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The Road to Extraction of S-Matrix Poles from Experimental Data ⋆

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Abstract. By separating data points close to a resonance into intervals, and fitting all possible intervals to a simple pole with constant coherently added background, we obtained a substantial number of convergent fits. After a chosen set of statistical constraints was imposed, we calculated the average of a resonance pole position from the statistically acceptable results. We used this method to find pole positions of Z boson.

Breit-Wigner (BW) parameters are often used for the description of unstable particles (see *e.g.*, *Review of Particle Physics* [1]), although shortcomings of such choice have been pointed out on numerous occasions. For example, Sirlin showed that the BW parameters of the Z boson were gauge dependent [2]. To resolve this issue he redefined BW parameters, but also suggested usage of the S-matrix poles as an alternative, since poles are fundamental properties of the S-matrix and therefore gauge independent by definition. In a somewhat different study, Höhler advocated using S-matrix poles for characterization of nucleon resonances [3] in order to reduce confusion that arises when different definitions of BW parameters are used [4]. However, loosely defined [5] BW parameters of mesons and baryons are still being extracted from experimental analyses, compared among themselves [1], and used as input to QCD-inspired quark models [6] and as experiment-to-theory matching points for lattice QCD [7].

Our group has been very interested in reducing human and model dependence from resonance parameters' extraction procedures (from scattering matrices). We developed the regularization method for pole extraction from S-matrix elements [8]. Its main disadvantage is that it needs very dense data, one that is attainable only after an energy dependent partial-wave analysis. The other method was the K-matrix pole extraction method [9] which needed the whole unitary S-matrix to begin with, making it impossible to use on any single reaction. Both of those methods were purely mathematical, and the only assumption were that there is a pole in the complex energy plane of an S-matrix. We had no physical input into our procedures. Therefore, we proclaimed these procedures model-independent. The only thing missing, was a method which could be applied directly to the experimental data, *e.g.,* total cross sections.

In this proceeding, we illustrate a method for model-independent extraction of S-matrix pole positions directly from the data.

⋆ Talk delivered by S. Ceci

The first step in devising a method for extraction of the pole parameters from the experimental data is to set up an appropriate parameterization. The parameterization presented here is based on the assumption that close to a resonance, the T matrix will be well described with a simple pole and a constant background. The similar assumption was used in Höhler's speed plot technique [3]. The speed plot is a method used for the pole parameter extraction from the known scattering amplitudes. It is based on calculating the first order energy derivative of the scattering amplitude, with the key assumption that the first derivative of the background is negligible.

The T matrix with a single pole and constant background term is given by

$$T(W) = r_p \, \frac{\Gamma_p/2}{M_p - W - i\,\Gamma_p/2} + b_p, \tag{1}$$

where W is center-of-mass energy, r_b and b_b are complex, while M_p and Γ_p are real numbers. Total cross section is then proportional to $|T|^2/q^2$, where q is the initial center-of-mass momentum. Equation (1), as well as other similar forms (see e.g. [1]), are standardly called Breit-Wigner parameterizations, which can be somewhat misleading since M_p and Γ_p are generally not Breit-Wigner, but pole parameters (hence the index p). The square of the T matrix defined in Eq. (1) is given by

$$|T(W)|^2 = T_\infty^2 \, \frac{(W - M_z)^2 + \Gamma_z^2/4}{(W - M_p)^2 + \Gamma_p^2/4}, \tag{2}$$

where, for convenience, we simplified the numerator by combining the old parameters into three new real-valued ones: T_∞, M_z, and Γ_z. Pole parameters M_p and Γ_p are retained in the denominator.

With such a simple parameterization, it is crucial to use only data points close to the resonance peak. To avoid picking and choosing the appropriate data points by ourselves, we analyzed the data from a wider range around the resonance peak, and fitted localy the parameterization (2) to each set of seven successive data points (seven data points is minimum for our five-parameter fit). Then we increased the number of data points in the sets to eight and fitted again. We continued increasing the number of data points in sets until we fitted the whole chosen range. Such procedure allowed different background term for each fit, which is much closer to reality than assuming a single constant background term for the whole chosen data set (see e.g. discussion on the problems with speed plot in Ref. [8]). In the end, we imposed a series of statistical constraints to all fits to distinguish the good ones. The whole analysis was done in Wolfram Mathematica 8 using NonlinearModelFit routine [11].

Having defined the fitting strategy, we tested the method by applying it to the case of the Z boson. The data set we used is from the PDG compilation [1], and shown in Fig. 1. Extracted pole masses are shown in the same figure: filled histogram bins show pole masses from the good fits, while the empty histogram bins are stacked to the solid ones to show masses obtained in the discarded fits. Height of the pole-mass histogram in Fig. 1 is scaled for convenience.

Extracted S-matrix pole mass and width of Z boson are given in Table 1. The pole masses are in excelent agreement, while the pole widths are reasonably close.

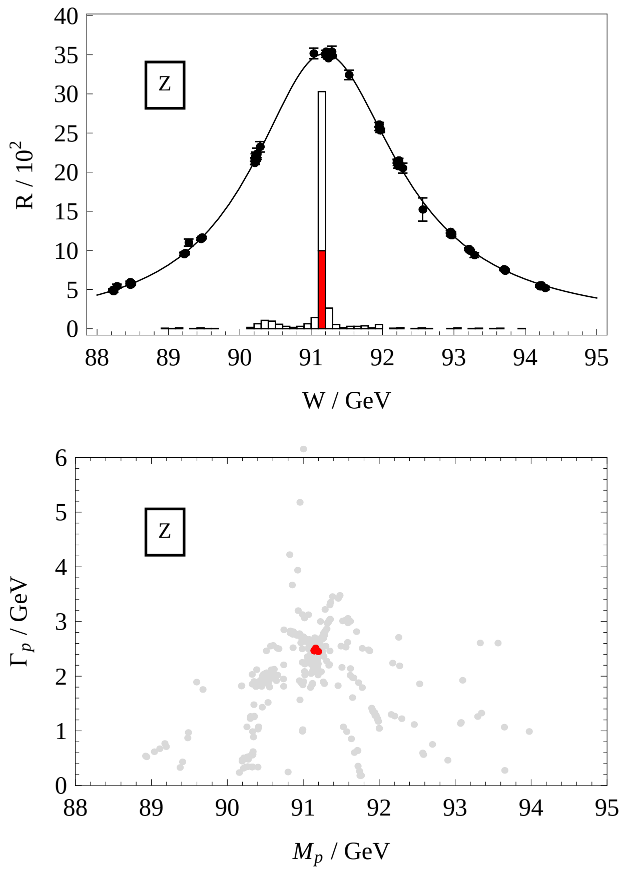

Fig. 1. [Upper figure] PDG compilation of Z data [1] and histogram of obtained pole masses. Line is the fit result with the lowest reduced χ^2 (just for illustration). Dark (red online) colored histogram bins are filled with statistically preferred results. [Lower figure] Pole masses vs. pole widths. Dark (red online) circles show statistically preferred results we use for averages.

It is important to stress that the difference between the pole and BW mass of the Z boson is fundamental and statistically significant. Distribution of discarded and good results is shown in the lower part of Fig. 1.

Table 1. Pole parameters of Z obtained in this work. PDG values of pole and BW parameters are given for comparison.

	Pole	Pole PDG [1]	BW PDG [1]
M/MeV	91159 ± 8	91162 ± 2	91188 ± 2
Γ/MeV	2484 ± 10	2494 ± 2	2495 ± 2

In conclusion, we have illustrated here a model-independent method for extraction of resonance pole parameters from total cross sections and partial waves. Very good estimates for Z boson pole position were obtained.

We are today witnessing the dawn of ab-initio calculations in low-energy QCD. In order to compare theoretical predictions with experimentally determined resonance states, we need first to establish proper point of comparison. We hope that our method, once it becomes fully operational, will help connecting experiment and theory.

References

1. J. Beringer *et al.*, (Particle Data Group), Phys. Rev. D**86**, 010001 (2012)
2. A. Sirlin, Phys. Rev. Lett. **67**, 2127 (1991).
3. G. Höhler, πN Newsletter **9**, 1 (1993).
4. R. E. Cutkosky *et al.*, Phys. Rev. D**20**, 2839 (1979); D. M. Manley and E. M. Saleski, Phys. Rev. D**45**, 4002 (1992).
5. G. Höhler, "A pole-emic" in *Review of particle properties*, D. E. Groom *et al.* Eur. Phys. J. C**15**, 1 (2000).
6. S. Capstick and W. Roberts Prog. Part. Nucl. Phys. **45**, S241-S331 (2000); T. Melde, W. Plessas, and B. Sengl, Phys. Rev. D**77**, 114002 (2008).
7. S. Dürr, *et al.*, Science **322**, 1224 (2008).
8. S. Ceci *et al.*, J. Stahov, A. Švarc, S. Watson, and B. Zauner, Phys. Rev. D**77**, 116007 (2008).
9. S. Ceci, A. Švarc, B. Zauner, D. M. Manley, and S. Capstick, Phys. Lett. **B659**, 228 (2008).
10. *http://gwdac.phys.gwu.edu/analysis/pin_analysis.html* Current PWA solution (June, 2010).
11. *http://reference.wolfram.com/mathematica/ref/NonlinearModelFit.html*