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Performance evaluation of Call-center with call redirection

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Annotation. The object of investigation is an analytical model of a Call-center functioning with a traffic distribution (call redirection) mechanism. Call-center functioning is described by the Markov process. A solution for stationary distribution is found and expressions for the main performance characteristics for the Call-center functioning are given.

Keywords: Call-center, call redirection, analytical model, performance evaluation

Ocena performanc klicnega centra s preusmerjanjem klicev

Povzetek. Objekt raziskave je analitični model klicnega centra z vgrajenim mehanizmom za preusmerjanje klicev. Delovanje klicnega centra sva opisala z Markovskim procesom. Za model sva podala rešitev za stacionarno porazdelitev, prikazala sva tudi rešitve za glavne performančne karakteristike.

Ključne besede: klicni center, preusmerjanje klicev, analitični model, ocena performanc

1 Model Description

Development of Call-center functioning schemes has given rise to investigations in new call-management schemes, changes in the number of agents and call forwarding [1-4]. Let's examine a Call-center that consists of two groups of agents: G1 with capacity C_1 and G2 with capacity C_2 (see Fig.1). The incoming calls to G1(2) are presented by Poisson arrivals with the $\lambda_1(\lambda_2)$ density. The times of call processing by any agent of G1(2) are independent random variables distributed according to the exponential law with the $\mu_1(\mu_2)$ parameter.

An incoming call to G1(2) call is redirected to G2(1) for processing with a probability of $g_{1,i}$ $(g_{2,j})$, which depends on $i\,(j)$ number of busy agents in G1(2). Otherwise the call is processed by its own group G1(2) with additional probability $\bar{g}_{1,i}$ $(\bar{g}_{2,j})$ if there are available agents in the G1(2) group. If all agents in groups are occupied, the call is lost and it won't be transferred again.

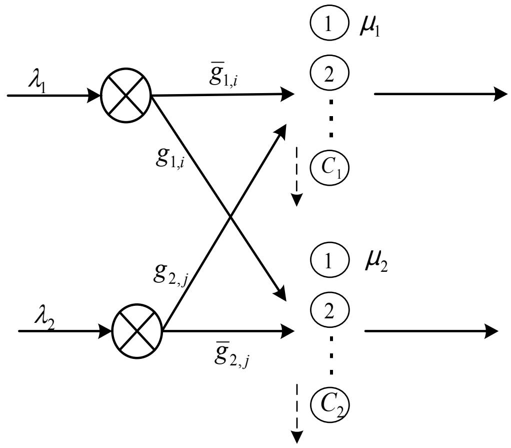

Figure 1. A generalized analytical model of a Call-center with call redirection.

Let $\{\nu_1(t),\nu_2(t)\}$ be a random variable describing the number of calls being processed in G1 and G2, respectively. We examine the Markov process $\{\nu_1(t),\nu_2(t),t\ge 0\}$ with the state space $X = X_1 \times X_2$, $X_1 = \{0,1,...,C_1\}$, $X_2 = \{0,1,...,C_2\}$. As all states of the process communicate and their number is finite, stationary probability distribution $p(i,j) = \lim_{t\to\infty} p_{i,j}(t)$, $p_{i,j}(t) = P\{\nu_1(t)=i,\nu_2(t)=j,t\ge 0\}$ exists [5] and it can be obtained via a system of equilibrium equations of $|X|$ dimension and $|X|-1$ rank of the form

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$$\vec{\mathbf{p}}^{\mathbf{T}}\mathbf{A} = \vec{\mathbf{0}}^{\mathbf{T}} \tag{1}$$

and normalizing condition

$$\vec{\mathbf{p}}^{\mathbf{T}}\vec{\mathbf{1}} = 1, \tag{2}$$

where

$\vec{\mathbf{p}}^{\mathbf{T}} = (\vec{\mathbf{p}}_0^{\mathbf{T}}, \vec{\mathbf{p}}_1^{\mathbf{T}}, \ldots, \vec{\mathbf{p}}_{C_1}^{\mathbf{T}})$, $\vec{\mathbf{p}}_i^{\mathbf{T}} = (p(i,0), p(i,1), \ldots, p(i,C_2))$.

Matrix $\mathbf{A}$ is of the following form:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_0 & \mathbf{D}_0 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{B}_1 & \mathbf{A}_1 & \mathbf{D}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_2 & \mathbf{A}_2 & \mathbf{D}_2 & \mathbf{0} & \mathbf{0} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{C_1-1} & \mathbf{A}_{C_1-1} & \mathbf{D}_{C_1-1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{C_1} & \mathbf{A}_{C_1} \end{bmatrix},$$

where

$$\mathbf{A}_n = \left\| a_{i,j}^n \right\|_{i,j=\overline{0,C_2}}, n = \overline{0,C_1},$$

$$\mathbf{B}_n = \left\| b_{i,j}^n \right\|_{i,j=\overline{0,C_2}}, n = \overline{1,C_1},$$

$$\mathbf{D}_n = \left\| d_{i,j}^n \right\|_{i,j=\overline{0,C_2}}, n = \overline{0,C_1-1}$$

and $\mathbf{0}$ is a zero quadratic matrix of the C_2 order.

Let $\gamma_{n,j} = \bar{g}_{1,n}\lambda_1 + g_{2,j}\lambda_2, \theta_{n,j} = g_{1,n}\lambda_1 + \bar{g}_{2,j}\lambda_2.$

Forms of elements of the $\mathbf{A}_n$, $\mathbf{B}_n$, $\mathbf{D}_n$ matrices are:

$$a_{i,j}^n = \begin{cases} -\gamma_{n,j}^{u(C_1-n)} - \theta_{n,j}^{u(C_2-j)} - n\mu_1 - j\mu_2, \ j = i, \\ \quad i = \overline{0,C_2}, \\ \theta_{n,j}, j = i+1, i = \overline{0,C_2-1}, \\ j\mu_2, \ j = i-1, i = \overline{1,C_2}, \\ 0, \text{otherwise}, \end{cases}$$

$$b_{i,j}^n = \begin{cases} n\mu_1, \ j = i, i = \overline{0,C_2}, \\ 0, \text{otherwise}, \end{cases}$$

$$d_{i,j}^n = \begin{cases} \gamma_{n,j}, j = i, i = \overline{0,C_2}, \\ 0, \text{otherwise}. \end{cases}$$

Here $u(x) = \begin{cases} 1, x > 0, \\ 0, x \le 0. \end{cases}$

The solution of the system in Eq. (1) is presented in the form of $\vec{\mathbf{p}}_{n+1}^{\mathbf{T}} = \vec{\mathbf{p}}_0^{\mathbf{T}}\tilde{\mathbf{A}}_n$, where

$$\tilde{\mathbf{A}}_n = \begin{cases} -\frac{1}{\mu_1}\mathbf{A}_0, \ n = 0, \\ -\frac{1}{2\mu_1}(\mathbf{D}_0 + \tilde{\mathbf{A}}_0\mathbf{A}_1), \ n = 1, \\ -\frac{1}{(n+1)\mu_1}(\tilde{\mathbf{A}}_{n-2}\mathbf{D}_{n-1} + \tilde{\mathbf{A}}_{n-1}\mathbf{A}_n), \ n = \overline{2,C_1-1}. \end{cases}$$

The vector $\vec{\mathbf{p}}_0$ is determined through the equation system

$$\vec{\mathbf{p}}_0^{\mathbf{T}}(\tilde{\mathbf{A}}_{C_1-2}\mathbf{D}_{C_1-1} + \tilde{\mathbf{A}}_{C_1-1}\mathbf{A}_{C_1-1}) = \vec{\mathbf{0}}^{\mathbf{T}},$$

and the normalizing condition in Eq. (2) obtaining the form

$$\vec{\mathbf{p}}_0^{\mathbf{T}}(\mathbf{I} + \sum_{n=1}^{C_1}\tilde{\mathbf{A}}_{n-1})\vec{\mathbf{1}} = 1.$$

It is also possible to find the solution with other methods, for example with *LU*-decomposition.

The model provides an opportunity to examine and investigate different schemes of traffic redirection. This is done just by setting the corresponding $g_{1,i}$ and $g_{2,j}$ probabilities distribution. If $g_{1,i} = 0$ and $g_{2,j} = 0$, we get a standard model of the Call-center functioning without traffic redirection (model 1, m_1). If $g_{1,i} = 0$, $i = \overline{0,C_1-1}$, $g_{1,C_1} = 1$ and $g_{2,j} = 0$, $j = \overline{0,C_2-1}$, $g_{2,C_2} = 1$, we get a model with partial traffic redirection for the cases of G1 and G2 overload (model 2, m_2).

The above proposed generalized model (model 3, m_3) enables investigation of different combinations of redirection mechanisms. We will use notation $m_l\_m_k$, which means that model m_l is implemented in G1, and model m_k is implemented in G2, $l,k = \overline{1,3}$. Model $m_l\_m_l$ corresponds to the case of a homogeneous model, and model $l \ne k$ corresponds to the case of a heterogeneous model.

2 Performance Evaluation Characteristics

Now that we know the probability distribution of a number of busy agents in G1 and G2 groups, we can calculate the necessary performance characteristics characterizing the effectiveness of the models under investigation.

Let π be the loss probability in the system. In 2_2, 2_3, 3_2 and 3_3 models call loss occurs due to occupation of all C_1 and C_2 agents in G1 and G2, i.e. $\pi = p(C_1, C_2)$.

In the 1_1 model the loss probability, π, is calculated according to the following formula

$$\pi = \frac{1}{\lambda_1 + \lambda_2}\left(\pi_1 \lambda_1 + \pi_2 \lambda_2\right),$$

where $\pi_1 = \sum_{j=0}^{C_2} p(C_1, j)$ ($\pi_2 = \sum_{i=0}^{C_1} p(i, C_2)$) is the stationary probability of the call loss in G1 (G2).

In 1_2 and 1_3 models the call loss occurs in case of occupation of all G1 agents. Thus the loss probability, π, for the 1_2 and 1_3 models is calculated according to the formula

$$\pi = \frac{1}{\lambda_1 + \lambda_2}\sum_{i=0}^{C_2} p(C_1, i)\left(\lambda_1 + g_{2,i}\lambda_2\right).$$

Similarly, the loss probability, π, for the 2_1 and 3_1 models is

$$\pi = \frac{1}{\lambda_1 + \lambda_2}\sum_{i=0}^{C_1} p(i, C_2)\left(g_{1,i}\lambda_1 + \lambda_2\right).$$

Let p_0 be the probability of an idle Call-center, i.e. the probability of absence of any call (it is valid for all models). Thus, p_0 is equal to the probability of $\nu_{1,2}(t) = \{\nu_1(t), \nu_2(t), t \geq 0\}$ process being in $(0,0)$ state.

The average number of busy agents, Q_1 (Q_2), in G1 (G2) group, respectively, is determined in the following way:

$$Q_1 = \sum_{i=1}^{C_1} i \sum_{j=0}^{C_2} p(i, j) \quad \left(Q_2 = \sum_{j=1}^{C_2} j \sum_{i=0}^{C_1} p(i, j)\right).$$

3 Case studies

Let's conduct a numerical analysis of the loss probability for the investigated model and its alternate versions provided the following parameters apply:

a) symmetrical case: $C_1 = 10$, $C_2 = 10$; $\mu_1 = 1$, $\mu_2 = 1$; $\lambda = \lambda_1 + \lambda_2$ is from 0 to 20 (λ_1 and λ_2 are from 0 to 10); for m_1 $g_{1,i}(g_{2,i}) = 0, \forall i = \overline{0, C_{1(2)}}$, for m_2 $g_{1,i}(g_{2,i}) = 0, \forall i = \overline{0, C_{1(2)} - 1}$, $g_{1,C_1}(g_{2,C_2}) = 1$ for m_3 $g_{1,i}(g_{2,i}) = 0.5, \forall i = \overline{0, C_{1(2)}}$

b) nonsymmetrical case: $C_1 = 5$, $C_2 = 15$; other parameters are identical to the previous case.

The diagram of relationship between the loss probability and the incoming traffic density for the homogeneous and heterogeneous models of the symmetrical case is shown in Figures 2-4.

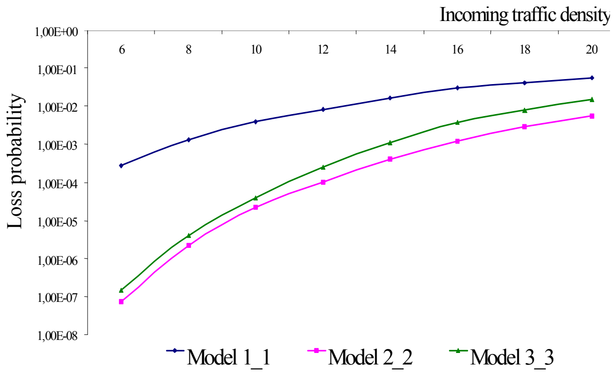

Figure 2. Relationship between π and λ for models $m_l\_m_l$, $l = \overline{1,3}$, case a).

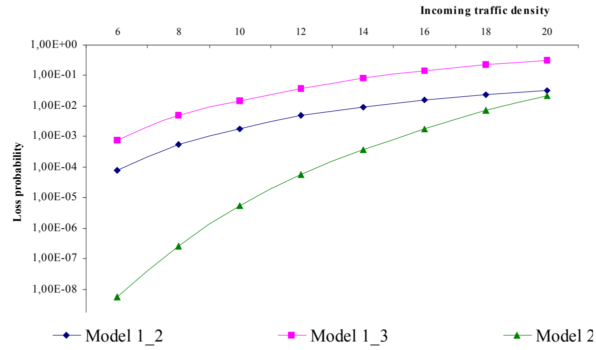

Figure 3. Relationship between π and λ for heterogeneous models, case a).

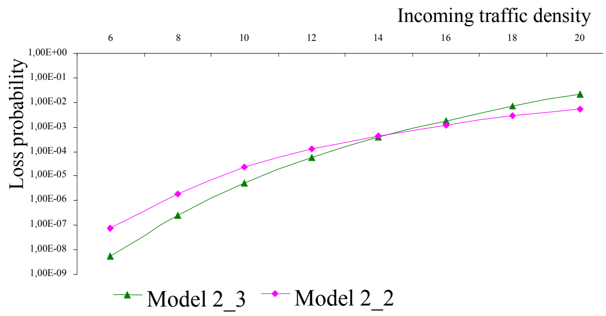

Figure 4. Relationship between π and λ for homogeneous and heterogeneous models, case a).

As expected, an increase in the incoming traffic density is followed by an increase of the loss probability. As shown in Figure 2, the greater loss probability corresponds to the 1_1 model regardless of incoming traffic density. It is caused by the loss in the system that becomes possible as soon as all the agents of any group are busy, regardless of idle devices availability in another group.

The smallest values of the loss probability correspond to the 2_2 model. Here call redirection is implemented only when all agents are busy in G1 or G2. In the 3_3 model calls can be redirected to another group even if one device is busy. This enables an additional load to be transmitted to a neighboring group.

Thus, in terms of the loss probability the most effective among the homogeneous models is the 2_2 model.

As for the heterogeneous models (see Figure 3), the greatest loss probability occurs in the 1_3 model. In the 1_2 and 1_3 models the traffic is redirected to G1, where the redistribution mechanism is not implemented. This causes an increase in the loss probability. As traffic redirection can be implemented earlier and more intensively in m_3 than it happens in m_2, then the loss probability, which corresponds to the 1_3 model, is higher than that of the 1_2 model. The least loss probability corresponds to the 2_3 model. It is caused by the redistribution mechanism implemented in both groups, and the loss becomes possible only after all the agents are busy in each of the groups.

Comparing the loss probabilities for the 2_2 and 2_3 models (see Figure 4), we can see that in case of low and medium incoming traffic density ($0 \le \lambda \le 12$) the loss probability in the 2_2 model is higher than that of the 2_3 model, but in case of high incoming traffic density ($14 \le \lambda$) the loss probability in the 2_2 model appears to be lower than that of the 2_3 model. In case of low load a relatively small call flow is redirected from G1 to G2 in the 2_3 model, but in case of a high load the number of redirected calls increases and causes an increase in the loss.

The diagrams of relationship between the loss probability and the incoming traffic density for homogeneous and heterogeneous models of the nonsymmetrical case are shown in Figures 5-9. In this case the i_j and j_i models are different and the diagrams for comparing the loss probability are given for both variants of models.

As shown in Figures 5-6, the loss probability for the 1_3 and 1_2 models significantly exceeds the loss probability for the 3_1 and 3_2 models. This is caused by the fact that in the first case (1_3 and 1_2) the traffic comes from another group to a group where the redistribution mechanism is not implemented and the number of agents is smaller.

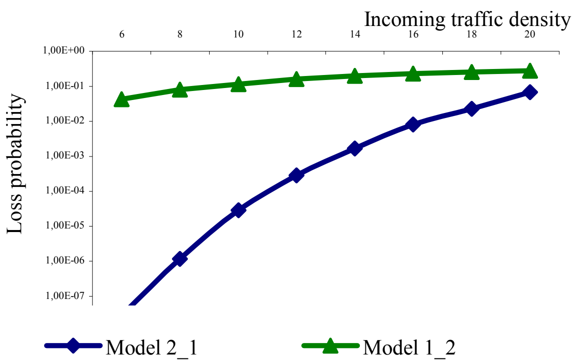

Figure 5. Relationship between π and λ for models 1_2, 2_1, case b).

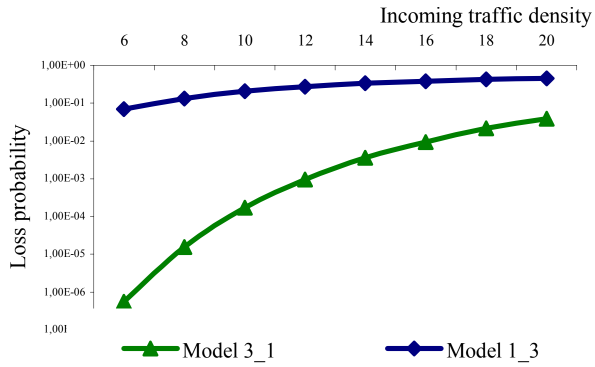

Fig. 6. Relationship between π and λ for models 1_3, 3_1, case b).

In case of low and medium incoming traffic density the loss probability in the 2_3 model is lower than that of the 3_2 model (see Figure 7), but in case of high incoming traffic density the loss probability in the 2_3 model exceeds the loss probability in the 2_3 model.

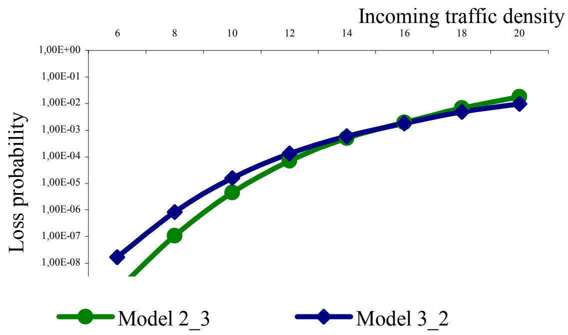

Figure 7. Relationship between π and λ for models 3_2, 2_3, case b).

Here the most effective among the homogeneous models is the 3_3 model (see Figure 8). Similarly to the symmetrical case, this can be explained through characteristics of the redistribution mechanism, implemented in m_3.

Comparing the loss probability for the 3_3, 3_2, 3_1, 2_1, 2_3 models (see Figure 9), we can conclude that in case of a low and medium incoming traffic density the least values of the loss probability correspond to the 2_3 model, and in case of a high incoming traffic density they correspond to the 2_3 model. It should be noted that in case of high values of λ the difference between the loss probability values for the 3_2 and 3_3 models is small (lower than 0.01).

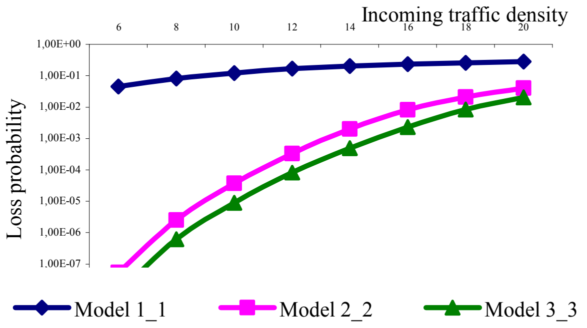

Figure 8. Relationship between π and λ for homogeneous models, case b).

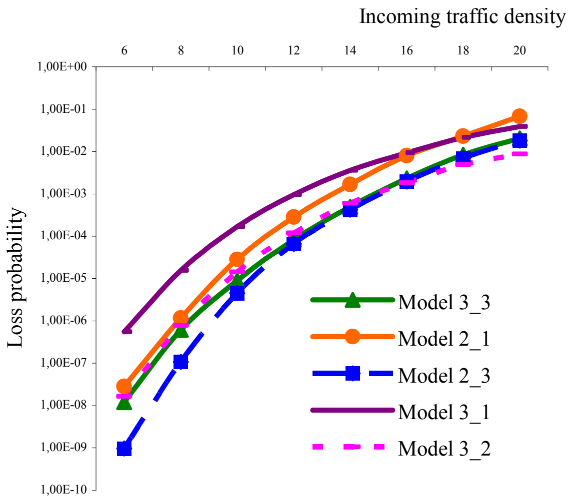

Figure 9. Relationship between π and λ for models 3_3, 3_2, 3_1, 2_1, 2_3, case b).

According to Figure 9 for the case of low and medium λ we can conclude that if m_3 is implemented in a group with a smaller number of agents, then the loss probability is greater compared to similar models, where m_2 is implemented in a group with a smaller number of agents. However, in case of a high traffic density smaller values of π correspond to models where m_3 is implemented in G1. It may be concluded that in case of implementing heterogeneous models in conditions of a high incoming traffic density it is advisable to implement m_3 in bottlenecks.

4 Conclusion

The proposed analytical model of Call-center operation with redirection of calls between groups of agents enables an investigation in different call traffic management schemes for the cases of agents overload as well as for the cases of agents underload.

The numerical analysis proved effectiveness of the applied call redirection procedure.

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