5 $\Delta F = 2$ in Neutral Mesons From a Gauged $SU(3)_F$ Family Symmetry

A. Hernandez-Galeana ⋆

Departamento de Física, ESFM - Instituto Politécnico Nacional.
U. P. "Adolfo López Mateos". C. P. 07738, Ciudad de México, México

Abstract. Within a broken local gauge vector-like $SU(3)_F$ family symmetry, we study some $\Delta F = 2$ processes induced by the tree level exchange of the new massive horizontal gauge bosons, which introduce flavor-changing couplings. We find out that some of the dangerous FCNC processes, like for instance; $K^o - \bar{K^o}$, $D^o - \bar{D^o}$ mixing, may be properly suppressed if the first stage of the Spontaneous Symmetry Breaking (SSB), $SU(3)_F \to SU(2)_F$, occurs at a high scale $\Lambda \sim 10^{11}$ GeV, with the $SU(2)_F$ gauge bosons acting on the light families. We provide a parameter space region where this framework can accommodate the hierarchical spectrum of quark masses and mixing and simultaneously suppress properly the contribution to $K^o - \bar{K^o}$ mixing as well as the $\mathcal{O}_{LL}$ and $\mathcal{O}_{RR}$ effective operators for the $\Delta C = 2$ processes.

Povzetek. Avtor obravnava procese, pri katerih se družinsko kvantno število spremeni za 2. Uporabi model, v katerem opiše družinsko kvantno število kvarkov in leptonov z grupo SU3, lokalna umeritvena polja grupe SU3 pa poskrbijo za interakcijo med fermioni, ki nosijo ustrezna kvantna števila. Masivni umeritveni bozoni dopuščajo sicer nevtralne prehode (FCNC) med fermioni iste družine, vendar so taki prehodi, kot primer navaja mešanje $K^o - \bar{K^o}$ ter $D^o - \bar{D^o}$, dovolj malo verjetni, če le pride do spontane zlomitve družinske simetrije $SU(3)_F \to SU(2)_F$ pri energiji $\Lambda \sim 10^{11}$ GeV. Poišče območje parametrov, v katerem imajo kvarki opazljive lastnosti.

Keywords: Quark and lepton masses and mixing, Flavor symmetry, $\Delta F = 2$ Processes.
Pacs: 14.60.Pq, 12.15.Ff, 12.60.-i

5.1 Introduction

Flavor physics and rare processes play an important role to test any Beyond Standard Model(BSM) physics proposal, and hence, it is crucial to explore the possibility to suppress properly these type of flavor violating processes.

Within the framework of a vector-like gauged $SU(3)_F$ family symmetry model[1,2], we study the contribution to $\Delta F = 2$ processes[3]-[6] in neutral mesons

⋆ E-mail: albino@esfm.ipn.mx

at tree level exchange diagrams mediated by the gauge bosons with masses of the order of some TeV's, corresponding to the lower scale of the $SU(3)_F$ family symmetry breaking.

The reported analysis is performed in a scenario where light fermions obtain masses from radiative corrections mediated by the massive bosons associated to the broken $SU(3)_F$ family symmetry, while the heavy fermions; top and bottom quarks and tau lepton become massive from tree level See-saw mechanisms. Previous theories addressing the problem of quark and lepton masses and mixing with spontaneously broken $SU(3)$ gauge symmetry of generations include the ones with chiral local $SU(3)_H$ family symmetry as well as other $SU(3)$ family symmetries. See for instance [7]-[20] and references therein.

5.2 $SU(3)_F$ flavor symmetry model

The model is based on the gauge symmetry

$$G \equiv SU(3)_F \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \tag{5.1}$$

where $SU(3)_F$ is a completely vector-like and universal gauged family symmetry. That is, the corresponding gauge bosons couple equally to Left and Right Handed ordinary Quarks and Leptons, with g_H, g_s, g and g' the corresponding coupling constants. The content of fermions assumes the standard model quarks and leptons:

$$\Psi_q^o = (3, 3, 2, \frac{1}{3})_L \quad , \quad \Psi_l^o = (3, 1, 2, -1)_L$$

$$\Psi_u^o = (3, 3, 1, \frac{4}{3})_R \quad , \quad \Psi_d^o(3, 3, 1, -\frac{2}{3})_R \quad , \quad \Psi_e^o = (3, 1, 1, -2)_R$$

where the last entry is the hypercharge Y, with the electric charge defined by $Q = T_{3L} + \frac{1}{2}Y$.

The model includes two types of extra fermions:

- Right Handed Neutrinos: $\Psi_{\nu_R}^o = (3, 1, 1, 0)_R$, introduced to cancel anomalies [21],
- and a new family of $SU(2)_L$ weak singlet vector-like fermions:

$$U_L^o, U_R^o = (1, 3, 1, \frac{4}{3}) \quad , \quad D_L^o, D_R^o = (1, 3, 1, -\frac{2}{3}) \tag{5.2}$$

Vector Like electrons: $E_L^o, E_R^o = (1, 1, 1, -2)$

and

New Sterile Neutrinos: $N_L^o, N_R^o = (1, 1, 1, 0)$, The particle content and gauge symmetry assignments are summarized in Table 5.1. Notice that all $SU(3)_F$ non-singlet fields transform as the fundamental representation under the $SU(3)_F$ symmetry.

	$SU(3)_F$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
ψ_q^o	3	3	2	$\frac{1}{3}$
ψ_{uR}^o	3	3	1	$\frac{4}{3}$
ψ_{dR}^o	3	3	1	$-\frac{2}{3}$
ψ_l^o	3	1	2	-1
ψ_{eR}^o	3	1	1	-2
$\psi_{\nu R}^o$	3	1	1	0
Φ^u	3	1	2	-1
Φ^d	3	1	2	+1
η_i	3	1	1	0
$U_{L,R}^o$	1	3	1	$\frac{4}{3}$
$D_{L,R}^o$	1	3	1	$-\frac{2}{3}$
$E_{L,R}^o$	1	1	1	-2
$N_{L,R}^o$	1	1	1	0

Table 5.1. Particle content and charges under the gauge symmetry

5.3 $SU(3)_F$ family symmetry breaking

To implement the SSB of $SU(3)_F$, we introduce the flavon scalar fields: $\eta_i, \ i = 2, 3$,

$$\eta_i = (3, 1, 1, 0) = \begin{pmatrix} \eta_{i1}^o \\ \eta_{i2}^o \\ \eta_{i3}^o \end{pmatrix} \quad , \quad i = 2, 3$$

with the "Vacuum ExpectationValues" (VEV's):

$$\langle \eta_2 \rangle^T = (0, \Lambda_2, 0) \quad , \quad \langle \eta_3 \rangle^T = (0, 0, \Lambda_3) \, . \tag{5.3}$$

It is worth to mention that these two scalars in the fundamental representation is the minimal set of scalars to break down completely the $SU(3)_F$ family symmetry. The interaction Lagrangian of the $SU(3)_F$ gauge bosons to the SM massless fermions is

$$i\mathcal{L}_{int,SU(3)_F} = g_H \ \left(\bar{f_1^o} \ \bar{f_2^o} \ \bar{f_3^o} \right) \gamma_\mu \begin{pmatrix} \frac{Z_1^\mu}{2} + \frac{Z_2^\mu}{2\sqrt{3}} & \frac{Y_1^{+\mu}}{\sqrt{2}} & \frac{Y_2^{+\mu}}{\sqrt{2}} \\ \frac{Y_1^{-\mu}}{\sqrt{2}} & -\frac{Z_1^\mu}{2} + \frac{Z_2^\mu}{2\sqrt{3}} & \frac{Y_3^{+\mu}}{\sqrt{2}} \\ \frac{Y_2^{-\mu}}{\sqrt{2}} & \frac{Y_3^{-\mu}}{\sqrt{2}} & -\frac{Z_2^\mu}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} f_1^o \\ f_2^o \\ f_3^o \end{pmatrix}$$

where g_H is the $SU(3)_F$ coupling constant, Z_1, Z_2 and $Y_j^{\pm} = \frac{Y_j^1 \mp i Y_j^2}{\sqrt{2}}$, $j = 1, 2, 3$ are the eight gauge bosons.

Thus, the contribution to the horizontal gauge boson masses from the VEV's in Eq.(5.3) read

- $\langle \eta_2 \rangle$: $\frac{g_{H_2}^2 \Lambda_2^2}{2}(Y_1^+ Y_1^- + Y_3^+ Y_3^-) + \frac{g_{H_2}^2 \Lambda_2^2}{4}(Z_1^2 + \frac{Z_2^2}{3} - 2Z_1 \frac{Z_2}{\sqrt{3}})$
- $\langle \eta_3 \rangle$: $\frac{g_{H_3}^2 \Lambda_3^2}{2}(Y_2^+ Y_2^- + Y_3^+ Y_3^-) + g_{H_3}^2 \Lambda_3^2 \frac{Z_2^2}{3}$

The "Spontaneous Symmetry Breaking" (SSB) of $SU(3)_F$ occurs in two stages

$$SU(3)_F \times G_{SM} \xrightarrow{\langle \eta_3 \rangle} SU(2)_F \ ? \times G_{SM} \xrightarrow{\langle \eta_2 \rangle} G_{SM}$$

FCNC ?

Λ_3: 5 very heavy boson masses ($\geq 100\,\mathrm{TeV's}$)

Λ_2: 3 heavy boson masses (may be a few $\mathrm{TeV's}$).

Notice that the hierarchy of scales $\Lambda_3 \gg \Lambda_2$ define an "approximate $SU(2)$ global symmetry" in the spectrum of $SU(2)_F$ gauge boson masses. To suppress properly the FCNC like, for instance: $\mu \to e\gamma$, $\mu \to e\,e\,e$, $K^o - \bar{K^o}$, and $D^o - \bar{D^o}$, it is crucial to choose properly the $SU(2)_F$ symmetry at the lower scale.

Therefore, neglecting tiny contributions from electroweak symmetry breaking, we obtain the gauge boson mass terms.

$$M_2^2\, Y_1^+ Y_1^- + M_3^2\, Y_2^+ Y_2^- + (M_2^2 + M_3^2)\, Y_3^+ Y_3^- + \frac{1}{2} M_2^2\, Z_1^2 + \frac{1}{2} \frac{M_2^2 + 4M_3^2}{3}\, Z_2^2 - \frac{1}{2}(M_2^2) \frac{2}{\sqrt{3}}\, Z_1\, Z_2 \quad (5.4)$$

$$M_2^2 = \frac{g_H^2 \Lambda_2^2}{2} \quad , \quad M_3^2 = \frac{g_H^2 \Lambda_3^2}{2} \quad , \quad y \equiv \frac{M_3}{M_2} = \frac{\Lambda_3}{\Lambda_2} \quad (5.5)$$

	Z_1	Z_2
Z_1	M_2^2	$-\frac{M_2^2}{\sqrt{3}}$
Z_2	$-\frac{M_2^2}{\sqrt{3}}$	$\frac{M_2^2 + 4M_3^2}{3}$

Table 5.2. $Z_1 - Z_2$ mixing mass matrix

Diagonalization of the $Z_1 - Z_2$ squared mass matrix yield the eigenvalues

$$M_-^2 = \frac{2}{3}\left(M_2^2 + M_3^2 - \sqrt{(M_3^2 - M_2^2)^2 + M_2^2 M_3^2}\right) \tag{5.6}$$

$$M_+^2 = \frac{2}{3}\left(M_2^2 + M_3^2 + \sqrt{(M_3^2 - M_2^2)^2 + M_2^2 M_3^2}\right) \tag{5.7}$$

and finally

$$M_2^2\, Y_1^+ Y_1^- + M_3^2\, Y_2^+ Y_2^- + (M_2^2 + M_3^2)\, Y_3^+ Y_3^- + M_-^2\, \frac{Z_-^2}{2} + M_+^2\, \frac{Z_+^2}{2}\,, \tag{5.8}$$

where

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} Z_- \\ Z_+ \end{pmatrix} \tag{5.9}$$

$$\cos\phi\ \sin\phi = \frac{\sqrt{3}}{4}\, \frac{M_2^2}{\sqrt{M_2^4 + M_3^2(M_3^2 - M_2^2)}}$$

$$Z_1 = \cos\phi\ Z_- - \sin\phi\ Z_+ \quad , \quad Z_2 = \sin\phi\ Z_- + \cos\phi\ Z_+ \tag{5.10}$$

5.4 Electroweak symmetry breaking

For electroweak symmetry breaking we introduction two triplets of $SU(2)_L$ Higgs doublets, namely;

$$\Phi^u = (3, 1, 2, -1) \qquad , \qquad \Phi^d = (3, 1, 2, +1)\,,$$

and the VEV?s

$$\Phi^u\rangle = \begin{pmatrix} \langle\Phi_1^u\rangle \\ \langle\Phi_2^u\rangle \\ \langle\Phi_3^u\rangle \end{pmatrix} \quad , \quad \langle\Phi^d\rangle = \begin{pmatrix} \langle\Phi_1^d\rangle \\ \langle\Phi_2^d\rangle \\ \langle\Phi_3^d\rangle \end{pmatrix} \quad ,$$

where

$$\Phi_i^u\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_{ui} \\ 0 \end{pmatrix} \quad , \quad \langle\Phi_i^d\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{di} \end{pmatrix} \ .$$

The contributions from $\langle\Phi^u\rangle$ and $\langle\Phi^d\rangle$ yield the W and Z_o gauge boson masses and mixing with the $SU(3)_F$ gauge bosons

$$\frac{g^2}{4}\,(v_u^2 + v_d^2)\, W^+ W^- + \frac{(g^2 + {g'}^2)}{8}\,(v_u^2 + v_d^2)\, Z_o^2$$

$+$ tiny contribution to the $SU(3)_F$ gauge boson masses and mixing with the gauge boson Z_o ,

$v_u^2 = v_{1u}^2 + v_{2u}^2 + v_{3u}^2$, $v_d^2 = v_{1d}^2 + v_{2d}^2 + v_{3d}^2$. So, if we define $M_W = \frac{1}{2} g v$, we may write $v = \sqrt{v_u^2 + v_d^2} \approx 246$ GeV.

5.5 Fermion masses

5.5.1 Dirac See-saw mechanisms

The scalars and fermion content allow the gauge invariant Yukawa couplings

$$H_u \, \overline{\psi_q^o} \, \Phi^u \, U_R^o \; + \; h_{iu} \, \overline{\psi_{uR}^o} \, \eta_i \, U_L^o \; + \; M_U \, \overline{U_L^o} \, U_R^o \; + h.c \tag{5.11}$$

$$H_d \, \overline{\psi_q^o} \, \Phi^d \, D_R^o \; + \; h_{id} \, \overline{\psi_{dR}^o} \, \eta_i \, D_L^o \; + \; M_D \, \overline{D_L^o} \, D_R^o \; + h.c \tag{5.12}$$

$$H_e \, \overline{\psi_l^o} \, \Phi^d \, E_R^o \; + \; h_{ie} \, \overline{\psi_{eR}^o} \, \eta_i \, E_L^o \; + \; M_E \, \overline{E_L^o} \, E_R^o \; + h.c \tag{5.13}$$

$$H_\nu \, \overline{\psi_l^o} \, \Phi^u \, N_R^o \; + \; h_{i\nu} \, \overline{\psi_{\nu R}^o} \, \eta_i \, N_L^o \; + \; M_{N_D} \, \overline{N_L^o} \, N_R^o \; + h.c \tag{5.14}$$

$$h_L \, \overline{\psi_l^o} \, \Phi^u \, (N_L^o)^c \; + \; m_L \, \overline{N_L^o} \, (N_L^o)^c \; + h.c \tag{5.15}$$

$$h_{iR} \, \overline{\psi_{\nu R}^o} \, \eta_i \, (N_R^o)^c \; + \; m_R \, \overline{N_R^o} \, (N_R^o)^c \; + h.c \tag{5.16}$$

M_U , M_D , M_E , M_{N_D} , m_L , m_R are free mass parameters and H_u, H_d, H_e, H_ν, h_{iu}, h_{id}, h_{ie}, $h_{i\nu}$, h_L, h_{iR} are Yukawa coupling constants. When the involved scalar fields acquire VEV's, we get in the gauge basis ${\psi_{L,R}^o}^T = (e^o, \mu^o, \tau^o, E^o)_{L,R}$, the mass terms $\bar{\psi}_L^o \mathcal{M}^o \psi_R^o + h.c$, where

$$\mathcal{M}^o = \begin{pmatrix} 0 & 0 & 0 & h\,v_1 \\ 0 & 0 & 0 & h\,v_2 \\ 0 & 0 & 0 & h\,v_3 \\ 0 & h_2\Lambda_2 & h_3\Lambda_3 & M \end{pmatrix} \equiv \begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ 0 & b_2 & b_3 & M \end{pmatrix} . \tag{5.17}$$

$\mathcal{M}^o$ is diagonalized by applying a biunitary transformation $\psi_{L,R}^o = V_{L,R}^o \, \chi_{L,R}$. Using the possible parametrizations for the orthogonal matrices V_L^o and V_R^o are written explicitly in the Appendix A, Using one obtains

$${V_L^o}^T \mathcal{M}^o \, V_R^o = Diag(0, 0, -\lambda_3, \lambda_4) \tag{5.18}$$

$${V_L^o}^T \mathcal{M}^o {\mathcal{M}^o}^T \, V_L^o = {V_R^o}^T {\mathcal{M}^o}^T \mathcal{M}^o \, V_R^o = Diag(0, 0, \lambda_3^2, \lambda_4^2) \, . \tag{5.19}$$

where λ_3 and λ_4 are the nonzero eigenvalues defined in Eqs.(5.56-5.58), λ_4 being the fourth heavy fermion mass, and λ_3 of the order of the top, bottom and tau mass for u, d and e fermions, respectively. We see from Eqs.(5.18,5.19) that from tree level the See-saw mechanism yields two massless eigenvalues associated to the light fermions:

*I*t is worth to mention that the Yukawa couplings in Eqs.5.11–5.16 are invariant under the global symmetry $U(1)_B \times U(1)_Y \times U(1)_\alpha \times U(1)_\beta$, where B is the baryon number, Y is the hypercharge, and $U(1)_\alpha$, $U(1)_\beta$ are two additional symmetries, and one of them could play the role of a Peceei-Quinn symmetry to address the strong CP problem[22].

5.6 One loop contribution to fermion masses

After tree level contributions the first two generations remain massless. Therefore, in this scenario light fermion masses, including neutrinos, may get small masses from radiative corrections mediated by the $SU(3)_F$ heavy gauge bosons.

The one loop diagram of Fig. 1 gives the generic contribution to the mass term $m_{ij}\ \bar{e}^o_{iL} e^o_{jR}$, where

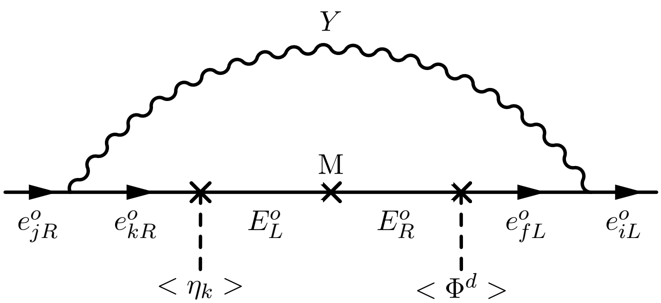

Fig. 5.1. Generic one loop diagram contribution to the mass term $m_{ij}\ \bar{e}^o_{iL} e^o_{jR}$

$$m_{ij} = c_Y \frac{\alpha_H}{\pi} \sum_{k=3,4} m^o_k\ (V^o_L)_{ik} (V^o_R)_{jk} f(M_Y, m^o_k) \qquad , \qquad \alpha_H \equiv \frac{g^2_H}{4\pi} , \tag{5.20}$$

M_Y being the mass of the gauge boson, c_Y is a factor coupling constant, Eq.(5.3), $m^o_3 = -\lambda_3$ and $m^o_4 = \lambda_4$ are the See-saw mass eigenvalues, Eq.(5.18), $f(x, y) = \frac{x^2}{x^2-y^2} \ln \frac{x^2}{y^2}$, and

$$\sum_{k=3,4} m^o_k\ (V^o_L)_{ik} (V^o_R)_{jk} f(M_Y, m^o_k) = \frac{a_i\ b_j\ M}{\lambda^2_4 - \lambda^2_3} F(M_Y) , \tag{5.21}$$

$i = 1, 2, 3$, $j = 2, 3$, and $F(M_Y) \equiv \frac{M^2_Y}{M^2_Y - \lambda^2_4} \ln \frac{M^2_Y}{\lambda^2_4} - \frac{M^2_Y}{M^2_Y - \lambda^2_3} \ln \frac{M^2_Y}{\lambda^2_3}$. Adding up all possible the one loop contributions, we get the mass terms $\bar{\psi^o_L} \mathcal{M}^o_1\ \psi^o_R + h.c.$,

$$\mathcal{M}_1^o = \begin{pmatrix} D_{11} & D_{12} & D_{13} & 0 \\ 0 & D_{22} & D_{23} & 0 \\ 0 & D_{32} & D_{33} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \frac{\alpha_H}{\pi} \,, \tag{5.22}$$

$$D_{11} = \frac{1}{2}(\mu_{22}F_1 + \mu_{33}F_2) \,, \; D_{12} = \mu_{12}(-\frac{F_{Z_1}}{4} + \frac{F_{Z_2}}{12}) \,, \; D_{13} = -\mu_{13}(\frac{F_{Z_2}}{6} + F_m) \,,$$

$$D_{22} = \mu_{22}(\frac{F_{Z_1}}{4} + \frac{F_{Z_2}}{12} - F_m) + \frac{1}{2}\mu_{33}F_3 \quad , \quad D_{23} = -\mu_{23}(\frac{F_{Z_2}}{6} - F_m) \,,$$

$$D_{32} = -\mu_{32}(\frac{F_{Z_2}}{6} - F_m) \quad , \quad D_{33} = \mu_{33}\frac{F_{Z_2}}{3} + \frac{1}{2}\mu_{22}F_3 \,,$$

$$\alpha_H = \frac{g_H^2}{4\pi} \quad , \quad F_1 \equiv F(M_{Y_1}) \quad , \quad F_2 \equiv F(M_{Y_2}) \quad , \quad F_3 \equiv F(M_{Y_3})$$

$$F_{Z_1} = \cos^2\phi \, F(M_-) + \sin^2\phi \, F(M_+) \,, \; F_{Z_2} = \sin^2\phi \, F(M_-) + \cos^2\phi \, F(M_+)$$

$$F_m = \frac{\cos\phi \sin\phi}{2\sqrt{3}} \left[F(M_-) - F(M_+) \right] .$$

F_{Z_1} , F_{Z_2} are the contributions from the diagrams mediated by the Z_1 , Z_2 gauge bosons, F_m comes from the $Z_1 - Z_2$ mixing diagrams, with M_2, M_3, M_-, M_+ the horizontal boson masses, Eqs.(5.5-5.7),

$$\mu_{ij} = \frac{a_i \, b_j \, M}{\lambda_4^2 - \lambda_3^2} = \frac{a_i \, b_j}{a \, b} \lambda_3 \, c_\alpha \, c_\beta \,, \tag{5.23}$$

with $c_\alpha = \cos\alpha, c_\beta = \cos\beta, s_\alpha = \sin\alpha, s_\beta = \sin\beta$ the mixing angles coming from the diagonalization of $\mathcal{M}^o$. Therefore, up to one loop corrections the fermion masses are

$$\bar{\psi}_L^o \mathcal{M}^o \, \psi_R^o + \bar{\psi}_L^o \mathcal{M}_1^o \, \psi_R^o = \bar{\chi}_L \, \mathcal{M} \, \chi_R \,, \tag{5.24}$$

where $\psi_{L,R}^o = V_{L,R}^o \, \chi_{L,R}$, and $\mathcal{M} \equiv \left[Diag(0, 0, -\lambda_3, \lambda_4) + V_L^{o\,T} \mathcal{M}_1^o \, V_R^o \right]$ can be written as:

$$\mathcal{M} = \begin{pmatrix} m_{11} & m_{12} & c_\beta \, m_{13} & s_\beta \, m_{13} \\ m_{21} & m_{22} & c_\beta \, m_{23} & s_\beta \, m_{23} \\ c_\alpha \, m_{31} & c_\alpha \, m_{32} & (-\lambda_3 + c_\alpha c_\beta \, m_{33}) & c_\alpha s_\beta \, m_{33} \\ s_\alpha \, m_{31} & s_\alpha \, m_{32} & s_\alpha c_\beta \, m_{33} & (\lambda_4 + s_\alpha s_\beta \, m_{33}) \end{pmatrix} \,, \tag{5.25}$$

The explicit expression for the m_{ij} mass terms depends on the used parametrization for V^o_L, V^o_R.
The diagonalization of $\mathcal{M}$, Eq.(5.25) gives the physical masses for u and d quarks, e charged leptons and ν Dirac neutrino masses.
Using a new biunitary transformation

$$\chi_{L,R} = V^{(1)}_{L,R}\,\Psi_{L,R};\ \bar{\chi}_L\,\mathcal{M}\,\chi_R = \bar{\Psi}_L\,{V^{(1)}_L}^T \mathcal{M}\,V^{(1)}_R\,\Psi_R,$$

with ${\Psi_{L,R}}^T = (f_1, f_2, f_3, F)_{L,R}$ the mass eigenfields, that is

$${V^{(1)}_L}^T \mathcal{M}\,\mathcal{M}^T\,V^{(1)}_L = {V^{(1)}_R}^T \mathcal{M}^T\,\mathcal{M}\,V^{(1)}_R = Diag(m_1^2, m_2^2, m_3^2, M_F^2)\,, \qquad (5.26)$$

$m_1^2 = m_e^2$, $m_2^2 = m_\mu^2$, $m_3^2 = m_\tau^2$ and $M_F^2 = M_E^2$ for charged leptons. So, the rotations from massless to mass fermions eigenfields in this scenario reads

$$\psi^o_L = V^o_L\,V^{(1)}_L\,\Psi_L \qquad \text{and} \qquad \psi^o_R = V^o_R\,V^{(1)}_R\,\Psi_R \qquad (5.27)$$

5.6.1 Quark Mixing Matrix V_{CKM}

We recall that vector like quarks, Eq.(5.2), are $SU(2)_L$ weak singlets, and they do not couple to W boson in the interaction basis. In this way, the interaction of L-handed up and down quarks; ${f^o_{uL}}^T = (u^o, c^o, t^o)_L$ and ${f^o_{dL}}^T = (d^o, s^o, b^o)_L$, to the W charged gauge boson is

$$\frac{g}{\sqrt{2}}\,\bar{f^o}_{uL}\gamma_\mu f^o_{dL} W^{+\mu} =$$
$$\frac{g}{\sqrt{2}}\,\bar{\Psi}_{uL}\,[(V^o_{uL}\,V^{(1)}_{uL})_{3\times4}]^T\,(V^o_{dL}\,V^{(1)}_{dL})_{3\times4}\,\gamma_\mu \Psi_{dL}\,W^{+\mu}\,, \qquad (5.28)$$

Hence, in this scenario the non-unitary V_{CKM} of dimension 4×4 is identified as

$$(V_{CKM})_{4\times4} = [(V^o_{uL}\,V^{(1)}_{uL})_{3\times4}]^T\,(V^o_{dL}\,V^{(1)}_{dL})_{3\times4} \qquad (5.29)$$

5.7 Numerical results for quark masses and mixing

As an example of the possible spectrum of quark masses and mixing from this scenario, we consider the following set of parameters at the M_Z scale [23]

Using the input values for the horizontal boson masses, Eq.(5.5), and the coupling constant of the $SU(3)_F$ symmetry:

$$M_2 = 6.0\,\text{TeV} \quad , \quad M_3 = 1.5\times10^8\,\text{TeV} \quad , \quad \frac{\alpha_H}{\pi} = 0.2\,, \qquad (5.30)$$

we show in the interaction basis the following tree level $\mathcal{M}^o_q$, and one loop $\mathcal{M}^o_{q\,1}$ quark mass matrices, and the corresponding mass eigenvalues and mixing:

u-quarks:

Tree level see-saw mass matrix:

$$\mathcal{M}_u^o = \begin{pmatrix} 0 & 0 & 0 & 5573.43 \\ 0 & 0 & 0 & 23883.8 \\ 0 & 0 & 0 & 397346. \\ 0 & -1.931\times 10^8 & 5.193\times 10^6 & 2.470\times 10^8 \end{pmatrix} \text{MeV}\,, \tag{5.31}$$

the mass matrix up to one loop corrections:

$$\mathcal{M}_{u\,1}^o = \begin{pmatrix} 1.42 & -220.786 & 34.6742 & 0 \\ 0 & -944.713 & 148.589 & 0 \\ 0 & -91921.3 & 11631.6 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \text{MeV} \tag{5.32}$$

the u-quark mass eigenvalues

$$(m_u\,,\, m_c\,,\, m_t\,,\, M_U) = (1.382\,,\, 633.289\,,\, 172968\,,\, 313.606\times 10^6)\ \text{MeV} \tag{5.33}$$

and the mixing matrices:

$V_{u\,L} = V_{u\,L}^o\, V_{u\,L}^{(1)}$:

$$\begin{pmatrix} 0.973838 & -0.226464 & -0.0188217 & 0.0000144353 \\ -0.227244 & -0.970491 & -0.080663 & 0.0000618569 \\ 9.34208\times 10^{-7} & 0.0828299 & -0.996563 & 0.0011792 \\ -2.14837\times 10^{-9} & -0.0000343726 & 0.00118041 & 0.999999 \end{pmatrix} \tag{5.34}$$

$V_{u\,R} = V_{u\,R}^o\, V_{u\,R}^{(1)}$:

$$\begin{pmatrix} 1. & 0.000507791 & 1.54519\times 10^{-7} & 0 \\ -7.6088\times 10^{-6} & 0.0147444 & 0.787788 & -0.61577 \\ 0.000507462 & -0.999362 & 0.0316481 & 0.0165598 \\ -0.0000166153 & 0.0325336 & 0.615132 & 0.787752 \end{pmatrix} \tag{5.35}$$

d-quarks:

$$\mathcal{M}_d^o = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3102.75 \\ 0 & 0 & 0 & 61977.5 \\ 0 & -9.805\times 10^7 & 2.837\times 10^6 & 6.046\times 10^8 \end{pmatrix} \text{MeV} \tag{5.36}$$

$$\mathcal{M}_{d\,1}^o = \begin{pmatrix} 2.82 & 0 & 0 & 0 \\ 0 & -130.851 & 10.43 & 0 \\ 0 & -7200.83 & 664.801 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \text{MeV} \tag{5.37}$$

the d-quark mass eigenvalues

$$(m_d \, , \, m_s \, , \, m_b \, , \, M_D \,) = (\, 2.82 \, , \, 52.087 \, , \, 2861.96 \, , \, 612.541 \times 10^6 \,) \text{ MeV} \tag{5.38}$$

the mixing matrices:

$V_{d\,L} = V^o_{d\,L} \, V^{(1)}_{d\,L}$:

$$\begin{pmatrix} 1. & 0 & 0 & 0 \\ 0 & 0.991883 & -0.127155 & 5.03428 \times 10^{-6} \\ 0 & -0.127155 & -0.991883 & 0.000101762 \\ 0 & 7.94612 \times 10^{-6} & 0.000101576 & 1. \end{pmatrix} \tag{5.39}$$

$V_{d\,R} = V^o_{d\,R} \, V^{(1)}_{d\,R}$:

$$\begin{pmatrix} 1. & 0 & 0 & 0 \\ 0 & -0.127762 & 0.9788 & -0.160083 \\ 0 & 0.99148 & 0.130175 & 0.00463162 \\ 0 & -0.0253722 & 0.158128 & 0.987093 \end{pmatrix} \tag{5.40}$$

and the quark mixing matrix

$$V_{CKM} = \begin{pmatrix} 0.97383 & 0.2254 & 0.02889 & -1.14 \times 10^{-6} \\ -0.22646 & 0.97314 & 0.04124 & 3.54 \times 10^{-6} \\ -0.01882 & -0.04670 & 0.99873 & -0.00010 \\ 1.44 \times 10^{-5} & 8.85 \times 10^{-5} & -0.00117 & 1.20 \times 10^{-7} \end{pmatrix} \tag{5.41}$$

5.8 ΔF = 2 Processes in Neutral Mesons

Here we study the tree level FCNC interactions that contribute to $K^o - \bar{K^o}$, $D^o - \bar{D^o}$ mixing via Z_1 , $Y_1^{\pm}$ exchange from the depicted diagram in Fig. 2.

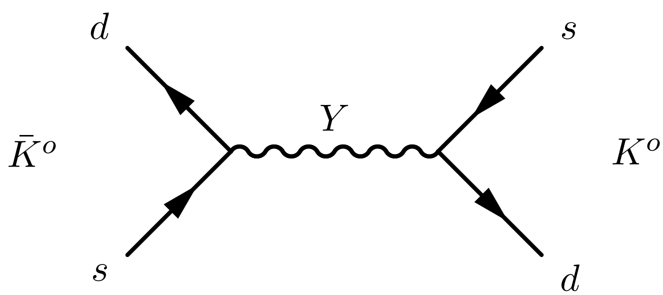

Fig. 5.2. Generic tree level exchange contribution to $K^o - \bar{K^o}$ from the $SU(3)$ horizontal gauge bosons.

The Z_1 , $Y_1^{\pm}$ gauge bosons have flavor changing couplings in both left- and right-handed fermions, and then contribute the $\Delta S = 2$ effective operators

$$\mathcal{O}_{LL} = (\bar{d}_L\gamma_\mu s_L)(\bar{d}_L\gamma^\mu s_L) \quad , \quad \mathcal{O}_{RR} = (\bar{d}_R\gamma_\mu s_R)(\bar{d}_R\gamma^\mu s_R) \tag{5.42}$$

$$\mathcal{O}_{LR} = (\bar{d}_L\gamma_\mu s_L)(\bar{d}_R\gamma^\mu s_R) \tag{5.43}$$

The $SU(3)_F$ couplings to fermions, Eq.5.3, when written in the mass basis yield the gauge couplings

$$\mathcal{L}_{int,Z_1} = \frac{g_H}{2}\left(C_{L\,Z_1}\,\bar{d}_L\gamma_\mu s_L + C_{R\,Z_1}\,\bar{d}_R\gamma_\mu s_R\right) Z_1^\mu \tag{5.44}$$

$$\mathcal{L}_{int,Y_1^1} = \frac{g_H}{2}\left(C_{L\,Y_1^1}\,\bar{d}_L\gamma_\mu s_L + C_{R\,Y_1^1}\,\bar{d}_R\gamma_\mu s_R\right) Y_1^{1\,\mu} \tag{5.45}$$

$$\mathcal{L}_{int,Y_1^2} = \frac{g_H}{2}\left(C_{L\,Y_1^2}\,\bar{d}_L\gamma_\mu s_L + C_{R\,Y_1^2}\,\bar{d}_R\gamma_\mu s_R\right) i\,Y_1^{2\,\mu} \tag{5.46}$$

with the coefficients

$$\begin{aligned}
C_{L\,Z_1} &= L_{11}\,L_{12} - L_{21}\,L_{22} \quad , \quad C_{R\,Z_1} = R_{11}\,R_{12} - R_{21}\,R_{22} \\
C_{L\,Y_1^1} &= L_{12}\,L_{21} + L_{11}\,L_{22} \quad , \quad C_{R\,Y_1^1} = R_{12}\,R_{21} + R_{11}\,R_{22} \\
C_{L\,Y_1^2} &= (L_{12}\,L_{21} - L_{11}\,L_{22})\,, \quad C_{R\,Y_1^2} = (R_{12}\,R_{21} - R_{11}\,R_{22})
\end{aligned} \tag{5.47}$$

where $L_{ij} = V_{L\,ij}$ and $R_{ij} = V_{R\,ij}$. For each gauge boson, the effective four-fermion hamiltonian at the scale of the gauge boson mass is

$$\mathcal{H}_{Z_1} = \frac{g_H^2}{4M_{Z_1}^2}\left(C_{L\,Z_1}^2\,\mathcal{O}_{LL} + 2\,C_{L\,Z_1}C_{R\,Z_1}\mathcal{O}_{LR} + C_{R\,Z_1}^2\,\mathcal{O}_{RR}\right) \tag{5.48}$$

$$\mathcal{H}_{Y_1^1} = \frac{g_H^2}{4M_2^2}\left(C_{L\,Y_1^1}^2\,\mathcal{O}_{LL} + 2\,C_{L\,Y_1^1}C_{R\,Y_1^1}\mathcal{O}_{LR} + C_{R\,Y_1^1}^2\,\mathcal{O}_{RR}\right) \tag{5.49}$$

$$\mathcal{H}_{Y_1^2} = -\frac{g_H^2}{4M_2^2}\left(C_{L\,Y_1^2}^2\,\mathcal{O}_{LL} + 2\,C_{L\,Y_1^2}C_{R\,Y_1^1}\mathcal{O}_{LR} + C_{R\,Y_1^2}^2\,\mathcal{O}_{RR}\right) \tag{5.50}$$

with $M_{Y_1} = M_{Y_2} = M_2$. Therefore, the total four-fermion hamiltonian $\mathcal{H}_{SU(2)} = \mathcal{H}_{Z_1} + \mathcal{H}_{Y_1^1} + \mathcal{H}_{Y_1^2}$ can be written as

$$\begin{aligned}
\mathcal{H}_{SU(2)} = \frac{g_H^2}{4M_2^2}\Big[&(C_{L\,Z_1}^2 + C_{L\,Y_1^1}^2 - C_{L\,Y_1^2}^2)\mathcal{O}_{LL} + (C_{R\,Z_1}^2 + C_{R\,Y_1^1}^2 + C_{R\,Y_1^2}^2)\mathcal{O}_{RR} \\
&+2(C_{L\,Z_1}C_{R\,Z_1} + C_{L\,Y_1^1}C_{R\,Y_1^1} - C_{L\,Y_1^2}C_{R\,Y_1^2})\mathcal{O}_{LR}\Big] \\
&+ \frac{g_H^2}{4}\left(\frac{1}{M_{Z_1}^2} - \frac{1}{M_2^2}\right)\left[C_{L\,Z_1}^2\mathcal{O}_{LL} + C_{R\,Z_1}^2\mathcal{O}_{RR} + 2C_{L\,Z_1}C_{R\,Z_1}\mathcal{O}_{LR}\right]
\end{aligned} \tag{5.51}$$

From the coefficients in Eq.5.47 we get:

$$C^2_{L\,Z_1} + C^2_{L\,Y^1_1} - C^2_{L\,Y^2_1} = \delta^2_L \quad , \quad C^2_{R\,Z_1} + C^2_{R\,Y^1_1} - C^2_{R\,Y^2_1} = \delta^2_R \quad , \quad ,$$

$$C_{L,Z_1}\, C_{R,Z_1} + C_{L,Y^1_1}\, C_{R,Y^1_1} - C_{L,Y^2_1}\, C_{R,Y^2_1} = \delta_L\, \delta_R + 2(L_{11}\, R_{21} - L_{21}\, R_{11})(L_{22}\, R_{12} - L_{12}\, R_{22})\,,$$

and finally we can write

$$\mathcal{H}_{SU(2)} = \frac{g_H^2}{4M_1^2}\,\left[\delta^2_L\, \mathcal{O}_{LL} + \delta^2_R\, \mathcal{O}_{RR} + \delta^2_{LR}\, \mathcal{O}_{LR}\right] \tag{5.52}$$

$$+ \frac{g_H^2}{4}\left(\frac{1}{M^2_{Z_1}} - \frac{1}{M^2_1}\right)\,\left[(L_{11}L_{12} - L_{21}L_{22})^2\, \mathcal{O}_{LL} + (R_{11}R_{12} - R_{21}R_{22})^2\, \mathcal{O}_{RR} + 2(L_{11}L_{12} - L_{21}L_{22})(R_{11}R_{12} - R_{21}R_{22})\, \mathcal{O}_{LR})\right] \tag{5.53}$$

with

$$\delta_L = L_{11}\, L_{12} + L_{21}\, L_{22} \quad , \quad \delta_R = R_{11}\, R_{12} + R_{21}\, R_{22}$$

$$\delta_{LR} = \sqrt{2(\delta_L\, \delta_R + 2(L_{11}\, R_{21} - L_{21}\, R_{11})(L_{22}\, R_{12} - L_{12}\, R_{22}))}$$

5.8.1 $K^o - \bar{K^o}$ meson mixing

The numerical fit of parameters provided in section 7 yield the mixing angles $V_{d\,12} = V_{d\,21} = 0$ for left- and right-handed d-quarks, and then all the contributions to the effective operators, Eqs.5.42–5.43, for $\Delta S = 2$ vanish.

5.8.2 $D^o - \bar{D^o}$ meson mixing

The reported parameter space region in section 7 generate $M_{Z_1} = M_2$ with very good approximation, and then only the four-fermion Hamiltonian in Eq.5.52 contribute. For this case we compute the numerical values

$$\begin{aligned} \delta_L &= -7.73804 \times 10^{-8} \quad , \quad \frac{M_2}{\frac{g_H}{2}\,\delta_L} = -5.51894 \times 10^7\ \text{TeV} \\ \delta_R &= 5.07679 \times 10^{-4} \quad , \quad \frac{M_2}{\frac{g_H}{2}\,\delta_R} = 8411.97\ \text{TeV} \\ \delta_{LR} &= 0.0508636 \quad , \quad \frac{M_2}{\frac{g_H}{2}\,\delta_{LR}} = 83.9614\ \text{TeV} \end{aligned} \tag{5.54}$$

Accordingly to the Review "The CKM quark - mixing matrix" in PDG2016[24], the $\Delta C = 2$ effective operators $\mathcal{O}_{LL}$ and $\mathcal{O}_{RR}$ are within suppression limits.

5.9 Conclusions

Horizontal gauge bosons from the local $SU(3)_F$ introduce flavor changing couplings, and in particular mediate $\Delta F = 2$ processes at tree level. We reported the analytic contribution to $K^o - \bar{K^o}$ and $D^o - \bar{D^o}$ meson mixing from tree level exchange diagrams mediated by the $SU(2)_F$ gauge bosons Z_1 , $Y_1^{\pm}$ with masses in the TeV region. We provide a particular parameter space region in in section 7 where this scenario can accommodate the hierarchy spectrum of quark masses and simultaneously suppress properly the $K^o - \bar{K^o}$ meson mixing, and the effective operators $\mathcal{O}_{LL}$ and $\mathcal{O}_{RR}$ for the $\Delta C = 2$ processes.

Acknowledgements

It is my pleasure to thank the organizers N.S. Mankoc-Borstnik, H.B. Nielsen, M. Y. Khlopov, and participants for the stimulating Workshop at Bled, Slovenia. The author is grateful for the warm hospitality at the APC Laboratory, Paris, France, during sabbatical staying, and the financial support of the Cosmology group to attend this conference. This work was partially supported by the "Instituto Politécnico Nacional", (Grants from EDI and COFAA) and "Sistema Nacional de Investigadores" (SNI) in Mexico.

5.10 APPENDIX: Diagonalization of the generic Dirac See-saw mass matrix

$$\mathcal{M}^o = \begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ 0 & b_2 & b_3 & c \end{pmatrix} \tag{5.55}$$

The tree level $\mathcal{M}^o$ 4×4 See-saw mass matrix is diagonalized by a biunitary transformation $\psi^o_L = V^o_L \chi_L$ and $\psi^o_R = V^o_R \chi_R$. The diagonalization of $\mathcal{M}^o \mathcal{M}^{o\,T}$ ($\mathcal{M}^{o\,T} \mathcal{M}^o$) yield the nonzero eigenvalues

$$\lambda_3^2 = \frac{1}{2}\left(B - \sqrt{B^2 - 4D}\right) \quad , \quad \lambda_4^2 = \frac{1}{2}\left(B + \sqrt{B^2 - 4D}\right) \tag{5.56}$$

and rotation mixing angles

$$\begin{aligned} \cos\alpha &= \sqrt{\frac{\lambda_4^2 - a^2}{\lambda_4^2 - \lambda_3^2}} \quad , \quad \sin\alpha = \sqrt{\frac{a^2 - \lambda_3^2}{\lambda_4^2 - \lambda_3^2}} \, , \\ \cos\beta &= \sqrt{\frac{\lambda_4^2 - b^2}{\lambda_4^2 - \lambda_3^2}} \quad , \quad \sin\beta = \sqrt{\frac{b^2 - \lambda_3^2}{\lambda_4^2 - \lambda_3^2}} \, . \end{aligned} \tag{5.57}$$

$$B = a^2 + b^2 + c^2 = \lambda_3^2 + \lambda_4^2 \quad , \quad D = a^2 b^2 = \lambda_3^2 \lambda_4^2 \; , \tag{5.58}$$

$$a^2 = a_1^2 + a_2^2 + a_3^2 \qquad , \qquad b^2 = b_1^2 + b_2^2 + b_3^2$$

The rotation matrices V_L^o, V_R^o admit several parametrizations related to the two zero mass eigenstates.

5.10.1 Parametrization P12

$$V_L^o = \begin{pmatrix} c_1 & c_2\, s_1 & s_1\, s_2\, c_\alpha & s_1\, s_2\, s_\alpha \\ -s_1 & c_1\, c_2 & c_1\, s_2\, c_\alpha & c_1\, s_2\, s_\alpha \\ 0 & -s_2 & c_2\, c_\alpha & c_2\, s_\alpha \\ 0 & 0 & -s_\alpha & c_\alpha \end{pmatrix} \quad , \quad V_R^o = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_r & s_r\, c_\beta & s_r\, s_\beta \\ 0 & -s_r & c_r\, c_\beta & c_r\, s_\beta \\ 0 & 0 & -s_\beta & c_\beta \end{pmatrix}$$

$$a_p = \sqrt{a_1^2 + a_2^2} \; , \; b_p = \sqrt{b_1^2 + b_2^2} \; , \; a = \sqrt{{a_p}^2 + a_3^2} \; , \; b = \sqrt{{b_p}^2 + b_3^2} \; ,$$

$$s_1 = \frac{a_1}{a_p} \; , \; c_1 = \frac{a_2}{a_p} \; , \; s_2 = \frac{a_p}{a} \; , \; c_2 = \frac{a_3}{a} \; , \; s_r = \frac{b_2}{b} \; , \; c_r = \frac{b_3}{b}$$

5.10.2 Parametrization P13

$$V_L^o = \begin{pmatrix} c_1 & -s_1\, s_2 & s_1\, c_2\, c_\alpha & s_1\, c_2\, s_\alpha \\ 0 & c_2 & s_2\, c_\alpha & s_2\, s_\alpha \\ -s_1 & -c_1\, s_2 & c_1\, c_2\, c_\alpha & c_1\, c_2\, s_\alpha \\ 0 & 0 & -s_\alpha & c_\alpha \end{pmatrix} \quad , \quad V_R^o = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_r & s_r\, c_\beta & s_r\, s_\beta \\ 0 & -s_r & c_r\, c_\beta & c_r\, s_\beta \\ 0 & 0 & -s_\beta & c_\beta \end{pmatrix}$$

$$a_n = \sqrt{a_1^2 + a_3^2} \; , \; b_n = \sqrt{b_1^2 + b_3^2} \; , \; a = \sqrt{a_n^2 + a_2^2} \; , \; b = \sqrt{b_n^2 + b_2^2} \; ,$$

$$s_1 = \frac{a_1}{a_n} \; , \; c_1 = \frac{a_3}{a_n} \; , \; s_2 = \frac{a_2}{a} \; , \; c_2 = \frac{a_n}{a} \; , \; s_r = \frac{b_2}{b} \; , \; c_r = \frac{b_3}{b} \tag{5.59}$$

5.10.3 Parametrization P23

$$V_L^o = \begin{pmatrix} c_1 & 0 & s_1\, c_\alpha & s_1\, s_\alpha \\ -s_1\, s_2 & c_2 & c_1\, s_2\, c_\alpha & c_1\, s_2\, s_\alpha \\ -s_1\, c_2 & -s_2 & c_1\, c_2\, c_\alpha & c_1\, c_2\, s_\alpha \\ 0 & 0 & -s_\alpha & c_\alpha \end{pmatrix} \quad , \quad V_R^o = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_r & s_r\, c_\beta & s_r\, s_\beta \\ 0 & -s_r & c_r\, c_\beta & c_r\, s_\beta \\ 0 & 0 & -s_\beta & c_\beta \end{pmatrix}$$

$$a_n = \sqrt{a_2^2 + a_3^2} \; , \; b_n = \sqrt{b_2^2 + b_3^2} \; , \; a = \sqrt{a_n^2 + a_1^2} \; , \; b = \sqrt{b_n^2 + b_1^2} \; ,$$

$$s_1 = \frac{a_1}{a} \; , \; c_1 = \frac{a_n}{a} \; , \; s_2 = \frac{a_2}{a_n} \; , \; c_2 = \frac{a_3}{a_n} \; , \; s_r = \frac{b_2}{b} \; , \; c_r = \frac{b_3}{b} \tag{5.60}$$

References

1. A. Hernandez-Galeana, Rev. Mex. Fis. **Vol. 50(5)**, (2004) 522. hep-ph/0406315.
2. A. Hernandez-Galeana, Bled Workshops in Physics, (ISSN:1580-4992), **Vol. 17, No. 2**, (2016) Pag. 36; arXiv:1612.07388[hep-ph]; **Vol. 16, No. 2**, (2015) Pag. 47; arXiv:1602.08212[hep-ph]; **Vol. 15, No. 2**, (2014) Pag. 93; arXiv:1412.6708[hep-ph]; **Vol. 14, No. 2**, (2013) Pag. 82; arXiv:1312.3403[hep-ph]; **Vol. 13, No. 2**, (2012) Pag. 28; arXiv:1212.4571[hep-ph]; **Vol. 12, No. 2**, (2011) Pag. 41; arXiv:1111.7286[hep-ph]; **Vol. 11, No. 2**, (2010) Pag. 60; arXiv:1012.0224[hep-ph]; Bled Workshops in Physics,**Vol. 10, No. 2**, (2009) Pag. 67; arXiv:0912.4532[hep-ph];
3. E. Golowich, J. Hewett, S. Pakvasa, and A. Petrov, Phys. Rev. D **76**, 095009 (2007).
4. E. Golowich, J. Hewett, S. Pakvasa, and A. Petrov, Phys. Rev. D **79**, 114030 (2009).
5. M. Kirk, A. Lenz, and T. Rauh, arXiv:1711.02100[hep-ph]; T. Jubb, M. Kirk, A. Lenz, and G. Tetlalmatzi-Xolocotzi, arXiv:1603.07770[hep-ph];
6. C. Bobeth, A. J. Buras, A. Celis, and M. Junk, arXiv:1703.04753[hep-ph]; A. J. Buras, arXiv:1611.06206[hep-ph]; arXiv:1609.05711[hep-ph];
7. Z. Berezhiani and M. Yu.Khlopov: Theory of broken gauge symmetry of families, Sov.J.Nucl.Phys. **51**, 739 (1990).
8. A. S. Sakharov and M. Yu. Khlopov: Horizontal unification as the phenomenology of the theory of everything, Phys.Atom.Nucl. **57**, 651 (1994).
9. M. Yu. Khlopov: *Cosmoparticle physics*, World Scientific, New York -London-Hong Kong - Singapore, 1999.
10. Maxim Khlopov, *Fundamentals of Cosmic Particle physics*, CISP-SPRINGER, Cambridge, 2012.
11. M. Yu. Khlopov: Fundamental Particle Structure in the Cosmological Dark Matter, J. of Mod. Phys. A 28 1330042 (60 pages) (2013).
12. Z. Berezhiani and M. Yu.Khlopov: Physical and astrophysical consequences of breaking of the symmetry of families, Sov.J.Nucl.Phys. **51**, 935 (1990).
13. Z. Berezhiani, M. Yu.Khlopov and R. R. Khomeriki: On the possible test of quantum flavor dynamics in the searches for rare decays of heavy particles, Sov.J.Nucl.Phys. **52**, 344 (1990).
14. T. Appelquist, Y. Bai and M. Piai: SU(3) Family Gauge Symmetry and the Axion, Phys. Rev. D **75**, 073005 (2007).
15. T. Appelquist, Y. Bai and M. Piai: Neutrinos and SU(3) family gauge symmetry, Phys. Rev. D **74**, 076001 (2006).
16. T. Appelquist, Y. Bai and M. Piai: Quark mass ratios and mixing angles from SU(3) family gauge symmetry, Phys. Lett. B **637**, 245 (2006).
17. J. L. Chkareuli: Quark-Lepton families: From SU(5) to SU(8) symmetry, JETP Lett. **32**, 671 (1980).
18. Z. G. Berezhiani, J. L. Chkareuli: Mass of the T quark and the number of quark lepton generations, JETP Lett. **35**, 612 1982.
19. Z. G. Berezhiani, J. L. Chkareuli: Neutrino oscillations in grand unified models with a horizontal symmetry, JETP Lett. **37**, 338 1983.
20. Z.G. Berezhiani: The weak mixing angles in gauge models with horizontal symmetry: A new approach to quark and lepton masses, Phys. Lett. B **129**, 99 (1983).
21. T. Yanagida, Phys. Rev. D **20**, 2986 (1979).
22. A. Hernandez-Galeana and M. Yu.Khlopov, work in progress.
23. Zhi-zhong Xing, He Zhang and Shun Zhou, Phys. Rev. D **86**, 013013 (2012).
24. The Review of Particle Physics (2016), C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016).
25. See PDG review: The CKM quark-mixing matrix, by A. Ceccucci (CERN), Z. Ligeti (LBNL), and Y. Sakai (KEK).