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The genus crossing number

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Abstract

Pach and Tóth [5] introduced a new version of the crossing number parameter, called the degenerate crossing number, by considering proper drawings of a graph in the plane and counting multiple crossing of edges through the same point as a single crossing when all pairwise crossings of edges at that point are transversal. We propose a related parameter, called the genus crossing number, where edges in the drawing need not be represented by simple arcs. This relaxation has two important advantages. First, the genus crossing number is invariant under taking subdivisions of edges and is also a minor-monotone graph invariant. Secondly, it is "computable" in many instances, which is a rare phenomenon in the theory of crossing numbers. These facts follow from the proof that the genus crossing number is indeed equal to the non-orientable genus of the graph. It remains an open question if the genus crossing number can be strictly smaller than the degenerate crossing number of Pach and Tóth. A relation to the minor crossing number introduced by Bokal, Fijavž, and Mohar [1] is also discussed.

Keywords: Crossing number, genus.

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Rodno presečno število

Povzetek

Pach in Tóth [5] sta vpeljala novo različico presečno-številskega parametra, t.i. *degenerirano presečno število*: na risbah grafa v ravnini sta štela večkratna presečišča povezav skozi isto točko kot eno samo presečišče, kadar so se vsi pari povezav v tej točki sekali transverzalno. Predlagamo soroden parameter, t.i. *rodno presečno število*, pri katerem povezave na risbi niso nujno predstavljene z enostavnimi loki. Ta sprostitev ima dve pomembni prednosti. Prvič, rodno presečno število je invarianta za subdividiranje povezav, pa tudi minorsko-monotona grafovska invarianta. Drugič, v veliko primerih "se da izračunati", kar je v teoriji presečnih števil redek pojav. Omenjeni dejstvi sledita iz dokaza, da je rodno presečno število dejansko enako ne-orientabilnemu rodu grafa. Vprašanje, ali je rodno presečno število strogo manjše od degeneriranega presečnega števila Pacha in Tótha, ostaja odprto. Obravnavana je tudi zveza z minorskim presečnim številom, ki so ga vpeljali Bokal, Fijavž, and Mohar [1].

Ključne besede: Presečno število, rod.