror" properties of the baryons. Then we found that one does not need any $U_A(1)$ symmetry breaking to describe the nucleon mass spectrum, provided one uses a complete set of interactions.

4 Results

In the following discussion, it is convenient to group the four nucleon fields as follows; $\Psi = (N^+_m,\ N^{'-}_n)$ for the pair of the single Abelian charge (the single-Abelian doublet), and $\Phi = (N^+_n,\ N^{'-}_m)$ for that of the triple Abelian charge (the triple-Abelian doublet). We emphasize that the two nucleons in each of these pairs are in "mirror" relations to each other, with regard to both the Abelian and non-Abelian chiral symmetries. Manifestly, the identification of fields, or their admixtures, with actual resonances *viz.* $N(940)$, $R(1440)$, $N^*(1535)$ and $N^*(1650)$ is not unique. In this brief review we consider only one choice; another scenario is considered in Ref. [12]. A substantial body of QCD sum rule evidence is pointing towards $N(940)$ being the "Ioffe current" N^+_{1m}. Together with the lowest negative parity nucleon $N(1535)$ in the partner of the parity doublet, we have $\Psi = (N^+_m, N'^-_n) = (N(940), N(1535))$ and consequently $\Phi = (N^+_n, N'^-_m) = (N(1440), N(1650))$.

The nucleon mass matrix is already in a simple block-diagonal form when the nucleon fields form the following 1×4 row/column "vector":

$$(\Psi, \Phi) = (N^+_m, N^{'-}_n, N^+_n, N^{'-}_m) \to (N^+_m, \gamma_5 N^{'-}_n, N^+_n, \gamma_5 N^{'-}_m)\ ,$$

$$\lim_{U_A(1)\text{symm.}} M = \begin{pmatrix} g_1 f_\pi & m_{12}\gamma_5 & 0 & g_5 f_\pi \gamma_5 \\ m_{12}\gamma_5 & g_2 f_\pi & g_6 f_\pi \gamma_5 & 0 \\ 0 & g_6 f_\pi \gamma_5 & g_3 f_\pi & m_{34}\gamma_5 \\ g_5 f_\pi \gamma_5 & 0 & m_{34}\gamma_5 & g_4 f_\pi \end{pmatrix}. \tag{8}$$

Note that only the parity-changing interaction $g_{5,6}$ mixes these two new equal parity doublets. Without inter-doublet interactions ($g_{5,6} = 0$) one can immediately read off the eigenvalues following Ref. [5]. We determine the coupling and mass parameters and show them in Table 3 and Fig. 1.

Table 3. Coupling constants obtained from the nucleon masses with doublets $(N(940), N^*(1535))$, $(R(1440), N^*(1650))$ and the decay widths $N^*(1535) \to \pi N(940)$ and $N^*(1650) \to \pi R(1440)$.

constant	g_1	g_2	m_{12}	g_3	g_4	m_{34}
value	10.4	16.8	270 MeV	14.6	16.8	503 MeV

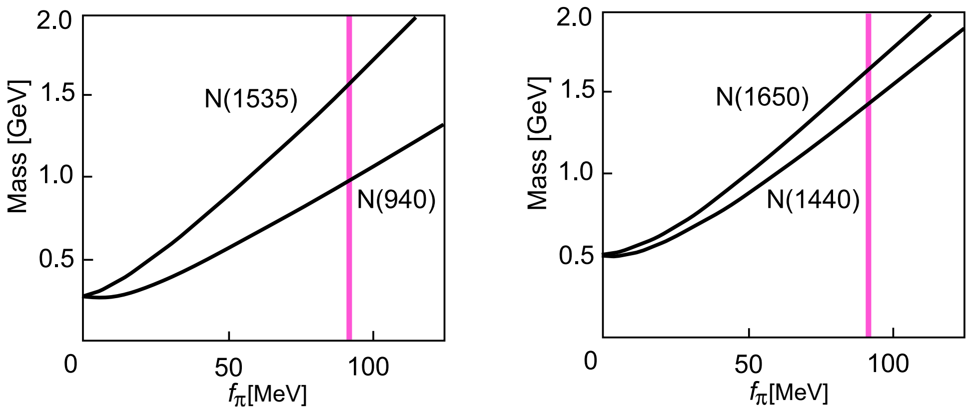

Fig. 1. The nucleon masses as functions of $\langle\sigma\rangle_0$.

Manifestly, the good $U_A(1)$ symmetry limit is sufficient to reproduce the nucleon spectrum. Thence our **main conclusion**: *the mass degeneracy of opposite-parity nucleon resonances is not a consequence of the explicit* $U_A(1)$ *symmetry (non) breaking*. This conclusion was also reached by Christos [8], albeit for one parity doublet and without mirror fields, which means that his $N^*(1535)$ can not decay into $N(940)$ by π emission.

5 Summary and Discussion

We have analyzed the $U_A(1)$ symmetry in the nucleon-Roper-two-odd-parity-nucleon-resonances system, under the assumption that the above four nucleon states are described by a particular set of independent interpolating fields. The four nucleon fields naturally split into two "parity doublets" due to their $U_A(1)$ symmetry transformation properties.

Our analysis has been based on the Born approximation: Higher-order (one-, two-, etc. meson loop) corrections belong to the $\mathcal{O}(1/N_c)$ corrections, that have been studied only intermittently in chiral quark models of the nucleon and then only in certain simple models with one kind of nucleon. In principle, instanton effects are expected to vanish in the large-N_c limit, which justifies our assumption of good $U_A(1)$ symmetry, *ex post facto*. The extracted value of the "bare mirror" nucleon mass (m_{12}=270 MeV, see also Ref. [6]) is something that can be checked on the lattice, now that the interpolating fields have been specified for the mirror nucleons.

The insight that the nucleon and the Roper fields may form two different representations of the $U_A(1)$ symmetry, and that their mass difference can be viewed as a consequence of $U_A(1)$ symmetry conservation and not of the symmetry breaking, are the main results of this work. A corollary of this result is that the parity-doublet mass splittings are not entirely determined by the $U_A(1)$ symmetry breaking, as was conjectured in the literature [3]. Moreover, the nucleon-Roper mass difference in some calculations, such as the one of Ref. [10] in the NJL model, are not a consequence of the broken $U_A(1)$ symmetry in that model.

$U_A(1)$ symmetry in nucleon spectra has been discussed before, most notably by Christos [8], who used only one parity doublet (N(940) and N*(1535)), however. He argued that the parity doublet mass difference is proportional to a particular $\eta N N^*$ coupling constant, which is in close agreement with our results. He did not try to relate other mass differences, such as the Roper-nucleon one, to this mechanism, as he did not know of an alternative ("mirror") set of fields, which is a novel feature/contribution of our paper. Consequently his $N^*(1535)$ can not decay into $N(940)$ by π emission, in blatant conflict with experiment.

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