BLED WORKSHOPS
IN PHYSICS
VOL. 8, NO. 1
p. 25

Proceedings of the Mini-Workshop
Hadron Structure and Lattice QCD
Bled, Slovenia, July 9-16, 2007

Covariant description of the few-nucleon systems from chiral dynamics*

L. Girlanda[a], M. Viviani[a] and W.H. Klink[b]

[a] INFN, Sez. di Pisa, Largo Bruno Pontecorvo, 56127 Pisa, Italy
[b] Department of Physics and Astronomy and Department of Mathematics
University of Iowa, Iowa City, Iowa 52242

Abstract. We discuss some aspects of our relativistic framework for the few-nucleon systems (Ref. [1] to which we refer for further details), which were discussed at the Workshop, particularly the issue of renormalization.

The understanding of the few-nucleon systems based on Chiral Perturbation Theory (ChPT, see Ref. [2] for a review), provides the link of nuclear physics with QCD: as a matter of fact the low-energy constants, in terms of which the chiral nuclear forces are expressed, are QCD Green functions, in principle calculable on the lattice. The ChPT setting is perturbative, in the sense that it is a low-energy expansion, the small parameter being the typical momentum p divided by the hadronic scale. This type of ordering is the only justification, from first principles, of the hierarchy of nuclear forces. Indeed ChPT predicts that 3-nucleon forces are suppressed by a factor $O(p^2)$ compared to 2-nucleon forces, 4-nucleon forces by a factor $O(p^4)$, and so on. In order to mantain the power counting a non relativistic expansion of the ChPT Lagrangian is usually performed, referred to as heavy baryon ChPT (HBChPT). Moreover, in the original Weinberg's definition of a nucleon-nucleon effective potential, a non relativistic setting was used, based on old-fashioned (time ordered) perturbation theory. By these two steps relativistic corrections and chiral corrections get mixed together and are treated on the same footing. However relativity and chiral symmetry are symmetries on a completely different status: chiral symmetry (which is always approximate) can be useful in this context as an ordering criterium, whereas Poincaré invariance is required by Nature. There are several instances where one might want to have relativity exactely. Most importantly, a relativistic scheme would allow to describe particle production, which is out of the scope of non-relativistic quantum mechanics. Our aim is therefore to devise a scheme which satisfies all requirements of relativity and use chiral symmetry merely as a bookkeeping device to order terms, in order to have a systematic expansion. This is why we have considered the point-form formulation of relativistic quantum mechanics proposed in [3]. It relies on a Bakamjian-Thomas construction, which is a way (although not the most

* Talk delivered by L. Girlanda

general one) to solve Dirac's covariance problem in the construction of a dynamical theory of interacting particles. The problem constists of finding expressions for the generators of the Poincaré group, H, $\mathbf{P}$, $\mathbf{J}$ and $\mathbf{K}$ in terms of the coordinates of the particles. In the usual formulation (instant-form of the dynamics) the interactions are contained in the Hamiltonian H and in the boost generators $\mathbf{K}$, while the other operators are said kinematical, they are the same as in the non-interacting theory. In the point-form the interactions are contained in all components of the four-momentum, whereas Lorentz transformations are kinematical. In the Bakamjian-Thomas construction this is accomplished by introducing auxiliary operators, the mass operator $M_0 = \sqrt{P_0^\mu P_{0\mu}}$ and a four-velocity operator V^μ such that $P_0^\mu = M_0 V^\mu$ (the subscripts 0 refer to the non-interacting theory); one then adds the interactions only to the mass operator $M = M_0 + M_I$, and reconstructs the interacting four-momentum as $P^\mu = MV^\mu$. Poincaré commutation relations are then satisfied provided the interacting mass operator is a Lorentz scalar which commutes with the four-velocity V^μ. It is therefore particularly convenient to consider the "velocity states" $|v\rangle$ [3]: these are linear combinations of multiparticle momentum states which are eigenstates of the four-velocity operator. However, starting from a quantum-field theoretical Lagrangian the interacting four-momentum is

$$P_I^\mu = \int d^4x \frac{\partial F(x)}{\partial x^\mu} \delta(F(x) - \tau^2) \mathcal{H}_{\mathcal{I}}(x), \tag{1}$$

where in the point-form $F(x) = x^2$. This operator is not diagonal in the four-velocity,

$$\langle v|P_I^\mu|v'\rangle = \langle v|\mathcal{H}_I(0)|v'\rangle \int d^4x \delta(x^2 - \tau^2) 2x^\mu \theta(x_0) e^{-i(mv - m'v')x} \tag{2}$$

and therefore it is not of the Bakamjian-Thomas type. In order to enforce that, one has to introduce a velocity-conserving delta-function by hand, such that the interacting four-momentum has matrix elements of the form

$$\langle v|P_I^\mu|v'\rangle = (2\pi)^3 \delta^3(\boldsymbol{v} - \boldsymbol{v}') v^\mu \frac{f(m, m')}{\sqrt{m^3 m'^3}} \langle v|\mathcal{H}_I(0)|v'\rangle. \tag{3}$$

The form factor $f(m, m')$, depending on the relativistic energies, is meant to compensate somehow for the neglect of the off-diagonal elements in the velocity, and also to regulate the ultraviolet behaviour. The square-root factor in the denominator is included so that one recovers the quantum-field theoretical result when $v = v'$ and $m = m'$ with $f = 1$. We have taken for f a real symmetric function of its arguments, further specified as a Gaussian function centered around zero with cutoff Λ,

$$f(m, m') = \exp\left[-\frac{(m - m')^2}{2\Lambda^2}\right] \xi. \tag{4}$$

The cutoff Λ is to be understood as the scale at which new physics starts to become relevant. There will be one such form factor for each vertex of the interaction Hamiltonian. For some vertices the gaussian alone is not enough to regulate all integrals, so one has to include an additional cutoff ξ, function of the relativistic invariants.

For illustration purposes, we consider the simple case of a scalar nucleon field Ψ interacting with a pion field ϕ, with the interactions provided by a Hamiltonian density of the form $\mathcal{H}(x) = g\Psi^\dagger(x)\Psi(x)\phi(x)$. Creation of nucleon-anti-nucleon pairs is neglected and a truncation of the Fock space to a given maximum number of pions is considered from the beginnning. In the 1-nucleon sector, truncating the states containing two or more pions, the mass operator takes the form

$$M = \begin{pmatrix} m_N + \delta_1^{ren} & gK \\ gK^\dagger & D_{1+1} \end{pmatrix}, \tag{5}$$

where m_N is the physical nucleon mass, and D_{1+1} is the relativistic 1-nucleon + 1-pion free particle energy. The counterterm δ_1^{ren} is needed for the mass renormalization. Due to the form of $\mathcal{H}(x)$, the interactions show up as off-diagonal entries in the mass operator. The nucleon mass renormalization and pion-nucleon scattering are described as eigenvalue-eigenvector problems for this mass operator. For instance, for the eigenvalue m_N, the physical nucleon mass, one finds an equation for the counterterm

$$\delta_1^{ren} = g^2 K^\dagger (D_{1+1} - m_N)^{-1} K, \tag{6}$$

with $D_{1+1} = \omega_\mathbf{k} + \omega_\mathbf{k}^\pi$, having defined $\omega_\mathbf{k} \equiv \sqrt{m_N^2 + \mathbf{k}^2}$ and $\omega_\mathbf{k}^\pi = \sqrt{M_\pi^2 + \mathbf{k}^2}$. Taking the expectation value of the above equation between 1-nucleon states and inserting a complete set of velocity states in the subspace of 1-nucleon + 1-pion states one arrives at the nucleon mass renormalization due to the "pion cloud",

$$\delta_1^{ren} = \frac{g^2}{2m_N} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{4\omega_\mathbf{k}\omega_\mathbf{k}^\pi} \frac{|f^{(1)}(m_N, \omega_\mathbf{k} + \omega_\mathbf{k}^\pi)|^2}{\omega_\mathbf{k} + \omega_\mathbf{k}^\pi - m_N}. \tag{7}$$

The superscript $^{(1)}$ refers to the sector of the Fock space with baryon number 1: the mass operator commutes with the baryon number, and there is the freedom to choose a different structure function f for each sector of the Fock space.

In the 2-nucleon sector, an analogous equation describes the deuteron,

$$(D_2 + \delta_2^{ren})\, \phi_2^D + g^2 K^\dagger (m_D - D_{2+1})^{-1} K \phi_2^D = m_D \phi_2^D, \tag{8}$$

where ϕ_2^D is a state vector in the subspace of 2-nucleon states, and the operators D_2 and D_{2+1} are respectively the relativistic 2-nucleon and 2-nucleon + 1-pion energy. As in the 1-nucleon sector, a counterterm δ_2^{ren} is introduced in the corresponding diagonal element of the mass operator, in order to properly renormalize the 2-particle states. By left-multiplying Eq. (8) with the bra $\langle v, \mathbf{k}, -\mathbf{k}|$ representing a 2-nucleon state with four velocity v and relative momentum (in the center-of-mass system) $2\mathbf{k}$, one arrives, after insertion of a complete set of states in the subspace of 2-nucleon + 1-pion states, to an eigenvalue wave equation for the center-of-mass wave function $\phi_2^D(\mathbf{k}) = \langle v = (1, \mathbf{0}), \mathbf{k}, -\mathbf{k}|\phi_2^D\rangle$,

$$(2\omega_\mathbf{k} + \delta_2^{ren}(\mathbf{k}))\, \phi_2^D(\mathbf{k}) + 2\omega_\mathbf{k} A(\mathbf{k}) \phi_2^D(\mathbf{k}) + \int \frac{d^3\mathbf{q}}{(2\pi)^3} B(\mathbf{k}, \mathbf{q}) \phi_2^D(\mathbf{q}) = m_D \phi_2^D(\mathbf{k}), \tag{9}$$

The term proportional to $A(\mathbf{k})$ represents a wave function renormalization of the two-nucleon state: it describes diagrams in which the nucleon lines are disconnected and dressed with pion loops. Its explicit expression reads

$$A(\mathbf{k}) = \int \frac{d^3\mathbf{q}}{(2\pi)^3} \left\{ \frac{g^2}{16\omega_{\mathbf{k}}^2 \omega_{\mathbf{q}} \omega_{\mathbf{k}+\mathbf{q}}^{\pi}} \frac{\left| f^{(2)}(2\omega_{\mathbf{k}}, \omega_{\mathbf{k}} + \omega_{\mathbf{q}} + \omega_{\mathbf{k}+\mathbf{q}}^{\pi}) \right|^2}{m_D - \omega_{\mathbf{k}} - \omega_{\mathbf{q}} - \omega_{\mathbf{k}+\mathbf{q}}^{\pi}} + \mathbf{q} \leftrightarrow -\mathbf{q} \right\}. \tag{10}$$

We can choose the counterterm δ_2^{ren} so as to cancel the disconnected kernel, $\delta_2^{\text{ren}}(\mathbf{k}) = -2\omega_{\mathbf{k}} A(\mathbf{k})$. Correspondingly, the NN scattering is described by the Lippmann-Schwinger equation for the scattering amplitude,

$$T(\mathbf{q}, \mathbf{k}) = V(\mathbf{q}, \mathbf{k}) + \int \omega_{\mathbf{p}} \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{V(\mathbf{q}, \mathbf{p}) T(\mathbf{p}, \mathbf{k})}{\sqrt{s} - 2\omega_{\mathbf{p}} + i\epsilon}, \tag{11}$$

where the potential consists only of the connected kernel B,

$$V(\mathbf{q}, \mathbf{k}) = g^2 \langle \nu, \mathbf{q}, -\mathbf{q} | \, K^\dagger \left[\sqrt{s} - D_{2+1} \right]^{-1} K \Big|_{\text{conn}} | \nu, \mathbf{k}, -\mathbf{k} \rangle = B(\mathbf{q}, \mathbf{k}). \tag{12}$$

The renormalization of the 2-nucleon lines describing NN scattering, realized by the choice of the counterterm $\delta_2^{\text{ren}}(\mathbf{k}) = -2\omega_{\mathbf{k}} A(\mathbf{k})$, and of the 1-nucleon line, Eq. (7), correspond to the same physical processes, as can be seen diagrammatically. Physical considerations would require that, when the two nucleons are far apart and at rest, their energies should be renormalized as their respective masses. This implies the condition

$$\delta_2^{\text{ren}}(\mathbf{0}) = 2\delta_1^{\text{ren}}, \tag{13}$$

which can be regarded as the manifestation of the cluster decomposition principle in the simple case of two particles. We can see by direct inspection, replacing in Eq. (10) m_D by $\sqrt{s} = 2m_N$, since we are considering the case of two widely separated nucleons at rest, that the equation is fulfilled provided $f^{(1)} = f^{(2)} = f$, with f depending on $m - m'$ as in Eq. (4), independently of the baryon number sector. Notice that this would not happen had we chosen the original formulation of Ref. [3]: the crucial point was the inclusion of a different normalization for the matrix elements of the interacting mass operator, Eq. (3), which in turn was dictated by a proper matching to the quantum field theory. The cluster decomposition principle, satisfied by local quantum field theories, could in general be violated by a truncation of the full quantum field theory to a relativistic quantum mechanics. In view of the above consideration, we can drop the superscripts and use the same structure function f for all sectors of the Fock space.

Having identified the general features of the construction of the interacting mass operator from a vertex Lagrangian, one can proceed to make full use of the constraints given by chiral symmetry. Most importantly, the Goldstone theorem requires that the coupling between pion and nucleons be of derivative type (suppressed at low energy). This provides a power-counting justification for the truncation of the Fock space, since the creation of pions brings more and more powers of momentum. The complete combined analysis of πN and NN systems at the leading order of the chiral counting can be found in Ref. [1]. By comparison with the non relativistic limit (realized in our framework as $m_N \to \infty$), the (all order) relativistic effects are found to be smaller than the NLO chiral corrections, in the NN case, while they are sizeable in the πN case.

I thank Mitja Rosina, Bojan Golli and Simon Sirca for the very nice Workshop.

References

1. L. Girlanda, M. Viviani and W. H. Klink, Phys. Rev. C **76**, 044002 (2007).
2. E. Epelbaum, Prog. Part. Nucl. Phys. **5**7 (2006) 654.
3. W. H. Klink, Nucl. Phys. A **7**16 (2003) 123; Phys. Rev. C **5**8 (1998) 3617.