

9th Slovenian International Conference on Graph Theory BLED'19

Abstracts of the
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Bled, Slovenia, June 23 – 29, 2019



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WELCOME

We thank all of you for coming to the 9th Slovenian International Conference on Graph Theory, Bled'19, and wish you a pleasant and successful meeting in Bled.

This conference has come a long way from its first meeting in Dubrovnik (now in Croatia) in 1985. The second meeting of the series *Slovenian (International) Conference on Graph Theory* was held at Lake Bled in 1991, following by the subsequent meetings at the same location in 1995, 1999, 2003, 2007, and 2011. In 2015 the meeting took place in Kranjska Gora. For this edition we are back to Bled.

The conference has seen a substantial growth, from 30 participants at the 1991 meeting to well over 300 participants at the present *9th Slovenian Conference on Graph Theory*. We are very happy to see participants that have attended previous editions, always a clear indicator of successful previous meetings, and we welcome the newcomers.

The growth of the conference has been parallel to the growth of graph theory in Slovenia, two achievements to be proud of. Our international colleagues, and friends, are largely responsible for this success. We thank you for this.

In this edition we have 11 plenary speakers and 16 invited special sessions. We believe that the quality of the plenary speakers and the invited special sessions play a key role in the success of the conference. This booklet contains the abstracts of the 287 talks to be delivered at our conference.

Similar to the last edition, the conference is linked to the *Meeting of the International Academy of Mathematical Chemistry*, and the *9th PhD Summer School in Discrete Mathematics* will take place on Rogla the week after the conference. Looking to the future, an event deserves special attention: the *8th European Congress of Mathematics* to take place in Portorož, Slovenia, from July 5 to July 11, 2020. This will be a challenge and a great opportunity to promote Discrete Mathematics at large and in particular Slovenian Discrete Mathematics. We hereby warmly invite you to participate in the event.

The organization of this meeting would not have been possible without financial and technical support from the Institute of Mathematics, Physics and Mechanics, Ljubljana (IMFM); University of Ljubljana, Faculty of Mathematics and Physics (UL FMF); University of Maribor, Faculty of Natural Sciences and Mathematics (UM FNM); University of Primorska, Faculty of Mathematics, Natural Sciences and Information Technologies (UP FAMNIT), and Andrej Marušič Institute (UP IAM); the Society of Mathematicians, Physicists and Astronomers of Slovenia (DMFA); the Slovenian Discrete and Applied Mathematics Society (SDAMS); and Abelium d.o.o.

We hope that you enjoy this instance of our conference and wish you a fruitful, pleasant week devoted to graph theory.

Bled, June 2019

Sandi Klavžar
Dragan Marušič
Bojan Mohar
Tomaž Pisanski



CONTENTS

Bled 2019	5
Welcome	5
Contents	7
General Information	17
Past Conferences	19
Abstracts	21
Plenary Invited Talks	21
Noga Alon: <i>Random and quasi-random Cayley sum graphs with applications</i>	22
Marco Buratti: <i>Cyclic designs: some selected topics</i>	22
Gareth A. Jones: <i>Paley, Carlitz and the Paley Graphs</i>	22
Gábor Korchmáros: <i>Permutation groups in the study of geometric structures</i>	23
Daniel Král': <i>Profiles of graphs and tournaments</i>	23
Daniela Kühn: <i>Resolution of the Oberwolfach problem</i>	24
Sergei Lando: <i>Integrability property of polynomial graph invariants</i>	24
János Pach: <i>Many Pairwise Crossing Edges</i>	24
Cheryl E. Praeger: <i>Limited geodesic transitivity for finite regular graphs</i>	24
Zsolt Tuza: <i>Parity colorings in graphs and hypergraphs</i>	25
Xuding Zhu: <i>List colouring and Alon-Tarsi number of planar graphs</i>	25
Association Schemes	27
Harvey Blau: <i>Nilpotent commutative standard integral table algebras of order p^3</i>	28
Christopher French: <i>Hypergroup rings</i>	28
Allen Herman: <i>Rationality for irreducible representations of association schemes of rank 6 and 7</i>	28
Stephen Humphries: <i>Schur rings over infinite groups</i>	29
Akihiro Munemasa: <i>Krein parameters of fiber-commutative coherent configurations</i>	29
Misha Muzychuk: <i>Non-commutative schemes of rank six and related objects</i>	29
Ilia Ponomarenko: <i>A characterization of some equivalenced association schemes</i>	30
Grigory Ryabov: <i>Infinite family of non-schurian separable association schemes</i>	30
Gábor Somlai: <i>New family of CI-groups</i>	30
Sho Suda: <i>Linked systems of group divisible designs</i>	31
Andrey Vasil'ev: <i>Closures of permutation groups and the isomorphism problem for schurian coherent configurations</i>	31
Janoš Vidali: <i>On tight 4-designs in Hamming association schemes</i>	32
Paul-Hermann Zieschang: <i>Residually Thin Hypergroups</i>	32
Biomathematics and Bioinformatics	33
Daniel Doerr: <i>On the Computational Hardness of Ancestral Genome Reconstructions</i>	34
André Fujita: <i>Network Statistics on biological data analyses</i>	34
Marc Hellmuth: <i>Best Match Graphs</i>	35
Lina Herbst: <i>Trees on scales – measures of balance for rooted binary trees</i>	35
Remie Janssen: <i>Navigating the space of phylogenetic networks</i>	36

Manuel Lafond: <i>Will we ever find a forbidden subgraph characterization of leaf powers?</i>	36
Nikolai Nøjgaard: <i>Partial Homology Relations - Satisfiability in terms of Di-Cographs</i>	37
Guillaume Scholz: <i>Un-folding and folding-up phylogenetic networks</i>	37
Carsten R. Seemann: <i>The Matroid Structure of Representative Triple Sets and Triple-Closure Computation</i>	37
Kristina Wicke: <i>Phylogenetics meets classic graph theory – Connections between Hamiltonicity, GSP graphs and treebased networks</i>	38
Chemical graph theory	39
Simon Brezovnik: <i>Resonance graphs of catacondensed even ring systems (CERS)</i>	40
Zhongyuan Che: <i>Resonance graphs of plane elementary bipartite graphs</i>	40
Haiyan Chen: <i>The average Laplacian polynomial of a graph</i>	40
Peter Dankelmann: <i>On the Wiener Index of Eulerian Graphs</i>	41
Tomislav Došlić: <i>Packing stars in fullerenes</i>	41
Boris Furtula: <i>Novel method for measuring sensitivity of topological descriptors on structural changes</i>	41
Xian’an Jin: <i>On the existence of some strong traces of graphs</i>	42
Martin Knor: <i>Bounding the Graovac-Pisanski index</i>	42
Xueliang Li: <i>The asymptotic value of graph energy for random graphs with degree-based weights</i>	43
Jelena Sedlar: <i>On combining Zagreb and Forgotten index to obtain better predictive power</i>	43
Niko Tratnik: <i>Computing Distance-Based Topological Indices from Quotient Graphs</i>	43
Jianfeng Wang: <i>Median eigenvalues and HOMO-LUMO index of graphs</i>	44
Heping Zhang: <i>Anti-Kekulé number of graphs</i>	44
Petra Žigert Pleteršek : <i>Two topological indices applied on hydrocarbons</i>	45
Configurations	47
Leah Wrenn Berman: <i>Eventually, 5-configurations exist for all n</i>	48
Jürgen Bokowski: <i>On the Finding of Symmetric Polyhedral Realizations without Self-Intersections of Hurwitz’s Regular Map $(3,7)_{18}$ of Genus 7</i>	48
Gábor Gévay: <i>Exotic configurations</i>	48
Harald Gropp: <i>On the early history of configurations</i>	49
Milagros Izquierdo: <i>Configurations and Dessins d’Enfants</i>	49
Jurij Kovič: <i>Platonic configurations</i>	49
Vito Napolitano: <i>Configurations and some classes of finite linear spaces</i>	49
Tomaž Pisanski: <i>The remarkable rhombic dodecahedron graph</i>	49
Michael Raney: <i>Trilateral matroids</i>	50
Metod Saniga: <i>Doily – A Gem of the Quantum Universe</i>	50
Klara Stokes: <i>Dualities, trialities, configurations and graphs</i>	50
Arjana Žitnik: <i>Chiral astral realizations of cyclic 3-configurations</i>	51
Designs	53
Sara Ban: <i>New extremal Type II \mathbb{Z}_4-codes of length 32 obtained from Hadamard designs</i>	54
Simone Costa: <i>Relative Heffter arrays and biembeddings</i>	54

Dean Crnković: <i>Self-orthogonal codes from block designs and association schemes</i>	55
Ronan Egan: <i>Complementary sequences and combinatorial structures</i>	55
Vladislav Kabanov: <i>Deza graphs with parameters $(v,k,k-2,a)$</i>	55
Francesca Merola: <i>Cycle systems of the complete multipartite graph</i>	56
Oktay Olmez: <i>Partial Geometric Designs and Their Links</i>	56
Anita Pasotti: <i>Odd sun systems of the complete graph</i>	57
Mario Osvin Pavčević: <i>Constructions of some new t-designs acted upon non-abelian automorphism groups</i>	57
Alexander Pott: <i>Homogeneous cubic bent functions</i>	58
John R. Schmitt: <i>Distinct partial sums in cyclic groups</i>	58
Andrea Švob: <i>Self-orthogonal codes from orbit matrices of Seidel and Laplacian matrices of strongly regular graphs</i>	58
Tommaso Traetta: <i>Pyramidal Steiner and Kirkman triple systems</i>	59
Tanja Vučićić: <i>Groups $S_n \times S_m$ in construction of flag-transitive block designs</i>	59
Alfred Wassermann: <i>On q-analogues of group divisible designs</i>	59
Discrete and computational geometry	61
Drago Bokal: <i>Bounded degree conjecture holds precisely for c-crossing-critical graphs with $c \leq 12$</i>	62
Sergio Cabello: <i>Maximum Matchings in Geometric Intersection Graphs</i>	62
Éric Colin de Verdière: <i>Multicuts in planar and surface-embedded graphs</i>	62
Cyril Gavoille: <i>Shorter Implicit Representation for Planar Graphs</i>	63
Petr Hliněný: <i>On Colourability of Polygon Visibility Graphs</i>	63
Yulia Kempner: <i>Spanoids, Greedoids and Violator Spaces</i>	63
Giuseppe Liotta: <i>Bend-minimum Orthogonal Drawings</i>	64
Dömötör Pálvölgyi: <i>Polychromatic coloring of geometric hypergraphs</i>	64
Maria Saumell: <i>Hamiltonicity for convex shape Delaunay and Gabriel graphs</i>	65
Hans Raj Tiwary: <i>Lower bounds for semantic read-once BDDs using Extension Complexity</i>	65
Alen Vegi Kalamar: <i>Counting Hamiltonian cycles in 2-tiled graphs</i>	65
Distance-regular graphs	67
Robert F. Bailey: <i>On distance-regular graphs on 486 vertices</i>	68
Sarah Bockting-Conrad: <i>Tridiagonal pairs of Racah type and the universal enveloping algebra $U(\mathfrak{sl}_2)$</i>	68
Blas Fernández: <i>On the Terwilliger Algebra of Locally Pseudo-Distance-Regular Graphs</i>	68
Štefko Miklavič: <i>Irreducible T-modules with endpoint 1 of almost 1-homogeneous distance-regular graph</i>	68
Arnold Neumaier: <i>t-point counts in distance-regular graphs</i>	69
Safet Penjić: <i>A combinatorial approach to the Terwilliger algebra modules of a bipartite distance-regular graph</i>	69
Hiroshi Suzuki: <i>The universal C-cover of a completely regular clique graph</i>	70
Paul Terwilliger: <i>Leonard pairs, spin models, and distance-regular graphs</i>	70
Domination in graphs	71
Boštjan Brešar: <i>Graphs with a unique maximum open packing</i>	72
Paul Dorbec: <i>Reconfiguring and enumerating dominating sets.</i>	72

Tanja Gologranc: <i>On graphs with equal total domination and Grundy total domination number</i>	72
Michael A. Henning: <i>The independent domatic number and the total domatic number</i>	73
Lisa Hernandez Lucas: <i>Dominating sets in finite generalized quadrangles</i> . . .	73
Marko Jakovac: <i>Relating the annihilation number and two domination invariants</i>	73
Sandi Klavžar: <i>The Maker-Breaker domination game</i>	74
Tim Kos: <i>On the Grundy dominating sequences</i>	74
Tadeja Kraner Šumenjak: <i>On k-rainbow independent domination in graphs</i> . .	74
Aparna Lakshmanan S.: <i>Double Roman Domination Number</i>	74
Berenice Martínez-Barona: <i>Identifying codes in line digraphs</i>	75
Iztok Peterin: <i>$[1, k]$-domination number of lexicographic product of graphs</i> . . .	75
Aleksandra Tepeh: <i>On k-rainbow total domination in graphs</i>	76
Finite Geometries	77
Sam Adriaensen: <i>An Investigation into Small Weight Code Words of Projective Geometric Codes</i>	78
Simeon Ball: <i>Maximum Distance Separable Codes: Recent advances and applications</i>	78
Zoltán L. Blázsik: <i>Balanced upper chromatic number of $PG(2, q)$</i>	79
Bence Csajbók: <i>Generalising KM-arcs</i>	80
Maarten De Boeck: <i>Cameron-Liebler classes for finite geometries</i>	80
Lins Denaux: <i>Small weight code words arising from the incidence of points and hyperplanes in $PG(n, q)$</i>	81
Jozefien D'haeseleer: <i>Projective solids pairwise intersecting in at least a line</i> . .	81
Stephen Glasby: <i>Classical groups and transitive actions on subspaces</i>	82
Ferdinand Ihringer: <i>New SDP Bounds on Subspace Codes</i>	82
György Kiss: <i>On resolving sets for the point-line incidence graph of $PG(n, q)$</i> . . .	83
Michel Lavrauw: <i>MDS codes, arcs and tensors</i>	83
Giuseppe Marino: <i>Subspace code constructions</i>	84
Sam Mattheus: <i>Bipartite Ramsey numbers: C_4 versus the star</i>	85
Valentina Pepe: <i>A construction for clique-free pseudorandom graphs</i>	85
Tamás Szőnyi: <i>On the stability of Baer subplanes</i>	85
Games on graphs	87
Csilla Bujtás: <i>Domination and transversal games: Conjectures and perfectness</i> .	88
Simone Dantas: <i>The Graceful Game</i>	88
Jarosław Grytczuk: <i>Choosability Games</i>	89
Vesna Iršič: <i>Some results on the connected domination game</i>	89
Tijo James: <i>Domination game on split graphs</i>	89
Daniel Pinto: <i>Replicator equations on graphs</i>	91
András Pongrácz: <i>Non-deterministic decision making on finite graphs</i>	91
Gregory J. Puleo: <i>Online Sum-Paintability: Slow-Coloring of Trees</i>	91
Éric Sopena: <i>A connected version of the graph coloring game</i>	92
Daša Štesl: <i>Indicated coloring game on Cartesian products of graphs</i>	92
Douglas B. West: <i>The Slow-Coloring Game on a Graph</i>	92
General Graph Theory	95

Marién Abreu: <i>Orthogonal Array Configurations</i>	96
Kiyoshi Ando: <i>Some new local conditions for k-contractible edges</i>	96
Suzana Antunović: <i>Exponential generalised network descriptors</i>	97
Gábor Bacsó: <i>The Equal-Sum-Free Subset problem</i>	97
Fernando I. Becerra López: <i>Integer invariants of a graph manifold using Hirzenbruch-Jung continued fractions on the linking matrix.</i>	97
Simona Bonvicini: <i>A variant of orthogonality for symmetric Latin squares</i>	98
Sylwia Cichacz: <i>Group distance magic hypercubes</i>	98
Jacob Cooper: <i>Finitely forcible graph limits are universal</i>	99
Michał Dębski: <i>Edge colorings avoiding patterns</i>	99
Mark Ellingham: <i>The structure of 4-connected $K_{2,t}$-minor-free graphs</i>	99
Igor Fabrici: <i>Circumference of essentially 4-connected planar graphs</i>	100
Gašper Fijavž: <i>Forbidding prism immersions</i>	100
Hanna Furmańczyk: <i>Equitable list vertex colourability and arboricity of grids</i>	101
Jerzy Garus: <i>Underwater vehicle trajectory planning using the graph theory in dynamic environments with obstacles</i>	101
Jan Goedgebeur: <i>Graphs with few Hamiltonian Cycles</i>	101
Mariusz Grech: <i>Wreath product of permutation groups as the automorphism group of a graph.</i>	102
Dan Hawtin: <i>s-Elusive Codes in Hamming Graphs</i>	103
Michael Hecht: <i>Tight Localisations of Minimal Feedback Sets in Cubic Time</i>	103
Florian Hoersch: <i>Highly vertex-connected orientations of regular eulerian graphs</i>	103
Szabolcs Horvát: <i>Exact random sampling of connected graphs with a given degree sequence</i>	104
Gyula Y. Katona: <i>Complexity questions for minimally t-tough graphs</i>	104
Grzegorz Kubicki: <i>An optimal algorithm for stopping on the element closest to the center of an interval</i>	104
Mariusz Kwiatkowski: <i>Tree structured z-knotted triangulations of a sphere.</i>	105
Domenico Labbate: <i>Non bipartite regular 2-factor isomorphic graphs: an update</i>	105
Florian Lehner: <i>On the cop-number of toroidal graphs</i>	105
Christian Lindorfer: <i>Ends of graphs and the language of self-avoiding walks</i>	106
Mária Maceková: <i>Acyclic coloring of graphs with prescribed maximum average degree</i>	106
Snježana Majstorović: <i>Graphs preserving total distance upon vertex removal</i>	107
Davide Mattiolo: <i>Some results regarding upper and lower bounds on the circular flow number of snarks</i>	107
Giuseppe Mazzuoccolo: <i>Reduction of the Berge-Fulkerson Conjecture to cyclically 5-edge-connected snarks</i>	107
Maria Chiara Molinari: <i>Colored even cycle decompositions</i>	108
Atsuhiko Nakamoto: <i>Vertex-face structures in quadrangulations on surfaces</i>	108
Gábor Nyul: <i>Various generalizations of Bell numbers</i>	109
Deryk Osthus: <i>Decompositions into isomorphic rainbow spanning trees</i>	109
Silvia Pagani: <i>Discrete tomography: a graph-theoretical formulation of local uniqueness for two directions</i>	109
Vincenzo Pallozzi Lavorante: <i>AG codes from the second generalization of the GK maximal curve</i>	110
Mark Pankov: <i>Face z-monodromies in triangulations of surfaces</i>	110
Balázs Patkós: <i>On the general position problem on Kneser graphs</i>	110

Sofia J. Pinheiro: <i>Erdős-Gallai type results on weighted degree sequences of graphs</i>	111
Miguel Pizaña: <i>Cliques, Logic and Games</i>	111
Gabriella Rácz: <i>Generalized Lah numbers and their graph theoretical interpretation</i>	112
Murat Sahin: <i>Core partitions with d-distinct parts</i>	112
Binod Kumar Sahoo: <i>Vertex connectivity of the power graph of a finite cyclic group</i>	112
Yusuke Suzuki: <i>Partially Broken Orientations of Eulerian Plane Graphs</i>	113
Adam Tyc: <i>Z-knotted triangulations of surfaces</i>	113
Stefan Veldsman: <i>Congruences and subdirect representations of graphs</i>	113
Aleksander Vesel: <i>Packing colorings of Sierpiński-type and H graphs</i>	114
Mate Vizer: <i>On the Turán number of edge ordered graphs</i>	114
Chuanqi Xiao: <i>The Turán number of the square of a path</i>	114
Dong Ye: <i>Circuit Covers of Signed Graphs</i>	114
Bogdan Žak: <i>Determination of the optimal ship trajectory in a collision situation with the use of directed graphs</i>	115
Jean Paul Zerafa: <i>Hamiltonian cycles containing a prescribed perfect matching</i>	115
Graph coloring	117
Marcin Anholcer: <i>Majority coloring of infinite digraphs</i>	118
János Barát: <i>Decomposition of cubic graphs related to Wegner’s conjecture</i>	118
Ewa Drgas-Burchardt: <i>Consecutive colouring of oriented graphs</i>	118
Jasmina Ferme: <i>Graphs that are critical for the packing chromatic number</i>	119
Ewa Kubicka: <i>Total coloring of graphs with minimum sum of colors; existence of T-strong graphs and trees.</i>	119
Xueliang Li: <i>Monochromatic disconnection of graphs</i>	119
Borut Lužar: <i>Homogeneous Coloring of Graphs</i>	120
Jakub Przybyło: <i>The 1–2–3 Conjecture almost holds for regular graphs</i>	120
Douglas Rall: <i>Packing chromatic vertex-critical graphs</i>	120
Ingo Schiermeyer: <i>Polynomial χ-binding functions and forbidden induced subgraphs - a survey</i>	121
Riste Škrekovski: <i>Some results and problems on unique-maximum colorings of plane graphs</i>	121
Małgorzata Śleszyńska-Nowak: <i>Strong cliques in graphs</i>	121
Metric Graph Theory	123
Bhaskar DasGupta: <i>Computational complexities of three problems related to computing the metric anti-dimension of a graph.</i>	124
Ville Junnila: <i>On the Solid-Metric Dimension of a Graph</i>	124
Cong X. Kang: <i>The connected metric dimension at a vertex of a graph</i>	125
Aleksander Kelenc: <i>Hausdorff Distance Between Trees in Polynomial Time</i>	125
Dorota Kuziak: <i>Metric and strong metric dimensions of direct product graphs</i>	125
Tero Laihonen: <i>On Resolving Several Vertices in a Graph</i>	126
Mercè Mora: <i>k-Fault-tolerant resolving sets in graphs</i>	126
María Luz Puertas: <i>Bounding the determining number of a graph by removing twins</i>	127
Yunior Ramírez-Cruz: <i>Constrained incremental resolvability in dynamic graphs: a case study in active re-identification attacks on social networks</i>	127
Rinovia Simanjuntak: <i>Centroidal dimensions of product graphs</i>	129

Andrej Taranenko: <i>On graphs achieving the trivial upper bound for edge metric dimension</i>	129
Ismael G. Yero: <i>Uniquely identifying the vertices of a graph by means of distance multisets</i>	129
Eunjeong Yi: <i>The fractional k-metric dimension of graphs</i>	130
Polytopes	131
Javier Bracho: <i>A family of finite quiral polyhedra in S^3</i>	132
Marston Conder: <i>Chiral polytopes of arbitrarily large rank</i>	132
Maria Elisa Fernandes: <i>Locally spherical hypertopes</i>	132
Isabel Hubard: <i>Geometric chiral polyhedra in 3-dimensional spaces</i>	132
Dimitri Leemans: <i>Rank reduction of string C-group representations</i>	133
Tilen Marc: <i>Oriented matroids as graphs</i>	133
Elias Mochan: <i>2- orbit polytopes</i>	133
Luis Montejano: <i>On the Geometric Banach Conjecture</i>	134
Antonio Montero: <i>Highly symmetric toroidal polytopes</i>	134
Daniel Pellicer: <i>Tight chiral abstract polytopes</i>	134
Claudio Alexandre Piedade: <i>Possible degrees of Toroidal Regular Maps</i>	134
Moshe Rosenfeld: <i>Branko Grünbaum, the mathematician who beat the odds</i>	135
Asia Ivić Weiss: <i>Realizations of regular abstract polytopes in euclidean spaces</i>	135
Gordon Williams: <i>Geometric realizability of simplicial 3-spheres</i>	135
Steve Wilson: <i>Rotary 4-Maniflexes having one facet</i>	136
 Spectral Graph Theory	 137
Aida Abiad: <i>A characterization and an application of weight-regular partitions of graphs</i>	138
Ambat Vijayakumar: <i>The Distance Spectra of Some Graph Classes — A Survey</i>	138
Milica Andelic: <i>Tridiagonal matrices and spectral properties of some graph classes</i>	140
Francesco Belardo: <i>On some recent results of Slobodan K. Simić (1948-2019)</i>	140
Maurizio Brunetti: <i>On Some Spectral Properties of Signed Circular Caterpillars</i>	140
Cristina Dalfo: <i>A new general method to obtain the spectrum and local spectra of a graph from its regular partitions</i>	141
Alexander Farrugia: <i>Generating Pairs of Generalized Cosppectral Graphs from Controllable Graphs</i>	141
Alexander Gavriljuk: <i>On the multiplicities of digraph eigenvalues</i>	141
Wilfried Imrich: <i>The Structure of Quartic Graphs with Minimal Spectral Gap</i>	142
Bojana Mihailović: <i>Some examples of transformations that preserve $\text{sgn}(\lambda_2 - r)$</i>	142
Bojan Mohar: <i>About the smallest eigenvalue of non-bipartite graphs</i>	142
Kamal Lochan Patra: <i>Laplacian eigenvalues of the zero divisor graph of the ring \mathbb{Z}_n</i>	142
Soňa Pavlíková: <i>Inverting non-invertible labeled trees</i>	143
Paula Rama: <i>Spectral and combinatorial properties of lexicographic polynomials of graphs</i>	143
Irene Sciriha: <i>On graphs with the same main eigenspace</i>	144
Zoran Stanić: <i>Notes on spectra of signed graphs</i>	144
Tetsuji Taniguchi: <i>A generalization of Hoffman Graph</i>	144
Jianfeng Wang: <i>On graphs whose spectral radius does not exceed the Hoffman limit value</i>	145

Pepijn Wissing: <i>The negative tetrahedron and the first infinite family of connected digraphs that are strongly determined by the Hermitian spectrum</i>	145
Structural and algorithmic graph theory	147
Pierre Aboulker: <i>Subgraphs of directed graphs with large dichromatic number</i> .	148
Jesse Beisegel: <i>Maximum Weighted Clique in Hole-Cyclically Orientable Graphs</i>	148
Michael Hecht: <i>Topological Invariants of Elementary Cycles and a Generalization of Biggs Theorem</i>	148
Tony Huynh: <i>Unavoidable minors for graphs with large ℓ_p-dimension</i>	148
Eunjung Kim: <i>Erdős-Pósa Property of Chordless Cycles and its Applications</i> . . .	149
Miklós Krész: <i>Uniquely restricted (g, f)-factors</i>	149
Matjaž Krnc: <i>On the End-Vertex Problem of Graph Searches</i>	150
William Lochet: <i>Immersion of transitive tournaments</i>	150
Martin Milanič: <i>Avoidable Vertices, Avoidable Edges, and Implications for Highly Symmetric Graphs</i>	151
Alantha Newman: <i>Explicit 3-colorings for exponential graphs</i>	151
Shmuel Onn: <i>Deciding and Optimizing over Degree Sequences</i>	152
Nevena Pivač: <i>Minimal separators in graph classes dened by small forbidden induced subgraphs</i>	152
Miguel Pizaña: <i>Algorithmic Aspects of the Finite Extension Problem</i>	152
Ni Luh Dewi Sintuari: <i>Layered wheels</i>	153
Symmetries of graphs and maps	155
Martin Bachraty: <i>Skew morphisms of simple groups</i>	156
Antonio Breda d’Azevedo: <i>Regular bi-oriented maps of negative prime characteristic</i>	156
Domenico Catalano: <i>Strong map-symmetry of $SL(3, K)$ and $PSL(3, K)$ for any finite field K</i>	156
Marston Conder: <i>Observations and answers to questions about edge-transitive maps</i>	156
Ted Dobson: <i>Every 2-closed group of degree qp^2 has a semiregular element</i> . . .	157
Ben Fairbairn: <i>Some non-Beauville groups: Why you should always pay attention to what is said at wine receptions</i>	157
Michael Giudici: <i>Arc-transitive bicirculants</i>	157
Robert Jajcay: <i>Generalized Cayley maps</i>	157
Gareth A. Jones: <i>Realisation of groups as automorphism groups of maps and hypermaps</i>	158
Rafał Kalinowski: <i>Symmetry breaking in claw-free graphs of small maximum degree</i>	158
Carlisle King: <i>Edge-primitive 3-arc-transitive graphs</i>	159
István Kovács: <i>Groups all of whose Haar graphs are Cayley graphs</i>	159
Young Soo Kwon: <i>Square roots of automorphisms of cyclic groups</i>	159
Hoi Ping Luk: <i>Tilings of the Sphere by Almost Equilateral Pentagons</i>	160
Martin Mačaj: <i>On external symmetries of Wilson maps</i>	160
Aleksander Malnič: <i>Covers of digraphs</i>	160
Adnan Melekoğlu: <i>Patterns of edge-transitive maps</i>	161
Rögnvaldur G. Möller: <i>Infinite vertex-transitive graphs and their arc-types</i> . .	161

Luke Morgan: <i>The distinguishing number of 2-arc-transitive graphs and permutation groups</i>	162
Roman Nedela: <i>Reductions of maps preserving the isomorphism relation I</i> . . .	162
Monika Pilšniak: <i>Distinguishing colorings of maps</i>	163
Daniel Pinto: <i>Duality and Chiral-Duality</i>	163
Primož Potočnik: <i>Generalised voltage graphs</i>	163
Nemanja Poznanovic: <i>Normal quotients of four-valent G-oriented graphs</i> . . .	164
Marina Šimac: <i>LDPC codes constructed from cubic symmetric graphs</i>	164
Primož Šparl: <i>On loosely attached tetravalent half-arc-transitive graphs</i>	164
Micael Toledo: <i>Cubic graphs with long orbits</i>	165
Thomas Tucker: <i>Surface Symmetry: Kulkarni Revisited</i>	165
Peter Zeman: <i>Reductions of maps preserving the isomorphism relation II</i>	165
Speaker index	167

GENERAL INFORMATION

Bled'19 – 9th Slovenian International Conference on Graph Theory

Bled, Slovenia, June 23 – 29, 2019

ORGANIZED BY:

IMFM (*Institute of Mathematics, Physics and Mechanics*)

IN COLLABORATION WITH:

DMFA (*Society of Mathematicians, Physicists and Astronomers of Slovenia*) SDAMS (*Slovenian Discrete and Applied Mathematics Society*) UL FMF (*University of Ljubljana, Faculty of Mathematics and Physics*),

UM FNM (*University of Maribor, Faculty of Natural Sciences and Mathematics*),

UP FAMNIT (*University of Primorska, Faculty of Mathematics, Natural Sciences and Information Technologies*),

UP IAM (*University of Primorska, Andrej Marušič Institute*),

Abelium d.o.o.

SCIENTIFIC COMMITTEE:

Sandi Klavžar, Dragan Marušič, Bojan Mohar (chair), Tomaž Pisanski

ORGANIZING COMMITTEE:

Boštjan Brešar, Sergio Cabello, Ademir Hujdurović, Rok Požar

CONFERENCE VENUES:

Hotel Kompas Bled, Cankarjeva Cesta 2, SI-4260, Bled, Slovenia

Best Western Premier Hotel Lovec, Ljubljanska cesta 6, SI-4260, Bled, Slovenia

Rikli Balance Hotel, Cankarjeva Cesta 4, SI-4260, Bled, Slovenia

CONFERENCE WEBSITE:

<https://conferences.matheo.si/e/bled19>

PLENARY SPEAKERS:

Noga Alon, *Princeton University, USA, and Tel Aviv University, Israel*

Marco Buratti, *Università di Perugia, Italy*

Gareth A. Jones, *University of Southampton, UK*

Gábor Korchmáros, *Università della Basilicata, Italy*

Daniel Král', *Masaryk University, Czech Republic, and University of Warwick, UK*

Daniela Kühn, *University of Birmingham, UK*

Sergei Lando, *University Higher School of Economics, Skolkovo Institute of Science and Technology, Moscow, Russia*

János Pach, *EPFL, Switzerland, and Rényi Institute, Hungary*

Cheryl E. Praeger, *University of Western Australia*

Zsolt Tuza, *Rényi Institute and University of Pannonia, Hungary*

Xuding Zhu, *Zhejiang Normal University, China*

INVITED SPECIAL SESSIONS AND THEIR ORGANIZERS

Association Schemes (*Mikhail Muzychuk*)

Biomathematics and Bioinformatics (*Marc Hellmuth*)

Chemical graph theory (*Xueliang Li*) – This session is associated with the meeting of the International Academy of Mathematical Chemistry, IAMC 2019

Configurations (*Gábor Gévay*)

Designs (*Dean Crnković*)

Discrete and computational geometry (*Sergio Cabello*)

Distance-regular graphs (*Štefko Miklavič*)

Domination in graphs (*Michael A. Henning*) – This session celebrates the 70th birthday of Douglas F. Rall.

Finite Geometries (*Tamás Szőnyi*)

Games on graphs (*Csilla Bujtás*)

Graph coloring (*Ingo Schiermeyer*)

Metric Graph Theory (*Ismael G. Yero*)

Polytopes (*Asia Ivić Weiss*) – This session celebrates the life and work of Branko Grünbaum

Spectral Graph Theory (*Francesco Belardo*)

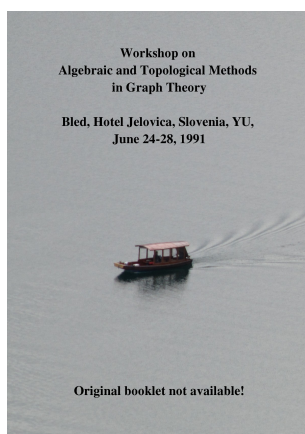
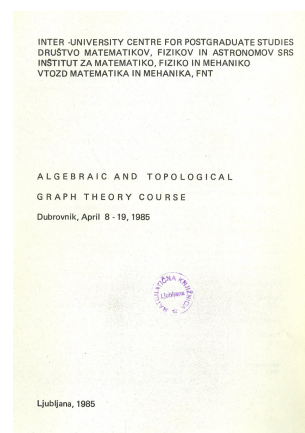
Structural and algorithmic graph theory (*Pierre Aboulker*)

Symmetries of graphs and maps (*Marston Conder*)

PAST CONFERENCES

1ST SLOVENIAN INTERNATIONAL CONFERENCE ON GRAPH THEORY

a. k. a. Algebraic and Topological Graph Theory Course. Dubrovnik (now Croatia), April 8–19, 1985. 47 participants, 10 invited speakers, 37 talks. Scientific Committee: W. Imrich, T. D. Parsons, T. Pisanski.

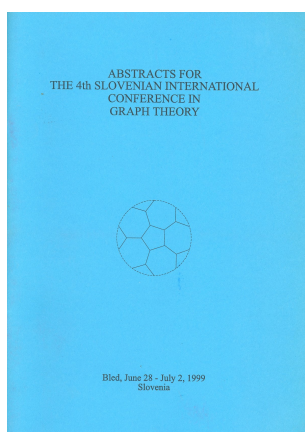
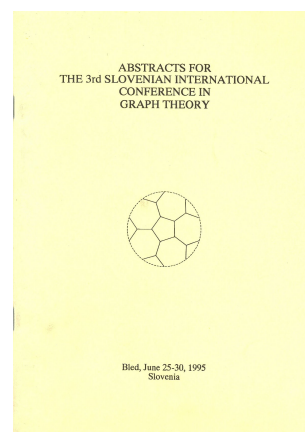


2ND SLOVENIAN INTERNATIONAL CONFERENCE ON GRAPH THEORY

a. k. a. Workshop on Algebraic and Topological Methods in Graph Theory. Bled, Slovenia, YU, June 24–28, 1991. Around 30 participants, 7 invited speakers. The meeting started in FLR Yugoslavia and ended in independent Slovenia. Scientific Committee: V. Batagelj, D. Marušič, B. Mohar and T. Pisanski. Conference proceedings printed in a special issue of Discrete Math, Vol 134 (1994).

3RD SLOVENIAN INTERNATIONAL CONFERENCE ON GRAPH THEORY

Bled, June 25–30, 1995. 8 invited speakers, 79 talks, general session and 6 special sessions. Scientific Committee: D. Marušič and B. Mohar. Conference proceedings printed in a special issue of Discrete Math, Vol 182 (1998).

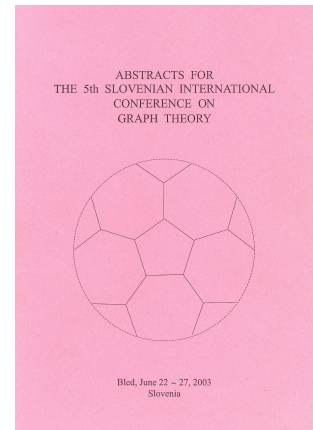


4TH SLOVENIAN INTERNATIONAL CONFERENCE ON GRAPH THEORY

Bled, June 28–July 2, 1999. 18 invited speakers, 92 talks. Scientific Committee: S. Klavžar, D. Marušič and B. Mohar. Proceedings in a special issue of Discrete Math, Vol. 244 (2002).

5TH SLOVENIAN INTERNATIONAL CONFERENCE ON GRAPH THEORY

Bled, June 22–27, 2003. 87 participants, 13 invited speakers, 72 talks. Scientific Committee: B. Brešar, M. Juvan, S. Klavžar, D. Marušič, A. Malnič, B. Mohar, T. Pisanski. Conference proceedings printed in a special issue of Discrete Math, Vol. 307 (2007).



6TH SLOVENIAN INTERNATIONAL CONFERENCE ON GRAPH THEORY

Bled, June 24–30, 2007. 15 invited speakers, 189 registered contributions, general session, 14 minisymposia and 2 satellite conferences. Scientific Committee: S. Klavžar, D. Marušič, B. Mohar, T. Pisanski. Conference proceedings printed in special issues of Discrete Math, Vol. 310 (2010) and Ars Mathematica Contemporanea, Vol. 1 (2008).

7TH SLOVENIAN INTERNATIONAL CONFERENCE ON GRAPH THEORY

Bled, June 19–25, 2011. 10 keynote speakers, 222 registered contributions, general session, 17 minisymposia and 4 satellite events. Scientific Committee: S. Klavžar, D. Marušič, B. Mohar, T. Pisanski. Conference proceedings printed in special issue of Ars Mathematica Contemporanea, Vol. 6 (2013).



8TH SLOVENIAN CONFERENCE ON GRAPH THEORY

Kranjska Gora, June 21–27, 2015. 9 plenary speakers, 247 registered contributions, general session, 13 minisymposia and 3 satellite events. Scientific Committee: S. Klavžar, D. Marušič, B. Mohar, T. Pisanski.

PLENARY INVITED TALKS

Random and quasi-random Cayley sum graphs with applications

Noga Alon

Princeton University and Tel Aviv University

For a finite abelian group A and a subset S of A , the Cayley sum graph $G(A, S)$ is the graph whose vertex set is A , where a, b are adjacent iff $a + b$ lies in S and a, b are distinct. I will describe several old and new applications of random and quasi-random Cayley sum graphs in extremal graph theory, additive number theory, geometry and coding theory, and will mention several intriguing open problems.

Cyclic designs: some selected topics

Marco Buratti

Università di Perugia

A combinatorial structure is said to be *cyclic* if it admits an automorphism that cyclically permutes all its points or vertices depending on the type of the structure. In this talk I will select some problems on cyclic *designs* as, for instance, how to determine a significant lower bound on the number of cyclic *Steiner triple systems* of an assigned order or how to construct cyclic *graph decompositions over a finite field*. In particular, I would like to speak about some collateral (probably new) problems concerning *difference sets*.

Paley, Carlitz and the Paley Graphs

Gareth A. Jones

University of Southampton, UK

Anyone who seriously studies algebraic graph theory or finite permutation groups will, sooner or later, come across the Paley graphs and their automorphism groups. The most frequently cited sources for these are respectively Paley's 1933 paper for their discovery, and Carlitz's 1960 paper for their automorphism groups. It is remarkable that neither of those papers uses the concepts of graphs, groups or automorphisms. Indeed, one cannot find these three terms, or any synonyms for them, in those papers: Paley's paper is entirely about the construction of what are now called Hadamard matrices, while Carlitz's is entirely about permutations of finite fields.

The aim of this talk is to explain how this strange situation came about, by describing the background to these two papers and how they became associated with the Paley graphs. This involves links with other branches of mathematics, such as matrix theory, number theory, design theory, coding theory, finite geometry, polytope theory and group theory, reaching back to 1625. I will summarise the life and work of these two great mathematicians, together with important contributions from Coxeter and Todd, Sachs, and Erdős and Rényi. I will also briefly cover some recent developments concerning generalised Paley graphs. A preprint is available at <https://arxiv.org/abs/1702.00285>.

Permutation groups in the study of geometric structures

Gábor Korchmáros

Università della Basilicata, Italy

The studies of permutation groups developed into a sophisticated theory over the years, also stimulated by investigations concerning Finite geometries, Graph theory, and, in recent years, algebraic curves over finite fields. The interplay between permutation groups and finite geometric (and/or combinatorial) structures occurs by means of symmetries, or automorphisms in modern terminology. The saying that *the larger its automorphism group, the richer its geometry* is especially appropriate in finite geometric structures. The huge amount of work done in this area has moved around the following questions.

- (i) Construction of nice finite geometric structure from permutation group.
- (ii) Characterizations of geometric structures by their automorphism groups.
- (iii) How large can be the automorphism group of a geometric structure.

In our talk we focus on finite geometric structures on which a large automorphism group can act. A key issue is to understand the constraints imposed by the underlying geometry on the structure and action of its automorphism groups since even relevant geometric structures may happen to impose drastic restrictions so that their automorphism groups are either trivial or of limited order. We illustrate geometric structures embedded in a projective space where the interplay between geometry and permutation groups works well and gives substantial contributes to the questions (i) and (ii).

We also give an overview of the recent developments in the study of algebraic curves over a finite field which have many automorphisms with respect to their genera and Hasse-Witt invariants. Geometry and combinatorics together with deeper results from Group theory (especially on permutation groups) have been an essential tool in improving previous results obtained by classical methods based on Algebraic geometry and Function field theory.

Profiles of graphs and tournaments

Daniel Král'

Masaryk University and University of Warwick

The interaction between substructure densities is one of the central themes in extremal combinatorics. For example, the famous Erdős-Rademacher Problem asks for determining the minimum possible triangle density in a graph with a given edge density. In this talk, we survey recent developments concerning tools for studying such problems. In particular, we provide an overview of analytic tools, which have been applied with great success in the graph setting. At the end of the talk, we focus on problems concerning tournament profiles, where the analytic tools fail to provide satisfactory results, and present recent results on cycle profiles obtained using methods from linear algebra.

Resolution of the Oberwolfach problem

Daniela Kühn

University of Birmingham

The Oberwolfach problem, posed by Ringel in 1967, asks for a decomposition of the complete graph of order $2n + 1$ into edge-disjoint copies of a given 2-factor. We show that this can be achieved for all large n . We actually prove a significantly more general result, which allows for decompositions into more general types of factors. In particular, this also resolves the Hamilton-Waterloo problem for large n .

This is joint work with Stefan Glock, Felix Joos, Jaehoon Kim and Deryk Osthus.

Integrability property of polynomial graph invariants

Sergei Lando

*University Higher School of Economics, Skolkovo Institute of Science and Technology,
Moscow, Russia*

Stanley's symmetrized chromatic polynomial, which generalizes the conventional chromatic polynomial, was discovered in middle 90'ies independently by R. Stanley on one side and S. Chmutov, S. Duzhin, and S. Lando (under the name of *weighted chromatic polynomial*) on the other side. We show that the generating function for the symmetrized chromatic polynomial of all connected graphs satisfies (after appropriate scaling change of variables) the Kadomtsev–Petviashvili integrable hierarchy of mathematical physics. Moreover, we describe a large family of polynomial graph invariants giving the same solution of the KP. The key point here is a Hopf algebra structure on the space spanned by graphs and the behavior of the invariants on its primitive space.

It is interesting that similar Hopf algebras of other combinatorial objects seem to be not related to integrable hierarchies of partial differential equations.

The talk is based on a joint work with S. Chmutov and M. Kazarian [arxiv:1803.09800](https://arxiv.org/abs/1803.09800).

Many Pairwise Crossing Edges

János Pach

EPFL, Lausanne and Rényi Institute, Budapest

We show that any set of n points in general position in the plane determines at $n^{1-o(1)}$ pairwise crossing segments. The best previously known lower bound, $\Omega(\sqrt{n})$, was proved more than 25 years ago by Aronov, Erdős, Goddard, Kleitman, Klugerman, Pach, and Schulman. Our proof is fully constructive, and extends to dense geometric graphs. We also discuss several related extremal problems for geometric graphs.

This is joint work with Natan Rubin, and Gábor Tardos.

Limited geodesic transitivity for finite regular graphs

Cheryl E. Praeger

University of Western Australia

For vertex transitive graphs, transitivity on t -arcs, t -geodesics, or distance t vertex pairs, for $t \leq s$, all give symmetry measures of the graph in balls of radius s about the vertices. If the graph has girth g and $s \leq g/2$, then the sets of t -arcs and t -geodesics are the same for each $t \leq s$, and so the conditions of s -arc transitivity and s -geodesic transitivity are equivalent. The next cases where $s = (g + 1)/2$ and $s = (g + 2)/2$ are interesting. For these values there are s -geodesic transitive examples that are not s -arc transitive. Those which have $s = 2$ and $g = 3$ are collinearity graphs of point-line incidence geometries. However there is no nice general description for the cases where $s = 3$ and g is 4 or 5. Our approach to analysing these cases (this is joint work with Wei Jin) has required us to classify, as a by product, all 2-arc transitive strongly regular graphs, and to examine their normal covers. We have lots to describe, as well as several open problems to pose.

Most of what I discuss is joint work with Wei Jin.

Parity colorings in graphs and hypergraphs

Zsolt Tuza

Rényi Institute and University of Pannonia, Hungary

We study vertex colorings of graphs and set systems, in which the parity of the multiplicity of colors is restricted in the specified subsets. Similarities and remarkable differences occur in comparison to the coloring theory of mixed hypergraphs.

This is joint work with Július Czap.

List colouring and Alon-Tarsi number of planar graphs

Xuding Zhu

Zhejiang Normal University, China

It is well-known that there are planar graphs that are not 4-choosable, although every planar graph is 4-colourable. The problem we are interested is how far can a planar graph from being 4-choosable? One measure of the distance of a graph G from being 4-choosable is "what subgraphs need to be removed so that the resulting graph is 4-choosable". Another measure is "what proportion of vertices need to have 5-permissible colours in an assignment L so that G is L -colourable". We say every planar graph is $(4 + \epsilon)$ -choosable, if every planar graph G has a subset X of vertices with $|X| \leq \epsilon|V(G)|$ such that for any list assignment L of G for which $|L(v)| = 5$ for $v \in X$ and $|L(v)| = 4$ for $v \notin X$, G is L -colourable. We are interested what is the smallest ϵ such that every planar graph is $(4 + \epsilon)$ -choosable.

For the first question, it was proved by Cushing and Kierstead that every planar graph is 1-defective 4-choosable. In other words, for any 4-assignment L of G , there is

a matching M such that $G - M$ is L -colourable. However, in Cuhsing and Kierstead's proof, the choice of M depends on L . We shall show that every planar graph G has a matching M such that $G - M$ is 4-choosable. I.e., there is a matching M that works for all 4-assignment L . The difference may seem small. However, we observe that every planar graph is 2-defective 3-choosable, but there are planar graphs G such that for any subgraph H of G of maximum degree at most 3, $G - E(H)$ is not 3-choosable. For the second problem, we show that every planar graph is $(4 + 1/2)$ -choosable. An earlier result of Mirzakhani shows that the smallest such ϵ is at least $1/63$. Our results are proved by using Combinatorial Nullstellensatz. We prove that every planar graph G has a matching M such that $G - M$ has Alon-Tarsi number at most 4, and also there is an orientation of the matching M so that G is f -Alon-Tarsi, where $f(v) = 5$ if v is the tail of an edge in M and $f(v) = 4$ otherwise. This result also implies that every planar graph is 1-defective 4-painytable, and improves the result that every planar graph has Alon-Tarsi number at most 5.

This is a joint work with Grytczuk.

Invited Special Session

ASSOCIATION SCHEMES

Organized by MIKHAIL MUZYCHUK

Nilpotent commutative standard integral table algebras of order p^3

Harvey Blau

Northern Illinois University

We determine up to exact isomorphism (that is, we find the explicit structure constants) for the class three nilpotent commutative standard integral table algebras of order p^3 , for an arbitrary prime p . We discuss the consequences for p -valenced association schemes and for p -Schur rings over abelian groups of this order. This is joint work with Caroline Kettlestrings.

Hypergroup rings

Christopher French

Grinnell College

The notion of a hypergroup is a natural extension of that of a group, allowing us to study structures where an ordered pair of elements determine not a single product, but a set of elements, which we call their hyperproduct. Many interesting structures have underlying hypergroups, association schemes being a notable example.

A growing body of research on association schemes has focused on the so-called "metathin" schemes, those which contain a thin closed subset with thin quotient. Two features of such metathin schemes naturally arise: first, for each relation p , we have $pp^*p = \{p\}$. This condition implies that p^*p is a closed subset, and, for metathin association schemes, one can show that this closed subset is a normal closed subset of the thin residue.

In this talk, I will present some recent work, in which we have studied hypergroups (not necessarily metathin) satisfying the two properties described above. We have found that these hypergroups determine a hypergroup ring; if the underlying hypergroup of an association scheme satisfies the two properties above, then the scheme ring coincides with the hypergroup ring. In particular, our construction generalizes that of a group ring.

This talk is based on joint work with Paul-Hermann Zieschang.

Rationality for irreducible representations of association schemes of rank 6 and 7

Allen Herman

University of Regina

We study the question of when an irreducible complex representation of the adjacency algebra of a finite association scheme is realized in rational matrices. We will show that irreducible representations are always rational for noncommutative association schemes of rank 6 and 7. For noncommutative association schemes of rank 6, rationality of the degree 2 irreducible representation can be deduced from its explicit construction in the recent article of Muzychuk, Xu, and the speaker. In fact this same construction

yields rationality for irreducible representations of noncommutative integral table algebras of rank 6 over their field of definition. In rank 7, we do find examples of table algebras with irrational irreducible representations. Nevertheless we can appeal to p -adic methods to show such table algebras can never be integral, and so association schemes of this kind will not occur.

Schur rings over infinite groups

Stephen Humphries
Brigham Young University

We define non-traditional Schur rings over various infinite groups, including some Schur rings over surface groups, some free products with amalgamation, $PSL(2, \mathbb{Z})$, and some groups related to crystallographic groups. Here non-traditional means Schur rings that are not direct products or wedge products or orbit Schur rings.

Krein parameters of fiber-commutative coherent configurations

Akihiro Munemasa
Tohoku University

In this talk, I would like to convince the audience that fiber-commutative coherent configurations form a more natural class of configurations containing the class of commutative association schemes, than the class of non-commutative association schemes. To support this view, we show that, for fiber-commutative coherent configurations, eigenmatrices and Krein parameters can be defined essentially uniquely. Precisely speaking, Krein parameters for commutative association schemes generalize to matrices, and the system of inequalities called the Krein condition reduces to positive semidefiniteness of matrices of Krein parameters. This results in a simplification of the absolute bound using the matrices of Krein parameters.

This is joint work with Keiji Ito. Preprint available at <https://arxiv.org/abs/1901.11484>.

Non-commutative schemes of rank six and related objects

Misha Muzychuk
Ben-Gurion University of the Negev

Non-commutative schemes of rank six are of special interest, since they have the smallest rank among non-commutative schemes. An intensive study of these schemes was started by Hanaki and Zieschang and continued by other authors. In my talk I'm going to present some recent developments in the area and discuss connections with other well-known combinatorial objects like strongly regular graphs, directed strongly regular graphs etc.

This is joint work with M. Klin and S. Reichard.

A characterization of some equivalenced association schemes

Ilia Ponomarenko

St.Petersburg Department of Steklov Mathematical Institute

An association scheme is said to be equivalenced if all the valencies except for the trivial one are the same. We present a characterization in terms of multidimensional intersection numbers, for two exceptional families of equivalenced schemes. Based on joint work with G.Chen and J.He

Infinite family of non-schurian separable association schemes

Grigory Ryabov

Novosibirsk State University

A coherent configuration is defined to be schurian if all its basis relations are 2-orbits of an appropriate permutation group and it is defined to be separable if every algebraic isomorphism of this coherent configuration to another one is induced by a combinatorial isomorphism. Each separable coherent configuration is determined up to an isomorphism only by the tensor of its intersection numbers. The problems of determining whether given a coherent configuration is schurian and determining whether given a coherent configuration is separable are among the most fundamental problems in the theory of coherent configurations. It is known that there exist infinite families of coherent configurations which are: (1) schurian and separable; (2) schurian and non-separable; (3) non-schurian and non-separable. The following question was asked, in fact, in [1]: whether there exists an infinite family of non-schurian separable coherent configurations? We give an affirmative answer to this question.

Theorem. There exists an infinite family of non-schurian separable association schemes.

Bibliography

- [1] G. Chen, I. Ponomarenko, Lectures on Coherent Configurations (2019), <http://www.pdmi.ras.ru/~inp/ccNOTES.pdf>.

New family of CI-groups

Gábor Somlai

Eötvös Loránd University

The investigation of CI-groups started in 1967 with a seemingly innocent question of Ádám [1] and was later generalized by Babai [2]. A group G is called a CI-group if and only if two isomorphic Cayley graphs of G are isomorphic via an isomorphism of G as well. Many of the most important results concerning CI-groups deals with elementary abelian p -groups which are the Sylow p -subgroups of finite CI-groups if $p > 3$. Recently, the study of direct sums of elementary abelian CI-groups was initiated

by Kovács and Muzychuk [3]. They proved that $\mathbb{Z}_p^2 \times \mathbb{Z}_q$ is a CI-group if p and q are primes. We extend this result by proving that $\mathbb{Z}_p^3 \times \mathbb{Z}_q$ is also a CI-group. The techniques involved are based on Schur ring methods and one of the key observations uses a recent result of Kovács and Ryabov [4]. The proof also gives a description of certain type of Schur rings.

Joint work with Mikhail Muzychuk.

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- [1] A. Ádám, Research Problem 2-10, J. Combin. Theory, 2 (1967), 393.
- [2] L. Babai, Isomorphism problem for a class of point-symmetric structures, Acta Math. Acad. Sci. Hungar. 29 (1977), 329-336.
- [3] I. Kovács, M. Muzychuk, The group $\mathbb{Z}_p^2 \times \mathbb{Z}_q$ is a CI-group, Comm. Alg. 37 (2009), 3500–3515.
- [4] I. Kovács, G. Ryabov, CI-property for decomposable Schur rings over an abelian group, Algebra Colloquium 26, No. 01 (2019), 147–160.

Linked systems of group divisible designs

Sho Suda

Aichi University of Education

As a generalization of linked systems of symmetric designs, the concepts of linked systems of symmetric group divisible designs are introduced. The connection with association schemes is established, and as a consequence we obtain an upper bound on the number of symmetric group divisible designs which are linked. Several examples of linked systems of symmetric group divisible designs are provided. This talk is based on joint work with Hadi Kharaghani.

Closures of permutation groups and the isomorphism problem for schurian coherent configurations

Andrey Vasil'ev

Sobolev Institute of Mathematics and Novosibirsk State University

There is a natural Galois correspondence between subgroups of symmetric group on a set Ω and coherent configurations defined on Ω . The closed objects with respect to that correspondence are *2-closed* permutation groups and *schurian* coherent configurations. In our talk, we will address the question how one can effectively solve the following problems.

2-Closure Problem. *Given a finite permutation group, find the 2-closure of it.*

Isomorphism Problem for Schurian Coherent Configurations. *Given two schurian coherent configurations, find the set of isomorphisms between them.*

On tight 4-designs in Hamming association schemes

Janoš Vidali

University of Ljubljana

We complete the classification of tight 4-designs in Hamming association schemes $H(n, q)$, i.e., that of tight orthogonal arrays of strength 4, which had been open since a result by Noda (1979). To do so, we construct an association scheme attached to a tight 4-design in $H(n, q)$ and analyze its triple intersection numbers to conclude the non-existence in all open cases.

Joint work with Alexander Gavriljuk and Sho Suda.

Preprint available at <https://arxiv.org/abs/1809.07553>.

Residually Thin Hypergroups

Paul-Hermann Zieschang

University of Texas Rio Grande Valley, Edinburg (U.S.A.)

The notion of a hypergroup (in the sense of [1], but without commutativity) provides a far reaching and meaningful generalization of the concept of a group. Specific classes of hypergroups have given rise to challenging questions and interesting connections to geometric and group theoretic topics; cf. [2], [3], and [5]. In the present article, we investigate residually thin hypergroups, that is hypergroups S which contain closed subsets T_0, T_1, \dots, T_n such that $T_0 = \{1\}$, $T_n = S$, and, for each element i in $\{1, \dots, n\}$, $T_{i-1} \subseteq T_i$ and $T_i//T_{i-1}$ is thin. In our first main result, we analyze the normal structure of residually thin hypergroups. Our second main result says that hypergroups are residually thin if all of their elements s satisfy $ss^*s = \{s\}$.

The results are obtained jointly with Chris French and can be considered as a contribution to an abstract approach to hypergroups; cf. [4].

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Invited Special Session

BIOMATHEMATICS AND BIOINFORMATICS

Organized by MARC HELLMUTH

On the Computational Hardness of Ancestral Genome Reconstructions

Daniel Doerr

Bielefeld University, Germany

We are interested in the problem of computing an ancestral genome that is a median of a set of given genomes. The median problem consists of finding a genome, called the *median genome*, such that the sum of the rearrangement distances between this new genome and each given genome is minimized. For each rearrangement distance that one considers we have a distinct median problem. The simplest model represents genomes as a set of chromosomes, where each chromosome can be linear or circular and composed of a sequence of oriented genes. Furthermore, for a given set of genes \mathcal{G} , this model assumes that each gene $g \in \mathcal{G}$ occurs exactly once in each genome, that is, g occurs once in exactly one chromosome of each genome.

It is well known that the similarities and differences between two circular genomes under the described model can be represented by a bipartite graph composed of even-length cycles only. A distance between the two genomes can be directly derived from this graph. Let c be the total number of cycles and let c_i , for $i = 2, 4, 6, \dots$, be the number of cycles of length i in the graph. Let the *breakpoint distance* be $|\mathcal{G}| - c_2$, and let the *general distance* be $|\mathcal{G}| - c$. While computing the median for the breakpoint distance is polynomial, computing it for the general distance is NP-hard, even for an input set of only three genomes. Our goal is to determine the boundary in which the complexity changes. To this end, we first concentrate on studying the complexity of the median for the c_4 -distance, that is $|\mathcal{G}| - c_2 - c_4$. We conjecture that the complexity for the median in this case follows the one for the general distance and is NP-hard.

This work is of interest to theoretical bioinformatics, since neither the polynomial approach to solve the median problem considering the breakpoint distance can be extended to the median of c_4 -distance, nor the hardness proof of the median problem considering the general distance can be adapted to the median of the c_4 -distance. We devise a completely new strategy, by re-modeling the problem in graph-theoretical terms: we study the equivalent problem of maximizing the number of bicolored cycles of length 2 or 4 in a 4-colored Berge graph, for which a 3-colored subgraph is already given.

In this talk, we present a general overview of the problem and its relevance in ancestral reconstruction and outline a strategy for proving our conjecture.

Joint work with Marília D. V. Braga, Cedric Chauve, Fábio V. Martinez, Diego Rubert, and Jens Stoye.

Network Statistics on biological data analyses

André Fujita

University of São Paulo

Graphs have been used as tools to study the interaction among biological components. For example, in molecular biology, biomedical researchers are interested in analyzing the interactions among genes and their products. In neuroscience, researchers wish to understand how is the relationship between brain connectivity and phenotype. Traditional graph theory assumes that the graph is deterministic, and consequently, the algorithms usually aim at identifying patterns (motifs), colors, and isomorphism. More recent approaches are based on measures that characterize complex networks, such as centrality and clustering coefficients. However, empirical networks present two often ignored characteristics, i.e., heterogeneity and randomness, which make the use of these approaches very limited. Notice that even complex

network measures do not explain the underlying biological mechanisms due to the lack of a mathematical model that generates the graph with a given set of characteristics (e.g. centrality measure). Thus, a natural solution for this problem could be the use of a framework based on formal statistical methods on random graphs, such as parameter estimation, model selection, comparison, correlation, and causality. In this talk, I will present some formal statistical tools we have developed over the last years to analyze biological networks.

Best Match Graphs

Marc Hellmuth

University Greifswald, Germany

Best matches play an important role in numerous applications in computational biology, in particular as the basis of many widely used tools for orthology detection. Let T be a phylogenetic (gene) tree and σ an assignment of leaves of T to species. The best match graph (BMG) is a vertex-colored digraph (G, σ) that contains an arc from x to y if the genes x and y reside in different species and y is one of possibly many (evolutionary) closest relatives of x compared to all other genes contained in the species $\sigma(y)$. In this talk, we investigate the structure of BMGs and provide polynomial time algorithms for the recognition of BMGs and the reconstruction of their unique least resolved trees. In addition, we will investigate the structure of reciprocal best match graphs, i.e., the symmetric part of best match graphs.

Articles:

Best Match Graphs, Geiß et al., *Journal Math. Biology* (doi.org/10.1007/s00285-019-01332-9), 2019

Reciprocal Best Match Graphs, Geiß, Stadler, Hellmuth, (arxiv.org/abs/1903.07920v1), 2019

Trees on scales – measures of balance for rooted binary trees

Lina Herbst

University of Greifswald

Trees are graph theoretical objects, which are used in different areas of research ranging from computer science to evolutionary biology, where they are used to represent the evolutionary relationships among different species. Often it is of interest to study the structure and shape of a tree, in particular its degree of balance. In order to measure the balance of a tree, several balance indices have been introduced and have lately gained considerable interest in the literature. The *Sackin index* and the *Colless index* are two of the most popular indices for rooted binary trees. In my talk, we study the extremal properties of both indices. Additionally, I show differences and similarities between both of them. While the so-called caterpillar tree is the unique tree with maximal Sackin and maximal Colless index, there are instances where there exists more than one tree with minimal Sackin or minimal Colless index.

Navigating the space of phylogenetic networks

Remie Janssen

Delft University of Technology

A phylogenetic network is a representation of evolutionary history involving complex interactions such as hybrid speciation and horizontal gene transfer. Finding a network that best represents evolutionary history often consists of solving hard optimization problems. Hence, good solutions are often found using heuristics. A frequently used technique is local search through the space of phylogenetic networks. The efficacy of such methods depends on the allowed steps through this space, which are called rearrangement moves in the study of phylogenetic networks. Recently, many rearrangement moves for phylogenetic trees have been generalized for phylogenetic networks. This results in different spaces of phylogenetic networks. These spaces are graphs in which each vertex represents a phylogenetic network and there is an edge between two networks if one can be transformed into the other with one rearrangement move. Hence, the generalized moves come with a myriad of questions about the spaces they define: Is the space of phylogenetic networks connected? If so, what is the diameter? Can we find bounds on the shortest sequence of moves between a pair of networks? Is it hard to find a shortest sequence? Are certain restrictions of the space of networks isometrically embedded into the whole space of networks? In this talk, I will show solutions to several of these problems and state some open problems.

Will we ever find a forbidden subgraph characterization of leaf powers?

Manuel Lafond

Université de Sherbrooke

A common task in computational biology is to find an evolutionary tree that explains proximity relationships between species. Representing these relationships as a graph motivates the notion of leaf powers: a graph $G = (V, E)$ is a leaf power if there exist a tree T on leafset V and a threshold k such that $uv \in E$ if and only if the distance between u and v in T is at most k . Deciding whether a given graph is a leaf power can be useful to validate whether the proximity relationships inferred by a biologist can be explained by some evolutionary tree.

Characterizing leaf powers is a challenging open problem, along with determining the algorithmic complexity of their recognition. In this talk, I will briefly survey some recent progress on forbidden induced subgraph characterizations of this graph class. In particular, we will discuss how leaf powers were thought to possibly coincide with strongly chordal graphs, then how it was believed that there could be only finitely many (minimal) non-strongly chordal leaf powers, and finally how this was disproved very recently. During the process, we will establish some connections between leaf powers and quartet compatibility, a popular topic in phylogenetics.

A preprint is available at <https://arxiv.org/abs/1703.08018>.

Partial Homology Relations - Satisfiability in terms of Di-Cographs

Nikolai Nøjgaard

University of Greifswald and University of Southern Denmark

Directed cographs (di-cographs) play a crucial role in the reconstruction of evolutionary histories of genes based on homology relations which are binary relations between genes. A variety of methods based on pairwise sequence comparisons can be used to infer such homology relations (e.g. orthology, paralogy, xenology). They are satisfiable if the relations can be explained by an event-labeled gene tree, i.e., they can simultaneously co-exist in an evolutionary history of the underlying genes. Every gene tree is equivalently interpreted as a so-called cotree that entirely encodes the structure of a di-cograph. Thus, satisfiable homology relations must necessarily form a di-cograph. The inferred homology relations might not cover each pair of genes and thus, provide only partial knowledge on the full set of homology relations. Moreover, for particular pairs of genes, it might be known with a high degree of certainty that they are not orthologs (resp. paralog, xenolog) which yields forbidden pairs of genes. Motivated by this observation, we characterize (partial) satisfiable homology relations with or without forbidden gene pairs, provide a quadratic-time algorithm for their recognition and for the computation of a cotree that explains the given relations. Joint work by: Nikolai Nøjgaard, Nadia El-Mabrouk, Daniel Merkle, Nicolas Wieseke, Marc Hellmuth.

Un-folding and folding-up phylogenetic networks

Guillaume Scholz

*Laboratoire d'Informatique, de Robotique et de Microélectronique de Montpellier (LIRMM),
Montpellier, France*

Phylogenetic networks are becoming of increasing interest to evolutionary biologists due to their ability to capture complex non-treelike evolutionary processes. From a combinatorial point of view, such networks are certain types of rooted directed acyclic graphs whose leaves are labelled by, for example, species. Reflecting to some extent the different evolutionary contexts within which such processes can arise has led to the introduction of a number of classes of such structures. These include the class of stable phylogenetic networks which, in addition to being biologically relevant, turn out to be interesting from a mathematical point of view.

Roughly speaking, stable phylogenetic networks are defined via certain "fold-up" and "un-fold" operations. The former turns a phylogenetic network into a certain type of rooted tree, called a MUL-tree, whereas the latter does the opposite. A phylogenetic network N is called stable if N is isomorphic to the phylogenetic network $F(U(N))$ obtained by successively applying the un-fold and fold-up operation to N . We will start this talk with formally introducing these operations, and provide a characterization for deciding whether a given phylogenetic network is stable.

Interestingly, the fold-up and un-fold operations allow one to study stable phylogenetic networks from the perspective of their associated MUL-tree, thus offering new ways to investigate additional properties these networks may enjoy. We will illustrate this in the second part of the talk with three such properties, all three of which are popular in phylogenetics: the tree-displaying, tree-based and tree-child properties.

Joint work with Katharina T. Huber, University of East Anglia, Norwich, UK.

The Matroid Structure of Representative Triple Sets and Triple-Closure Computation

Carsten R. Seemann

Max Planck Institute for Mathematics in the Sciences; and Leipzig University

The closure $\text{cl}(R)$ of a consistent set R of triples (rooted binary trees on three leaves) provides essential information about tree-like relations that are shown by any supertree that displays all triples in R . In this contribution, we are concerned with representative triple sets, that is, subsets R' of R with $\text{cl}(R') = \text{cl}(R)$. In this case, R' still contains all information on the tree structure implied by R , although R' might be significantly smaller. We show that representative triple sets that are minimal w.r.t. inclusion form the basis of a matroid. This in turn implies that minimal representative triple sets also have minimum cardinality. In particular, the matroid structure can be used to show that minimum representative triple sets can be computed in polynomial time with a simple greedy approach. For a given triple set R that “identifies” a tree, we provide an exact value for the cardinality of its minimum representative triple sets. In addition, we utilize the latter results to provide a novel and efficient method to compute the closure $\text{cl}(R)$ of a consistent triple set R that improves the time complexity $\mathcal{O}(|R| \cdot |L_R|^4)$ of the currently fastest known method proposed by Bryant and Steel (1995). In particular, if a minimum representative triple set for R is given, it can be shown that the time complexity to compute $\text{cl}(R)$ can be improved by a factor up to $|R| \cdot |L_R|$. As it turns out, collections of quartets (unrooted binary trees on four leaves) do not provide a matroid structure, in general.

This is a joint work with Marc Hellmuth; and the published version is available at <https://doi.org/10.1016/j.jejc.2018.02.013>.

Phylogenetics meets classic graph theory – Connections between Hamiltonicity, GSP graphs and treebased networks

Kristina Wicke

University of Greifswald

In biology, phylogenetic trees and networks are used to represent the evolutionary history and relationships of different species. While trees and networks are in general different concepts (the latter may contain cycles, the former are acyclic), a special class of networks strongly related to trees – the class of treebased networks – has recently gained considerable interest in the literature. Roughly speaking, treebased networks are networks that can be obtained from phylogenetic trees by adding additional interior vertices and edges. Interestingly, many questions that arise in the study of treebased networks are directly related to classic graph theory. It has for example been shown that deciding whether a network is treebased or not is an NP-complete problem by relating this problem to the planar Hamiltonian circuit problem. On the other hand, there are classes of networks that are guaranteed to be treebased and one of them, namely the class of so-called edgebased networks, is surprisingly related to GSP graphs (generalized series parallel graphs). In this talk, I will introduce the concept of treebased networks in full detail and illustrate some of its properties. In particular, I will highlight the interplay between classic graph theory and phylogenetic research by showing how graph-theoretical concepts help to understand the class of treebased networks.

Joint work with Mareike Fischer, Michelle Galla, Lina Herbst and Yangjing Long.

Invited Special Session

CHEMICAL GRAPH THEORY

Organized by XUELIANG LI

This session is associated with the meeting of the International Academy
of Mathematical Chemistry, IAMC 2019

Resonance graphs of catacondensed even ring systems (CERS)

Simon Brezovnik
University of Maribor

In this talk I will present the generalization of the binary coding procedure of perfect matchings from catacondensed benzenoid graphs to catacondensed even ring systems (also called CERS). Next, we study CERS with isomorphic resonance graphs. For this purpose, we define resonantly equivalent CERS. Finally, we investigate CERS whose resonance graphs are isomorphic to the resonance graphs of catacondensed benzenoid graphs. As a consequence we show that for each phenylene there exists a catacondensed benzenoid graph such that their resonance graphs are isomorphic.

Resonance graphs of plane elementary bipartite graphs

Zhongyuan Che
Penn State University, Beaver Campus

Let G be a plane elementary bipartite graph. The resonance graph of G , denoted by $Z(G)$, is the graph whose vertices are the perfect matchings of G , and two vertices M_1 and M_2 of $Z(G)$ are adjacent if and only if their symmetric difference is the periphery of a finite face of G . It is known that the resonance graph $Z(G)$ is a median graph.

In this talk, we present a decomposition structure of $Z(G)$ with respect to a reducible face of G using the Djoković-Winkler relation Θ and structural characterizations of a median graph. A necessary and sufficient condition can be obtained on when $Z(G)$ has a peripheral convex expansion structure with respect to a reducible face of G . In particular, if G is a 2-connected outerplane bipartite graph, then $Z(G)$ can be obtained from an edge by a sequence of peripheral convex expansions with respect to a reducible face decomposition of G , and the induced graph on the Djoković-Winkler relation Θ -classes of $Z(G)$ is a tree and isomorphic to the inner dual of G . Our results generalized the peripheral convex expansion structure of $Z(G)$ given by Klavžar et al. in 2002 for the case when G is a catacondensed even ring system, as well as the corresponding characterization of the resonance graph of a catacondensed hexagonal graph given by Vesel in 2005.

Articles available at

<https://www.sciencedirect.com/science/article/pii/S0166218X18301641>,

<https://www.sciencedirect.com/science/article/pii/S0166218X18306449>.

The average Laplacian polynomial of a graph

Haiyan Chen
Jimei University, China

Let $G = (V(G), E(G))$ be a graph of order $n = |V(G)|$ and size $m = |E(G)|$, and let $\Sigma(G) = \{\sigma : E(G) \rightarrow \{-1, +1\}\}$ be the set of sign functions defined on the edges of G . Then, it is well known that

$$\mu(G, x) = \frac{1}{2^m} \sum_{\sigma \in \Sigma(G)} \det(xI_n - A(G_\sigma)),$$

where $\mu(G, x)$ is the matching polynomial of G , and $A(G_\sigma)$ is the adjacency matrix of signed

graph $G_\sigma = (G, \sigma)$. Motivated by this result, in this talk, we introduce the average Laplacian polynomial of G , which is defined as:

$$\bar{\psi}(G, x) = \frac{1}{2^m} \sum_{\sigma \in \Sigma(G)} \det(xI_n - L(G_\sigma)),$$

where $L(G_\sigma)$ is the Laplacian matrix of signed graph G_σ . We find that this polynomial is closely related to the structure of the graph. For example, its coefficients can be expressed naturally in terms of the TU-subgraphs of G , and the multiplicity of 0 as a root is equal to the number of tree components in G . The relations between the average Laplacian polynomial of G and other polynomials, especially, the matching polynomial, are also investigated in this paper. Based on the relations, we prove that the roots of the average Laplacian polynomial of any graph are non-negative real numbers.

On the Wiener Index of Eulerian Graphs

Peter Dankelmann

University of Johannesburg

The Wiener index of a connected graph G is defined as the sum of the distances between all unordered pairs of vertices of G . Several bounds on the Wiener index of graphs are known. The most basic bound states that the Wiener index of a connected graph G of order n is between $\binom{n}{2}$ and $\binom{n+1}{3}$. It is known that these bounds can be improved if we restrict ourselves to graphs from certain graph classes, such as trees, planar graphs, maximal planar graphs, graphs with given minimum degree or connectivity.

In our talk we present results on the Wiener index of Eulerian graphs. It is easy to prove that the cycle is the unique graph maximising the Wiener index among all Eulerian graph of given order. An open conjecture by Gutman, Cruz and Rada [Wiener index of Eulerian graphs. *Discrete Applied Mathematics* 162, (2014), 247-250] concerns the graphs with second largest Wiener index among Eulerian graphs of given order. We will prove this conjecture in our talk. We will also pose the problem of determining Eulerian graphs with minimum Wiener index among Eulerian graphs of given order and size.

Packing stars in fullerenes

Tomislav Došlić

University of Zagreb

Let G and H be two simple connected graphs. An H -**packing** of G (or a packing of H in G) is a collection of vertex-disjoint subgraphs of G such that each component is isomorphic to H . If a packing is a spanning subgraph of G , we say that the packing is **perfect**.

A $(k, 6)$ -fullerene graph is a planar, 3-regular and 3-connected graph with k -gonal and hexagonal faces. For $k = 5$ we have ordinary fullerene graphs, while for $k = 3$ or 4 we speak of generalized fullerenes.

In this contribution, we investigate the existence and properties of perfect packings of $K_{1,3}$ (and also some other small graphs) in ordinary and generalized fullerenes.

Joint work with Meysam Taheri Dehkordi

Novel method for measuring sensitivity of topological descriptors on structural changes

Boris Furtula

University of Kragujevac

The gradual changes in the values of a molecular descriptor with the gradual alternations in the structure of the underlying molecule is an important quality that descriptor should have. Until today, only one algorithm was proposed for quantifying and ranking molecular descriptors using this feature. Here, the advantages and the pitfalls of the existing algorithm will be discussed. Further on, a novel algorithm will be presented that aims to improve the shortcomings of the existing one.

On the existence of some strong traces of graphs

Xian'an Jin

Xiamen University

Recently, the notion of strong trace was introduced as a mathematical support for self-assembly of polypeptide. Graphs which admit parallel strong traces and antiparallel strong traces were then characterized. In this talk, we introduce the notion of F -strong trace, i.e. a strong trace whose corresponding antiparallel edges are exactly edges in $F \subseteq E$, which includes parallel strong trace ($F = \emptyset$) and antiparallel strong trace ($F = E$) as two extreme cases. Given a graph $G = (V, E)$ and $F \subseteq E$, we study the problem whether G admits an F -strong trace. We solve it when (V, F) is acyclic by proving that in this case G admits an F -strong trace if and only if $G \setminus F$ is even. We provide examples to show that this condition is not always true when (V, F) contains cycles.

Bounding the Graovac-Pisanski index

Martin Knor

Slovak University of Technology in Bratislava

Let G be a graph. Its Graovac-Pisanski index, originally called a modified Wiener index, is defined as

$$\text{GP}(G) = \frac{|V(G)|}{2|\text{Aut}(G)|} \sum_{u \in V(G)} \sum_{\alpha \in \text{Aut}(G)} \text{dist}(u, \alpha(u)),$$

where $\text{Aut}(G)$ is the group of automorphisms of G and $\text{dist}(u, v)$ is the distance from u to v in G . This index was introduced in 1991 in [1], and its advantage is in combining the distances with the symmetries of a graph. By $W(G)$ we denote the Wiener index of a graph, i.e., the sum of all distances in G . Already in [1] it was proved that $\text{GP}(G) = W(G)$ if G is vertex-transitive, that is if there is just one orbit of the group of automorphisms in G . On the other hand, it is easy to see that $\text{GP}(G) \geq 0$ and equality holds if and only if the group of automorphisms of G is trivial, that is if $|\text{Aut}(G)| = 1$. We prove that $\text{GP}(T) \leq W(T)$ if T is a tree, but there are graphs G for which $\text{GP}(G) > W(G)$. We also show that there are graphs which are not vertex-transitive and though $\text{GP}(G) = W(G)$. Further, we determine all values ℓ for which there is a tree T with $W(T) - \text{GP}(T) = \ell$. Next, in the class of trees on n vertices we find those with the maximum value of Graovac-Pisanski index. We find extremal graphs also in the class of unicyclic graphs

on n vertices. This research was partially supported by Slovak research grants VEGA 1/0142/17, VEGA 1/0238/19, APVV-15-0220 and APVV-17-0428, and also by Slovenian research agency ARRS, program no. P1-00383, project no. L1-4292.

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The asymptotic value of graph energy for random graphs with degree-based weights

Xueliang Li

Nankai University, Tianjin, China

In this paper, we investigate the energy of weighted random graphs $G_{n,p}(f)$, in which each edge ij takes the weight $f(d_i, d_j)$, where d_v is a random variable, the degree of vertex v in the random graph $G_{n,p}$ of Erdős–Rényi model, and f is a symmetric real function on two variables. Suppose $f(d_i, d_j) \leq Cn^m$ for some constants $C, m > 0$, and $f((1 + o(1))np, (1 + o(1))np) = (1 + o(1))f(np, np)$. Then, for almost all graphs G in $G_{n,p}(f)$, the energy of G is $(1 + o(1))f(np, np) \frac{8}{3\pi} \sqrt{p(1-p)} \cdot n^{3/2}$. Consequently, with this one basket we get the asymptotic values of various kinds of graph energies of chemical use, such as Randić energy, ABC energy, and energies of random matrices obtained from various kinds of degree-based chemical indices.

This is a joint work with Dr. Yiyang Li and Jiarong Song

On combining Zagreb and Forgotten index to obtain better predictive power

Jelena Sedlar

University of Split

Graph theory is a branch of mathematics which can be applied in chemistry for modeling chemical compounds to predict their chemical properties. For that purpose many topological indices were proposed, among them the first Zagreb index $M_1(G)$ and the Forgotten index $F(G)$ of a graph G . By testing these two indices on the benchmark dataset of 18 octane isomers recommended by the International Academy of Mathematical Chemistry it was established they correlate well with the same two chemical properties, while for eleven other properties the correlation is not satisfactory. Yet, their linear combination $M_1(G) + \lambda F(G)$ for $\lambda = -0.140$ correlates well with the third property, namely the octanol-water partition coefficient. The optimal value $\lambda = -0.140$ was obtained by varying the free parameter λ from -20 to 20 . In order to eliminate free parameter λ we introduce a new index under the name Lanzhou index and denoted by $Lz(G)$, which depends only on the properties of the graph G . Comparing these two indices we establish they both correlate well with the octanol-water partition coefficient of octanes, while for nonanes $Lz(G)$ way outperforms $M_1(G) + \lambda F(G)$ for $\lambda = -0.140$. Finally, we prove several mathematical properties of the newly introduced index.

Joint work with D. Vukičević, Q. Li and T. Došlić.

Computing Distance-Based Topological Indices from Quotient Graphs

Niko Tratnik

University of Maribor

Recently, various methods for computing distance-based topological indices were proposed. By using such methods, a topological index of a connected graph G can be computed as the sum of corresponding indices of weighted quotient graphs, which are obtained from a partition of the edge set $E(G)$ that is coarser than the Θ^* -partition. Usually, such a method is called a cut method.

In this talk, we will present a method for computing the edge-Wiener index. Moreover, we expect to mention also methods for calculating some other topological indices, for example the degree distance, the Mostar index, the weighted (edge-)Szeged index, and the weighted (vertex-)PI index. Finally, we will show how our methods can be applied to efficiently calculate the mentioned indices for interesting families of (molecular) graphs.

Median eigenvalues and HOMO-LUMO index of graphs

Jianfeng Wang

Shandong University of Technology

In this report, we will introduce the results about the median eigenvalues and HOMO-LUMO index of graphs. Moreover, for the threshold graphs we will introduce our recent results in this direction.

Anti-Kekulé number of graphs

Heping Zhang

Lanzhou University

Perfect matchings in graphs correspond to Kekulé structures in some organic molecular structures. In 2007, D. Vukičević and N. Trinajstić introduced the concept of the anti-Kekulé number of a graph G , which is defined as the smallest number of edges of G whose deletion results in a connected spanning subgraph without perfect matchings. The anti-Kekulé numbers of hexagonal systems, and infinite hexagonal (resp. triangular and square) lattices have been obtained. It was known that the fullerenes have the anti-Kekulé number 4. In this talk we report some new progresses made on this direction, including the anti-Kekulé number of (4, 5, 6)-fullerenes, cubic graphs, as well as general regular graphs, computational complexity of the anti-Kekulé problem, and some close relations with matching preclusion and conditional matching preclusion proposed as measures of robustness in the event of edge failure in interconnection networks.

Two topological indices applied on hydrocarbons

Petra Žigert Pleteršek

University of Maribor

We consider two topological indices, the first one is the Graovac-Pisanski index, which is also called the modified Wiener index, defined by Graovac and Pisanski in 1991; and the second one is the fifth geometric–arithmetic index, introduced by Graovac et al. in 2011. Graovac-Pisanski index, which is a variation of the classical Wiener index, takes into account the symmetries of a graph. More precisely, the Graovac-Pisanski index of a graph G is defined as

$$\widehat{G} = \frac{|V(G)|}{2|\text{aut}(G)|} \sum_{u \in V(G)} \sum_{\alpha \in \text{Aut}(G)} d_G(u, \alpha(u)),$$

where $\text{aut}(G)$ is the automorphism group of G in d_G is the distance between two vertices of G . We show that it can be used for prediction of melting points of some families of hydrocarbon molecules, such as alkanes and polyaromatic hydrocarbons.

The fifth geometric–arithmetic index is one of the variation of geometric–arithmetic indices and is defined as

$$GA_5(G) = \sum_{uv \in E(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v},$$

where $S_u = \sum_{uv \in E(G)} \deg(v)$, $\deg(v)$ being the degree of vertex v . We establish the correlation

of this index with the well known atom–bond connectivity index in the case of alkanes and polyaromatic hydrocarbons, what results in the connection of GA_5 with the heat of formation of these two families of hydrocarbons.

Joint work with Matevž Črepnjak and Niko Tratnik.

Invited Special Session

CONFIGURATIONS

Organized by GÁBOR GÉVAY

Eventually, 5-configurations exist for all n

Leah Wrenn Berman

University of Alaska Fairbanks

In a series of papers and in his 2009 book on configurations, Branko Grünbaum described a sequence of operations to produce new (n_4) configurations from various input configurations. These operations were later called the “Grünbaum Calculus”. In this talk, we provide an overview of known construction methods for 5-configurations and we generalize the Grünbaum Calculus to 5-configurations to show that there exists some integer N_5 so that for any $n \geq N_5$, there exists at least one (n_5) configuration; we can show that $N_5 \leq 481$. This is joint work with Gábor Gévay and Tomaž Pisanski.

On the Finding of Symmetric Polyhedral Realizations without Self-Intersections of Hurwitz’s Regular Map $(3, 7)_{18}$ of Genus 7

Jürgen Bokowski

Technical University Darmstadt

The flag-transitive automorphism groups of the Platonics have led to its abstract and topological generalization: *regular maps*. Famous examples are connected with mathematicians of the 19th century like Felix Klein, Adolf Hurwitz, and Walther Dyck. For only very few of these regular maps, we have polyhedral realizations with flat n -gons and without self-intersections. For all of these regular maps, we have realizations with a geometric symmetry as well. The only example in which such a geometric symmetry of its polyhedral realization is missing concerns the Hurwitz surface $\{3, 7\}_{18}$ of genus 7, published in 1893. It has a high combinatorial symmetry of order 1008. A non-symmetric realization of this regular map was a recent result of M. Cuntz and the author; it was published in the first volume of the ADAM journal, here in Slovenia. It seemed to be natural to ask of whether or not this is the first example in which no geometric symmetry is possible for a polyhedral realization. The talk reports that most subgroups of the combinatorial symmetry group of order 1008 cannot occur as a geometric subgroup and that it is very likely that we cannot have in this case any geometric symmetry at all. When we insist on a geometric symmetry, it remains to provide some symmetric Kepler-Poinsot type realizations of this regular map of Hurwitz.

Joint work with Gábor Gévay.

Exotic configurations

Gábor Gévay

University of Szeged, Szeged, Hungary

Traditionally, geometric configurations consist of points and lines. In recent years, research has been extended to configurations of points and circles as well as to other non-traditional configurations. In this talk we deal with point-conic configurations. We give simple conditions for distinguishing various kinds of geometric configurations on combinatorial level. We present sporadic examples and some infinite series of point-conic configurations. We also touch some realizability problems.

Joint work with Tomaž Pisanski and is based partly on joint work with Nino Bašić and Jurij Kovič.

On the early history of configurations

Harald Gropp
Universität Heidelberg

In my talk I want to give a short survey on the early history of configurations, say between 1876 and circa 1935. If time allows, I shall also discuss some open problems in the existence of configurations and spatial configurations and possibly the existence of orbital matrices.

Configurations and Dessins d'Enfants

Milagros Izquierdo
Linköping University

In this talk we give an overview, with many examples, of how dessins d'enfants (maps and hypermaps) on Riemann surfaces produce point-circle realisations of combinatorial configurations.

Platonic configurations

Jurij Kovič
University of Primorska and IMFM, Slovenia

We present a method for constructing highly symmetric configurations of points and lines in space, starting from any of the five Platonic solids and placing identical copies of configurations along each of their faces. The obtained configurations, under certain natural symmetry conditions, inherit the symmetry groups of the Platonic solids.

Based on joint work with Aleksander Simonič.

Configurations and some classes of finite linear spaces

Vito Napolitano
Università degli Studi della Campania

In this talk I will show a connection between symmetric configurations and a class of finite linear spaces. Moreover, some existence results concerning full configurations (named also stopping sets) of smallest size in a finite projective plane of order q will be presented.

The remarkable rhombic dodecahedron graph

Tomaž Pisanski
IMFM and University of Primorska

The rhombic dodecahedron graph can be obtained from $Q_4 = K_2 \square K_2 \square K_2 \square K_2$ by removing a pair of vertices at maximal distance. Its significance can be best seen from the fact that it can be viewed as a skeleton graph of two combinatorially equivalent but geometrically distinct

zonohedra and, in totally different context, connected with a pair of isometric point-circle configurations. Its connection to some other graphs, such as $K_{3,4}$ and the Hoffman graph will be explained. In particular, rhombic dodecahedron graph may serve as an example of a bipartite double cover that is not canonical.

This is a joint work in progress with Izidor Hafner and Gábor Gévay.

Trilateral matroids

Michael Raney

Georgetown University

Let $\mathcal{C} = (P, B, I)$ be a combinatorial n_3 -configuration having point set P , block set B and incidence relation I . A *trilateral* of \mathcal{C} is a cyclically ordered set $\{p_0, b_0, p_1, b_1, p_2, b_2\}$ of pairwise distinct points p_i and pairwise distinct blocks b_i such that p_i is incident with b_{i-1} and b_i for each $i \in \mathbb{Z}_3$. Without ambiguity, this notation may be shortened to the point triple $\{p_0, p_1, p_2\}$. Let \mathcal{T} denote the collection of all point triples of P which form trilaterals within \mathcal{C} . We study when \mathcal{T} determines a rank-3 matroid $M_{tri}(\mathcal{C})$ on P , in the sense that if $p_1, p_2, p_3 \in P$ are distinct, then $\{p_1, p_2, p_3\}$ is a base of the matroid if and only if $\{p_1, p_2, p_3\} \notin \mathcal{T}$. Particular attention in this context is paid to configurations of points and lines.

Doily – A Gem of the Quantum Universe

Metod Saniga

Astronomical Institute, Slovak Academy of Sciences

Among finite geometries relevant for the theory of quantum information, the unique *triangle-free* 15_3 -configuration – the doily – has been recognized to play the foremost role. First, being isomorphic to the symplectic polar space of type $W(3, 2)$, it underlies the commutation relations between the elements of the two-qubit Pauli group and provides us with simplest settings (namely $GQ(2, 1)$'s) for observable proofs of quantum contextuality. Second, being isomorphic to a non-singular quadric of type $Q(4, 2)$, it also lies in the heart of a remarkable magic three-qubit Veldkamp line of form theories of gravity and its four-qubit extensions. Finally, being a sub-quadrangle of a generalized quadrangle of type $GQ(2, 4)$, it enters in an essential way certain black-hole entropy formulas and the so-called black-hole/qubit correspondence. I will highlight the most essential features of the above-outlined doily-settings, pointing out also physical relevance of other distinguished configurations, like the Fano plane, the split Cayley hexagon of order two and the Steiner-Plücker $(20_3, 15_4)$ -configuration.

Dualities, trialities, configurations and graphs

Klara Stokes

Maynooth University

In projective geometry, a duality is a correlation sending points to lines and lines to points. A triality is a correlation of order three in a geometry of at least three types. This talk will be about some configurations and incidence geometries with interesting dualities and trialities and their related graphs.

Chiral astral realizations of cyclic 3-configurations

Arjana Žitnik

University of Ljubljana and IMFM

A cyclic (n_3) configuration is a configuration in which the automorphism group contains a cyclic permutation of the points of the configuration; that is, the points of the configuration may be considered to be elements of \mathbb{Z}_n , and the lines of the configuration as cyclic shifts of a single fixed starting block $[0, a, b]$, where $a, b \in \mathbb{Z}_n$. We denote such configurations as $\text{cyc}_n(0, a, b)$. In the case where $n = 2m$, it is combinatorially possible to divide the points and lines of the configuration into two classes according to parity. We provide methods for producing geometric realizations of these configurations with two symmetry classes under the geometric symmetry group (that is, realizations as *astral* 3-configurations) where possible, and we also show that there is an infinite family configurations $\text{cyc}_{2(k+1)}(0, 1, k)$, which cannot be geometrically realized with cyclic symmetry using only two symmetry classes.

This is joint work with Leah Wrenn Berman, István Kovács and Tomaž Pisanski.

Invited Special Session

DESIGNS

Organized by DEAN CRNKOVIĆ

New extremal Type II \mathbb{Z}_4 -codes of length 32 obtained from Hadamard designs

Sara Ban

University of Rijeka

The subject of this talk is a construction of new extremal Type II \mathbb{Z}_4 -codes of length 32 from Hadamard designs.

For every Hadamard design with parameters $2-(n-1, \frac{n}{2}-1, \frac{n}{4}-1)$ having a skew-symmetric incidence matrix we give a construction of 54 Hadamard designs with parameters $2-(4n-1, 2n-1, n-1)$. Moreover, for the case $n=8$ we construct doubly-even self-orthogonal binary linear codes from the corresponding Hadamard matrices of order 32. From these binary codes we construct five new extremal Type II \mathbb{Z}_4 -codes of length 32. The constructed codes are the first examples of extremal Type II \mathbb{Z}_4 -codes of length 32 and type $4^{k_1}2^{k_2}$, $k_1 \in \{7, 8, 9, 10\}$, whose residue codes have minimum weight 8. Further, correcting the results from the literature we construct 5147 extremal Type II \mathbb{Z}_4 -codes of length 32 and type $4^{14}2^4$.

Joint work with Dean Crnkovic, Matteo Mravic and Sanja Rukavina.

Relative Heffter arrays and biembeddings

Simone Costa

Università degli Studi di Brescia

Heffter arrays are combinatorial objects introduced by D.S. Archdeacon in 2015. Here we propose the following generalization.

We set $v = 2nk + t$ and let $J = \frac{v}{t}\mathbb{Z}_v$ be the subgroup of \mathbb{Z}_v of order t . A (square) *Heffter array over \mathbb{Z}_v relative to J* , denoted by $H_t(n; k)$, is an $n \times n$ partially filled array with elements in $\mathbb{Z}_v \setminus J$ such that:

- (a) each row and each column contains k filled cells;
- (b) for every $x \in \mathbb{Z}_{2nk+t} \setminus J$, either x or $-x$ appears in the array;
- (c) the elements in every row and column sum to 0.

We remark that, for $t=1$, we meet again the classical definition of (square) Heffter array.

The main result we will present in this talk concerns the existence problem of integer (i.e. the entries are chosen in $\pm\{1, \dots, \lfloor \frac{2nk+t}{2} \rfloor\}$ and the sums are zero in \mathbb{Z}) relative Heffter arrays. In particular, if $k \neq 5$, an integer $H_k(n; k)$ exists if and only if one of the following holds:

- k is odd and $n \equiv 0, 3 \pmod{4}$;
- $k \equiv 2 \pmod{4}$ and n is even;
- $k \equiv 0 \pmod{4}$.

In case $k=5$, we have been able to obtain the class $n \equiv 3 \pmod{4}$ leaving open the existence problem only for $n \equiv 0 \pmod{4}$.

Using the relative difference families method, introduced by M. Buratti, our existence results of a (simple) $H_k(n; k)$ for $k \leq 9$ leads us to get a pair of orthogonal cyclic k -cycle decompositions of the complete multipartite graph with $2n+1$ parts of size k . Moreover, generalizing a construction of D.S. Archdeacon, under certain conditions (that are satisfied by our $H_k(n; k)$ for $k \in \{3, 5, 7, 9\}$)

and $n \equiv 3 \pmod{4}$) we can provide a face 2-colorable embedding (i.e. a biembedding) of those pairs of orthogonal decompositions on an orientable surface.

Joint work with F. Morini, A. Pasotti and M.A. Pellegrini.

Self-orthogonal codes from block designs and association schemes

Dean Crnković

University of Rijeka

In this talk we will describe a construction of self-orthogonal codes from orbit matrices of 2-designs and strongly regular graphs, and also a generalization of these methods to a construction of self-orthogonal codes from quotient matrices of association schemes. Further, we propose a notion of self-orthogonal subspace codes and give a method of constructing such subspace codes using quotient matrices of association schemes.

Complementary sequences and combinatorial structures

Ronan Egan

National University of Ireland, Galway

Pairs of complementary sequences such as Golay pairs have zero sum autocorrelation at all non-trivial phases. Several generalizations are known where conditions on either the autocorrelation function, or the entries of the sequences are altered. In this talk I will unify many of these ideas and introduce autocorrelation functions that apply to sequences with entries in a set equipped with a ring-like structure which is closed under multiplication and contains multiplicative inverses. Depending on the elements of the chosen set, the resulting complementary pairs may be used to construct a variety of combinatorial structures such as Hadamard matrices, complex generalized weighing matrices, and signed group weighing matrices.

Deza graphs with parameters $(v,k,k-2,a)$

Vladislav Kabanov

Krasovskii Institute of Mathematics and Mechanics

A Deza graph with parameters (v, k, b, a) is a k -regular graph on v vertices in which the number of common neighbors of two distinct vertices takes two values a or b ($a \leq b$) and both cases exist. Deza graphs include $(0, \lambda)$ -graphs [5], when λ more than 1, strongly regular graphs and divisible design graphs. The concept of a Deza graph was introduced by M. Erickson, S. Fernando, W. Haemers, D. Hardy, and J. Hemmeter in 1999 [1]. In particular, they studied Deza graphs with restrictions of parameters $k = b$. In 2011 W.H. Haemers, H. Kharaghani, M. Meulenberg introduced divisible design graphs [3]. In the paper of V.V. Kabanov, N.V. Maslova, L.V. Shalaginov [4] and in the paper of S. Goryainov, W.H. Haemers, V.V. Kabanov, L. Shalaginov [2] Deza graphs with parameters (v, k, b, a) where $k - b = 1$ were characterized. In this work we characterised Deza graphs with $k - b = 2$ and divisible design graphs with $k - \lambda_1 = 2$.

Joint work with Leonid Shalaginov. Both authors are partially supported by RFBR according to the research project 17-51-560008.

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Cycle systems of the complete multipartite graph

Francesca Merola
Roma Tre University

A *cycle system* for a graph Γ is a set \mathcal{B} of cycles of Γ whose edges partition the edge set of Γ ; if the cycles in \mathcal{B} all have the same length ℓ , we speak of a ℓ -cycle system of Γ . An ℓ -cycle system is *regular* if there is an automorphism group G of the graph Γ acting sharply transitively on the vertices of Γ and permuting the cycles of \mathcal{B} ; it is called *cyclic* if G is the cyclic group.

If Γ is the complete graph on v vertices (v odd), or the so called cocktail party graph $K_v - F$ (v even and F a 1-factor of K_v), the problem of determining for which values of ℓ and v there exists a ℓ -cycle system has been completely solved.

When Γ is $K_m[n]$, the complete multipartite graph with m parts each of size n , only partial results on the existence of ℓ -cycle systems are known, and little is known on regular cycle systems. In the talk we discuss new existence results for cycle systems of $K_m[n]$, concentrating on cyclic systems.

Joint work with Andrea Burgess and Tommaso Traetta

Partial Geometric Designs and Their Links

Oktay Olmez
Ankara University

Combinatorial designs have fruitful links to graph theory, coding theory and study of Boolean functions. Its deep connections to other fields attracted many researchers from different fields of application problems. Difference set methods provide a powerful tool to construct designs and other combinatorial objects with large automorphism groups. For example, symmetric difference sets can be used to construct symmetric designs; relative difference sets can be used to construct divisible designs; partial difference sets can be used to construct strongly regular graphs; Hadamard difference sets can be used to construct bent functions.

In this talk, we will focus on partial geometric designs (PGD) and their connections to other combinatorial objects. Well-known examples of partial geometric designs include 2-designs, partial geometries and transversal designs. We will provide some introductory outline for

PGD and focus on a difference set method known as partial geometric difference sets. By this method we will also investigate the links between partial geometric difference sets and other combinatorial structures.

Odd sun systems of the complete graph

Anita Pasotti
University of Brescia

A k -cycle with a pendant edge attached to each vertex is called a k -sun. The existence problem for k -sun systems of the complete graph of order v , with k odd, has been solved only when $k = 3$ or 5 . Here we present a complete solution whenever k is an odd prime. Furthermore, we reduce the above problem to the orders v in the range $2k < v < 6k$ satisfying the obvious necessary conditions.

Joint work with M. Buratti and T. Traetta.

Constructions of some new t -designs acted upon nonabelian automorphism groups

Mario Osvin Pavčević
University of Zagreb

Although different methods have been developed and combined to construct t -designs, it still remains a great challenge to run successfully a construction procedure for given design parameters t - (v, k, λ) for which the existence is still in question. In our talk we shall explain and implement the famous Kramer-Mesner method of construction assuming in addition an automorphism group to be acting on the design we want to construct. We shall show how the construction depends on the choice of the automorphism group (in our case semidirect products), how the search space has been reduced by introducing that assumption, and provide a few new examples of t -designs, according to the last version of the t -design tables by Laue and Khosrovshahi from the Handbook of Combinatorial Designs, proving in such a way their existence by explicit construction. We point out that most of the ideas we use here have been known before, but we could apply them efficiently to solve some existence questions for relatively small unknown t -designs.

Joint work with Vedran Krčadinac

Homogeneous cubic bent functions

Alexander Pott

Otto von Guericke University Magdeburg

In this talk we prove the existence of bent functions which have simultaneously the following properties: cubic, homogeneous, no affine derivatives and not in the completed Maiorana-McFarland class. We also show, that in opposite to the cases of 6 and 8 variables the original Maiorana-McFarland construction does not describe the whole class of cubic bent functions in n variables for all $n \geq 16$.

This is joint work with Alexandr Polujan.

Distinct partial sums in cyclic groups

John R. Schmitt

Middlebury College (VT, USA)

Let $(G, +)$ be an abelian group and consider a subset $A \subseteq G$ with $|A| = k$. Given an ordering (a_1, \dots, a_k) of the elements of A , define its *partial sums* by $s_0 = 0$ and $s_j = \sum_{i=1}^j a_i$ for $1 \leq j \leq k$. Alspach conjectured that for any cyclic group \mathbf{Z}_n and any subset $A \subseteq \mathbf{Z}_n \setminus \{0\}$ with $s_k \neq 0$, it is possible to find an ordering of the elements of A such that no two of its partial sums s_i and s_j are equal for $0 \leq i < j \leq k$.

We address this conjecture (and a weakening of it due to Archdeacon) in the case that n is *prime* and do the following. We show how Noga Alon's Combinatorial Nullstellensatz can be used to frame the conjecture. Further, in the case that n is prime, we verify computationally that Alspach's Conjecture is true for small values of $|A|$. In the case that n is prime, we show that a sequence of length k having distinct partial sums exists in any subset of $\mathbf{Z}_n \setminus \{0\}$ of size at least $2k - \sqrt{8k}$ in all but at most a bounded number of cases.

Joint work with Jacob Hicks (U. Georgia) and Matt Ollis (Marlboro College, VT, USA)

Self-orthogonal codes from orbit matrices of Seidel and Laplacian matrices of strongly regular graphs

Andrea Švob

University of Rijeka

In this talk we introduce the notion of orbit matrices of integer matrices such as Hadamard matrices, Seidel and Laplacian matrices of some strongly regular graphs with respect to their permutation automorphism groups. We further show that under certain conditions these orbit matrices yield self-orthogonal codes over finite fields \mathbb{F}_q , where q is a prime power and over finite rings \mathbb{Z}_m . As a case study, we construct codes from orbit matrices of Seidel, Laplacian and signless Laplacian matrices of strongly regular graphs. In particular, we construct self-orthogonal codes from orbit matrices of Seidel and Laplacian matrices of the Higman-Sims and McLaughlin graphs.

Joint work with Dean Crnković (Department of Mathematics, University of Rijeka, Croatia) and Ronan Egan (National University of Ireland, Galway)

Pyramidal Steiner and Kirkman triple systems

Tommaso Traetta
University of Brescia

Steiner (STS) and Kirkman (KTS) triple systems have been widely studied over the past 150 years. However, very little is known about the existence of the f -pyramidal ones, namely, those having an automorphism group fixing f points and acting sharply transitively on the remaining.

In this talk, we present the most recent results on this subject, in particular when $f = 3$. In this case, we provide a complete solution for STSs and partial existence results for KTSs.

Joint work with Simona Bonvicini, Marco Buratti, Martino Garonzi, Gloria Rinaldi.

Groups $S_n \times S_m$ in construction of flag-transitive block designs

Tanja Vučičić
University of Split

Our recent research has been focused on the construction of flag-transitive block designs. These are the designs that have an automorphism group acting transitively on the set of ordered pairs of incident points and blocks. An equivalent form of the well-known result by Cameron and Praeger [1, Proposition 1.1] states that flag-transitive block design can appear as a substructure only within an overstructure which is a flag-transitive design itself. Therefore the procedure of finding all nontrivial flag-transitive designs with the chosen point set Ω can start by considering transitive maximal subgroups of $Sym(\Omega)$ or $Alt(\Omega)$ as possible automorphism groups of the designs and subsets $B \subseteq \Omega$ as possible base blocks.

In [2] we considered the designs with $\Omega = \{1, 2, \dots, n\} \times \{1, 2, \dots, n\}$ and wreath product $S_n \wr S_2$ as an automorphism group. This group acts primitively and it is a maximal subgroup of $Sym(\Omega)$ for $n \geq 5$. The wide range ($n \leq 63$) of successful constructions is accomplished due to a specific approach to the construction by using smaller, appropriately defined flag-transitive or weakly flag-transitive incidence structures in obtaining base blocks.

In this talk we present how a similar idea works in determining whether group $S_n \times S_m$ acts as a flag-transitive automorphism group of a block design with point set $\{1, \dots, n\} \times \{1, \dots, m\}$. We show the problem to be equivalent to the existence of the specific smaller flag-transitive incidence structures. This equivalence enabled us to solve the existence problem for the desired designs with nm points in the range $4 \leq n \leq m \leq 70$. By developing and applying several algorithms for construction of the introduced incidence structures, in the vast majority of the cases with confirmed existence we obtained all possible structures up to isomorphism.

In case $n = m$ the observed group S_n^2 is a subgroup of $S_n \wr S_2$ of index 2 and the obtained results are related to those in [2]; we include a comparison between them.

All constructions are performed using software package Magma.

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Joint work with Snježana Braić, Joško Mandić, and Tanja Vojković.

On q -analogs of group divisible designs

Alfred Wassermann

University of Bayreuth

Group divisible designs are well-studied objects in combinatorics. Recently, q -analogs of group divisible designs (q -GDDs) have been introduced.

Let v, g, k , and λ be sets of positive integers and let q be a positive integer. The q -analog of a group divisible design of index λ and order v with parameters $(v, g, k, \lambda)_q$ is a triple $(\mathcal{V}, \mathcal{G}, \mathcal{B})$, where \mathcal{V} is a vector space of dimension v over $\text{GF}(q)$, \mathcal{G} is a partition of \mathcal{V} into g -dimensional subspaces (groups), and \mathcal{B} is a family of k -dimensional subspaces (blocks) of \mathcal{V} such that every 2-dimensional subspace of \mathcal{V} occurs in exactly λ blocks or one group, but not both.

q -analogs of group divisible designs are connected to scattered subspaces in finite geometry, q -analogs of Steiner systems, subspace design packings and more.

After an introduction to the subject, recent results are presented with special attention on cases where the vector space partition is a non-Desarguesian spread. For example, there is no $(8, 4, 4, 7)_2$ GDD and a $(8, 4, 4, 14)_2$ GDD exists only for the Desarguesian spread.

Joint work with Jan de Beule, Michael Kiermaier, Sascha Kurz.

Invited Special Session

DISCRETE AND COMPUTATIONAL GEOMETRY

Organized by **SERGIO CABELLO**

Bounded degree conjecture holds precisely for c -crossing-critical graphs with $c \leq 12$

Drago Bokal

University of Maribor

c -crossing-critical graphs are the minimal graphs that require at least c edge-crossings when drawn in the plane. Richter conjectured that such graphs have maximum degree bounded in c and Mohar and Dvořák disproved this conjecture existentially. In the talk we present arguments that show the conjecture holds precisely for each $c \leq 12$ and give explicit construction of counterexamples with $c \geq 13$. Using zip product, these examples yield c -crossing-critical graphs with k vertices of arbitrarily large degree for every $c \geq 13k$. We sketch an approach that may combine the embedding method of lower bounds for crossing number with our constructions to yield c -crossing-critical graphs with k vertices of arbitrarily large degree for at least some $13 \leq c < 13k$.

This is joint work with Zdeněk Dvořák, Petr Hliněný, Jesús Leaños, and Bojan Mohar.

Preprint is available at <http://arxiv.org/abs/1903.05363>.

Maximum Matchings in Geometric Intersection Graphs

Sergio Cabello

University of Ljubljana and IMFM

We show that a maximum matching in the intersection graph of n geometric objects in the plane can be computed in $O(\rho^{3\omega/2} n^{\omega/2})$ time with high probability, where ρ is the density of the geometric objects and ω is any constant such that $\omega > 2$ and any two $n \times n$ matrices can be multiplied in $O(n^\omega)$ time. The same result holds for a subgraph of the intersection graph, assuming that a geometric representation is at hand. For this, we combine algebraic methods, namely Gaussian elimination to compute the rank of a matrix, and the existence of small separators in geometric intersection graphs.

Joint work with Wolfgang Mulzer.

Multicuts in planar and surface-embedded graphs

Éric Colin de Verdière

CNRS, Université Paris-Est, Marne-la-Vallée, France

The Multicut problem is defined as follows. Given an edge-weighted graph G and pairs of vertices $(s_1, t_1), \dots, (s_k, t_k)$, compute a minimum-weight subset of edges whose removal disconnects each pair (s_i, t_i) .

This problem is NP-hard, APX-hard, and W[1]-hard in the number of pairs of terminals, even in very simple cases, such as planar graphs.

We will survey some recent results on this problem, on planar graphs and more generally on graphs embedded on a fixed surface: An exact algorithm, whose running time is a polynomial in the genus and the number of terminals; a matching lower bound assuming ETH; and an approximation scheme with running time $O(n \log n)$ if the approximation factor, the genus, and the number of terminals are fixed.

All these results rely on topological methods: The subgraph of the dual of G , made of the edges dual to a multicut, have nice properties, which can be exploited using classical tools from

algebraic topology such as homotopy, homology, and universal covers of surfaces.

Based on joint works with Vincent Cohen-Addad, Arnaud de Mesmay, and Dániel Marx.

Shorter Implicit Representation for Planar Graphs

Cyril Gavoille

University of Bordeaux

For a graph family \mathcal{F} , a *labeling scheme* is an algorithm that given a graph G of \mathcal{F} , assigns bit strings (labels) to vertices of G so that for any two vertices u, v , whether u and v are adjacent can be determined by a fixed procedure that examines only their labels. It is known that planar graphs admit a labelling scheme with labels of $2 \log n$ bits asymptotically. In this work we improve this result by constructing a new labeling scheme with labels of length $\frac{4}{3} \log n + O(\log \log n)$.

In graph-theoretical terms, this implies that there exists a graph on $n^{4/3+o(1)}$ vertices that contains all planar graphs on n vertices as induced subgraphs, improving the best known upper bound of $n^{2+o(1)}$.

This is a joint work with Marthe Bonamy and Michał Pilipczuk.

On Colourability of Polygon Visibility Graphs

Petr Hliněný

Masaryk University

We study the problem of colouring visibility graphs of polygons. In particular, for visibility graphs of simple polygons, we provide a polynomial algorithm for 4-colouring, and prove that the 5-colourability question is already NP-complete for them. For visibility graphs of polygons with holes, we prove that the 4-colourability question is NP-complete.

Joint work with Onur Cagirici and Bodhayan Roy. Preliminary version published at FSTTCS 2017.

Spanoids, Greedoids and Violator Spaces

Yulia Kempner

Holon Institute of Technology, Israel

The main goal of this talk is to make connections between four independently developed theories: spanoids, greedoids, closure spaces, and violator spaces.

Closure spaces appear in many disciplines, including topology, algebra, logic, geometry, convex analysis, graph theory and the theory of ordered sets (Jamison, 2019, in press). It also goes by the names of span, hull, and envelope. The convex hull operator on the Euclidean space E^n is a classic example of a closure operator. Spanoids were recently introduced as logical inference structures with applications in coding theory (Dvir, Goppy, Gu, Wigderson, 2019). We prove that each spanoid is a closure space, while every closure space may be represented as a spanoid.

In 1996, LP-type problems was introduced and analyzed by Matoušek, Sharir and Welzl as a combinatorial framework that encompasses linear programming and other geometric optimization problems. Further, (Matoušek et al. 2008) defined a more general framework, namely,

violator spaces, which constituted a proper generalization of LP-type problems. Originally, violator spaces were defined for the set of constraints H , where each subset $A \subseteq H$ is associated with $V(A)$ - the set of all constraints violating A . For instance, a violator space is naturally revealed, when one computes the smallest enclosing ball of a finite set of points in R^d . Here the set H is a set of points in R^d , and the violated constraints of some subset of the points A are exactly the points lying outside the smallest enclosing ball of A . We investigated interrelations between violator spaces and closure spaces and showed that a violator mapping may be defined by a weakened version of a closure operator (Kempner & Levit, 2019, in press). Thus the family of closure spaces may be considered as a sub-family of violator spaces. In this work we show that the family of greedoids is also a sub-family of violator spaces.

Joint work with Vadim E. Levit.

Bend-minimum Orthogonal Drawings

Giuseppe Liotta

University of Perugia, Italy

One of the oldest and most studied problems in graph drawing is about computing an orthogonal drawing of a planar graph such that the total number of bends is minimized. This problem can be solved in polynomial time in the so-called fixed embedding setting, that is when the input is a plane graph and the orthogonal drawing maintains the given planar embedding. It is NP-hard in the variable embedding setting, that is when the drawing algorithm is allowed to choose the best planar embedding to minimize the total number of bends. Polynomial time solutions are known if the vertex degree of the input is at most three or if the tree-width of the input is two. In this talk, I will revisit some of the known techniques about the bend-minimization problem of orthogonal drawings and shortly describe some new findings. In particular, I will sketch a solution to a long standing open problem that asks to establish the best possible computational upper bound to construct a bend-minimum orthogonal drawing for a planar graph of maximum vertex degree three in the variable embedding setting. I will also sketch a polynomial-time algorithm for the bend-minimization problem in the variable embedding setting when the input graph has degree four and bounded tree-width.

Note: The linear-time algorithm to compute bend-minimum orthogonal drawings of graphs with maximum degree three is the result of a joint work with W. Didimo, G. Ortali, and M. Patrignani.

The polynomial-time algorithm for bend-minimum orthogonal drawings of graphs having degree four and bounded tree-width is the result of a joint work with E. Di Giacomo and F. Montecchiani.

Polychromatic coloring of geometric hypergraphs

Dömötör Pálvölgyi

MTA-ELTE CoGe

What is the minimum number of colors that always suffice to color every planar set of points such that any disk that contains enough points contains two points of different colors? It is known that the answer to this question is either three or four. We show that three colors always suffice if the condition must be satisfied only by disks that contain a fixed point. (Joint work with Eyal Ackerman and Balázs Keszegh.) We also survey related recent results and pose some

mouth-watering open questions about abstract hypergraphs as well.

Hamiltonicity for convex shape Delaunay and Gabriel graphs

Maria Saumell

The Czech Academy of Sciences & Czech Technical University in Prague

We study Hamiltonicity for some of the most general variants of Delaunay and Gabriel graphs. Instead of defining these proximity graphs using circles, we use an arbitrary convex shape \mathcal{C} . Let S be a point set in the plane. The k -order Delaunay graph of S , denoted $k-DG_{\mathcal{C}}(S)$, has vertex set S and edge pq provided that there exists some homothet of \mathcal{C} with p and q on its boundary and containing at most k points of S different from p and q . The k -order Gabriel graph $k-GG_{\mathcal{C}}(S)$ is defined analogously, except for the fact that the homothets considered are restricted to be smallest homothets of \mathcal{C} with p and q on its boundary.

We provide upper bounds on the minimum value of k for which $k-GG_{\mathcal{C}}(S)$ and $k-DG_{\mathcal{C}}(S)$ are Hamiltonian. In particular, we give upper bounds of 24 for every \mathcal{C} and 15 for every point-symmetric \mathcal{C} . We also improve the bound for even-sided regular polygons. These constitute the first general results on Hamiltonicity for convex shape Delaunay and Gabriel graphs.

Joint work with Prosenjit Bose, Pilar Cano and Rodrigo I. Silveira.

Lower bounds for semantic read-once BDDs using Extension Complexity

Hans Raj Tiwary

Charles University, Prague

Binary decision diagrams are representations of sequential computation as directed acyclic graphs. Lower bounds on the size of BDDs provide a space/time lower bound on the computation and are therefore interesting. Lower bounds are specially difficult to obtain in presence of nondeterminism in the computation and so further restrictions - such as the number of times an input bit is read - are placed on the BDD. In this talk I will present a superpolynomial lower bound on the size of read-once nondeterministic BDDs for an easy-to-compute function. These lower bounds are obtained using recent lower bounds on the sizes of polytopes corresponding to some standard problems.

Counting Hamiltonian cycles in 2-tiled graphs

Alen Vegi Kalamar

University of Maribor, Slovenia

Kuratowski has shown that $K_{3,3}$ and K_5 are the only two minor-minimal non-planar graphs. Robertson and Seymour extended finiteness of the set of forbidden minors for any surface. On the other hand, Širan and Kochol showed that studying minimal forbidden subgraphs, there are infinitely many k -crossing-critical graphs for any $k \geq 2$, even if restricted to simple 3-connected graphs. Recently, 2-crossing-critical graphs have been completely characterized in a lengthy paper by Bokal, Oporowski, Richter and Salazar. We present a simplified description of large 2-crossing-critical graphs and use this simplification to count all Hamiltonian cycles in large

2-crossing-critical graphs. We generalize this approach to an algorithm counting Hamiltonian cycles in all 2-tiled graphs.

Joint work with Drago Bokal and Tadej Žerak.

Invited Special Session

DISTANCE-REGULAR GRAPHS

Organized by ŠTEFKO MIKLAVIČ

On distance-regular graphs on 486 vertices

Robert F. Bailey

Grenfell Campus, Memorial University

The Koolen–Riebeek graph is a bipartite graph on 486 vertices, and is the unique distance-regular graph with intersection array $\{45, 44, 36, 5; 1, 9, 40, 45\}$. It exists inside the coherent configuration arising from rank-9 permutation representation of the group $3^5 : (2 \times M_{10})$. However, this coherent configuration also contains several other distance-regular graphs: some more interesting than others. We present the results of an investigation into these graphs, including some observations which we believe are new.

This is joint work with Daniel Hawtin (Rijeka).

Tridiagonal pairs of Racah type and the universal enveloping algebra $U(\mathfrak{sl}_2)$

Sarah Bockting-Conrad

DePaul University

Let \mathbb{F} denote a field and let V denote a vector space over \mathbb{F} with finite positive dimension. Let A, A^* denote a tridiagonal pair of Racah type with diameter $d \geq 1$. Let $\{U_i\}_{i=0}^d$ (resp. $\{U_i^\downarrow\}_{i=0}^d$) denote the first (resp. second) split decomposition of A, A^* . In an earlier paper, we associated with A, A^* a linear transformation $\psi : V \rightarrow V$ such that $\psi U_i \subseteq U_{i-1}$ and $\psi U_i^\downarrow \subseteq U_{i-1}^\downarrow$ for $0 \leq i \leq d$. One of the relations involving ψ was reminiscent of a defining relation for the universal enveloping algebra $U(\mathfrak{sl}_2)$. We explore this connection further. In doing so, we will give two natural $U(\mathfrak{sl}_2)$ -module structures for V and discuss how they are related. This leads to a number of interesting relations involving the operator ψ and other operators associated with A, A^* .

On the Terwilliger Algebra of Locally Pseudo-Distance-Regular Graphs

Blas Fernández

University of Primorska, Koper, Slovenia.

The concept of pseudo-distance-regularity around a vertex of a graph is a natural generalization of the standard distance-regularity around a vertex. Given a vertex x of a connected graph Γ , let $T = T(x)$ denote the Terwilliger algebra of Γ with respect to x . Let W denote the unique irreducible T -module with endpoint 0. In this talk we show that the following (i), (ii) are equivalent:

(i) Γ is pseudo distance-regular around vertex x ,

(ii) T -module W is thin.

Joint work with Štefko Miklavič.

Irreducible T -modules with endpoint 1 of almost 1-homogeneous distance-regular graph

Štefko Miklavič

University of Primorska

Let Γ denote a distance-regular graph with valency $k \geq 3$, and diameter $D \geq 3$. Let x denote a vertex of Γ and let $T = T(x)$ denote the corresponding Terwilliger algebra. If Γ is 1-homogeneous, then the structure of irreducible T -modules with endpoint 1 was determined by Curtin and Nomura. In this talk we assume that Γ is (in a certain sense) “close” to being 1-homogeneous. We will show that in this case all irreducible T -modules are thin. Joint work with Mark MacLean.

t-point counts in distance-regular graphs

Arnold Neumaier

University of Vienna

A t -point count $[\Delta]$ of a distance-regular graph Γ is a normalized count of the number of ordered subsets isomorphic to a template Δ with certain specified distances between the t vertices of Δ . It is shown that many well-known inequalities for the intersection array can be derived in a uniform way using t -point counts.

A combinatorial approach to the Terwilliger algebra modules of a bipartite distance-regular graph

Safet Penjić

University of Primorska

Let Γ denote a bipartite distance-regular graph with diameter $D \geq 4$ and valency $k \geq 3$. Let X denote the vertex set of Γ , and let $\Gamma_i(x)$ denote the set of vertices in X that are distance i from vertex x . Let $V = \mathbb{C}^X$ denote the vector space over \mathbb{C} consisting of column vectors whose coordinates are indexed by X and whose entries are in \mathbb{C} , and for $z \in X$ let \widehat{z} denote the element of V with a 1 in the z coordinate and 0 in all other coordinates.

Fix $x \in X$, $u \in \Gamma_2(x)$, $v \in \Gamma_2(x) \cap \Gamma_2(u)$ and abbreviate $\Gamma_{ij}^h = \Gamma_h(x) \cap \Gamma_i(u) \cap \Gamma_j(v)$. For $0 \leq i \leq D$, define

$$s_i = \sum_{z \in \Gamma_{i-2,i}^i} \widehat{z} - \sum_{z \in \Gamma_{i,i-2}^i} \widehat{z} \quad \text{and} \quad t_i = \sum_{z \in \Gamma_{i+2,i}^i} \widehat{z} - \sum_{z \in \Gamma_{i,i+2}^i} \widehat{z}.$$

In this paper we study a particular family of a bipartite distance-regular graphs for which we show that

$$W = \text{span}\{s_2, s_3, \dots, s_D, t_2, \dots, t_{D-2}\}$$

is a T -module of endpoint 2, and we give the action of A on this generating set. We use this T -module to construct combinatorially-defined bases for all isomorphism classes of irreducible T -modules of endpoint 2 for the doubled Odd graphs, the double Hoffman-Singleton graph, Tutte’s 12-cage graph, and the Foster graph. We show the double Grassmannian (for $q = 2$) has at least one thin irreducible T -module of endpoint 2, and give a basis for this module.

Moreover, for any bipartite distance-regular graph with diameter $D \geq 4$ and valency $k \geq 3$, we construct a set $\{W_1, W_2, \dots, W_{k-1}\}$ of orthogonal thin irreducible T -modules of endpoint 1, with a basis defined combinatorially in a similar fashion to the vectors s_i above.

Joint work with Mark S. MacLean.

The universal \mathcal{C} -cover of a completely regular clique graph

Hiroshi Suzuki

International Christian University

In this talk, we discuss a point-line incidence structure of a distance-regular graph and its universal cover.

A connected graph Γ is said to be a completely regular clique graph with parameters (s, c) for positive integers s and c , if there is a collection \mathcal{D} of completely regular cliques of size $s + 1$ such that every edge is contained in exactly c members of \mathcal{D} . By regarding \mathcal{D} as the set of lines, Γ affords a point-line incidence structure of order (s, t) with a suitable choice of a positive integer t . All known families of distance-regular graphs with unbounded diameter are completely regular clique graphs except the Doob graphs, the twisted Grassmann graphs and the Hermitean forms graphs. Every distance-regular completely regular clique graph is a bipartite half of the associated incidence graph which is distance-semiregular.

Let \mathcal{C} be a collection of closed walks of a connected graph Γ . We study coverings of Γ under the condition that every member of \mathcal{C} can be lifted through covering morphisms. Since they depend on \mathcal{C} , we call them \mathcal{C} -coverings. We introduce conditions that a finite graph is \mathcal{C} -simply connected, i.e., the graph itself is a universal \mathcal{C} -cover, and a finiteness condition of a universal \mathcal{C} -cover of a class of connected bipartite graphs when \mathcal{C} is the collection of closed paths of minimal length.

We apply the theory to a distance-semiregular graph and a distance-regular completely regular clique graph with a suitable choice of \mathcal{C} .

Leonard pairs, spin models, and distance-regular graphs

Paul Terwilliger

U. Wisconsin-Madison

The work of Caughman, Curtin, and Nomura shows that for a distance-regular graph G affording a spin model, the irreducible modules for the subconstituent algebra T take a certain form. We show that the converse is true: whenever all the irreducible T -modules take this form, then G affords a spin model. We explicitly construct this spin model when G has q -Racah type. This is joint work with Kazumasa Nomura.

Invited Special Session

DOMINATION IN GRAPHS

Organized by MICHAEL A. HENNING

This session celebrates the 70th birthday of Douglas F. Rall.

Graphs with a unique maximum open packing

Boštjan Brešar

University of Maribor

A set S of vertices in a graph is an open packing if (open) neighborhoods of any two distinct vertices in S are disjoint. The cardinality of a maximum open packing presents a natural lower bound for the total domination number of a graph. In this talk, we consider the graphs that have a unique maximum open packing. We present a characterization of the trees with this property by using four local operations such that any nontrivial tree with a unique maximum open packing can be obtained by a sequence of these operations starting from P_2 . We also discuss the recognition complexity of the graphs with a unique maximum open packing, and the question of computing the size of a maximum open packing in an arbitrary graph.

Joint work with Kirsti Kuenzel and Douglas Rall.

Reconfiguring and enumerating dominating sets.

Paul Dorbec

Greyc, University of Caen-Normandy.

Given a graph, the question of enumerating all minimal dominating set has been carefully studied (see e.g. . The original idea behind this work was to consider the same problem for minimum dominating sets. A natural strategy for enumerating those would be to start from a minimum dominating set and to reconfigure it into all the others. This raises a number of other questions. What are the natural operation to reconfigure a dominating set? Under such a rule, is it possible to reach all minimum dominating set from one? What is the complexity of doing so?

During this talk, we will present some results in that direction, in particular on the token sliding reconfiguration, which consist of selecting a vertex in the dominating set and move it along an edge to a neighbor, so that the resulting set is again a dominating set.

Based on joint work with M. Bonamy and P. Ouvrard.

On graphs with equal total domination and Grundy total domination number

Tanja Golobranec

University of Maribor and Institute of Mathematics, Physics and Mechanics

A sequence (v_1, \dots, v_k) of vertices in a graph G without isolated vertices is called a total dominating sequence if every vertex v_i in the sequence totally dominates at least one vertex that was not totally dominated by $\{v_1, \dots, v_{i-1}\}$ and $\{v_1, \dots, v_k\}$ is a total dominating set of G . The length of a shortest such sequence is the total domination number of G ($\gamma_t(G)$), while the length of a longest such sequence is the Grundy total domination number of G ($\gamma_{gr}^t(G)$). In this talk we characterize bipartite graphs with both total and Grundy total domination number equal to 4. Since it is conjectured that all connected graphs with $\gamma_t(G) = \gamma_{gr}^t(G) = 4$ are bipartite, we show that the conjecture holds in the class of chordal graphs. We also give a characterization of regular bipartite graphs with $\gamma_t(G) = \gamma_{gr}^t(G) = 6$ which is proved by establishing a correspondence between existence of such graphs and the existence of certain finite projective planes.

This is joint work with Marko Jakovac, Tim Kos and Tilen Marc.

The independent domatic number and the total domatic number

Michael A. Henning
University of Johannesburg

The idomatic number, $\text{idom}(G)$, of a graph G is the maximum number of vertex-disjoint independent dominating sets of G , while the total domatic number, $\text{tdom}(G)$, of G is the maximum number of disjoint total dominating sets. For a family \mathcal{F} of sets, the multiplicity, $m(\mathcal{F})$, is the maximum number of times an element appears in \mathcal{F} . The fractional idomatic number, $\text{FIDOM}(G)$, is the supremum of the ratio $|\mathcal{F}|/m(\mathcal{F})$ over all families \mathcal{F} of independent dominating sets of G , while the fractional total domatic number, $\text{FTD}(G)$, is the supremum of the ratio $|\mathcal{F}|/m(\mathcal{F})$ over all families \mathcal{F} of total dominating sets of G . We present various results on these parameters. For example, we show that every planar graph G of order n has $\text{FIDOM}(G) \geq 2$ and $i(G) \leq n/2$, where $i(G)$ denotes the independent domination number. We show that every planar graph G has $\text{tdom}(G) \leq 4$ and $\text{FTD}(G) \leq 5 - \frac{12}{n}$, and every toroidal graph G has $\text{tdom}(G) \leq 5$. We show that if G is a cubic graph, then $\text{FTD}(G) \geq 2$, which is in contrast to the fact that $\text{tdom}(G) = 1$ for infinitely many connected cubic graphs G .

Dominating sets in finite generalized quadrangles

Lisa Hernandez Lucas
Vrije Universiteit Brussel

A dominating set in a graph is a set D of vertices such that each vertex of the graph is either an element of D , or is adjacent to an element of D . In general it is desirable to find the smallest dominating sets. This talk focuses on the smallest dominating sets in the incidence graph of a finite generalized quadrangle $GQ(s, t)$. It seems obvious that when an ovoid and a spread of $GQ(s, t)$ exist, the union of these two is the smallest dominating set, having size $2st + 2$.

However, this turns out not to be the case at all. To our surprise, we found that the smallest dominating set in $GQ(q, q)$ has size $2q^2 + 1$. In this talk I tell the story of how we made this discovery, and present some other observations regarding the problem of the smallest dominating sets in finite generalized quadrangles. This is joint work with Tamás Héger.

Relating the annihilation number and two domination invariants

Marko Jakovac
University of Maribor and IMFM

The annihilation number $a(G)$ of G is the largest integer k such that there exist k different vertices in G with degree sum of at most $|E(G)|$. The total domination number $\gamma_t(G)$ of a graph G is the order of a smallest set $D \subseteq V(G)$ such that each vertex of G is adjacent to some vertex in D . The 2-domination number $\gamma_2(G)$ of a graph G is the order of a smallest set $D \subseteq V(G)$ such that each vertex of $V(G) \setminus D$ is adjacent to at least two vertices in D . It has been conjectured that the inequalities $\gamma_t(G) \leq a(G) + 1$ and $\gamma_2(G) \leq a(G) + 1$ hold for every nontrivial connected graph G . In this presentation we will talk about the correctness of both conjectures.

Joint work with Csilla Bujtás.

The Maker-Breaker domination game

Sandi Klavžar

University of Ljubljana, University of Maribor, IMFM

The Maker-Breaker domination game is played on a graph G by Dominator and Staller. The players alternatively select a vertex of G that was not yet chosen in the course of the game. Dominator wins if at some point the vertices he has chosen form a dominating set. Staller wins if Dominator cannot form a dominating set. The Maker-Breaker domination number $\gamma_{\text{MB}}(G)$ of G is the minimum number of moves of Dominator to win the game provided that he has a winning strategy and is the first to play. If Staller plays first, then the corresponding invariant is denoted $\gamma'_{\text{MB}}(G)$. It will be demonstrated that these invariants behave much differently than the related game domination numbers. Using the Erdős-Selfridge Criterion a large class of graphs G will be presented for which $\gamma_{\text{MB}}(G) > \gamma(G)$ holds. Residual graphs will be introduced and used to bound/determine $\gamma_{\text{MB}}(G)$ and $\gamma'_{\text{MB}}(G)$. Using residual graphs, $\gamma_{\text{MB}}(T)$ and $\gamma'_{\text{MB}}(T)$ will be determined for an arbitrary tree.

This is a joint work with Valentin Gledel and Vesna Iršič.

On the Grundy dominating sequences

Tim Kos

IMFM

In a graph G a sequence (v_1, v_2, \dots, v_k) of distinct vertices is closed neighborhood sequence if $N[v_i] \not\subseteq \cup_{j=1}^{i-1} N[v_j]$, is open neighborhood sequence if $N(v_i) \not\subseteq \cup_{j=1}^{i-1} N(v_j)$, is Z-sequence if $N(v_i) \not\subseteq \cup_{j=1}^{i-1} N[v_j]$, and is L-sequence if $N[v_i] \not\subseteq \cup_{j=1}^{i-1} N(v_j)$ for all $2 \leq i \leq k$. The length of a longest such sequence of a graph is called Grundy domination number (resp. Grundy total domination number, Z-Grundy domination number or L-Grundy domination number). We will compare properties of all four concepts and present some new results for trees, split graphs, interval graphs and Kneser graphs.

On k -rainbow independent domination in graphs

Tadeja Kraner Šumenjak

University of Maribor

A function $f : V(G) \rightarrow \{0, 1, \dots, k\}$ is called a k -rainbow independent dominating function of G if $V_i = \{x \in V(G) : f(x) = i\}$ is independent for $1 \leq i \leq k$, and for every $x \in V_0$ it follows that $N(x) \cap V_i \neq \emptyset$, for every $i \in [k]$. The k -rainbow independent domination number, $\gamma_{\text{rik}}(G)$, of a graph G is the minimum of $w(f) = \sum_{i=1}^k |V_i|$ over all such functions. We will determine some bounds and exact values concerning this domination concept. We will show that the problem of determining whether a graph has a k -rainbow independent dominating function of a given weight is NP-complete for bipartite graphs and that there exists a linear-time algorithm to compute $\gamma_{\text{rik}}(G)$ of trees. We will also focus on the k -rainbow independent domination number of the lexicographic product of graphs and present some new bounds.

Based on joint work with S. Brezovnik and joint work with D. F. Rall and A. Tepeh.

Double Roman Domination Number

Aparna Lakshmanan S.

St. Xavier's College for Women, Aluva, Kerala, India

Given a graph $G = (V, E)$, a function $f : V \rightarrow \{0, 1, 2, 3\}$ having the property that if $f(v) = 0$, then there exist $v_1, v_2 \in N(v)$ such that $f(v_1) = f(v_2) = 2$ or there exists $w \in N(v)$ such that $f(w) = 3$, and if $f(v) = 1$, then there exists $w \in N(v)$ such that $f(w) \geq 2$ is called a double Roman dominating function (DRDF). The weight of a DRDF f is the sum $f(V) = \sum_{v \in V} f(v)$, and the minimum weight of a DRDF on G is the double Roman domination number, $\gamma_{dR}(G)$ of G . In this talk, we present the impact of the Mycielskian graph operator on double Roman domination number of a graph. For any two positive integers a and b we construct a graph G and an induced subgraph H of G such that $\gamma_{dR}(G) = a$ and $\gamma_{dR}(H) = b$ and conclude that there is no relation between double Roman domination number of a graph and its induced subgraph. We also present the impact of edge addition and some graph operations such as Corona with complete graph, cartesian product and addition of twins, on the double Roman domination number.

This is a joined work with my research scholar Anu V.

Identifying codes in line digraphs

Berenice Martínez-Barona

Universitat Politècnica de Catalunya

Given an integer $\ell \geq 1$, a $(1, \leq \ell)$ -identifying code in a digraph is a dominating subset C of vertices such that all distinct subsets of vertices of cardinality at most ℓ have distinct closed in-neighbourhood within C . In this work, we prove that every k -iterated line digraph of minimum in-degree at least 2 and $k \geq 2$, or minimum in-degree at least 3 and $k \geq 1$, admits a $(1, \leq \ell)$ -identifying code with $\ell \leq 2$, and in any case it does not admit a $(1, \leq \ell)$ -identifying code for $\ell \geq 3$. Moreover, we find that the identifying number of a line digraph is lower bounded by the size of the original digraph minus its order. Furthermore, this lower bound is attained for oriented graphs of minimum in-degree at least 2.

Joint work with Camino Balbuena and Cristina Dalfo Preprint available at <http://arxiv.org/abs/1905.05083>.

$[1, k]$ -domination number of lexicographic product of graphs

Iztok Peterin

University of Maribor and IMFM

A subset D of the vertex set $V(G)$ of a graph G is called an $[1, k]$ -dominating set if every vertex from $V - D$ is adjacent to at least one vertex and at most k vertices of D . A $[1, k]$ -dominating set with minimum number of vertices is called a $\gamma_{[1, k]}(G)$ -set and number of its vertices is called the $[1, k]$ -domination number of G and denoted by $\gamma_{[1, k]}(G)$. We express $[1, k]$ -domination number of lexicographic products $G \circ H$ as an optimization problem over minimum over certain partitions of $V(G)$. In several cases this can be described with different properties of G .

Coauthors of this work are Narges Ghareghani, Wilfried Imrich and Pouyeh Sharifani.

On k -rainbow total domination in graphs

Aleksandra Tepoh
University of Maribor

In this talk a new domination invariant on a graph G will be presented. It is called the k -rainbow total domination number and denoted by $\gamma_{krt}(G)$. This concept generalizes the total domination on G , and for $k \geq 2$ it coincides with the total domination number of generalized prism $G \square K_k$. Various properties of this domination invariant will be presented, including, inter alia, that $\gamma_{krt}(G) = n$ for a nontrivial graph G of order n as soon as $k \geq 2\Delta(G)$. It will be demonstrated how a weight-redistribution technique can be used to derive closed formulas for the k -rainbow total domination number of paths and cycles, for every k . The mentioned results complement recently proved partial observations on total domination number of prisms.

Invited Special Session

FINITE GEOMETRIES

Organized by TAMÁS SZŐNYI

An Investigation into Small Weight Code Words of Projective Geometric Codes

Sam Adriaensen

Vrije Universiteit Brussel

Let $\text{PG}(n, q)$ denote the n -dimensional Desarguesian projective space over \mathbb{F}_q , where $q = p^h$, with p prime. Choose natural numbers $0 \leq j < k < n$. Define $\mathcal{C}_{j,k}(n, q)$ as the span of the rows of the p -ary incidence matrix of j -spaces (to which the columns correspond) and k -spaces (to which the rows correspond) in $\text{PG}(n, q)$. It is known that code words of $\mathcal{C}_{0,n-1}(n, q)$ up to weight $2q^{n-1}$ are linear combinations of at most two (characteristic vectors of) hyperplanes. For larger values of q , stronger characterizations are known. We prove that code words up to weight $2q^k$ of $\mathcal{C}_{0,k}(n, q)$ are linear combinations of at most two k -spaces. For larger values of q , we can push the bound on the weight and make it sharp. We can generalize this to prove that code words up to weight almost $3 \binom{k+1}{j+1}_q^{-1}$ of $\mathcal{C}_{j,k}(n, q)^\perp$ are linear combinations of at most two k -spaces (for larger values of q). This determines the minimum weight of $\mathcal{C}_{j,k}(n, q) \cap \mathcal{C}_{j,n-k+j}(n, q)^\perp$, in most cases, which was yet generally unknown.

We also investigate the dual codes $\mathcal{C}_{j,k}(n, q)^\perp$, and reduce the problem of finding their minimum weight to finding the minimum weight of the codes $\mathcal{C}_{0,1}(n, q)^\perp$. This allows us to characterize the minimum weight code words in case q is prime, solving a conjecture of B. Bagchi & S. P. Inamdar. It also allows us to determine the minimum weight of $\mathcal{C}_{j,k}(n, q)^\perp$ in case q is even, and to generalize bounds on the minimum weight of $\mathcal{C}_{0,1}(n, q)^\perp$ to bounds on the minimum weight of $\mathcal{C}_{j,k}(n, q)^\perp$.

Keywords: linear codes, small weight code words

Joint work with Lins Denaux — Ghent University.

Maximum Distance Separable Codes: Recent advances and applications

Simeon Ball

Universitat Politècnica Catalunya

Let A be a finite set and let C be a subset of A^n .

Let d be minimal such that any two codewords (elements of C) differ in at least d coordinates. Fixing any $n - d + 1$ coordinates, one obtains the (Singleton) bound

$$|C| \leq |A|^{n-d+1},$$

since if C was larger then the pigeon-hole principle would imply that two codewords agree on these $n - d + 1$ coordinates and therefore differ on at most $d - 1$ coordinates.

A **maximum distance separable code** (MDS code) is a code C for which $|C| = |A|^{n-d+1}$. Thus, C has the property that for any k -tuple ($k = n - d + 1$) of elements of A on any k coordinates, there is a unique codeword of C which agrees with the k -tuple on these k coordinates.

Two important applications of MDS codes are to distributed storage systems and to error-correcting communication (particularly to channels susceptible to burst-errors).

For the main part of the talk, I will consider the geometrical object (known as an **arc**) which one obtains by taking the set of columns of a generator matrix of a linear MDS code

${}^1 \binom{k+1}{j+1}_q$ is the Gaussian coefficient, giving the number of j -spaces in $\text{PG}(k, q)$.

over a finite field \mathbb{F}_q and considering this set of columns as a set of points in $\text{PG}(k-1, q)$, the $(k-1)$ -dimensional projective space.

The main conjecture for linear MDS codes (also known as the MDS conjecture) states, in terms of arcs, that if $4 \leq k \leq q-2$ then an arc in $\text{PG}(k-1, q)$ has size at most $q+1$. This would imply that there are no (linear) MDS codes which outperform Reed-Solomon codes. If k is outside this range then we know how large an arc can be and therefore how many errors one can correct with a k -dimensional linear MDS code.

The MDS conjecture was proven for q prime in 2012. I will detail all results since then and before then which prove the MDS conjecture for ranges of k when q is not prime.

These results can be found in [2], [3] (with an additional hypothesis) and [4]. in [1]

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Balanced upper chromatic number of $\text{PG}(2, q)$

Zoltán L. Blázsik

MTA-ELTE Geometric and Algebraic Research Group

Consider the hypergraph formed by the points and the lines of $\text{PG}(2, q)$. The balanced upper chromatic number of $\text{PG}(2, q)$ is the most number of colors that can be used to color the points of this hypergraph so that every line contains at least two points of the same color and the size of the color classes are almost the same, namely they can differ by at most 1.

In this talk, we are going to improve the known lower bounds of the balanced upper chromatic number of the projective plane of order q if $q \not\equiv 1 \pmod{3}$. We will show two constructions such that the color classes have size 3 or 4. Moreover if $q \equiv 0 \pmod{3}$ then our construction is sharp in a sense that it certifies the exact value of the balanced upper chromatic number due to a result of Araujo-Pardo, Kiss and Montejano.

Joint work with Aart Blokhuis and Tamás Szőnyi.

Generalising KM-arcs

Bence Csajbók

*MTA–ELTE Geometric and Algebraic Combinatorics Research Group &
ELTE Eötvös Loránd University, Budapest, Hungary*

A KM-arc of type $(0, 2, t)$ is a point set \mathcal{S} of a finite projective plane of order q such that each point Q of \mathcal{S} is incident with a unique line meeting \mathcal{S} in t points and the other lines incident with Q meet \mathcal{S} in 2 points. These objects have been studied first by Korchmáros and Mazzocca in 1990, that is why nowadays they are called KM-arcs. KM-arcs exist only for q and t even and they have been studied mostly in Desarguesian planes, where Gács and Weiner proved that the t -secants of a KM-arc are concurrent. In this talk, I will introduce the following generalisations.

A *generalised KM-arc of type $(0, m, t)$* is a point set \mathcal{S} of a finite projective plane of order q such that each point Q of \mathcal{S} is incident with a unique line meeting \mathcal{S} in t points and the other lines incident with Q meet \mathcal{S} in m points.

A *mod p generalised KM-arc of type $(0, m_p, t_p)$* is a point set \mathcal{S} of a finite projective plane of order $q = p^n$ such that each point Q of \mathcal{S} is incident with a unique line meeting \mathcal{S} in $t \bmod p$ points and the other lines incident with Q meet \mathcal{S} in $m \bmod p$ points.

Theorem 1. Let \mathcal{S} be a mod p generalized KM-arc of type $(0, m_p, t_p)$ in $\text{PG}(2, q)$, $q > 17$. Assume that $t \not\equiv m \pmod{p}$. If there are no 0-secants of \mathcal{S} or $m = 0$, then lines meeting \mathcal{S} in $t \bmod p$ points are concurrent.

Theorem 2. A generalized KM-arc \mathcal{S} of type $(0, m, t)$ is either one of the following: a set of t collinear points; the union of m lines incident with a point P , minus P ; an oval a maximal arc with at most one point removed; a unital; complement of a Baer subplane; complement of one point; or $m \equiv t \equiv 0 \pmod{p}$.

The proofs rely on a stability result of Szőnyi and Weiner regarding $k \bmod p$ multisets; and other polynomial techniques which ensure that in case of $t \not\equiv m \pmod{p}$ the $t \bmod p$ secants meeting a fixed $m \bmod p$ secant in \mathcal{S} are concurrent. Connections with the Dirac–Motzkin conjecture regarding the number of lines meeting a point set of $\text{PG}(2, \mathbb{R})$ in two points will be discussed as well.

Joint work with Zsuzsa Weiner.

Cameron-Liebler classes for finite geometries

Maarten De Boeck

Ghent University

In [2] Cameron and Liebler studied the orbits of the projective groups $\text{PGL}(n+1, q)$. For this purpose they introduced line classes in the projective space $\text{PG}(3, q)$ with a specific property, which afterwards were called Cameron-Liebler line classes. Many equivalent characterisations of these Cameron-Liebler classes are known, relating them to line spreads, to the row space of the point-line incidence matrix, to the eigenspaces of the Grassmann scheme, ... or describing them combinatorially. The two main questions for Cameron-Liebler sets are proving several equivalent characterisations, and the classification problem, i.e. describing all Cameron-Liebler sets (with a small parameter).

In the past decades Cameron-Liebler classes were introduced for several combinatorial structures, in particular geometries. In the first part of this talk I will discuss the article [1] where we introduced Cameron-Liebler classes of k -spaces in $\text{PG}(n, q)$, generalising work from

[5] (line classes in $\text{PG}(n, q)$) and [6] (classes of k -spaces in $\text{PG}(2k + 1, q)$). We proved several classification and characterisation results.

In the second part of this talk I will discuss Cameron-Liebler classes of generators in polar spaces, which were introduced in [4] and studied further in [3]. I will present several characterisations of these Cameron-Liebler classes, which vary dependent on the polar space. Moreover I will present the classification of Cameron-Liebler classes of generators in polar spaces with a small parameter.

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Small weight code words arising from the incidence of points and hyperplanes in $\text{PG}(n, q)$

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This topic concerns small weight code words of the code $C_{n-1}(n, q)$, the vector space generated by the incidence matrix of points and hyperplanes of $\text{PG}(n, q)$ ($n \in \mathbb{N} \setminus \{0, 1\}$, $q = p^h$, p prime, $h \in \mathbb{N}^\times$). Polverino and Zullo proved that the second minimum weight of $C_{n-1}(n, q)$ is $2q^{n-1}$. Code words matching this weight are precisely the scalar multiples of the difference of the incidence vectors of two hyperplanes. Szonyi and Weiner have characterised all code words up to weight $4q - 22$ in the code of points and lines in the plane. Such small weight code words are linear combinations of the incidence vectors of at most three lines, or a specific, ‘odd’ code word independently discovered by Bagchi on the one hand and De Boeck and Vandendriessche on the other hand. Based on these results, we have characterised all code words up to weight $4q^{n-1} - \Theta(q^{n-2}\sqrt{q})$ as being linear combinations of the incidence vectors of hyperplanes having a fixed $(n - 3)$ -dimensional subspace in common. Furthermore, we will discuss some other results related to codes arising from substructures in projective spaces.

Joint work with Sam Adriaensen, Leo Storme and Zsuzsa Weiner. Preprint available at <https://arxiv.org/abs/1905.04978>.

Projective solids pairwise intersecting in at least a line

Jozefien D'haeseleer
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In the last decades, projective subspaces, pairwise intersecting in at least a t -space were investigated [2]. The special case with $t = 0$ (the so called Erdős-Ko-Rado-sets), received special attention [1, 3]. Let $\text{PG}(n, q)$ be the projective space of dimension n , over the finite field of order q . In this talk, I discuss the structure of maximal sets of 3-spaces of $\text{PG}(n, q)$, $n \geq 5$, pairwise intersecting in at least a line, and give an overview of the largest examples of these sets. We also generalize these results to a maximal set of k -dimensional spaces, $k > 3$, that mutually intersect in at least a $(k - 2)$ -dimensional space in $\text{PG}(n, q)$, where $n \geq k + 2$. (The presentation will be based on joint work with Giovanni Longobardi, Ago-Erik Riet, Leo Storme)

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Classical groups and transitive actions on subspaces

Stephen Glasby
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Christoph Hering classified all subgroups H of semilinear transformations a finite vector space $V = (\mathbb{F}_q)^n$ that act transitively on the nonzero vectors of V . This led to the classification of the 2-transitive permutation groups of affine type.

In joint work with Giudici and Praeger, we classify all subgroups H of a finite classical group on V that act on a set \mathcal{U} of subspaces of V . This has many applications, including the classification of the point-transitive subgroups acting on finite thick classical generalised quadrangles.

New SDP Bounds on Subspace Codes

Ferdinand Ihringer
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We can define a metric on the subspaces of \mathbb{F}_q^n by saying that two subspaces U and V have distance $d(U, V) := \dim(U + V) - \dim(U \cap V)$. In the context of network coding theory, the parallelized many-to-many transmission of data, codes with respect to this metric were investigated in recent years. They are also of interest in the context of the existence of q -analogs of designs such as the q -Fano plane. In 2013 Bachoc et. al. determined several semidefinite programming (SDP) bounds for subspace codes with $q = 2$ and odd minimum distance δ . We will discuss several improvements and generalizations of their results.

This is joint work with Daniel Heinlein.

On resolving sets for the point-line incidence graph of $\text{PG}(n, q)$

György Kiss

ELTE Budapest & University of Primorska, Koper

For a simple, connected, finite graph $\Gamma = (V, E)$ and $x, y \in V$ let $d(x, y)$ denote the length of a shortest path joining x and y . A vertex $v \in V$ is *resolved* by $S = \{s_1, s_2, \dots, s_r\} \subset V$ if the ordered sequence $(d(v, s_1), d(v, s_2), \dots, d(v, s_r))$ is unique. S is a *resolving set* in Γ if it resolves all the elements of V . The *metric dimension* of Γ , denoted by $\mu(\Gamma)$, is the size of the smallest example of resolving set in it.

Let $\Gamma_{n,q}$ denote the point-line incidence graph of the finite projective space $\text{PG}(n, q)$. It was shown by Héger and Takáts [1] that $\mu(\Gamma_{2,q}) = 4q - 4$ if $q \geq 23$. In this talk we show the following natural generalizations of this planar result.

If $n > 2$ is fixed, then the metric dimension of $\Gamma_{n,q}$ is asymptotically $2q^{n-1}$.

Joint work with *Daniele Bartoli, Stefano Marcugini and Fernanda Pambianco.*

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MDS codes, arcs and tensors

Michel Lavrauw

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A *Maximum Distance Separable code* (or *MDS code*) is a code meeting the Singleton bound. This means that an MDS code corrects the maximum number of errors given its size and length. Examples of MDS codes include the (extended) Reed-Solomon codes which have many applications. MDS codes have been studied since the 1950's and, according to MacWilliams and Sloane [3], already in 1977, they formed "one of the most fascinating chapters in all of coding theory". It is believed that a linear MDS code cannot have length larger than the extended Reed-Solomon code (except in some very special cases which are well-understood). This is known as the *MDS conjecture* (sometimes referred to as the *main conjecture*). Many mathematicians have contributed to a possible solution and several instances of the MDS conjecture have been solved in the last 50 years. However the conjecture in its full generality is still open. In 2012, Simeon Ball proved the conjecture for MDS codes which are linear over prime fields. We will report on recent progress, in particular on recent results from [1] and [2].

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Subspace code constructions

Giuseppe Marino

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Let V be an n -dimensional vector space over $\text{GF}(q)$, q any prime power. The set $S(V)$ of all subspaces of V , or subspaces of the associated projective space $\text{PG}(V)$, forms a metric space with respect to the *subspace distance* defined by $d(U, U') = \dim(U + U') - \dim(U \cap U')$. In the context of subspace codes, the main problem is to determine the largest possible size of codes in the space $(S(V), d)$ with a given minimum distance, and to classify the corresponding optimal codes. The interest in these codes is a consequence of the fact that codes in the projective space and codes in the Grassmannian over a finite field referred to as subspace codes and constant-dimension codes, respectively, have been proposed for error control in random linear network coding, see [3].

An $(n, M, 2\delta; k)_q$ constant-dimension subspace code (CDC) is a set \mathcal{C} of k -subspaces of V with $|\mathcal{C}| = M$ and minimum subspace distance $d(\mathcal{C}) = \min\{d(U, U') \mid U, U' \in \mathcal{C}, U \neq U'\} = 2\delta$. In the terminology of projective geometry, an $(n, M, 2\delta; k)_q$ CDC, $\delta > 1$, is a set \mathcal{C} of $(k - 1)$ -dimensional projective subspaces of $\text{PG}(n - 1, q)$ such that $|\mathcal{C}| = M$ and every $(k - \delta)$ -dimensional projective subspace of $\text{PG}(n - 1, q)$ is contained in at most one member of \mathcal{C} or, equivalently, any two distinct codewords of \mathcal{C} intersect in at most a $(k - \delta - 1)$ -dimensional projective space. The maximum size of an $(n, M, 2\delta; k)_q$ CDC subspace code is denoted by $\mathcal{A}_q(n, 2\delta; k)$ and several upper bounds are known [7], [2].

As for lower bounds, in [4] there is a construction of CDC obtained by using maximum rank distance codes, which yields the bound $\mathcal{A}_q(n, 2\delta; k) \geq q^{(n-k)(k-\delta+1)}$.

In this talk, we construct a set of $q^{12} + 2q^8 + 2q^7 + q^6 + q^5 + q^4 + 1$ planes of $\text{PG}(8, q)$ mutually intersecting in at most a point providing an improvement on the known lower bound. Also we present some constructions of *orbit-codes*, i.e. constant dimension codes which admit a certain automorphism group and whose members form an orbit under the action of such a group [5, 6, 1], with parameters $(6, (q^3 - 1)(q^2 + q + 1), 4; 3)_q$.

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(Joint work with Antonio Cossidente and Francesco Pavese)

Bipartite Ramsey numbers: C_4 versus the star

Sam Mattheus

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Finite projective planes can be found in abundance in extremal graph theory, their properties making them an excellent source to construct extremal graphs from. Starting from the point-line incidence graph, we can build bipartite graphs, not containing C_4 as a subgraph and with high minimal degree. These graphs will be extremal, having the largest possible number of vertices with respect to these two properties, and prove the exact value of the bipartite Ramsey number $R_b(C_4, K_{1,n})$ for several values of n . This idea has been applied by several authors, and we will further explore it. For the Desarguesian plane $PG(2, q)$, we will show an extension of the classical De Bruijn-Erdős theorem, revealing the limitations of the construction method.

Joint work with Tamás Héger.

A construction for clique-free pseudorandom graphs

Valentina Pepe

Sapienza University of Roma

A construction of Alon and Krivelevich gives highly pseudorandom K_k -free graphs on n vertices with edge density equal to $\Theta(n^{-1/(k-2)})$. We explore the construction of dense clique free pseudorandom graphs arising from polar spaces, obtaining an infinite family of highly pseudorandom K_k -free graphs with a higher edge density of $\Theta(n^{-1/(k-1)})$.

Joint work with A. Bishnoi and F. Ihringer.

On the stability of Baer subplanes

Tamás Szőnyi

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A blocking set in a projective plane is a point set intersecting each line. The smallest blocking sets are lines. The second smallest minimal blocking sets are Baer subplanes (subplanes of order \sqrt{q}). Our aim is to study the stability of Baer subplanes in $PG(2, q)$. If we delete $\sqrt{q} + 1 - k$ points from a Baer subplane, then the resulting set has $\Delta = (\sqrt{q} + 1 - k)(q - \sqrt{q})$ skew lines (0-secants). Erdős and Lovász proved that a set of $q + k$ points, $0 \leq k \leq \sqrt{q} + 1$ either contains a large collinear subset or has at least Δ skew lines.

If we have somewhat more 0-secants, then our main theorem says that this point set can be obtained from a Baer subplane or from a line by deleting somewhat more than k points and adding some points. The motivation for this theorem comes from planes of square orders, but our main result is valid also for non-square orders. In this case the point set contains a relatively large collinear subset. The precise statement is the following.

Theorem

Let B be a point set in $PG(2, q)$, with cardinality $q + k$, $0 \leq k \leq 0.6\sqrt{q}$ and $1600 \leq q$. Assume that the number of skew lines of B is less than $(q - \sqrt{q} - c)(\sqrt{q} - k + c + 1)$, where $0 \leq c \leq 0.05\sqrt{q} - 2$. Then B contains more than $q + 1 - (\sqrt{q} - k + c + 1)$ points from a line or $q + \sqrt{q} + 1 - (\sqrt{q} - k + c + 1)$ points from a Baer subplane.

For $0.6\sqrt{q} \leq k \leq \sqrt{q}$, earlier work of the authors on the stability of small blocking sets implies a stability result for Baer subplanes, in which the bound on the number of skew lines is roughly

$(\frac{1}{2} - \varepsilon)q^{3/2}$.

This is a joint work with Zsuzsa Weiner.

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Invited Special Session

GAMES ON GRAPHS

Organized by CSILLA BUIJTÁS

Domination and transversal games: Conjectures and perfectness

Csilla Bujtás

University of Ljubljana

The domination game [1] and the total domination game [3] are two-person optimization games played on a graph by a ‘slow’ and a ‘fast’ player. The transversal game [2], which is played on hypergraphs, offers a more general frame to study domination and total domination games.

First, we survey upper bounds on the corresponding graph and hypergraph invariants that are the game domination number $\gamma_g(G)$, the game total domination number $\gamma_{tg}(G)$, and the game transversal number $\tau_g(H)$. In the second part of the talk, based on a joint work with V. Iršič and S. Klavžar, we discuss recent results on (γ, γ_g) -perfect and (γ_t, γ_{tg}) -perfect graphs.

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The Graceful Game

Simone Dantas

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One of the most studied graph labelings is the *graceful labeling*, so named by S. W. Golomb [1] and initially introduced by A. Rosa [2] in 1966. A *graceful labeling* of a graph G with m edges is an injective function $f: V(G) \rightarrow \{0, 1, \dots, m\}$ such that, when each edge $uv \in E(G)$ is assigned the (*induced*) label $g(uv) = |f(u) - f(v)|$, all induced edge labels are distinct. Labeling problems are usually studied from the perspective of determining whether a given graph has a required labeling or not. An alternative perspective is to analyze labeling problems from the point of view of combinatorial games.

We investigate the Graceful Game, first proposed by Tuza [3]. The *Graceful Game* is defined in the following way: Alice and Bob alternately assign a previously unused label $f(v) \in \{0, \dots, m\}$ to a previously unlabeled vertex v of a given graph G . If both endpoints of an edge $uv \in E(G)$ are labeled, the *label* of uv is defined as $|f(u) - f(v)|$. A move (label assignment) is said to be *legal* if, after it, all edge labels are distinct. Alice *wins* the game if the whole graph G is gracefully labeled, and Bob *wins* if he can prevent this.

In this work, we study winning strategies for Alice and Bob in many well know graph classes, for example paths, complete graphs, cycles, complete bipartite graphs, caterpillars, helms, web graphs, gear graphs, prisms, hypercubes and powers of paths P_n^2 .

Joint work with Luisa Frickes (Fluminense Federal University, Brazil), Simone Dantas (Fluminense Federal University, Brazil), Atilio G. Luiz (Federal University of Ceará, Brazil). This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível

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Choosability Games

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A well known concept of graph choosability can be defined as a one-round game in which Lister assigns lists of colors to the vertices of a graph G , and then Painter colors G properly choosing a color for each vertex from its list. The *list chromatic number*, denoted by $\chi_\ell(G)$, is the minimum size of lists guaranteeing a win for Painter.

Suppose now that lists assigned by Lister are initially hidden from Painter. In each round Lister reveals one color in all lists containing it, and then Painter colors some of the vertices properly by this color. Each color can be used in only one round. Let $\chi_\ell^*(G)$ denote the minimum size of lists guaranteeing a win for Painter. Clearly, we have $\chi_\ell(G) \leq \chi_\ell^*(G)$ for every graph G .

It is known that every planar graph G satisfies $\chi_\ell^*(G) \leq 5$, which is an extension of the famous result of Thomassen. We prove that every planar graph contains a matching M such that $\chi_\ell^*(G - M) \leq 4$. This extends a result by Cushing and Kierstead on defective choosability of planar graphs. The proof uses Combinatorial Nullstellensatz of Alon and a result of Schauz asserting that $\chi_\ell^*(G)$ is at most the Alon-Tarsi number of G .

We also propose other choosability games that can be played on more general combinatorial structures, as well as choosability games involving other coloring concepts.

Joint work with Xuding Zhu. Preprint available at <https://arxiv.org/abs/1811.12012>.

Some results on the connected domination game

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The connected domination game was introduced by Borowiecki, Fiedorowicz and Sidorowicz in 2018 as another variation of the domination game. The rules are essentially the same, except that the set of played vertices must be connected at all stages of the game. In this talk, the relation between the number of moves in a game where Dominator/Staller starts the game is discussed. The connected game domination number on the lexicographic product graphs is determined, and the effect of predomination of a vertex is considered.

Preprint available at <https://arxiv.org/abs/1902.02087>.

Domination game on split graphs

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The domination game played on a finite, undirected graph G consists of two players, Dominator (D) and Staller (S), who alternately taking turns choosing a vertex from G such that whenever a vertex is chosen by either player, at least one additional vertex is dominated. Dominator uses a strategy to dominate the graph in as few steps as possible, and Staller uses a strategy to delay the process as much as possible. D game and S game are two variants of the domination game in which Dominator and Staller has the first move, respectively. The **game domination number**, denoted by $\gamma_g(G)$, the number of vertices chosen in a D game when both players play optimally. The **Staller-start game domination number**, denoted by $\gamma'_g(G)$, is the number of vertices chosen in an S game when both players play optimally.

The domination game and the game domination number γ_g in the class of split graphs and an upper bound that $\gamma_g(G) \leq n/2$ for any isolate free n -vertex split graph G . This strengthening the conjectured $3n/5$ general bound and supporting Rall's $\lceil n/2 \rceil$ -conjecture. Split graphs of even order with $\gamma_g(G) = n/2$ are also in the talk.

The above is a joint work with Sandi Klavžar and Ambat Vijayakumar.

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Replicator equations on graphs

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The original form of the replicator equation was the first important tool to connect game dynamics, where individuals change their strategy over time, with evolutionary game theory, created by Maynard Smith and Price to predict the prevalence of competing strategies in evolving populations. The replicator equation was initially developed for infinitely large and well-mixed populations. Later, in 2006, using the standard pair approximation, Ohtsuki and Nowak proved that moving evolutionary game dynamics from a well-mixed population (a complete graph) onto a regular non-complete graph is simply described by a transformation of the payoff matrix. Under the assumption of weak selection, and using a new closure method for the pair approximation technique, we build a modified replicator equation on infinitely large graphs, for birth-death updating (a player is chosen with probability proportional to its fitness, and the offspring of this player replaces a random neighbour). The closure method that we propose takes into account the probability of triangles in the graph. Using this new equation, we study how graph structure can affect cooperation in some games with two different strategies, namely the Prisoner's Dilemma, the Snow-Drift Game and the Coordination Game. We compare our results with the ones that were obtained in the past using the standard replicator equation and the Ohtsuki-Nowak replicator equation on graphs, and we discuss the advantages and flaws of both approaches.

Non-deterministic decision making on finite graphs

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Voting protocols, such as the push and the pull protocol, are designed to model the behavior of people during an election. These processes are also used to study social models of interaction, distributed computing in peer-to-peer networks, and to describe how viruses or rumors spread in a community.

In general, a connected graph $G = (V, E)$ is given together with a function $f : V \rightarrow \{0, 1\}$ that represents the initial opinion of the vertices. A voting process is a non-deterministic game such that in each round the opinions are altered according to some given, specific rules until a consensus is reached. We provide an asymptotic formula for the expected runtime of such protocols on cycle graphs, paths and star graphs, and the probability for each consensus to win in the end. The method can be used to estimate the expected time of joint Drunkard walks on these graphs, as well. Finally, we analyse the continuous variant of these processes, and obtain that the possible outcome of the game is a fractal on the unit interval.

Preprint available at <http://math.unideb.hu/pongacz-andras/publications.html>.

Online Sum-Paintability: Slow-Coloring of Trees

Gregory J. Puleo

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The *slow-coloring game* is played by Lister and Painter on a graph G . On each round, Lister marks a nonempty subset M of the remaining vertices, scoring $|M|$ points. Painter then gives a

color to a subset of M that is independent in G . The game ends when all vertices are colored. Painter's goal is to minimize the total score; Lister seeks to maximize it. The score that each player can guarantee doing no worse than is the *sum-color cost* of G , written $s(G)$. We develop a linear-time algorithm to compute $s(G)$ when G is a tree, enabling us to characterize the n -vertex trees with the largest and smallest values. Our algorithm also computes on trees the *interactive sum choice number*, a parameter recently introduced by Bonamy and Meeks.

This is joint work with Douglas B. West.

A connected version of the graph coloring game

Éric Sopena

University of Bordeaux

The graph coloring game is a two-player game in which, given a graph G and a set of k colors, the two players, Alice and Bob, take turns coloring properly an uncolored vertex of G , Alice having the first move. Alice wins the game if and only if all the vertices of G are eventually colored. The game chromatic number of a graph G is then defined as the smallest integer k for which Alice has a winning strategy when playing the graph coloring game on G with k colors.

In this talk, we introduce and study a new version of the graph coloring game by requiring that, after each player's turn, the subgraph induced by the set of colored vertices is connected. The connected game chromatic number of a graph G is then the smallest integer k for which Alice has a winning strategy when playing the connected graph coloring game on G with k colors.

We will discuss the connected game chromatic number of some graph classes and we will prove, in particular, that if $G = C_m \square C_n$, m, n even, then the connected game chromatic number of G is 2, while Bob has a winning strategy when playing the connected graph coloring game on G with three colors.

This is a joint work (in progress) with Clément Charpentier and Hervé Hocquard.

Indicated coloring game on Cartesian products of graphs

Daša Štesl

University of Maribor

Indicated coloring game is played on a simple graph G by two players, and a fixed set C of colors. In each round of the game Ann indicates an uncolored vertex, and Ben colors it using a color from C , obeying just the proper coloring rule. The goal of Ann is to achieve a proper coloring of the whole graph, while Ben is trying to prevent this. The minimum cardinality of the set of colors C for which Ann has a winning strategy is called the indicated chromatic number, $\chi_i(G)$, of a graph G . The concept was introduced by Grzesik in 2012. In this talk, we consider the indicated coloring game on Cartesian products of graphs. We prove that $G \square K_{m,n}$ is equal to 3 if and only if $\chi_i(G) = 3$. The indicated chromatic number in several other families of Cartesian products of graphs is also established. The investigations lead us to propose a Sabidussi-type equality as an open problem.

This is joint work with Boštjan Brešar and Marko Jakovac.

The Slow-Coloring Game on a Graph

Douglas B. West

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The *slow-coloring game* is played by Lister and Painter on a graph G . Initially, all vertices of G are uncolored. In each round, Lister marks a non-empty set M of uncolored vertices, and Painter colors a subset of M that is independent in G . The game ends when all vertices are colored. The score of the game is the sum of the sizes of all sets marked by Lister. The goal of Painter is to minimize the score, while Lister tries to maximize it; the score under optimal play is the *cost* of the graph.

A greedy strategy for Painter keeps the cost of G to at most $\chi(G)n$ when G has n vertices, which is asymptotically sharp for Turán graphs. On various classes Painter can do better. For n -vertex trees the maximum cost is $\lfloor 3n/2 \rfloor$. There is a linear-time algorithm and inductive formula to compute the cost on trees, and we know all the extremal n -vertex trees. Also, Painter can keep the cost to at most $(1 + 3k/4)n$ when G is k -degenerate, $7n/3$ when G is outerplanar, and $3.9857n$ when G is planar.

These results are joint work with various subsets of Grzegorz Gutowski, Tomasz Krawczyk, Thomas Mahoney, Gregory J. Puleo, Michal Zając, and Xuding Zhu. The slides and three papers are available at <https://faculty.math.illinois.edu/~west/pubs/publink.html>.

GENERAL GRAPH THEORY

Orthogonal Array Configurations

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An Orthogonal Array ($OA(n, k)$ of order n and depth k is a $k \times n^2$ array with entries from $N := \{1, \dots, n\}$ such that in any $2 \times n^2$ subarray all possible columns occur precisely once. The graph $\Gamma OA(n, k)$ of an $OA(n, k)$ has the n^2 columns of $OA(n, k)$ as vertices, two distinct vertices being adjacent if they have the same entry in exactly one coordinate position. A graph is *strongly regular* with parameters $SRG(n, d, \lambda, \mu)$ if it has n vertices, all of degree d , such that every pair of distinct vertices has λ common neighbours if they are adjacent, and μ common neighbours if they are not adjacent. $\Gamma OA(n, k)$ is a $SRG(n^2, (n-1)k, n-2+(k-1)(k-2), k(k-1))$.

A (v_k, b_r) -configuration C is an incidence structure consisting of v points and b lines such that there are k points in each line, each point lies in r lines and no pair of points belongs to two distinct lines of C . Given a (v_k, b_r) -configuration C we can build two complementary graphs: the *configuration graph* where the vertices are the points of C and two vertices are adjacent if together they do not lie in any line of C ; and the *point graph* where the vertices are the points of C and two vertices are adjacent if together they belong to some line of C . Note that the point graph of a configuration is not necessarily strongly regular.

We prove that an orthogonal array $OA(n, k)$ gives rise to a configuration C of type (n_k^2, nk_n) whose point graph is $\Gamma OA(n, k)$, hence strongly regular. Finally, we give some insight on the characterization of configurations arising from small orthogonal arrays.

Joint work with Martin Funk, Vedran Krčadinac and Domenico Labbate.

Some new local conditions for k -contractible edges

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For a graph G , let $V(G)$ and $E(G)$ denote the vertex set of G and the edge set of G , respectively. We denote the set of degree k vertices $V_k(G)$. We denote by $N_G(x)$ the neighborhood of x in G . Moreover, for a subset $S \subseteq V(G)$, let $N_G(S) = \cup_{x \in S} N(x) - S$. Let $\Delta(G)$ stand for the maximum degree of G . For $S \subseteq V(G)$, we let $G[S]$ denote the subgraph induced by S in G .

Let k be an integer such that $k \geq 2$ and we deal with k -connected graph G with $|V(G)| \geq k+2$. An edge e of G is said to be *k -contractible* if the contraction of the edge results in a k -connected graph. An edge $e = xy$ is *trivially k -noncontractible* if and only if $N_G(x) \cap N_G(y) \cap V_k(G) \neq \emptyset$. A vertex x of G is said to be *totally trivial* if every edge incident with x is trivially k -noncontractible. Let $\mathcal{T}(G)$ denote the set of totally trivial vertices of G and let $\xi(G)$ denote the order of maximum component of $G[\mathcal{T}(G)]$. A nontrivial edge e is said to be *bad 2-edge* if there is a 2-component A such that $V(e) \subseteq N(A)$ and $A \cap V_{k+1}(G) \neq \emptyset$. A vertex of G is said to be *bad vertex* if it is incident with a bad 2-edge.

We prove the following local conditions for G to have an k -contractible edge.

Theorem 1 *Let G be a k -connected graph. If $|E(G[N(x)])| \leq \lceil \frac{k}{2} \rceil - 1$ for every $x \in V(G)$, then G has a k -contractible edge.*

Theorem 2 *Let G be a k -connected graph with $\xi(G) \leq \lceil \frac{k+1}{2} \rceil - 1$. If $|E(G[N(x)])| \leq k-2$ for every $x \in V(G)$, then G has a k -contractible edge.*

Both of the inequalities in Theorems 1 and 2 concerning with the number of edges of the

graph induced by the neighborhood of a vertex are sharp.

Theorem 3 *Let G be a k -connected graph with $\xi(G) \leq \lceil \frac{k+1}{2} \rceil - 1$. If $\Delta(G[N(x)]) \leq \deg_G(x) - \lceil \frac{k+1}{2} \rceil$ for every $x \in V(G)$, then G has a k -contractible edge.*

Theorem 4 *Let k be an integer such that $k \geq 6$. Let G be a k -connected graph with $\mathcal{T}(G) = \emptyset$ and with at most $\lceil \frac{k+1}{2} \rceil - 1$ bad vertices. If $|E(G[N(x)])| \leq 2k - 9$ for every $x \in V(G)$, then G has a k -contractible edge.*

Exponential generalised network descriptors

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The study of networks, in the form of mathematical graph theory, is one of the fundamental pillars of discrete mathematics. Complex networks are extensively used to model objects and their relations. Throughout this paper we consider the representation of a complex network as a simple connected graph $G = (V, E)$. In communication networks theory the concepts of networkness and network surplus have recently been defined. Together with transmission and betweenness centrality, they were based on the assumption of equal communication between vertices. Generalised versions of these four descriptors are presented, taking into account that communication between vertices u and v is decreasing as the distance between them is increasing. Therefore, we weight the quantity of communication by $\lambda^{d(u,v)}$ where $\lambda \in (0, 1)$. Extremal values of these descriptors and the topology of graphs for which they are achieved are analysed.

Joint work with T. Kokan, T. Vojkovic and D. Vukicevic

Available at <https://arxiv.org/abs/1712.01245>.

The Equal-Sum-Free Subset problem

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In practice, competitions can occur, where the competitors have weights and we want to compare two groups with equal sum of weights. One of the theoretical problems involved here is the following:

Let w_1, w_2, \dots, w_n be a sequence of positive integers (repetition is allowed). A set $I \subseteq \{1, \dots, n\}$ is dependent if there exist nonempty subsets $J, K \subseteq I$ such that

- 1) The sum of the weights for $j \in J$ is equal to the sum for K .
- 2) The sets $\{w_j : j \in J\}$ and $\{w_k : k \in K\}$ are disjoint.

A set I is independent if it is not dependent. Given n , the question is, how large independent set we can guarantee for any sequence of length n .

Joint work with Zsolt Tuza.

Integer invariants of a graph manifold using Hirzenbruch-Jung continued fractions on the linking matrix.

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A graph manifold is a three-dimensional space which has only Seifert fibered pieces M^I , $I = 1, \dots, R$ in its decomposition along a set of incompressible tori (JSJ-decomposition). Using an adjacency matrix of the graph related to the manifold it is possible to calculate its linking matrix. This linking matrix is an important topological invariant of a graph manifold which is possible to interpret as a matrix of coupling constants of gauge interaction in Kaluza-Klein approach. Each Seifert fibered piece M^I is characterized by the rational Euler number $e^I = -\sum_{i=1}^{k_i} b_i^I/a_i^I$. We demonstrate that every edge of the graph can be expanded into a chain where the new vertices represent the integer information of the Hirzenbruch-Jung continued fraction of the Euler number of the vertices connected to that edge.

Joint work with Vladimir N. Efremov (efremov@cencar.udg.mx)

A variant of orthogonality for symmetric Latin squares

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A Latin square of order n is an $n \times n$ array of n symbols such that each symbol occurs exactly once in each row and column. Two Latin squares $L = [\ell_{ij}]$ and $M = [m_{ij}]$ of order n are said to be orthogonal (mates) if the n^2 ordered pairs (ℓ_{ij}, m_{ij}) are distinct. A set of $h > 2$ Latin squares is called a set of mutually orthogonal Latin squares (MOLS) if its squares are pairwise orthogonal.

It is known that a Latin square has an orthogonal mate if and only if it has a decomposition into disjoint transversals. We recall that a transversal of a Latin square of order n is a set of n cells that intersects each row and each column once and each symbol occurs exactly once.

Problems on the existence of MOLS led to consider alternative notions of orthogonality (see for instance [1]) and to generalise the notion of a transversal (such as partial transversal, near transversal or plexes).

We define and study a new sense of orthogonality arising from decompositions of symmetric Latin squares into transversal-like structures.

Joint work with Trent Marbach.

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Group distance magic hypercubes

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Let $G = (V, E)$ be a graph. A *distance magic* labeling of a graph G is a bijective assignment of labels from $\{1, 2, \dots, |V(G)|\}$ to the vertices of G such that the sum of labels on neighbors of u is the same for all vertices u . It is known that the n -dimensional hypercube Q has a distance magic labeling if and only if $n \equiv 2 \pmod{4}$.

Let Γ be an Abelian group of order $|V(G)|$. Analogously, a Γ -distance magic labeling of G is a bijection $\ell: V \rightarrow \Gamma$ for which the sum of labels on neighbors of u is the same for all vertices u .

In this paper we fully characterise a Γ -distance magic labeling of n -dimensional hypercubes Q . Namely prove that for n odd, there does not exist a Γ -distance magic labeling of Q for any Abelian group Γ of order $|V(Q)|$, whereas for n even there exists a Γ -distance magic labeling of Q for any Abelian group Γ of order $|V(Q)|$.

Finitely forcible graph limits are universal

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The theory of graph limits represents large graphs by an analytic object called a graphon. Graph limits determined by finitely many graph densities, which are represented by finitely forcible graphons, arise in various scenarios, particularly within extremal combinatorics. Lovász and Szegedy conjectured that all such graphons possess a simple structure, e.g., the space of their typical vertices is always finite dimensional; this was disproved by several ad hoc constructions of complex finitely forcible graphons. We prove that any graphon is a subgraphon of a finitely forcible graphon, which completely dismisses any hope for a result showing that finitely forcible graphons possess a simple structure. Moreover, since any finitely forcible graphon represents the unique minimizer of some linear combination of densities of subgraphs, our result also shows that such minimization problems may in fact have unique optimal solutions with arbitrarily complex structure. Joint work with Daniel Král' and Taisa Martins.

Edge colorings avoiding patterns

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We say that a pattern is a graph together with an edge coloring, and a pattern $P = (H, c)$ occurs in some edge coloring c' of G if c' , restricted to some subgraph of G isomorphic to H , is equal to c up to renaming the colors. Inspired by Matoušek's visibility blocking problem, we study edge colorings of cliques that avoid certain patterns.

We show that for every pattern P , such that the number of edges in P is at least the number of vertices in P plus the number of colors minus 2, there is an edge coloring of K_n that avoids P and uses linear number of colors; the same also holds for finite sets of such patterns.

The structure of 4-connected $K_{2,t}$ -minor-free graphs

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Guoli Ding has provided a rough structure theorem for $K_{2,t}$ -minor free graphs for all t . As a special case of his theorem, 4-connected $K_{2,t}$ -minor-free graphs are obtained by attaching *strips*, consisting of two paths joined by edges with restricted crossings, to a finite set of base graphs. The first value of t where this applies in a nontrivial way is $t = 5$. We give a characterization of 4-connected $K_{2,5}$ -minor-free graphs that shows that they can be obtained from a cyclic sequence

of four types of subgraph. Consequently, we can derive a generating function and asymptotic estimate for the number of nonisomorphic 4-connected $K_{2,5}$ -minor-free graphs of a given order. Our work extends to general t by providing a more precise description of the strips in Ding's result, suggesting a general asymptotic counting conjecture.

This is joint work with J. Zachary Gaslowitz.

Circumference of essentially 4-connected planar graphs

Igor Fabrici

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A 3-connected planar graph G is *essentially 4-connected* if, for every 3-separator S , one of the two components of $G - S$ is an isolated vertex. Let n denote the number of vertices of a graph. Jackson and Wormald (1992) proved that an essentially 4-connected planar graph contains a cycle of length at least $(2n + 4)/5$. For a cubic essentially 4-connected planar graph, Grünbaum with Malkevitch (1976) and Zhang (1987) showed the existence of a cycle of length at least $3n/4$.

We prove that every essentially 4-connected planar graph contains a cycle of length at least $5(n + 2)/8$ and every essentially 4-connected maximal planar graph contains a cycle of length at least $2(n + 4)/3$; moreover the second bound is best possible.

Joint work with Jochen Harant, Samuel Mohr, and Jens M. Schmidt.

Forbidding prism immersions

Gašper Fijavž

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A graph G contains an *immersion* of a graph H , if there exists an injective mapping $\varphi : V(H) \rightarrow V(G)$, so that for every edge $xy \in E(H)$ there exists a $\varphi(x) - \varphi(y)$ path P_{xy} in G , where the paths $P_{xy}, xy \in E(H)$, are edge disjoint.

Robertson and Seymour have proven that the relation of graph immersion is a WQO — in every infinite sequence of (finite) graphs there exists a pair of graphs so that one contains an immersion of the other.

The prism P is the cartesian product of a 3-cycle and the complete graph on two vertices. It is known that a 3-connected graph does not contain P as a minor if (1) it has at most 5 vertices, (2) is a wheel, or (3) has a vertex cover of order at most 3.

Building atop on the excluded minor theorem for P and using SPQR decompositions, we describe the structure of graphs that do not contain contain a P immersion.

Joint work with Matthias Kriesell.

Equitable list vertex colourability and arboricity of grids

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A graph G is equitably k -list arborable if for any k -uniform list assignment L , there is an equitable L -colouring of G whose each colour class induces an acyclic graph. The smallest number k admitting such a coloring is named equitable list vertex arboricity and is denoted by $\rho_l^-(G)$. Zhang in 2016 [2] posed the conjecture that if $k \geq \lceil (\Delta(G) + 1)/2 \rceil$ then G is equitably k -list arborable. We give [1] some new tools that are helpful in determining values of k for which a general graph is equitably k -list arborable. We use them to prove the Zhang's conjecture [2] for d -dimensional grids where $d \in \{2, 3, 4\}$ and give new bounds on $\rho_l^-(G)$ for general graphs and for d -dimensional grids with $d \geq 5$.

Joint work with E. Drgas-Burchardt, J. Dybizbański, E. Sidorowicz.

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Underwater vehicle trajectory planning using the graph theory in dynamic environments with obstacles

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For the underwater vehicle to solve an obstacle collision avoidance problem two tasks are necessary: the first - detecting collision threats, and the second - synthesizing a safe manoeuvre avoiding threatening obstacles. In the work a method for detecting a threat of collision is presented for the case of many obstacles being within the neighbourhood of the vehicle. The method for finding safe trajectory, based on by the graph theory, is proposed. A quality of the proposed solution using the several values of weighting factors are discussed. Simulations results of the vehicle movement in environments with obstacles are presented. Joint work with Bogdan Zak.

Graphs with few Hamiltonian Cycles

Jan Goedgebeur

Ghent University

We describe an algorithm for the exhaustive generation of non-isomorphic graphs with a given number $k \geq 0$ of hamiltonian cycles, which is especially efficient for small k . Our main findings, combining applications of this algorithm and existing algorithms with new theoretical results, revolve around graphs containing exactly one hamiltonian cycle – i.e. uniquely hamiltonian (UH) graphs – or exactly three hamiltonian cycles. Motivated by a classic result of Smith and recent work of Royle, we show that there exist nearly cubic UH graphs of order n iff $n \geq 18$ is even. This gives the strongest form of a theorem of Entringer and Swart, and sheds light on a question of Fleischner originally settled by Seamone.

We prove equivalent formulations of the conjecture of Bondy and Jackson that every planar UH graph contains two vertices of degree 2, verify it up to order 16, and show that its toric analogue does not hold.

We also treat Thomassen's conjecture that every hamiltonian graph of minimum degree at least 3 contains an edge such that both its removal and its contraction yield hamiltonian graphs. Furthermore, we verify up to order 21 the conjecture of Sheehan that there is no 4-regular UH graph. Finally, we answer a question of Chia and Thomassen on the number of longest cycles in cyclically 4-edge-connected planar cubic graphs.

This is joint work with Barbara Meersman and Carol T. Zamfirescu. Preprint available at <https://arxiv.org/abs/1812.05650>.

Wreath product of permutation groups as the automorphism group of a graph.

Mariusz Grech

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It is known that the automorphism group of a composition of graphs (digraphs, edge-colored graphs and digraphs) $G[H]$ contains the imprimitive wreath product $Aut(H) \wr Aut(G)$. Sabidussi [5] (Hemminger [4], Hahn [3], and latter Dobson and Morris [1]) described the conditions under which the equality holds.

On the other hand Grech, Jeż and Kisielewicz [2] showed that if $A = Aut(G_1)$ and $B = Aut(G_2)$, for any finite graphs (digraphs, edge-colored graphs and digraphs) G_1, G_2 , then there is a finite graph (digraph, edge-colored graph or digraph) G such that $Aut(G) = A \wr B$.

In fact the condition that $A = Aut(G_1)$ and $B = Aut(G_2)$ for some G_1, G_2 is sufficient, but not necessary for existence of such a graph. We establish the exact conditions under which the permutation group $A \wr B$ is the automorphism group of an edge-colored graph (digraph).

Some similar result are obtained for the product action of the wreath product of permutation groups $A \wr B$.

Based on joint work with A. Kisielewicz.

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***s*-Elusive Codes in Hamming Graphs**

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A *code* is a subset of the vertex set of a *Hamming graph*. The *set of s -neighbours* of a code is the set of all vertices at Hamming distance s from their nearest codeword. A code C is *s -elusive* if there exists a distinct code C' that is equivalent to C under the full automorphism group of the Hamming graph such that C and C' have the same set of s -neighbours.

I will present: an infinite family of 1-elusive and completely transitive codes, an infinite family of 2-elusive codes, a single example of a 3-elusive code, as well as several results on the parameters of s -elusive codes.

Tight Localisations of Minimal Feedback Sets in Cubic Time

Michael Hecht

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Based on the theoretical results of our previous work we present an $\mathcal{O}(|E|^3), \mathcal{O}(|V|^3)$ -heuristic of the classical NP-complete feedback arc set/feedback vertex set problem. That is, to identify a set of arcs/vertices $\varepsilon \subseteq E, \nu \subseteq V$ of a (weighted, directed) graph $G = (V, E, \omega)$ such that the deletion of ε, ν results in acyclic graphs $G \setminus \varepsilon, G \setminus \nu$ while the weighted sums $\sum_{e \in \varepsilon} \omega(e), \sum_{v \in \nu} \omega(v)$ are minimized, respectively. Though of cubic runtime complexity our implementation of the algorithm performs very efficiently in practice and yields very close solutions for several relevant instance classes as *planar graphs, sparse graphs, tournaments* and *random graphs*. Furthermore, the algorithm detects whether a given graph instance G belongs to the class of *resolvable graphs*, in which case the proposed solution is proven to be optimal. In fact, the class of *resolvable graphs* has a non-empty intersection with all previously mentioned graph classes, asserting the high accuracy of the algorithm. Numerical experiments and benchmarks comparisons with state of the art alternatives validate the high-quality performance of the algorithm.

Joint work with Krzysztof Gonczar and Szabolcs Horvát.

Highly vertex-connected orientations of regular eulerian graphs

Florian Hoersch

Grenoble INP

We are interested in characterizing graphs admitting highly connected orientations. In 1939, Robbins proved that all 2-edge-connected graphs admit a strongly connected orientation. Nash-Williams generalized this by showing that, for any positive integer k , all $2k$ -edge-connected graphs admit a k -edge-connected orientation. In general, characterizations of graphs admitting highly vertex-connected orientations turn out to be more complex. Thomassen provided a good characterization of graphs admitting a 2-vertex-connected orientation. On the other hand, Durand de Gevigney showed that deciding whether a graph admits a 3-vertex-connected orientation is NP-complete. For eulerian $2k$ -edge-connected graphs, in fact every eulerian orientation is k -arc-connected. Our aim is to find a similar result for vertex-connectivity. As this fails in general, we therefore focus on regular eulerian graphs and try to find classes for which the statement holds. In other words, we try to find families of $2k$ -regular graphs such that each of their eulerian orientations is k -vertex-connected for some positive integer k . Levit,

Chandran and Cheriyan proved the family of even hypercubes to be such a family. We detect two more such families: the line graphs of complete graphs and the line graphs of regular complete bipartite graphs. This answers a question raised by Joseph Cheriyan. This is joint work with Zoltán Szigeti.

Exact random sampling of connected graphs with a given degree sequence

Szabolcs Horvát

Center for Systems Biology Dresden

Sampling random graphs with various constraints is an essential tool for network analysis and modelling, but it is also challenging, each specific constraint requiring an individual approach. Here we consider random sampling from the set of connected realizations of a degree sequence. Most current methods for sampling with constrained degrees fall into two categories: rejection-based and Markov-chain Monte Carlo samplers, each with significant limitations and drawbacks. Recently, a new class of methods was proposed that can sample graphs with exact probabilities and generate the sampling weight together with each sample. However, the additional requirement of connectedness is important for many practical applications, such as computing certain network measures or simulating processes on the network. Unfortunately, imposing this requirement makes the classical approaches untenable. To solve this problem, we generalize and extend the newer class of sampling methods to effectively include the constraint of connectedness while maintaining its polynomial time efficiency. Further, we implement our new algorithm and demonstrate it on the sampling of connected networks with a power-law degree distribution.

This work was done jointly with Carl Modes.

Complexity questions for minimally t -tough graphs

Gyula Y. Katona

Budapest University of Technology and Economics

A graph G is minimally t -tough if the toughness of G is t and the deletion of any edge from G decreases the toughness. Kriesell conjectured that for every minimally 1-tough graph the minimum degree $\delta(G) = 2$. In the present talk we investigate different complexity questions related to this conjecture.

First we show that recognizing minimally t -tough graphs is a hard task for all t values. It is a DP-complete problem (implying that is probably even harder than being NP-hard). Does this change if the question is asked for some special graph classes like chordal, split, claw-free and $2K_2$ -free graphs and for special t values? The answers vary. In some cases there are no such graphs at all, so it is really easy to recognize them. In some other cases, we can characterize all the graphs. Yet in some particular case we can at least recognize it in polynomial time. Many open questions remain.

Based on joint work with Kitti Varga, István Kovács, Dániel Soltész.

An optimal algorithm for stopping on the element closest to the center of an interval

Grzegorz Kubicki

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Real numbers from the interval $[0, 1]$ are randomly selected with uniform distribution. There are n of them and they are revealed one by one. However, we do not know their values but only their relative ranks. We want to stop on recently revealed number maximizing the probability that that number is closest to $\frac{1}{2}$. We design an optimal stopping algorithm achieving our goal and prove that its probability of success is asymptotically equivalent to $\frac{1}{\sqrt{n}}\sqrt{\frac{2}{\pi}}$.

This is a report on joint work done with Ewa Kubicka, Małgorzata Kuchta, and Małgorzata Sulkowska.

Tree structured z-knotted triangulations of a sphere.

Mariusz Kwiatkowski

University of Warmia and Mazury

We investigate zigzags in triangulations of connected closed 2-dimensional surfaces and show that there is a one-to-one correspondence between triangulations with homogeneous zigzags and closed 2-cell embeddings of directed Eulerian graphs in surfaces.

A triangulation is called z-knotted if it has a single zigzag. A bipyramid with odd base is an example of a z-knotted triangulation of a sphere. Using bipyramids we construct a class of z-knotted triangulations of a sphere whose zigzags are homogeneous. These triangulations are tree structured in the sense that they can be described using rooted labeled trees.

If we allow triangulations with double edges we can construct a family of z-knotted maps for every surface of even Euler characteristic (not necessarily orientable)

This is joint work with Mark Pankov and Adam Tyc. Preprint is available at <https://arxiv.org/abs/1902.10788>.

Non bipartite regular 2-factor isomorphic graphs: an update

Domenico Labbate

Università degli Studi della Basilicata

A 2-factor of a graph G is a 2-regular spanning subgraph of G . A graph with a 2-factor is said to be 2-factor hamiltonian if all its 2-factors are hamiltonian cycles, and, more generally, 2-factor isomorphic if all its 2-factors are isomorphic. Examples of such graphs are K_4 , K_5 , $K_{3;3}$, the Heawood graph (which are all 2-factor hamiltonian) and the Petersen graph (which is 2-factor isomorphic). Several papers have addressed the problem of characterizing families of graphs (particularly regular graphs) which have these properties. We give an updated survey on results and open problems on the structure of 2-factors in (non bipartite) regular graphs obtained in the last few years by the author jointly with several other colleagues.

Joint papers with M. Abreu, R. Aldred, M. Funk, B. Jackson, J. Sheehan.

On the cop-number of toroidal graphs

Florian Lehner

TU Graz

Cops and Robber is a pursuit-evasion game played on a graph between two players which can be described as follows. Initially, the first player, places k cops on vertices of a graph G , then the second player places a robber on a vertex. Then the two players take turns. On the first player's turn each cop can either be moved to an adjacent vertex or left at the current position, on the second player's turn the robber can either be moved to an adjacent vertex or left where he is. The first player wins the game if at some point one of the cops is at the same vertex as the robber.

A central question related to this game is, whether for a given graph G the first player has a winning strategy using k cops. The smallest k for which this is the case is called the cop-number of G . This notion was introduced in 1984 by Aigner and Fromme, who showed that the cop-number of planar graphs is at most 3. In 1986, Andreae showed that for any fixed graph H , there is a constant upper bound (depending on H) for the cop-number of graphs with no H -minor and asked for the best possible bound for the cop-number of toroidal graphs.

We show that the cop-number of toroidal graphs is at most 3, thus answering Andreae's question and verifying a conjecture by Schroeder from 2001.

Ends of graphs and the language of self-avoiding walks

Christian Lindorfer

Graz University of Technology

Let $X = (V, E)$ be an infinite, locally finite, connected graph without loops or multiple edges. We consider the edges to be oriented, and E is equipped with an involution which inverts the orientation. Each oriented edge is labelled by an element of a finite alphabet Σ . The labelling is assumed to be deterministic: edges with the same initial (resp. terminal) vertex have distinct labels. Furthermore it is assumed that the group of label-preserving automorphisms of X acts quasi-transitively.

For any vertex o of X , consider the language of all words over Σ which can be read along self-avoiding walks starting at o . We characterize under which conditions on the graph structure this language is regular or context-free. This is the case if and only if the graph has more than one end, and the size of all ends is 1, or at most 2, respectively.

This is a joint work with Wolfgang Woess and a preprint is available at <https://arxiv.org/abs/1903.02368>.

Acyclic coloring of graphs with prescribed maximum average degree

Mária Maceková

P.J. Šafárik University in Košice

An acyclic edge coloring of a graph is a proper edge coloring without two-colored cycles. The acyclic chromatic index of a graph G , denoted by $\chi'_a(G)$, is the smallest possible number of colors in an acyclic edge coloring of G .

Alon, Sudakov, Zaks, and independently Fiamčík, conjectured that $\chi'_a(G) \leq \Delta(G) + 2$ for any graph G . The conjecture was confirmed for several classes of sparse graphs, defined by

conditions on maximum (average) degree and/or girth, as well for some classes of planar graphs. For some more particular graph classes it was even proved that they are acyclically $(\Delta + 1)$ - or Δ -edge-colorable, respectively.

We prove that graphs with $\Delta(G) \geq 14$ and $\text{mad}(G) < 4$ are acyclically $(\Delta + 1)$ -edge-colorable. Based on a joint work with František Kardoš and Roman Soták.

Graphs preserving total distance upon vertex removal

Snježana Majstorović

University of Osijek, Croatia

We consider the problem of determining graphs for which its total distance (Wiener index) remains unchanged after deletion of a vertex. This problem is related to Šoltés's problem in which this condition must hold for all its vertices. We show that there are infinitely many graphs with this property, even if we consider the removal of more than one vertex. Specially, we focus on unicyclic graphs and take them to be the basis for building general classes of such graphs. Additionally, we present some results concerning Cartesian product of graphs. (Joint work with Riste škrekovski, Martin Knor and Justin Schroeder)

Some results regarding upper and lower bounds on the circular flow number of snarks

Davide Mattiolo

University of Modena and Reggio Emilia

Given a real number $r \geq 2$, a *circular nowhere-zero r -flow*, or r -CNZF, in a graph $G = (V, E)$ is a function $f: E \rightarrow [1, r - 1]$ together with an orientation D of G , such that, at every $x \in V$, the sum of all incoming flow values equals the sum of all outgoing ones in the chosen orientation D . The *circular flow number* $\Phi_c(G)$ of G is the least r such that G admits an r -CNZF. One of the most important conjectures in the theory of flows in graphs is for sure the well known Tutte's 5-flow Conjecture claiming that every bridgeless graph admits a nowhere-zero 5-flow. The study of such a conjecture can be reduced to snarks. In the first part of this talk we present an infinite family of snarks whose circular flow numbers meet a known lower bound. Then we improve the best known upper bound for $\Phi_c(G_{2t+1})$, where G_{2t+1} is the Goldberg snark on $8(2t + 1)$ vertices.

Joint work with Jan Goedgebeur (Ghent University) and Giuseppe Mazzuoccolo (University of Verona)

Reduction of the Berge-Fulkerson Conjecture to cyclically 5-edge-connected snarks

Giuseppe Mazzuoccolo

University of Verona, Italy

The Berge-Fulkerson Conjecture belongs to one of the most prominent open problems in Graph theory. It suggests that the edges of any bridgeless cubic graph can be covered with six perfect matchings in such a way that each edge belongs to exactly two of them. Despite the fact that Berge and Fulkerson made this conjecture almost half a century ago, it has been verified

only for several explicitly defined families of graphs. Moreover, in contrast to what happens for other outstanding conjectures (Cycle Double Cover Conjecture, Tutte 5-flow Conjecture...), no additional restriction, except the trivial ones, is proved for a minimum possible counterexample. In the present talk, we prove that Berge-Fulkerson Conjecture can be reduced to cyclically 5-edge-connected cubic graphs.

Joint work with Edita Macajova (Comenius University)

Colored even cycle decompositions

Maria Chiara Molinari

University of Modena and Reggio Emilia

A *even cycle decomposition* of a simple graph G , briefly ECD, is a partition of the edge-set of G into even cycles. A necessary condition for the existence of an ECD of a graph G is that every vertex has even degree and every block of G has an even number of edges. For planar graphs the necessary condition is also sufficient [3]. For a non-planar graph G , it is known that, if G satisfies the necessary condition and has no K_5 -minor, then G possesses an ECD [4].

In a graph G having an ECD, we can color the cycles in such a way that cycles sharing a vertex receive distinct colors. If m is the minimum number of colors that are required in such a coloring, then we say that the ECD is an *even cycle decomposition of index m* . Each colored class is an even 2-regular subgraph of G , that is, a 2-regular subgraph whose connected components are cycle of even length.

An even cycle decomposition of G gives rise to a partition of $E(G)$ in m even 2-regular subgraphs. We are interested in determining the minimum number m of even 2-regular subgraphs that partition the edge-set of a graph G . The problem is suggested by a result on the palette index of a simple graph [1,2].

Joint work with Simona Bonvicini.

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Vertex-face structures in quadrangulations on surfaces

Atsuhiko Nakamoto

Yokohama National University

It is well-known that every 4-connected graphs on the sphere and the projective plane is Hamiltonian, and that the toroidal case has been conjectured by Nash-Williams but it is still open. Because of the toughness condition, this does not extend to any surface with negative Euler characteristic, and a counterexample can be constructed by a vertex-face structure in a quadrangulation on those surfaces. In our talk, we investigate the vertex-face structures in quadrangulations on surfaces.

Various generalizations of Bell numbers

Gábor Nyul

University of Debrecen

Bell numbers are fundamental objects in enumerative combinatorics, they count the number of partitions of finite sets. Various ways can be chosen to define variants of Bell numbers, we discuss three of them:

- We can require that certain pairs of elements have to belong to distinct blocks. For instance, if we have r distinguished elements, no two of which are allowed to share their blocks, then we obtain r -Bell numbers.
- It is possible to prescribe restrictions on the cardinality of the blocks. For example, if they are bounded from below, we arrive at associated Bell numbers.
- With a lattice theoretical background, one can introduce Dowling numbers related to Dowling lattices over finite groups. They can be described in a purely combinatorial manner by an interpretation using coloured partitions.

In our talk, we give an overview of these generalizations, then we present our recent results on combinations of the above directions.

(This is a joint work with Eszter Gyimesi.)

Decompositions into isomorphic rainbow spanning trees

Deryk Osthus

University of Birmingham

A subgraph of an edge-coloured graph is called rainbow if all its edges have distinct colours. Our main result implies that, given any optimal colouring of a sufficiently large complete graph K_{2n} , there exists a decomposition of K_{2n} into isomorphic rainbow spanning trees. This settles conjectures of Brualdi–Hollingsworth (from 1996) and Constantine (from 2002) for large graphs.

This is joint work with Stefan Glock, Daniela Kühn and Richard Montgomery.

Preprint available at [arXiv:1903.04262](https://arxiv.org/abs/1903.04262)

Discrete tomography: a graph-theoretical formulation of local uniqueness for two directions

Silvia Pagani

Università Cattolica del Sacro Cuore, Brescia

Discrete tomography is a branch of the inverse problems dealing with the reconstruction (i.e., the recovery) of the internal of an object, modeled as a function defined on a grid of pixels, by means of its projections. These are taken by summing the function values on the pixels lying on lines parallel to given directions. In the two-dimensional case, the object is usually called image and the domain is a rectangular subset of \mathbb{Z}^2 .

One of the main instances of discrete tomography is to achieve uniqueness of reconstruction. Because of the presence of the switching components, ensuring that the obtained solution equals the original image is not possible in general. In this talk, instead of considering the uniqueness

problem for the whole image, the focus is on a *local* notion of uniqueness, which considers the pixels whose function value is uniquely determined, and which is defined in two cases: structural and data-dependent. When projections are taken along two lattice directions, both kinds of local uniqueness have been characterized in a graph-theoretical approach. In particular, the well-studied region of uniqueness (ROU), which is a proper subset of the structural uniqueness, has a characterization in its bipartite graph counterpart.

AG codes from the second generalization of the GK maximal curve

Vincenzo Pallozzi Lavorante
Università di Modena e Reggio Emilia

The second generalized GK maximal curves $GK_{2,n}$ are maximal curves over finite fields with q^{2n} elements, where q is a prime power and $n \geq 3$ an odd integer. In this paper we determine the structure of the Weierstrass semigroup $H(P)$ where P is an arbitrary \mathbb{F}_{q^2} -rational point of $GK_{2,n}$. We show that these points are Weierstrass points and the Frobenius dimension of $GK_{2,n}$ is computed. A new proof of the fact that the first and the second generalized GK curves are not isomorphic for any $n \geq 5$ is obtained. AG codes and AG quantum codes from the curve $GK_{2,n}$ are constructed; in some cases, they have better parameters with respect to those already known.

Face z -monodromies in triangulations of surfaces

Mark Pankov
University of Warmia and Mazury

A *zigzag* in a map (a 2-cell embedding of a connected graph in a connected closed 2-dimensional surface) is a cyclic sequence of edges satisfying the following conditions: 1) any two consecutive edges lie on the same face and have a common vertex, 2) for any three consecutive edges the first and the third edges are disjoint and the face containing the first and the second edges is distinct from the face which contains the second and the third.

Using zigzags, for every face F we define the z -monodromy M_F which acts on the oriented edges of this face. If e is such an edge, then there is a unique zigzag coming out from F through e and we define $M_F(e)$ as the first oriented edge of F which occurs in this zigzag after e .

We investigate face z -monodromies in triangulations of (not necessarily orientable) closed surfaces. We show that there are precisely seven types of such z -monodromies and each of these types is realized.

We discuss the following problem: how many z -monodromies are of the same type?

There are four types of z -monodromies such that for each of these types there is a triangulation, where all z -monodromies are of this type. Next, we distinguish two types M_1 and M_2 with the following property: for every $i = 1, 2$ the subgraph of the dual graph formed by faces whose z -monodromies are of type M_i is a forest.

The remaining type is exceptional and the problem is open.

Based on joint work with Adam Tyc.

Preprint available at <https://arxiv.org/abs/1801.06585>.

On the general position problem on Kneser graphs

Balázs Patkós

Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences

In a graph G , a *geodesic* between two vertices x and y is a shortest path connecting x to y . A subset S of the vertices of G is *in general position* if no vertex of S lies on any geodesic between two other vertices of S . The size of a largest set of vertices in general position is the *general position number* that we denote by $gp(G)$. Recently, Ghorbani et al, proved that for any k if $n \geq k^3 - k^2 + 2k - 2$, then $gp(Kn_{n,k}) = \binom{n-1}{k-1}$, where $Kn_{n,k}$ denotes the Kneser graph. We improve on their result and show that the same conclusion holds for $n \geq 2k + 2$ and this bound is best possible. Our main tools are a result on cross-intersecting families and a slight generalization of Bollobás's inequality on intersecting set pair systems.

Erdős-Gallai type results on weighted degree sequences of graphs

Sofia J. Pinheiro

University of Aveiro

Extremal graph theory studies properties of graphs and their dependence on the values of some of its parameters. The Turán's theorem, which is associated to the very beginning of extremal graph theory, provides an upper bound on the size of graphs without q -cliques, the Moore bound on the order of graphs with a particular diameter and average degree, and the Erdős-Gallai theorem that gives a necessary and sufficient condition for a sequence of nonnegative integers be graphic are examples on extremal problems. Considering this results, bounds on the order and average degree of subgraphs of a given graph are established. Moreover, we extend the Erdős-Gallai theorem to the case of graphs with positive edge weights.

Keywords: Graph theory, extremal graph theory, graphical sequences.

A joint work with Domingos M. Cardoso.

Cliques, Logic and Games

Miguel Pizaña

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The *clique graph*, $K(G)$, of a graph G is the intersection graph of its (maximal) cliques. The *iterated clique graphs* of G are then defined by: $K^0(G) = G$ and $K^n(G) = K(K^{n-1}(G))$. We say that G is *clique-divergent* if the set of orders of its iterated clique graphs, $\{|K^n(G)| : n \in \mathbb{N}\}$ is unbounded. Clique graphs and iterated clique graphs have been studied extensively, but no characterization for clique-divergence has been found so far.

The class of (not necessarily finite) *automatic graphs* is the class of graphs that can be defined by finite automata: the vertices are the strings recognized by some finite automaton and the edges are the pairs of strings recognized by some (dual-input & synchronous) finite automaton.

Recently, it was proved that the clique-divergence is undecidable for the class of *automatic graphs*. This implies that clique-divergence is not first-order expressible for the same class.

Here we strengthened the latter result by proving that the clique-divergence property is not first-order expressible even for the class of finite graphs. The proof is based on the Ehrenfeucht-Fraïssé games. Logic expressibility has strong relations with complexity theory

and consequently, new avenues of research are opened for clique graph theory.

This is a joint work with Carmen Cedillo (Centro Universitario UAEM Nezahualcóyotl & Universidad Autónoma Metropolitana, MEXICO).

Generalized Lah numbers and their graph theoretical interpretation

Gabriella Rácz

University of Debrecen

In the middle 1950s, the Slovenian mathematician Ivo Lah introduced the numbers which were named after him. The Lah number $\left[\begin{smallmatrix} n \\ k \end{smallmatrix} \right]$ counts the number of partitions of a set with n elements into k ordered subsets. The r -Lah numbers are generalizations of these numbers, $\left[\begin{smallmatrix} n \\ k \end{smallmatrix} \right]_r$ counts the number of partitions of a set with $n+r$ elements into $k+r$ ordered subsets such that r distinguished elements belong to distinct ordered blocks. In the first part of our talk, we give a brief overview of the properties of these numbers.

In the second part, we present our main result, a new graph theoretical interpretation of r -Lah numbers in connection with matchings in complete bipartite graphs. We show several proofs of this observation, some of which use the previously mentioned properties of r -Lah numbers, but we also present a direct bijective proof. We note that this interpretation also works if the parameter r is a half-integer.

Finally, we introduce the r -Lah polynomials and study their properties. Among others, we prove that their roots are real and non-positive numbers. We derive bounds for the roots, compute the real magnitude of the smallest roots and study their asymptotic behaviour.

This is a joint work with Gábor Nyul.

Core partitions with d -distinct parts

Murat Sahin

Ankara University

In this talk, we define $(s, s+1)$ -core partitions with d -distinct parts. We present some results on the number and the largest size of such partitions, so we extend Xiong's paper in which the results are obtained about $(s, s+1)$ -core partitions with distinct parts.

Vertex connectivity of the power graph of a finite cyclic group

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NISER, Bhubaneswar, India

The *power graph* $\mathcal{P}(G)$ of a given finite group G is the simple undirected graph whose vertices are the elements of G , in which two distinct vertices are adjacent if and only if one of them can be obtained as an integral power of the other. The *vertex connectivity* $\kappa(\mathcal{P}(G))$ of $\mathcal{P}(G)$ is the minimum number of vertices which need to be removed from G so that the induced subgraph of $\mathcal{P}(G)$ on the remaining vertices is disconnected or has only one vertex.

For a positive integer n , let C_n denote the cyclic group of order n . Suppose that the prime power decomposition of n is given by $n = p_1^{n_1} p_2^{n_2} \cdots p_r^{n_r}$, where r, n_1, n_2, \dots, n_r are positive integers and p_1, p_2, \dots, p_r are prime numbers with $p_1 < p_2 < \cdots < p_r$. It is known that $\kappa(\mathcal{P}(C_n)) = p_1^{n_1} - 1$ if $r = 1$. For $r \geq 2$, we determine the exact value of $\kappa(\mathcal{P}(C_n))$ when $2\phi(p_1 p_2 \cdots p_{r-1}) \geq p_1 p_2 \cdots p_{r-1}$,

and give an upper bound for $\kappa(\mathcal{P}(C_n))$ when $2\phi(p_1 p_2 \cdots p_{r-1}) < p_1 p_2 \cdots p_{r-1}$, which is sharp for many values of n but equality need not hold always. As a consequence, we determine $\kappa(\mathcal{P}(C_n))$ for $r \leq 3$. For $r \geq 4$, we give a new upper bound for $\kappa(\mathcal{P}(C_n))$ and determine $\kappa(\mathcal{P}(C_n))$ when $n_r \geq 2$. We also determine $\kappa(\mathcal{P}(C_n))$ when n is a product of distinct prime numbers.

(Joint work with S. Chattopadhyay and K. L. Patra)

Preprint available at <https://arxiv.org/abs/1703.07149> & <https://arxiv.org/abs/1810.11316>.

Partially Broken Orientations of Eulerian Plane Graphs

Yusuke Suzuki

Niigata University, Japan

It is well-known that any Eulerian plane graph G is face 2-colorable and admits an orientation, which is an assignment of a direction to each edge of G , such that incoming edges and outgoing edges appear alternately around any $v \in V(G)$; we say that such an orientation is *good*. In this talk, we discuss whether a given Eulerian 1-plane graph G , or more generally, a properly drawn Eulerian graph on the plane has a good orientation or not, and give a characterization in terms of the *radial graph* of the planarization of G . (Joint work with Gen Kawatani.)

Z-knotted triangulations of surfaces

Adam Tyc

Institute of Mathematics, Polish Academy of Science

A zigzag in a map (a 2-cell embedding of a connected graph in a connected closed 2-dimensional surface) is an analogue of a Petrie polygon in a polytope. This is a cyclic sequence of edges satisfying the following conditions: 1) any two consecutive edges lie on the same face and have a common vertex, 2) for any three consecutive edges the first and the third edges are disjoint and the face containing the first and the second edges is distinct from the face which contains the second and the third. In contrast to Petrie polygons, zigzags admit self-intersections. A map is called *z-knotted* if it contains a single zigzag. *Z-knotted* maps have nice homological properties and closely connected to realizations of Gauss codes. We show that every triangulation of an arbitrary (not necessarily orientable) closed surface has a *z-knotted* shredding.

Joint work with Mark Pankov.

Preprint available at <https://arxiv.org/abs/1708.04296>.

Congruences and subdirect representations of graphs

Stefan Veldsman

Nelson Mandela University, South Africa

As in universal algebra, a congruence can be defined on a graph. This gives rise to the analogues of the algebraic isomorphism theorems for graphs. Subdirect products of graphs can then be characterized in terms of congruences. Subdirectly irreducible graphs are defined and explicitly determined leading to a graph theoretic version of Birkhoff's Theorem (every nontrivial graph is a subdirect product of subdirectly irreducible graphs).

Packing colorings of Sierpiński-type and H graphs

Aleksander Vesel

University of Maribor

For a nondecreasing sequence of integers $S = (s_1, s_2, \dots)$ an S -packing k -coloring of a graph G is a mapping from $V(G)$ to $\{1, 2, \dots, k\}$ such that vertices with color i have pairwise distance greater than s_i . By setting $s_i = d + \lfloor \frac{i-1}{n} \rfloor$ we obtain a (d, n) -packing coloring of a graph G . The smallest integer k for which there exists a (d, n) -packing coloring of G is called the (d, n) -packing chromatic number of G . In the special case when d and n are both equal to one we speak of the packing chromatic number of G . The packing chromatic number of the base-3 Sierpiński graphs S_k is determined and some results on (d, n) -packing colorings, $d \leq 2$, are provided for this class of graphs. The packing chromatic colorings of some general Sierpiński graphs and H graphs are also considered.

Based on joint work with Fei Deng, Zehui Shao and Danilo Korže.

On the Turán number of edge ordered graphs

Mate Vizer

MTA Alfréd Rényi Institute of Mathematics

In this talk we introduce Turán and Ramsey problems for edge ordered graphs. We call a simple graph *edge ordered*, if its edges are linearly ordered. An isomorphism between edge ordered graphs must respect the edge order. A subgraph of an edge ordered graph is itself an edge ordered graph with the induced edge order. We say that an edge ordered graph G *avoids* another edge ordered graph H if no subgraph of G is isomorphic to H . The Turán number $ex(n, \mathcal{H})$ of a family \mathcal{H} of edge ordered graphs is the maximum number of edges in an edge ordered graph on n vertices that avoids all elements of \mathcal{H} . We examine this parameter in general and also for several singleton families of edge orders of certain small specific graphs, like star forests, short paths and the cycle of length four.

In the same talk we also introduce Ramsey numbers of edge ordered graphs and talk about some results.

Joint work with M. Balko, D. Gerbner, A. Methuku, D. Nagy, D. Pálvölgyi, G. Tardos, M. Vizer.

The Turán number of the square of a path

Chuanqi Xiao

Central European University

The Turán number of a graph H , $ex(n, H)$, is the maximum number of edges in a graph on n vertices which does not have H as a subgraph. Let P_k be the path with k vertices, the square P_k^2 of P_k is obtained by joining the pairs of vertices with distance one or two in P_k . The powerful theorem of Erdős, Stone and Simonovits determines the asymptotic behavior of $ex(n, P_k^2)$. In the present paper, we determine the exact value $ex(n, P_5^2)$ and $ex(n, P_6^2)$ and pose a conjecture for the exact value of $ex(n, P_k^2)$. Joint work with Gyula O.H. Katona, Oscar Zamora Luna and Jimeng Xiao.

Circuit Covers of Signed Graphs

Dong Ye

Middle Tennessee State University

A signed graph is a graph associated with a map from its edge set to -1 or $+1$. A circuit of a signed graph is either a positive cycle or a barbell. A circuit cover of a signed graph is a family of circuits which cover all edges of the signed graph. It is known that a signed graph has a circuit cover if and only if it has a nowhere-zero integer flow. In this talk, we will present some recent developments in the area of circuit covers of signed graphs. This talk is based on joint work with Yezhou Wu.

Determination of the optimal ship trajectory in a collision situation with the use of directed graphs

Bogdan Żak

Polish Naval Academy

In ship's control systems, it is important to ensure the safety of their movement in collision situations. This task is carried out by ship anti-collision systems defining the trajectory of the ship based on information about the navigational situation in a given water area. The choice of trajectory is made on the basis of a certain function which is assessing the ship's safety. The value of this function depends on the changes of the relative positions of ships in the considered sea area. When we discretize the process, the collision situation can be described using a graphical description of connections between particular states, where the vertices of the graph constitute the relative position of objects, while the directed branches have assigned fuzzy values of the function assessing the ship's safety.

For the problem formulated in this way, a directed acyclic graph G consisting of a set of vertices $V = 1, 2, \dots, N$ and a set of ordered pairs named with directed edges is analyzed. Each edge is described by vertices (i, j) , where $i, j \in V$, for which a given edge is a connection and its weight is given by approximate values. It is assumed that the motion along the graph is possible only in the direction indicated by the edges, therefore the edge (i, j) is the connection from the vertices i to the vertices j . To solve the shortest path problem operators known from the fuzzy sets theory and the Dijkstra algorithm were used.

Hamiltonian cycles containing a prescribed perfect matching

Jean Paul Zerafa

Università di Modena e Reggio Emilia, Italy

A graph has the Perfect-Matching-Hamiltonian-property (for short the *PMH*-property) if every perfect matching can be extended to a hamiltonian cycle. Let G be a hamiltonian graph. It can be easily shown that the line graph of G , denoted by $L(G)$, is hamiltonian as well. We recently showed that if G is cubic and has an even number of edges, then, $L(G)$ also has the *PMH*-property. In this talk a brief overview of results and possible future directions will be discussed.

Joint work with Marién Abreu (University of Basilicata), John Baptist Gauci (University of

Malta), Domenico Labbate (University of Basilicata) and Giuseppe Mazzuocolo (University of Verona).

Invited Special Session

GRAPH COLORING

Organized by INGO SCHIERMEYER

Majority coloring of infinite digraphs

Marcin Anholcer

Poznań University of Economics and Business

Let D be a finite or infinite digraph. A *majority coloring* of D is a vertex coloring such that at least half of the out-neighbors of every vertex v have different color than v . Let $\mu(D)$ denote the least number of colors needed for a majority coloring of D . It is known that $\mu(D) \leq 4$ for any finite digraph D , and $\mu(D) \leq 2$ if D is acyclic. We prove that $\mu(D) \leq 5$ for any countably infinite digraph D , and $\mu(D) \leq 3$ if D does not contain finite directed cycles. We also state a twin supposition to the famous Unfriendly Partition Conjecture. This is joint work with Bartłomiej Bosek and Jarosław Grytczuk.

Decomposition of cubic graphs related to Wegner's conjecture

János Barát

MTA-ELTE Geometric and Algebraic Combinatorics Research Group

Wegner initiated the study of the square chromatic number of planar graphs in the 70's. We recall the case $\Delta \leq 3$ of his conjecture:

For any subcubic planar graph G , the square of G is 7-colorable.

Thomassen formulated the following attractive conjecture, which would imply Wegner's:

Every 3-connected cubic graph has a red-blue vertex coloring such that the blue subgraph has maximum degree 1 (that is, it consists of a matching and some isolated vertices) and the red subgraph has minimum degree at least 1 and contains no 3-edge path.

In Wegner's conjecture the Petersen graph is extremal. Therefore, we first prove the existence of the above red-blue colorings for generalised Petersen graphs.

We conjecture that all but a few very small subcubic graphs admit such a red-blue coloring. We confirm this statement for all subcubic trees.

If time permits, we mention various related problems in similar fashion.

Consecutive colouring of oriented graphs

Ewa Drgas-Burchardt

University of Zielona Góra, Poland

We consider arc colourings of oriented graphs in which the colours of all out-arcs incident with each vertex and the colours of all in-arcs incident with each vertex form intervals. We prove that the existence of such a colouring is NP-complete problem. We give the solution of the problem for circulant or r -regular oriented graphs, transitive tournaments, oriented graphs with small maximum degree, small order and some others. We state the conjecture that for each graph there exists its consecutive colourable orientation and confirm the conjecture for complete graphs, 2-degenerate graphs, planar graphs with girth at least 8, bipartite graphs with arboricity less than three including all planar bipartite graphs. Additionally, we prove that the conjecture is true for all perfect consecutively colourable graphs and for all forbidden graphs for the class of perfect consecutively colourable graphs.

Joint work with Marta Borowiecka-Olszewska, Nahid Yelene Javier Nol and Rita Esther Zuazua Vega

Graphs that are critical for the packing chromatic number

Jasmina Ferme

University of Maribor

Given a graph G and a positive integer i , an i -packing in G is a subset W of the vertex set of G such that the distance between any two distinct vertices from W is greater than i . The smallest integer k such that the vertex set of G can be partitioned into sets V_i , $i \in \{1, \dots, k\}$, where V_i is an i -packing, is called the *packing chromatic number* of a graph G and is denoted by $\chi_\rho(G)$. Recently, Klavžar and Rall studied packing chromatic vertex-critical graphs, i.e. the graphs G which satisfy the property that $\chi_\rho(G - v) < \chi_\rho(G)$ for every vertex $v \in V(G)$. In this talk, we consider another version of packing chromatic critical graphs, namely the graphs G with the property that $\chi_\rho(H) < \chi_\rho(G)$ for every proper subgraph H of a graph G . We characterize χ_ρ -critical graphs with small packing chromatic numbers, χ_ρ -critical graphs with diameter 2 and χ_ρ -critical block graphs with diameter 3. Additionally, we present some general properties of packing chromatic critical graphs, consider χ_ρ -critical trees and bound $\chi_\rho(G - e)$, where G is an arbitrary graph with an edge $e \in E(G)$.

Joint work with Boštjan Brešar.

Preprint available at <https://arxiv.org/abs/1904.10212>.

Total coloring of graphs with minimum sum of colors; existence of T-strong graphs and trees.

Ewa Kubicka

University of Louisville

The total chromatic sum of a graph is the minimum sum of colors (natural numbers) taken over all proper colorings of vertices and edges of a graph. We provide infinite families of graphs for which the minimum number of colors to achieve the total chromatic sum is larger than the total chromatic number; we call them T-strong graphs. We resolve an old conjecture about the existence of T-strong trees providing examples of such trees of maximum degree 3 and 4. This is a joint work with G. Kubicki, M. Małafiejski, and K. Ocetkiewicz.

Monochromatic disconnection of graphs

Xueliang Li

Nankai University, Tianjin, China

The new concept of monochromatic disconnection of graphs, recently introduced by us, is actually motivated from the concepts of rainbow disconnection and monochromatic connection of graphs. For an edge-colored connected graph G , we call an edge-cut M of G monochromatic if the edges of M are colored with a same color. The graph G is called monochromatically disconnected if any two distinct vertices of G are separated by a monochromatic edge-cut. For a connected graph G , the monochromatic disconnection number of G , denoted by $md(G)$, is the maximum number of colors that are allowed to make G monochromatically disconnected. In this talk, we will survey the main results along with the subject, and some unsolved problems are also presented.

This is a joint work with my student Ping Li.

Homogeneous Coloring of Graphs

Borut Lužar

Faculty of Information Studies, Slovenia

A k -homogeneous coloring of a graph G is a proper vertex-coloring such that the number of colors in the neighborhood of every vertex is equal to k . We present some general results and then focus to homogeneous coloring of regular and regular bipartite graphs. In particular, we are interested in the questions which d -regular bipartite graphs admit a k -homogeneous coloring for every k , $1 \leq k \leq d$. This question is strongly related to several variations of coloring uniform hypergraphs.

Based on joint work with T. Madaras, R. Soták, and M. Šurimová. Article available at <https://amc-journal.eu/index.php/amc/article/view/1083>.

The 1–2–3 Conjecture almost holds for regular graphs

Jakub Przybyło

AGH University of Science and Technology

The well-known 1–2–3 Conjecture asserts that the edges of every graph without isolated edges can be weighted with 1, 2 and 3 so that adjacent vertices receive distinct weighted degrees. This is open in general, while it is known to be possible from the weight set $\{1, 2, 3, 4, 5\}$. The conjecture also holds for 3-colourable graphs. Not much more is known in the case of regular graphs. We shall present a proof that for such a family of graphs it is sufficient to use weights 1, 2, 3, 4.

Packing chromatic vertex-critical graphs

Douglas Rall

Furman University

The *packing chromatic number* $\chi_\rho(G)$ of a graph G is the smallest integer k such that the vertex set of G can be partitioned into sets V_1, \dots, V_k , where vertices in V_i are pairwise at distance at least $i + 1$. *Packing chromatic vertex-critical* graphs, χ_ρ -critical for short, are introduced as the graphs G for which $\chi_\rho(G - x) < \chi_\rho(G)$ holds for every vertex x of G . If $\chi_\rho(G) = k$, then G is k - χ_ρ -critical. It is shown that if G is χ_ρ -critical, then the set $\{\chi_\rho(G) - \chi_\rho(G - x) : x \in V(G)\}$ can be almost arbitrary. The 3- χ_ρ -critical graphs are characterized, and 4- χ_ρ -critical graphs are characterized in the case when they contain an n -cycle, $n \geq 5$ and $n \not\equiv 0 \pmod{4}$. It is shown that for every integer $k \geq 2$ there exists a k - χ_ρ -critical tree and that a k - χ_ρ -critical caterpillar exists if and only if $k \leq 7$.

This is joint work with Sandi Klavžar

Polynomial *chi*-binding functions and forbidden induced subgraphs - a survey

Ingo Schiermeyer

Technische Universität Bergakademie Freiberg, Germany

A graph G with clique number $\omega(G)$ and chromatic number $\chi(G)$ is *perfect* if $\chi(H) = \omega(H)$ for every induced subgraph H of G . A family \mathcal{G} of graphs is called χ -bounded with binding function f if $\chi(G') \leq f(\omega(G'))$ holds whenever $G \in \mathcal{G}$ and G' is an induced subgraph of G . In this talk we will present a survey on polynomial χ -binding functions. Especially we will address perfect graphs, hereditary graphs satisfying the Vizing bound ($\chi \leq \omega + 1$), graphs having linear χ -binding functions and graphs having non-linear polynomial χ -binding functions. Thereby we also survey polynomial χ -binding functions for several graph classes defined in terms of forbidden induced subgraphs, among them $2K_2$ -free graphs, P_k -free graphs, *claw*-free graphs, and *diamond*-free graphs.

Keywords: Chromatic number, χ -binding function, forbidden induced subgraph

Some results and problems on unique-maximum colorings of plane graphs

Riste Škrekovski

University of Ljubljana & Faculty of Information Sciences in Novo mesto

A unique-maximum coloring of a plane graph G is a proper vertex coloring by natural numbers such that each face α of G satisfies the property: the maximal color that appears on α , appears precisely on one vertex of α (or shortly, the maximal color on every face is unique on that face). Fabrici and Göring proved that six colors are enough for any plane graph and conjectured that four colors suffice. Thus, this conjecture is a strengthening of the Four Color Theorem. Wendland later decreased the upper bound from six to five.

We first show that the conjecture holds for various subclasses of planar graphs but then we disprove it for planar graphs in general. Thus, the facial unique-maximum chromatic number of the sphere is not four but five. In the second part of the talk, we will consider various new directions and open problems.

Strong cliques in graphs

Małgorzata Śleszyńska-Nowak

Warsaw University of Technology

Let G be a simple graph. A coloring of the edges of G is *strong* if every path with at most three edges is rainbow. In other words, each color class in a strong coloring forms an induced matching. The least number of colors in a strong coloring of G is called the *strong chromatic index* of G , and is denoted by $\chi'_s(G)$. A well-known conjecture of Erdős and Nešetřil from 1985 states that $\chi'_s(G) \leq 1.25\Delta^2$ for every graph G with maximum degree Δ .

A *strong clique* in a graph G is a subset of edges whose every pair of elements belongs to a path with two or three edges in G . Every usual clique is a strong clique but notice that a complete bipartite graph or a blow-up of 5-cycle are also examples of strong cliques. Let $\omega_s(G)$ denote the maximum size of a strong clique in G . Since in a strong coloring of G every strong

clique must be rainbow, we have that $\omega_s(G) \leq \chi_s(G)$. So, conjectured upper bound for the strong clique number is $\omega_s(G) \leq 1.25\Delta^2$, while currently best result for the strong chromatic index is $\chi'_s(G) \leq 1.835\Delta^2$.

In our first general result we proved that $\omega_s(G) \leq 1.5\Delta^2$ for any graph with maximum degree Δ . This was recently improved to $\omega_s(G) \leq 1.33\Delta^2$. For some restricted classes of graphs better bounds are possible. For instance, it is known that $\omega_s(G) \leq \Delta^2$ for bipartite graphs, and we proved that $\omega_s(G) \leq \Delta^2 + 0.5\Delta$ for claw-free graphs.

Furthermore, we investigate a more general problem concerning *t-strong cliques* in graphs, defined by analogous condition involving paths with up to $t + 1$ edges. We prove that the size of a *t-strong* clique in a graph with maximum degree Δ is at most $1.75\Delta^t + O(\Delta^{t-1})$, and for bipartite graphs the upper bound is at most $\Delta^t + O(\Delta^{t-1})$. Additional results concern $K_{1,r}$ -free graphs and graphs with large girth.

This is joint work with Michał Dębski.

Invited Special Session

METRIC GRAPH THEORY

Organized by ISMAEL G. YERO

Computational complexities of three problems related to computing the metric anti-dimension of a graph.

Bhaskar DasGupta

University of Illinois at Chicago

The k -metric anti-dimension of a graph has important applications in quantifying (k,l) -anonymity privacy measure for social networks under active attack. Motivated by this application, in this talk we will discuss how to formalize three natural problems related to such a privacy measure for large networks and provide non-trivial theoretical computational complexity results for solving these problems. Our results show the first two problems can be solved efficiently, whereas the third problem is provably hard to solve within a logarithmic approximation factor. Furthermore, we also provide computational complexity results for the case when the privacy requirement of the network is severely restricted, including an efficient logarithmic approximation. Time permitting, we will also show empirical results that shed light on privacy violation properties of eight real social networks as well as a large number of synthetic networks generated by both the classical Erdos-Renyi model and the scale-free random networks generated by the Barabasi-Albert preferential-attachment model.

Based on joint works with Tanima Chatterjee, Nasim Mobasher, Venkatkumar Srinivasan and Ismael G. Yero. Preprints are available at <https://doi.org/10.1016/j.tcs.2018.12.013> and <https://doi.org/10.1016/j.ins.2018.09.023>.

On the Solid-Metric Dimension of a Graph

Ville Junnila

University of Turku

Let G be a finite, simple and undirected graph with vertices V and edges E . Consider a nonempty vertex set $S = \{s_1, s_2, \dots, s_k\}$. The *distance array* of $v \in V$ with respect to S is defined as $D_S(v) = (d(s_1, v), d(s_2, v), \dots, d(s_k, v))$. We say that S is a *resolving set* of G if each vertex of G has a unique distance array with respect to S . The smallest cardinality of a resolving set of G is called the *metric dimension* of G . By the definition, a single irregularity located in a vertex of a graph can be found solely based on the distances of vertices to the ones of S . However, problems can occur if the graph contains more than one irregularity: a wrong vertex may be located, and more disturbingly we might not even notice this wrong conclusion.

To avoid the previous scenario, we introduce so called solid-resolving sets. For the formal definition of solid-resolving sets, we first define the distance array of a vertex set X with respect to S as $D_S(X) = (d(s_1, X), \dots, d(s_k, X))$, where $d(s_i, X) = \min\{d(s_i, x) \mid x \in X\}$. Now we say that a set $S \subseteq V$ is *solid-resolving* if S is a resolving set and, in addition, $D_S(X) \neq D_S(v)$ for any vertex $v \in V$ and subset $X \subseteq V$ with $|X| \geq 2$. In other words, using a solid-resolving set, a single irregularity in the graph can be located and, furthermore, multiple irregularities can be detected. The smallest cardinality of a solid-resolving set of G is called the *solid-metric dimension* of G .

In this talk, we discuss the solid-resolving sets more closely and present some new results concerning them. The talk is based on joint work with Anni Hakanen and Tero Laihonen.

The connected metric dimension at a vertex of a graph

Cong X. Kang

Texas A&M University at Galveston

We begin a local analysis of metric dimension by introducing the connected metric dimension of a graph G at a vertex v : a set of vertices S of G is a *resolving set* if, for any pair of distinct vertices x and y of G , there is a vertex $z \in S$ such that the distance between z and x is distinct from the distance between z and y in G . We say that a *resolving set* S is *connected* if S induces a connected subgraph of G . The *connected metric dimension of G at a vertex v* , denoted by $\text{cdim}_G(v)$, is the minimum of the cardinalities of all connected resolving sets of G which contain the vertex v . The *connected metric dimension of G* , denoted by $\text{cdim}(G)$, is $\min\{\text{cdim}_G(v) : v \in V(G)\}$. It's clear that $\dim(G) \leq \text{cdim}(G) \leq \text{cdim}_G(v) \leq |V(G)| - 1$ for any vertex v of G . In this talk, we will consider the following aspects of the connected metric dimension: 1) the existence of a pair (G, v) such that $\text{cdim}_G(v)$ takes all positive integer values from $\dim(G)$ to $|V(G)| - 1$, as v varies in a fixed graph G ; 2) the characterization of graphs G and their vertices v satisfying $\text{cdim}_G(v) \in \{1, |V(G)| - 1\}$; 3) the planarity implication of the condition $\text{cdim}(G) = 2$. This talk is based on work joint with Linda Eroh and Eunjeong Yi; a preprint is available at <https://arxiv.org/abs/1804.08147>.

Hausdorff Distance Between Trees in Polynomial Time

Aleksander Kelenc

University of Maribor and IMFM

The Hausdorff distance is a relatively new measure of similarity of graphs. The notion of the Hausdorff distance considers a special kind of a common subgraph of the compared graphs which depends on the structural properties outside of the common subgraph. There was no known efficient algorithm on trees for this problem. In this talk we present a polynomial-time algorithm for Hausdorff distance between two trees. The algorithm is recursive and is based on the divide and conquer technique. As a subtask it uses the procedure that is based on well known graph algorithm of finding the maximum bipartite matching.

Metric and strong metric dimensions of direct product graphs

Dorota Kuziak

University of Cadiz

Given a connected graph G , a vertex $w \in V(G)$ distinguishes two distinct vertices u, v of G if $d(u, w) \neq d(v, w)$ where $d(x, y)$ represents the length of a shortest $x - y$ path. On the other hand, the vertex w strongly resolves the pair u, v if there exists some shortest $u - w$ path containing v or some shortest $v - w$ path containing u . A set of vertices $W \subset V(G)$ is a (strong) metric generator for G if every pair of vertices of G is (strongly resolved) distinguished by some vertex of W . The smallest cardinality of a (strong) metric generator for G is called the (strong) metric dimension of G . In this work, several results concerning the (strong) metric dimension of direct product of graphs are given.

Joint work with Iztok Peterin and Ismael G. Yero.

On Resolving Several Vertices in a Graph

Tero Laihonen

University of Turku

A resolving set $S \subseteq V$ helps us to locate one vertex in a graph $G = (V, E)$ using its distances to the elements of S . Let us now consider the situation, where we would like to locate simultaneously one or more vertices, say, up to ℓ vertices. We think of S as an ordered set $(s_1, s_2, \dots, s_{|S|})$ and define the *distance array* of a non-empty subset $X \subseteq V$ by

$$\mathcal{D}_S(X) = (d(s_1, X), d(s_2, X), \dots, d(s_{|S|}, X)),$$

where $d(s_i, X)$ is the shortest distance from s_i to a vertex of X , that is,

$$d(s_i, X) = \min_{x \in X} d(s_i, x), \quad \forall i = 1, 2, \dots, |S|.$$

The set S is called an $\{\ell\}$ -*resolving set in G* if for every pair of distinct subsets X and Y of V , with $1 \leq |X| \leq \ell$ and $1 \leq |Y| \leq \ell$, we have

$$\mathcal{D}_S(X) \neq \mathcal{D}_S(Y).$$

In other words, an $\{\ell\}$ -resolving set can locate up to ℓ vertices at the same time with the aid of the distance arrays. If a surveillance network is modelled by a simple and undirected graph where we place sensors to the vertices corresponding to the elements of an $\{\ell\}$ -resolving set S , then the sensors can locate up to ℓ intruders by sending signals to measure the distance.

In this talk, we consider new results on $\{\ell\}$ -resolving sets and also introduce ℓ -solid-resolving sets, which is a new related concept. This talk is based on joint work with Anni Hakanen, Ville Junnila and María Luz Puertas.

k-Fault-tolerant resolving sets in graphs

Mercè Mora

Universitat Politècnica de Catalunya

Coauthors: M. J. Souto-Salorio and A. D. Tarrío-Tobar.

In this work we generalize the concept of fault-tolerant resolving sets and fault-tolerant metric dimension [1]. Resolving sets can be used to distinguish the vertices of a graph G comparing distances to fixed vertices. Fault-tolerant resolving sets are defined to distinguish the vertices of G even though one of the vertices of the set fails. Here we consider a more general case, concretely, resolving sets that distinguish the vertices of the graph when any k vertices of the set fail, for a fixed $k \geq 1$.

The notion of resolving sets in graphs was defined independently by Harary and Melter [2] and Slater [3]. Let G be a simple connected graph. A vertex u of G *resolves* two vertices x and y if the distances from u to x and from u to y are distinct. A set of vertices S is a *resolving set* for G if for every pair of different vertices x and y of G there is a vertex u in S that resolves x and y . The minimum cardinality of a resolving set, denoted by $\beta(G)$, is the *metric dimension* of G , and a resolving set of size $\beta(G)$ is a *metric basis*.

We say that a resolving set S of G is *k-fault-tolerant* if it remains a resolving set after the removal of any k vertices of S . It is worth mentioning that, in contrast to resolving sets,

k -fault-tolerant resolving sets do not always exist for $k \geq 2$. If a graph G has at least one k -fault-tolerant resolving set, we define the k -fault-tolerant metric dimension of G , denoted by $\beta'_k(G)$, as the minimum cardinality of a k -fault-tolerant resolving set. After discussing some general properties of k -fault-tolerant resolving sets, we give exact values of $\beta'_k(G)$ for some families of graphs, including 2- and 3-dimensional grids of large enough order.

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Bounding the determining number of a graph by removing twins

María Luz Puertas

University of Almería, Spain

The problem of identifying the vertex set of a graph G by using a vertex subset S , has been addressed from different points of view.

The subset S is a resolving set if every vertex is identified by the distances to vertices of S . On the other hand, S is a locating (or distinguish) set if each vertex can be uniquely identified by its set of neighbours that are in S and, if S is also a dominating set, it is called a locating dominating set. Each of these sets is a determining set, that is, every automorphism of G is uniquely determined by its action on the vertices of S .

We present both lower and upper bounds of the determining number of a graph, which is the minimum size of its determining sets. These bounds allow us to compute this parameter in two graph families such are cographs and line graphs.

Joint work with Antonio González (University of Sevilla, Spain)

Constrained incremental resolvability in dynamic graphs: a case study in active re-identification attacks on social networks

Yunior Ramírez-Cruz

University of Luxembourg

Given a family of graphs $\mathcal{G} = \{G_1, G_2, \dots, G_t\}$ with a common vertex set V , a simultaneous metric generator for \mathcal{G} has been defined as a set of vertices $S \subseteq V$ such that, for every pair of different vertices u and v , and every graph $G \in \mathcal{G}$, there exists some vertex $x \in S$ such that $d_G(u, x) \neq d_G(v, x)$. A minimum cardinality simultaneous metric generator for \mathcal{G} is called a simultaneous metric basis of \mathcal{G} , and its cardinality the simultaneous metric dimension of \mathcal{G} . By analogy, the notions of adjacency generator/basis/dimension, local metric generator/basis/dimension and strong metric generator/basis/dimension were extended to account for simultaneity in a graph family with a common vertex set.

The assumption that $V(G_1) = V(G_2) = \dots = V(G_t)$ is reasonable when simultaneity is interpreted as the existence of multiple definitions of the edge set on a common vertex set, for example, a common set of beacons transmitting and receiving in multiple frequencies. A less natural environment for the notions of simultaneous resolvability is that of a dynamically changing graph. Firstly, in a dynamic graph it may be too strong to assume that the vertex set is common, since vertices may be created or deleted. Secondly, the notion of simultaneity assumes that the entire graph family is known, and assumes no order among the members of the family; whereas in the setting of a dynamic graph only a subsequence has been observed at any given moment, and the sequence has been observed in a fixed order. Finally, rather than guaranteeing resolvability simultaneously for the entire subfamily, practical applications are more likely to require only that resolvability is *maintained* as the graph evolves.

In consequence, in this talk we will introduce a notion of *incremental resolvability*. We define a *dynamic graph at time t* as a sequence $\mathcal{G}_t = G_0, G_1, \dots, G_t$, where every G_i , called the *snapshot* of \mathcal{G}_t at time i , has the form $G_i = (V_i, E_i)$. Although in principle there are no restrictions on the relations between V_i and V_{i-1} , or those between E_i and E_{i-1} , here we will assume that V_i and E_i are the result of applying a proportionally small number of editions in V_{i-1} and E_{i-1} , respectively. For a dynamic graph $\mathcal{G}_0 = G_0$, incremental resolvability simply consists in finding a minimum metric (adjacency/local metric/strong metric) generator for G_0 . For the general case, we define the incremental resolvability problem as that of finding, given a dynamic graph $\mathcal{G}_t = G_0, G_1, \dots, G_{t-1}, G_t$ and a solution S_{t-1} of the incremental resolvability problem for $\mathcal{G}_t = G_0, G_1, \dots, G_{t-1}$, a set S_t that resolves G_t and minimises $S_{t-1} \div S_t$.

We showcase the new notion in the context of privacy-preserving social graph publication. In particular, we define an active re-identification attack on periodically published dynamic social networks as a constrained version of the incremental resolvability problem. Given a social graph $G = (V, E)$, an active attacker transforms it into a graph $G^+ = (V \cup S, E \cup E')$, with $S \cap V = \emptyset$ and $E' \subseteq (S \times S) \cup (S \times V)$, by injecting a set S of vertices, called *sybils*, into G , and establishing unique connection patterns, or *fingerprints*, with a subset of legitimate users T , called *victims* or *targets*. Links are also established between the members of S in such a way that after a pseudonymised version φG^+ of G^+ is published, the image of the subgraph $\langle S \rangle_{G^+}$ can be unambiguously retrieved from φG^+ and the fingerprints of the victims used to re-identify them. In other words, S must be an adjacency generator for G^+ and no other subgraph of G^+ must be isomorphic to $\langle S \rangle_{G^+}$.

Here, we extend this attack strategy to the scenario of a dynamic social network, from which pseudonymised snapshots are periodically published. Given a dynamic graph $\mathcal{G}_t = G_0, G_1, \dots, G_t$ at time t , we define a *successful dynamic active attacker at time t* as a pair of vertex set sequences S_0, S_1, \dots, S_t and T_0, T_1, \dots, T_t satisfying the following properties for every $i \in \{0, \dots, t\}$:

- (i) $T_i \subseteq V_i$
- (ii) $S_i \cap V_i = \emptyset$
- (iii) S_i is an adjacency generator for $\langle T_i \cup S_i \rangle_{G_i^+}$
- (iv) there exists no vertex set $X \subseteq V_i \cup S_i$, $X \neq S_i$, such that $\langle X \rangle_{G_i^+} \cong \langle S_i \rangle_{G_i^+}$

Based on the previous definitions, we present an extension of the so-called *walk-based attack* to the scenario of periodically releasing pseudonymised snapshots of a dynamic social graph, which is viewed as a constrained incremental resolvability problem on $\mathcal{G}_t^+ = \langle T_0 \cup S_0 \rangle_{G_0^+}, \langle T_1 \cup S_1 \rangle_{G_1^+}, \dots, \langle T_t \cup S_t \rangle_{G_t^+}$. The purpose of this attack is to serve as an adversary model for privacy-preserving dynamic social graph publication methods. Thus, we will also discuss countermeasures to the new type of attacks.

The work covered in this talk has been conducted jointly with Ema Kepuska, Xihui Chen and Sjouke Mauw, from the University of Luxembourg.

Centroidal dimensions of product graphs

Rinovia Simanjuntak
Institut Teknologi Bandung

Let G be a connected graph, $W = \{w_1, w_2, \dots, w_k\}$ a set of vertices of G , and x a vertex in G . We denote by $r(x|W)$ the ordered distance partition of W , that is, the list of subsets of W in non-decreasing order by their distance from x . The set W is then called a centroidal locating set of G if $r(x|W) \neq r(y|W)$ for every pair of distinct vertices x and y . A centroidal basis of G is a centroidal locating set of minimum cardinality. The centroidal dimension of G , denoted by $CD(G)$, is the cardinality of a centroidal basis of G .

The concept of centroidal dimension was introduced by Foucaud, Klasing, and Slater in 2014, where they proved that for any n -node graph G with maximum degree at least 2, $(1 + o(1)) \frac{\ln n}{\ln \ln n} \leq CD(G) \leq n - 1$; however no follow-up results are known. In this talk we shall discuss sharp bounds for centroidal dimensions of some graph products.

Joint work with Tamaro Nadeak.

On graphs achieving the trivial upper bound for edge metric dimension

Andrej Taranenko
Institute of Mathematics, Physics and Mechanics / University of Maribor

An edge metric generator of a connected graph G is a vertex subset S for which every two distinct edges of G have distinct distance to some vertex of S , where the distance between a vertex v and an edge e is defined as the minimum of distances between v and the two endpoints of e in G . The smallest cardinality of an edge metric generator of G is the edge metric dimension, denoted by $\dim_e(G)$. It follows that $1 \leq \dim_e(G) \leq n - 1$ for any n -vertex graph G . A graph whose edge metric dimension achieves the upper bound is topful. In this talk, we present results about the structure of topful graphs, as well as many necessary and sufficient conditions for a graph to be topful. Using these results we design an $O(n^3)$ time algorithm which determines whether a graph of order n is topful or not. Moreover, we describe and address an interesting class of topful graphs whose super graphs obtained by adding one edge are not topful.

This is joint work with Enqiang Zhu, Zehui Shao and Jin Xu.

Uniquely identifying the vertices of a graph by means of distance multisets

Ismael G. Yero
University of Cadiz

Given a graph G and a subset of vertices $S = \{w_1, \dots, w_t\} \subseteq V(G)$, the multiset representation of a vertex $u \in V(G)$ with respect to S is the multiset $m(u|S) = \{d_G(u, w_1), \dots, d_G(u, w_t)\}$. A subset of vertices S such that $m(u|S) = m(v|S)$ if and only if $u = v$ for every $u, v \in V(G) \setminus S$ is said to be a multiset resolving set, and the cardinality of the smallest such set is the multiset dimension. Several results on the multiset dimension of graphs are given in this work. For instance, the exact value of this parameter for several graph families, and some bounds for other ones, are proved. It is also shown that computing the multiset dimension of arbitrary graphs is NP-hard,

and methods for efficiently handling particular cases are provided.

Joint work with R. Gil-Pons, Y. Ramírez-Cruz and R. Trujillo-Rasúa.

Preprint available at <https://arxiv.org/abs/1902.03017>.

The fractional k -metric dimension of graphs

Eunjeong Yi

Texas A&M University at Galveston

Let G be a graph with vertex set $V(G)$. For any two distinct vertices x and y of G , let $R\{x, y\}$ denote the set of vertices z such that the distance from x to z is not equal to the distance from y to z in G . For a function g defined on $V(G)$ and for $U \subseteq V(G)$, let $g(U) = \sum_{s \in U} g(s)$. Let $\kappa(G) = \min\{|R\{x, y\}| : x \neq y \text{ and } x, y \in V(G)\}$. For any real number $k \in [1, \kappa(G)]$, a real-valued function $g : V(G) \rightarrow [0, 1]$ is a k -resolving function of G if $g(R\{x, y\}) \geq k$ for any two distinct vertices $x, y \in V(G)$. The *fractional k -metric dimension*, $\dim_f^k(G)$, of G is $\min\{g(V(G)) : g \text{ is a } k\text{-resolving function of } G\}$; we note that the fractional k -metric dimension can be viewed as a generalization of "fractional metric dimension", as well as a fractionalization of " k -metric dimension". In this talk, we introduce the fractional k -metric dimension of graphs. For a connected graph G and $k \in [1, \kappa(G)]$, it's easy to see that $k \leq \dim_f^k(G) \leq \frac{k|V(G)|}{\kappa(G)}$; we characterize graphs G satisfying $\dim_f^k(G) = k$ and $\dim_f^k(G) = |V(G)|$, respectively. We show that $\dim_f^k(G) \geq k \dim_f(G)$ for any $k \in [1, \kappa(G)]$, and we give an example showing that $\dim_f^k(G) - k \dim_f(G)$ can be arbitrarily large for some $k \in (1, \kappa(G)]$; we also describe a condition for which $\dim_f^k(G) = k \dim_f(G)$ holds. We conclude with some open problems. This talk is based on joint work with Cong X. Kang and Ismael G. Yero. The preprint is available at <https://arxiv.org/abs/1706.05550>.

Invited Special Session

POLYTOPES

Organized by ASIA IVIĆ WEISS

This session celebrates the life and work of Branko Grünbaum

A family of finite quiral polyhedra in S^3

Javier Bracho

National University of Mexico

A new family of quiral polyhedra in the 3-sphere will be presented. They have helicoidal faces and are associated to the classic finite regular full-rank polytopes in R^4 . Some of them are the facets of finite quiral full-rank polytopes in R^4 , of which only one was previously known. Their construction and pictures will be presented.

Chiral polytopes of arbitrarily large rank

Marston Conder

University of Auckland

Regular abstract polytopes are well understood and relatively easy to construct. In contrast, chiral polytopes are much more challenging, and indeed less than 15 years ago, no finite chiral polytopes of rank $d \geq 5$ were known. That changed in the mid/late 2000s when some examples of ranks 5 to 8 were found, and then Daniel Pellicer (2010) proved the existence of finite chiral polytopes of arbitrarily large rank d . Daniel's examples were enormous, however, and it has been a challenge to find some smaller concrete examples. Gabe Cunningham recently proved that for $d \geq 8$, every chiral d -polytope has at least $48(d-2)(d-2)!$ flags, and it is no longer surprising that 'small' examples of large rank do not exist. Nevertheless, examples are now plentiful. In this talk I will describe a new construction (developed in joint work with Isabel Hubard, Eugenia O'Reilly-Regueiro and Daniel Pellicer) that can be used to show that for every $d \geq 5$, there exists a chiral d -polytope with automorphism group isomorphic to the alternating group A_n , and a chiral d -polytope with automorphism group isomorphic to the symmetric group S_n , for all but finitely many n .

Locally spherical hypertopes

Maria Elisa Fernandes

Universidade de Aveiro

Hypertope is a generalization of the concept of polytope. In this talk we give examples of regular hypertopes arising from the finite and euclidean Coxeter groups.

This is a joint work with Asia Weiss and Dimitri Leemans.

Geometric chiral polyhedra in 3-dimensional spaces

Isabel Hubard

Universidad Nacional Autónoma de México

In recent decades a lot of attention has been dedicated to the so-called skeletal polyhedra, where faces are not associated to membranes. The symmetry of a skeletal polyhedron is measured by the number of orbits of flags (triples of incident vertex, edge and face) under the action of the symmetry group. Those with only one flag-orbit are called regular, and in a combinatorial sense they have maximal symmetry by reflections. They have been the focus of a

lot of study.

Chiral polyhedra are those with maximal (combinatorial) symmetry by rotations but none by reflections and have two flag-orbits. In comparison to the regular ones, much less work has been done on chiral skeletal polyhedra. One of the main difficulties of working with chiral skeletal polyhedra is the lack of examples. They first appeared in 2005, when Schulte fully classified those living in \mathbb{E}^3 . In recent years the first chiral skeletal polyhedra in \mathbb{P}^3 appeared.

In this talk we will see some interesting aspects of geometrically chiral polyhedra living in \mathbb{E}^3 , \mathbb{P}^3 or \mathbb{H}^3 , and will give a construction that produces chiral polyhedra related to regular 4-polytopes.

This is work with Javier Bracho and Daniel Pellicer.

Rank reduction of string C-group representations

Dimitri Leemans

Université Libre de Bruxelles

We will present a rank reduction technique for string C-group representations that generalises a method first used by Fernandes and Leemans for the symmetric groups. The technique permits us, among other things, to prove that orthogonal groups defined on d -dimensional modules over fields of even order greater than 2 possess string C-group representations of all ranks $3 \leq n \leq d$. The broad applicability of the rank reduction technique provides fresh impetus to construct, for suitable families of groups, string C-groups of highest possible rank. It also suggests that the alternating group $\text{Alt}(11)$ —the only known group having ‘rank gaps’—is perhaps more unusual than previously thought.

This is joint work with Peter A. Brooksbank.

Oriented matroids as graphs

Tilen Marc

University of Ljubljana

Oriented matroids can be represented by their so called tope graph. These graphs have interesting properties such as being isometrically embeddable in hypercube graphs, having antipodal symmetries, and interesting metric properties. In the talk I will present a recent result characterizing tope graphs of oriented matroids and their generalization named complexes of oriented matroids in terms of their metric properties. I will discuss how approaching oriented matroids through graphs can give new views of open problems about oriented matroids and present some new results.

2- orbit polytopes

Elias Mochan

National University of Mexico

Pellicer, Potočnik and Toledo proved that for rank greater or equal than 5 there exist 2-orbit maniplexes with any possible symmetry type. They do this by giving a concrete voltage assignment to every possible symmetry type graph with two vertices.

We have proved a theorem that allows us to know what intersection properties must a voltage

group satisfy so that the derived graph from an admissible graph and a voltage assignment is the flag graph of a polytope. By applying this theorem to the voltage assignment constructed by Pellicer, Potočnik and Toledo, we attempt to prove that in some cases the derived graph is actually the flag graph of a polytope, showing that the corresponding graph is the symmetry type graph of a polytope.

Joint work with Isabel Hubbard. This is a work in progress.

On the Geometric Banach Conjecture

Luis Montejano

National University of Mexico at Queretaro

In 1932 Banach conjectured that if V is a Banach space all whose n -dimensional subspaces are isometric, then V is a Hilbert space, where $1 < n < \dim V$. Gromov proved in 1967 the conjecture for even integers n . The purpose of the talk is to give an sketch of the proof when n is an integer of the form $4k+1$ (n different from 133). The proof is a combination of ideas of convexity, algebraic topology and Lie groups.

Highly symmetric toroidal polytopes

Antonio Montero

National Autonomous University of Mexico

In the talk we will review a technique that has been used to classify several kinds of highly symmetric abstract polytopes lying on a toroidal ambient space. We will focus not only on the abstract approach of the classifications but on visualizing such objects.

Tight chiral abstract polytopes

Daniel Pellicer

National University of Mexico

In recent decades there has been much interest on combinatorial structures that admit almost all, but not all, possible symmetries. Chiral abstract polytopes are examples of such objects. They admit all symmetries by combinatorial rotations, but none by combinatorial reflections. The study of chiral polytopes has proven more difficult than the study of regular ones (those with all symmetries by combinatorial reflections). One major complication is that the sizes of chiral polytopes tend to grow very fast when we increase the rank.

When determining the smallest regular polytopes of every rank, Conder defined a tight (equivelar) polytope as one where the number of flags is twice the product of the entries of the Schläfli type. The smallest regular polytopes of ranks $n > 3$ are all tight.

In this talk we will show recent work on tight chiral polytopes. In particular, they exist only in ranks 3 and 4.

Joint work with Gabe Cunningham

Possible degrees of Toroidal Regular Maps

Claudio Alexandre Piedade

Universidade de Aveiro

CPR graphs, that are faithful permutation representations, are a powerful tool in the classification of abstract regular polytopes. In this talk we list all possible degrees of faithful permutation representations of the toroidal maps $\{4, 4\}$ and $\{3, 6\}$. This is a joint work with M. Elisa Fernandes. Preprint available at <https://arxiv.org/abs/1808.09705>.

Branko Grünbaum, the mathematician who beat the odds

Moshe Rosenfeld

University of Washington

Branko Grünbaum was born in Osijek, Yugoslavia in 1929. He spent his first 20 years in Yugoslavia, including four years under Nazi occupation. He started his mathematical journey in 1948 at the University of Zagreb, a journey that lasted 70 years and is still carried on by his many followers. He immigrated to Israel in July 1949 where he completed his MSc studies at the Hebrew University of Jerusalem in 1954 and got his Ph.D in 1957. His supervisor and mentor was Professor Arye Dvoretzki.

I will share with you the amazing life story of Branko and his wife Zdenka, his mathematical journeys through polytopes, arrangements, tilings, patterns and other lesser known topics. In Branko's tradition I will conclude with some open problems I had a chance to discuss with him.

Realizations of regular abstract polytopes in euclidean spaces

Asia Ivić Weiss

York University

We overview the realizations of regular polyhedra (as defined by Grünbaum) in euclidean 3-space. Using McMullen's theory of realizations of abstract polytopes in euclidean spaces we discuss the realizations of regular toroids. This is joint work with Barry Monson.

Geometric realizability of simplicial 3-spheres

Gordon Williams

University of Alaska Fairbanks

In 1909 Brückner enumerated the simple 4-polytopes with 8 facets. In 1967, Grünbaum and Sreedharan considered the equivalent problem of enumerating the simplicial 4-polytopes with 8 vertices; they found 37 but observed that one of Brückner's combinatorial types did not correspond to the boundary complex of a convex 4-polytope. Barnette found one more combinatorial type of simplicial 3-sphere that isn't convexly realizable and completed the enumeration of simplicial 3-spheres with 8 vertices in 1969. Since then a number of authors have considered questions related to the enumeration of combinatorial types of simplicial convex polytopes and simplicial spheres.

In this talk we will review our 2002 paper on the geometric realizability of Brückner's and

Barnettes 3-spheres, as well as some of the work that has been done on related problems and discuss open problems in this area.

This is joint work with Jed Mihalisin.

Rotary 4-Maniplexes having one facet

Steve Wilson

Northern Arizona University

The notion of 'maniplex' is slight generalization of that of 'polytope'. A maniplex, unlike a polytope, can have just one facet (highest-dimensional face). We will display several examples of such maniplexes in four dimensions, and we will boast, with little support for the claim, that we have determined precisely the conditions on a 3-maniplex (i.e., a map) for it to be the only facet of a rotary 4-maniplex. Moreover we determine when the resulting maniplex is orientable or not, reflexible or not. This is joint work with Daniel Pellicer.

Invited Special Session

SPECTRAL GRAPH THEORY

Organized by FRANCESCO BELARDO

A characterization and an application of weight-regular partitions of graphs

Aida Abiad

Maastricht University & Ghent University

A natural generalization of a regular (or equitable) partition of a graph, which makes sense also for non-regular graphs, is the so-called weight-regular partition, which gives to each vertex $u \in V$ a weight that equals the corresponding entry v_u of the Perron eigenvector v .

This paper contains three main results related to weight-regular partitions of a graph. The first is a characterization of weight-regular partitions in terms of double stochastic matrices. Inspired by a characterization of regular graphs by Hoffman, we also provide a new characterization of weight-regularity by using a Hoffman-like polynomial. As a corollary, we obtain Hoffman's result for regular graphs. In addition, we show an application of weight-regular partitions to study graphs that attain equality in the classical Hoffman's lower bound for the chromatic number of a graph, and we show that weight-regularity provides a condition under which Hoffman's bound can be improved.

The Distance Spectra of Some Graph Classes — A Survey

Ambat Vijayakumar

Cochin University of Science and Technology, Cochin, India

In this paper, of concern are the distance hereditary graphs and the two well studied graph classes, cographs and threshold graphs. Distance hereditary graphs are connected graphs in which all induced paths are isometric. These graphs were introduced by Howorka [8], who gave the first characterization and proved that all distance hereditary graphs are perfect. Ptolemaic graphs, cographs and block graphs are some well-known subclasses of distance hereditary graphs. Cographs (complement reducible graphs) are P_4 free graphs [1]. Distance spectrum, distance spectral radius and distance energy have been extensively studied, see [2, 4, 10, 11, 15]. It is well-known that the distance hereditary graphs admit a forbidden subgraph characterization. We obtain the necessary condition of this characterization using distance spectrum. Also, we obtain a new characterization of distance hereditary graphs using distance spectrum. Location of adjacency eigenvalues of cographs was studied recently by Mohammadian, Trevisan, Jacobs, Tura and Allem. We find that no distance eigenvalue lie in the interval $(-2, -1)$. Also, the multiplicities of the distance eigenvalues -1 and -2 in the distance spectrum of cographs are obtained. Threshold graphs form a subclass of cographs. Hence, the results on cographs proved in this paper are generalizations of some of the results proved by Huang and Lou [14]. Similar results on adjacency spectrum on threshold graphs are proved by Jacobs, Trevisan and Tura [7].

This is a joint work with Anu Varghese.

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Tridiagonal matrices and spectral properties of some graph classes

Milica Andelic

Kuwait University

A graph is called a chain graph if it is bipartite and the neighbourhoods of the vertices in each colour class form a chain with respect to inclusion. We give an explicit formula for the characteristic polynomial of any chain graph and we show that it can be expressed using the determinant of a particular tridiagonal matrix. Then this fact is applied to show that in a certain interval a chain graph does not have any non-zero eigenvalues. A similar result is provided for threshold graphs.

On some recent results of Slobodan K. Simić (1948-2019)

Francesco Belardo

University of Naples Federico II

In his long career Slobodan K. Simić published more than 160 papers and three monographs on Spectral Graph Theory. He had been active in research until his unexpected death. In this talk we survey some of his recent results and we describe a few open problems left from his latest papers.

On Some Spectral Properties of Signed Circular Caterpillars

Maurizio Brunetti

University of Naples Federico II

A circular caterpillars of girth n is a graph such that the removal of all pendant vertices yields a cycle C_n of order n .

A signed graph is a pair $\Gamma = (G, \sigma)$, where G is a simple graph and $\sigma : E(G) \rightarrow \{+1, -1\}$ is the sign function defined on the set $E(G)$ of edges of G . The signed graph Γ is said to be balanced if the number of negatively signed edges in each cycle is even, and it is said to be unbalanced otherwise. We determine some bounds for the first n Laplacian eigenvalues of any signed circular caterpillar. As an application, we prove that each signed spiked triangle $G(3; p, q, r)$, i. e. a signed circular caterpillars of girth 3 and degree sequence $\pi_{p,q,r} = (p+2, q+2, r+2, 1, \dots, 1)$, is determined by its Laplacian spectrum up to switching isomorphism. Moreover, in the set of signed spiked triangles of order N , we identify the extremal graphs with respect to the Laplacian spectral radius, the first two Zagreb indices and the Laplacian-energy-like invariant defined as the sum of square roots of the Laplacian eigenvalues. Whatever index we choose in such list, it turns out that the extremal signed spiked triangles are the same, the minimal being the balanced spike triangle $G(3; p, q, r)$ where p, q and r differ at most by 1. The maximal graph is instead the unbalanced spiked triangle with degree sequence $\pi_{N-3,0,0}$.

A new general method to obtain the spectrum and local spectra of a graph from its regular partitions

Cristina Dalfo

Universitat de Lleida

It is well known that, in general, a part of the spectrum of a graph can be obtained from the adjacency matrix of its quotient graph given by a regular partition. In this work, we propose a method that gives for the very first time all the spectrum, and also the local spectra, of a general graph from the quotient matrices of some of its regular (or equitable) partitions. Until now, this was known only for distance-regular graphs. Our main result yields a simple method to compute the local spectra of a vertex u in a given regular (or equitable) partition or, more generally, the crossed multiplicities between u and any other vertex v . Besides, with the union of the local spectra of the different classes of vertices according to their corresponding regular partitions (that is, we ‘hung’ the quotient graph from every one of the different classes of vertices), for the very first time all the spectrum of the original graph is obtained from regular partitions.

Joint work with Miquel Angel Fiol (Universitat Politècnica de Catalunya)

Preprint available at <https://arxiv.org/abs/1901.08048>

Generating Pairs of Generalized Cospectral Graphs from Controllable Graphs

Alexander Farrugia

University of Malta

Two graphs are said to be generalized cospectral if they have the same characteristic polynomials and so do their complements. A graph is controllable if its walk matrix is nonsingular; equivalently, if all the eigenvalues of its adjacency matrix are simple and main. A graph H on $(n+1)$ vertices is an overgraph of another graph G on n vertices if G is a vertex-deleted subgraph of H . We present methods that produce pairs of generalized cospectral graphs G' and H' starting from a pair of generalized cospectral, non-isomorphic, controllable graphs G and H . We also show that no two overgraphs of a controllable graph are generalized cospectral. This theorem strengthens an earlier result by Farrugia stating that no two overgraphs of a controllable graph are isomorphic.

On the multiplicities of digraph eigenvalues

Alexander Gavrilyuk

Pusan National University

In 1977, Delsarte, Goethals, and Seidel showed that a regular (simple) graph on n vertices, whose $(0,1)$ -adjacency matrix A has the smallest eigenvalue < -1 of multiplicity $n - d$, satisfies $n \leq \frac{1}{2}d(d+1) - 1$. The bound is sharp, and it is known as the absolute bound if a graph is strongly regular.

In 2003, Bell and Rowlinson extended this bound to any eigenvalue of A distinct from 0 or -1 , and showed that the graphs attaining equality are extremal strongly regular graphs (the only examples known are a pentagon, a complete multipartite graph, the Schläfli graph, the McLaughlin graph and their complements).

In this talk I will present the multiplicity bounds for eigenvalues of Hermitian adjacency matrices of digraphs. This talk is based on joint work with Sho Suda.

The Structure of Quartic Graphs with Minimal Spectral Gap

Wilfried Imrich

Montanuniversität Leoben

Motivated by a conjecture by Aldous and Fill we investigate regular graphs G with minimum spectral gap, that is, graphs for which the difference between the two largest eigenvalues of the adjacency matrix of G is minimal. This has already been done for cubic graphs by Brand, Guiduli and Imrich, who showed that these graphs must look like a path built from specific blocks. We extend this result to 4-regular graphs. We also conjecture that for graphs on at least eleven vertices the structure is unique.

This is joint work with Maryam Abdi, K.N. Toosi University of Technology, Teheran, Iran and Ebrahim Ghorbani, IPM - Institute for Research in Fundamental Sciences, Teheran, Iran

Some examples of transformations that preserve $\text{sgn}(\lambda_2 - r)$

Bojana Mihailović

University of Belgrade

We consider trees and cacti whose second largest eigenvalue does not exceed some given small boundaries. Some classes of such graphs can be easily described using mappings that preserve $\text{sgn}(\lambda_2 - r)$. We give examples of such mappings and determine certain classes of trees and cacti with given spectral property.

About the smallest eigenvalue of non-bipartite graphs

Bojan Mohar

Simon Fraser University & IMFM

How small could be the smallest eigenvalue of a non-bipartite graph whose spectral radius is ρ ? An answer in terms of the local odd girth of the graph will be provided. The proof of the main result is a simple combination of interlacing and fractional decompositions. This is joint work with Fiachra Knox.

Laplacian eigenvalues of the zero divisor graph of the ring \mathbb{Z}_n

Kamal Lochan Patra

NISER, Bhubaneswar, INDIA

Let $L(G)$ be the Laplacian matrix of a simple finite graph G . The second smallest and the largest eigenvalues of $L(G)$ are called the algebraic connectivity and the Laplacian spectral radius of G , respectively. A graph G is said to be Laplacian integral if all of its Laplacian eigenvalues are integers.

For a positive integer $n > 1$, let \mathbb{Z}_n denote the ring of integers modulo n . The *zero divisor graph* $\Gamma(\mathbb{Z}_n)$ of \mathbb{Z}_n is the simple graph whose vertex set consists of the zero divisors of \mathbb{Z}_n , and such that two distinct vertices x and y are adjacent if and only if $xy = 0$ in \mathbb{Z}_n . The graph $\Gamma(\mathbb{Z}_n)$ is the empty graph (i.e., there are no vertices) if n is prime.

In this talk, we consider the Laplacian eigenvalues of the zero divisor graph $\Gamma(\mathbb{Z}_n)$, and we

prove that $\Gamma(\mathbb{Z}_{p^t})$ is Laplacian integral for every prime p and positive integer $t \geq 2$. We also show that the Laplacian spectral radius and the algebraic connectivity of $\Gamma(\mathbb{Z}_n)$ for most values of n are, respectively, the largest and the second smallest eigenvalues of the vertex weighted Laplacian matrix of a graph, which is defined on the set of proper divisors of n . The values of n for which the algebraic connectivity and the vertex connectivity of $\Gamma(\mathbb{Z}_n)$ coincide are characterized, as well.

Inverting non-invertible labeled trees

Soňa Pavlíková

Slovak University of Technology Bratislava

If a graph with non-zero edge labels has a non-singular adjacency matrix, then one may use the inverse matrix to define a (labeled) graph that may be considered to be the inverse graph to the original one. It has been known that an adjacency matrix of a labeled tree is non-singular if and only if the tree has a unique perfect matching. In the opposite case one may use a generalized inverse (which, in the symmetric case, coincide with Moore-Penrose, Drazin, or group inverse) of the adjacency matrix to ‘invert’ a tree. A formula for entries of such a generalized inverse of a tree follows from the work of Britz, Olesky and van den Driessche (2004), based on a general formula for determining the Moore-Penrose inverse.

In our talk we will briefly introduce various approaches to ‘inverting’ non-invertible matrices, state a formula for a generalized inverse of (an adjacency matrix of) a labeled tree, and outline principles leading to a new proof of validity of this formula (based solely on considering eigenvectors).

This is a joint work with Jozef Siran.

Spectral and combinatorial properties of lexicographic polynomials of graphs

Paula Rama

University of Aveiro, Portugal

For a graph H and non-negative integers c_0, c_1, \dots, c_d ($c_d \neq 0$), the expression $p(H) = \sum_{k=0}^d c_k \cdot H^k$ is called the lexicographic polynomial in H of degree d , where H^k is the k -th power of H with respect to the lexicographic product, the sum of two graphs is their join and $c_k \cdot H^k$ is the join of c_k copies of H^k . Some combinatorial properties of the lexicographic polynomial graphs are deduced and their spectrum (when H is regular) and laplacian spectrum (in the general case) are determined.

Joint work with Domingos M. Cardoso¹, Paula Carvalho¹, Slobodan K. Simić², Zoran Stanić³

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On graphs with the same main eigenspace

Irene Sciriha

University of Malta

The main eigenvalues of a graph G are those eigenvalues of the $(0, 1)$ -adjacency matrix having at least one corresponding eigenvector not orthogonal to $j = (1, 1, \dots, 1)$. There is a close relationship between the main eigensystem of a graph and the number of walks. The number of walks in comain graphs with the same main eigenspace but different eigenvectors are related. We show that, among pairs of graphs with the same walk matrix, the bipartite double covering can distinguish between comain graphs that are two-fold isomorphic and those that are not.

Notes on spectra of signed graphs

Zoran Stanić

University of Belgrade

Given a graph $G = (V(G), E(G))$, let $\sigma: E(G) \rightarrow \{-1, +1\}$. Then $\hat{G} = (G, \sigma)$ is a *signed graph* derived from its *underlying graph* G . Signed graphs are intensively studied in the framework of social psychology or physics.

We expose some recent results concerning the spectra of signed graphs. In particular, we present some bounds for eigenvalues of the adjacency matrix or the Laplacian matrix of a signed graph, we give some results concerning signed graphs with a comparatively small number of (distinct) eigenvalues and we present a concept of strong regularity of signed graphs – a generalization of strong regularity of graphs.

A generalization of Hoffman Graph

Tetsuji Taniguchi

Hiroshima Institute of Technology

Hoffman graphs were introduced by Woo and Neumaier to study the graphs with smallest eigenvalue $\geq -1 - \sqrt{2}$ but < -2 . For a given value $\lambda (\leq -2)$, there exist graphs with smallest eigenvalue $\geq \lambda$ that they can not be represented by a sum of (usual) Hoffman graphs with smallest eigenvalue $\geq \lambda$. Therefore, we consider a further generalization of Hoffman graph by giving to the fat vertices a weight and by giving to the edges a sign “ \pm ”. By using this latest generalization, it becomes possible to such graphs to be represented by a sum of (new) Hoffman graphs with smallest eigenvalue $\geq \lambda$. In this talk we consider the above described generalization of Hoffman graphs and we give some results about them.

On graphs whose spectral radius does not exceed the Hoffman limit value

Jianfeng Wang

Shandong University of Technology

For a graph matrix M , the Hoffman limit value $\hbar(M)$ is the limit value of the sequence of M -spectral radii of cycles with a pendant edge, when the length of the cycles tends to infinity. The Hoffman program of graphs consists of determining the Hoffman limit value, and then characterize all connected graphs whose M -spectral radius does not exceed $\hbar(M)$. In this talk, we summarize the results about the Hoffman program for some well-known graph matrices. Furthermore, we will give some results about the Hoffman program of the A_α -matrices of graphs, where $A_\alpha(G) = \alpha D(G) + (1 - \alpha)A(G)$. This is a joint work with J. Wang, X.G. Liu and F. Belardo.

The negative tetrahedron and the first infinite family of connected digraphs that are strongly determined by the Hermitian spectrum

Pepijn Wissing

Tilburg University

Thus far, digraphs that are uniquely determined by their Hermitian spectra have proven elusive. Instead, researchers have turned to spectral determination of classes of switching equivalent digraphs, rather than individual digraphs. We consider the traditional notion: a digraph (or mixed graph) is said to be strongly determined by its Hermitian spectrum (abbreviated SHDS) if it is isomorphic to each digraph to which it is cospectral. Convincing numerical evidence to support the claim that this property is extremely rare is provided. Nonetheless, the first infinite family of connected digraphs that is SHDS is constructed. This family is obtained via the introduction of twin vertices into a structure that is named *negative tetrahedron*. This special digraph, that exhibits extreme spectral behavior, is contained in the surprisingly small collection of all digraphs with exactly one negative eigenvalue, which is determined as an intermediate result. Joint work with Edwin R. van Dam; Preprint available at <https://arxiv.org/abs/1903.09531>.

Invited Special Session

STRUCTURAL AND ALGORITHMIC
GRAPH THEORY

Organized by PIERRE ABOULKER

Subgraphs of directed graphs with large dichromatic number

Pierre Aboulker

Ecole Normale Supérieure

A k -dicolouring of a digraph is a partition of its vertex set into k acyclic subdigraphs. The *dichromatic number* of a digraph D is the minimum k such that D has a k -dicolouring. This notion has been introduced by Neumann-Lara in the 80's and has been proved to be a good generalization of chromatic number to directed graphs since then. In this talk, we will investigate the following questions: what can we say about subgraphs, induced subgraphs and topological minors of a digraph with large dichromatic number?

Maximum Weighted Clique in Hole-Cyclically Orientable Graphs

Jesse Beisegel

Brandenburg University of Technology

A graph is said to be *hole-cyclically orientable* if every induced cycle of size at least four can be oriented cyclically. These graphs form a generalization of 1-perfectly orientable graphs and, thus, of its subclasses chordal graphs and circular arc graphs. By showing that Lexicographic Breadth First Search computes a bisimplicial elimination ordering on these graphs, we use a divide and conquer approach to derive an algorithm which computes maximum weight clique in $\mathcal{O}(|V(G)|(|V(G)|\log|V(G)| + |E(G)|\log\log|V(G)|))$ time.

Based on joint work with Maria Chudnovsky, Vladimir Gurvich, Martin Milanič and Mary Servatius

Topological Invariants of Elementary Cycles and a Generalization of Biggs Theorem

Michael Hecht

TU Dresden & Max Planck Institute of Molecular Cell Biology and Genetics, Dresden

For a given multi-digraph $G = (V, E)$ we generalize Biggs Theorem to the case of directed cycles, which allows to compute the R -dimension, $R = \mathbb{Z}, \mathbb{Z}_2$, of the directed cycle space $\Lambda_R(G)$ of G in quadratic time. For a set of elementary directed (connected) cycles O we further consider the *CW complex of elementary cycles* $X(G, O)$, given by interpreting the vertices V as 0-cells, the edges E as 1-cells, and by attaching 2-cells σ whenever their boundary $\partial\sigma$ corresponds to a elementary cycle $c \in O$. We show that the associated homology groups $H_0(G, O, R)$, $H_1(G, O, R)$ and $H_2(G, O, R)$ are free R -modules. If $O = O(G)$ is given by all elementary directed or connected cycles then the dimensions of $H_0(G, O, R)$, $H_1(G, O, R)$ can be derived in $\mathcal{O}(|V||E|)$, $\mathcal{O}(|E|^2)$, while computing $\dim_R H_2(G, O, R)$ is $\#P$ -hard. To avoid $\#P$ -hard counting problems we present several partially filled CW complexes yielding efficiently computable cellular homologies. The thereby decoded topological structure of G enables us to shed new light on many problems in graph theory as *graph embedding*, *metric graph theory*, *spectral graph theory* and *discrete Morse theory*.

Preprint available at https://www.researchgate.net/publication/321833057_Topological_Invariants_of_Elementary_Cycles_and_a_Generalization_of_Biggs_Theorem

Unavoidable minors for graphs with large ℓ_p -dimension

Tony Huynh

Université libre de Bruxelles (ULB)

A *metric graph* is a pair (G, d) , where G is a graph and $d : E(G) \rightarrow \mathbb{R}_{\geq 0}$ is a distance function. Let $p \in [1, \infty]$ be fixed. An *isometric embedding* of the metric graph (G, d) in $\ell_p^k = (\mathbb{R}^k, d_p)$ is a map $\phi : V(G) \rightarrow \mathbb{R}^k$ such that $d_p(\phi(v), \phi(w)) = d(vw)$ for all edges $vw \in E(G)$. The ℓ_p -*dimension* of G is the least integer k such that there exists an isometric embedding of (G, d) in ℓ_p^k for all distance functions d such that (G, d) has an isometric embedding in ℓ_p^K for some K .

It is easy to show that ℓ_p -dimension is a minor-monotone property. We characterize the minor-closed graph classes \mathcal{C} with bounded ℓ_p -dimension, for $p \in \{2, \infty\}$. For $p = 2$, we give a simple proof that \mathcal{C} has bounded ℓ_2 -dimension if and only if \mathcal{C} has bounded treewidth. In this sense, the ℓ_2 -dimension of a graph is ‘tied’ to its treewidth.

For $p = \infty$, the situation is completely different. Our main result states that a minor-closed class \mathcal{C} has bounded ℓ_∞ -dimension if and only if \mathcal{C} excludes a graph obtained by joining copies of K_4 using the 2-sum operation, or excludes a Möbius ladder with one ‘horizontal edge’ removed.

This is joint work with Samuel Fiorini, Gwenaël Joret, and Carole Muller. Preprint available at <https://arxiv.org/abs/1904.02951>.

Erdős-Pósa Property of Chordless Cycles and its Applications

Eunjung Kim

CNRS, LAMSADE, Paris-Dauphine University

A chordless cycle, or equivalently a hole, in a graph G is an induced subgraph of G which is a cycle of length at least four. We prove that the Erdős-Pósa property holds for chordless cycles, which resolves the major open question concerning the Erdős-Pósa property. Our proof for chordless cycles is constructive: in polynomial time, one can find either $k + 1$ vertex-disjoint chordless cycles, or $ck^2 \log k$ vertices hitting every chordless cycle for some constant c . It immediately implies an approximation algorithm of factor $\mathcal{O}(\text{opt} \log \text{opt})$ for CHORDAL VERTEX DELETION. We complement our main result by showing that chordless cycles of length at least ℓ for any fixed $\ell \geq 5$ do not have the Erdős-Pósa property.

We also consider an edge version of Erdős-Pósa property for chordless cycles (this time, triangle is also considered to be a chordless cycle) of length at least ℓ . Partial results are obtained for different values of ℓ .

This talk is based on joint work with O-joung Kwon, Pierre Aboulker and Valia Mitsou. (Preprint is available at <https://arxiv.org/abs/1711.00667>.)

Uniquely restricted (g, f) -factors

Miklós Krész

InnoRenew CoE & UP IAM, Slovenia

University of Szeged, Hungary

The concept of (g, f) -factors is a classical generalization of matchings in graphs. Given an undirected graph G , let g and f be nonnegative integer-valued functions defined on the vertex set V of G with $g(v) \leq f(v) \leq \deg_G(v)$ for all $v \in V$, where $\deg_G(v)$ represents the degree of vertex

v in G . Then a (g, f) -factor is defined as a subgraph H of G with $g(v) \leq \deg_H(v) \leq f(v)$ for every $v \in V$.

One of the relevant characteristics of a (g, f) -factor is the so-called *degree pattern*: a vector p_H of the degrees of the vertices with a preliminary fixed order of the vertices. A (g, f) -factor H is *uniquely restricted* if the degree pattern of any other (g, f) -factor in G is different from p_H .

The above concept was originally introduced for matchings by M. C. Golumbic, T. Hirst and M. Lewenstein in 2001. They have proved that finding a maximum uniquely restricted matching is NP-hard; consequently research in the recent year mainly focused on special cases of this problem. Another approach was considered by V. E. Levit and E. Mandrescu in a paper from 2003, when posed the question whether it is polynomially solvable for a graph G , if all maximum matchings are uniquely restricted. For the above question Penso et al provided recently (Journal of Graph Theory, 2018, 89[.]) a positive answer.

In this talk we will show that the above results can be extended to (g, f) -factors.

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On the End-Vertex Problem of Graph Searches

Matjaž Krnc

University of Primorska, Koper, and Institute of Mathematics, Physics and Mechanics, Ljubljana, Slovenia

End vertices of graph searches can exhibit strong structural properties and are crucial for many graph algorithms. The problem of deciding whether a given vertex of a graph is an end-vertex of a particular search was first introduced by Corneil, Köhler, and Lanlignel in 2010. There they showed that this problem is in fact \mathcal{NP} -complete for LBFS on weakly chordal graphs. A similar result for BFS was obtained by Charbit, Habib and Mamcarz in 2014. Here, we prove that the end-vertex problem is \mathcal{NP} -complete for MNS on weakly chordal graphs and for MCS on general graphs. Moreover, building on previous results, we show that this problem is linear for various searches on split and unit interval graphs.

Preprint available at <https://arxiv.org/abs/1810.12253>.

Immersion of transitive tournaments

William Lochet

University of Bergen

A classical result by Mader shows the existence of a function g such that for every k , every graph with minimum degree at least $g(k)$ contains a subdivision of the complete graph on k vertices. An interesting topic is to understand possible generalisations of this result to digraphs. In 1985, Thomassen proved the existence of digraphs with arbitrarily large minimum outdegree and without even directed cycle. This implies in particular that these digraphs do not contain \overleftrightarrow{K}_3 as subdivision. In 1985 however, Mader asked the existence of a function f such that digraphs with minimum degree at least $f(k)$ contain a subdivision of the **transitive** tournament on k vertices. This conjecture remains completely open, as the existence of $f(5)$ is still unknown. In 2011, DeVos et al. proposed a weakening of this conjecture asking for immersions instead of subdivisions. In this talk, we will present a proof of this conjecture.

Avoidable Vertices, Avoidable Edges, and Implications for Highly Symmetric Graphs

Martin Milanič

University of Primorska

A vertex v in a graph G is said to be avoidable if every induced two-edge path with midpoint v is contained in an induced cycle. Generalizing Dirac's theorem on the existence of simplicial vertices in chordal graphs, Ohtsuki et al. proved in 1976 that every non-null graph has an avoidable vertex. In a different generalization, Chvátal et al. gave in 2002 a characterization of graphs without long induced cycles based on the concept of simplicial paths.

We introduce the concept of avoidable induced paths as a common generalization of avoidable vertices and simplicial paths. We propose a conjecture that would unify the results of Ohtsuki et al. and of Chvátal et al. The conjecture states that every graph that has an induced k -vertex path also has an avoidable k -vertex path.

We prove that every graph with an edge has an avoidable edge, thus establishing the case $k = 2$ of the conjecture. We point out a close relationship between avoidable vertices in a graph and its minimal triangulations, and observe that the proved cases of the conjecture have interesting consequences for highly symmetric graphs: in a vertex-transitive graph every induced two-edge path closes to an induced cycle, while in an edge-transitive graph every three-edge path closes to a cycle and every induced three-edge path closes to an induced cycle.

Joint work with Jesse Beisegel, Maria Chudnovsky, Vladimir Gurvich, and Mary Servatius.

Explicit 3-colorings for exponential graphs

Alantha Newman

CNRS and Université Grenoble-Alpes

For a graph H and integer $k \geq 1$, two functions f, g from $V(H)$ into $\{1, \dots, k\}$ are adjacent if for all edges uv of H , $f(u) \neq g(v)$. The graph of all such functions is the exponential graph K_k^H . El-Zahar and Sauer proved that if $\chi(H) \geq 4$, then K_3^H is 3-chromatic. Tardif showed that, implicit in their proof, is an algorithm for 3-coloring K_3^H whose time complexity is polynomial in the size of K_3^H . Tardif then asked if there is an "explicit" algorithm for finding such a coloring: Essentially, given a function f belonging to a 3-chromatic component of K_3^H , can we assign a color to this vertex in time polynomial in the size of H ? In this talk, we present such an algorithm, answering Tardif's question affirmatively. Our algorithm yields an alternative proof of the theorem of El-Zahar and Sauer that the categorical product of two 4-chromatic graphs is 4-chromatic.

Joint work with Adrien Argento and Pierre Charbit. Preprint available at: <https://arxiv.org/abs/1808.08691>.

Deciding and Optimizing over Degree Sequences

Shmuel Onn

Technion - Israel Institute of Technology

We consider the problem of deciding if a given vector is the degree sequence of some (hyper)graph and the more general problem of finding a (hyper)graph maximizing the sum of given functions at vertices evaluated at their degrees.

The decision problem for graphs is easy by a well known Erdos-Gallai theorem. For hypergraphs we show it is hard, solving a very long standing open problem.

The optimization problem for graphs is easy if all vertex functions are the same. But otherwise little is known. I will mention some of the many open problems and hope to stimulate research on this vastly open fresh area.

Partially based on the paper: SIAM J. Disc. Math. 32:2067–2079, 2018.

Minimal separators in graph classes dened by small forbidden induced subgraphs

Nevena Pivač

University of Primorska

Minimal separators in graphs are an important concept in algorithmic graph theory. In particular, many problems that are NP-hard for general graphs are known to become polynomial-time solvable for classes of graphs with a polynomially bounded number of minimal separators. Several well-know graph classes have this property, including chordal graphs, permutation graphs, circular-arc graphs, and circle graphs. We perform a systematic study of the question which classes of graphs defined by small forbidden induced subgraphs have a polynomially bounded number of minimal separators. We focus on sets of forbidden induced subgraphs with at most four vertices and obtain an almost complete dichotomy, leaving open only two cases.

Joint work with Martin Milanič, University of Primorska. Preprint available at <https://arxiv.org/abs/1903.04534>.

Algorithmic Aspects of the Finite Extension Problem

Miguel Pizaña

Universidad Autónoma Metropolitana

We say that a graph G is an *extension* of another graph H whenever $N_G(x) \cong H$ for every $x \in G$, i.e. when the subgraph induced by the open neighborhood of any vertex of G is isomorphic to H . For example, the octahedron is an extension of the 4-cycle and the icosahedron is an extension of the 5-cycle. It is easy to verify the the 3-path does not have any extension.

The *finite extension problem* (FEP, also known as the Trahtenbrot-Zykov problem) consist in deciding whether a given finite graph H has a finite extension. It is not known whether the FEP is algorithmically decidable, but several closely related problems are known to be undecidable, namely: the *infinite extension problem* (G must be infinite), the *extension problem* (G can be either finite or infinite), the *mixed finite extension problem* (neighborhoods are required to be isomorphic to some H_i in a given finite set of finite graphs $\mathcal{H} = \{H_1, H_2, \dots, H_r\}$) and the *2-mixed finite extension problem* (as before but with $|\mathcal{H}| = r = 2$). All these results where obtained by V. K. Bulitko.

In this talk we shall present procedure (we do not know whether it terminates on all inputs) that do find extensions of graphs for small enough graphs. For instance it is capable of finding an extension of the icosahedron of order 40 (the icosahedron has exactly 3 extensions of orders 40, 60 and 120). The procedure uses quite a range of algorithmic techniques including: backtracking, branch and bound, priority queues, dynamic programming, simplex method, isomorphism reduction, symmetry exploitation, and (exact) heuristic methods.

All the programming was done in GAP (<https://www.gap-system.org/>) plus YAGS (<http://xamanek.izt.uam.mx/yags/>). YAGS is an unofficial GAP package for graph manipulation developed by the authors of this work and others.

This is a joint work with R. Villarroel-Flores and N. García-Colín.

Layered wheels

Ni Luh Dewi Sintari
CNRS, LIP, ENS de Lyon

We present a construction called layered wheel. Layered wheels are graphs of arbitrarily large tree-width and girth. They might be an outcome for a possible theorem characterizing graphs with large tree-width in term of their induced subgraphs (while such a characterization is well understood in term of minors).

Layered wheels also provide examples of graphs of large tree-width in well studied classes, such as (theta, triangle)-free graphs and (even-hole, K_4)-free graphs (where a hole is a chordless cycle of length at least 4, a theta is a graph formed by three internally vertex-disjoint paths of length at least 2 between the same pair of distinct vertices so that the union of any two induces a hole, and K_4 is the complete graph on 4 vertices). For these two classes, we have a stronger result, that layered wheels also have arbitrarily large rank-width. For (even-hole, K_4)-free graphs, this answers a question by K. Cameron, S. Chaplick, and C.T. Hoàng (whether the tree-width or the clique-width of an even-hole-free graph is bounded by a function of its clique number). On the positive sides, we prove that the path-width of layered-wheels are bounded by some logarithmic function of the size of their vertex set.

Hence, we believe that layered wheels could be important in a structural description of these classes. To support this idea we prove that some classes have bounded tree-width, namely graphs with no theta, triangle, and k -wheel, and graphs with no even hole, K_4 , pyramid, and k -wheel (where a k -wheel is a graph formed by a hole and k vertices in which each of the vertices has at least 3 neighbors in the hole, and a pyramid is a graph formed by three internally vertex-disjoint paths between a triangle and a vertex so that the union of any two paths induces a hole).

Joint work with Nicolas Trotignon.

Invited Special Session

SYMMETRIES OF GRAPHS AND MAPS

Organized by MARSTON CONDER

Skew morphisms of simple groups

Martin Bachraty
University of Auckland

Skew morphisms, which generalise automorphisms for groups, are related to regular Cayley maps, and also to finite groups with a complementary factorisation $G = BC$, where C is cyclic and core-free in G . In this talk, I will focus on the case when B is monolithic (meaning that it has a unique minimal normal subgroup, and that subgroup is not abelian). In particular, we will give a complete classification when B is simple.

The author is grateful for the support from the VEGA Research Grant 1/0238/19.
Joint work with Marston Conder and Gabriel Verret.

Regular bi-oriented maps of negative prime characteristic

Antonio Breda d'Azevedo
University of Aveiro

Bi-oriented maps (or pseudo-oriented maps) were introduced by Wilson in the seventies to describe the non-orientable maps with the property that opposite orientations can consistently be assigned to adjacent vertices. In contrast to orientability, which is both a combinatorial and topological property, bi-orientability is only a combinatorial property. In this talk we speak about bi-oriented maps and the classification of the regular bi-oriented maps (those with automorphism groups acting regularly on darts) of negative prime Euler characteristic. Unlike other classification results for highly symmetric maps on such surfaces we do not use the Gorenstein-Walter result on the structure of groups with dihedral Sylow 2-subgroups.

Joint work with Domenico Catalano and Josef Shiran.

Print available at <https://link.springer.com/article/10.1007/s00026-019-00421-2>

Strong map-symmetry of $SL(3,K)$ and $PSL(3,K)$ for any finite field K .

Domenico Catalano
University of Aveiro

We discuss properties of regular oriented maps with automorphism group $SL(3,K)$ or $PSL(3,K)$, where K is a finite field.

Observations and answers to questions about edge-transitive maps

Marston Conder
University of Auckland, New Zealand

There are 14 classes of edge-transitive maps, determined by the effect of the automorphism group. In this talk I will make some observations about these classes, and answer three open questions from a 2001 paper by Širáň, Tucker and Watkins, by showing that (a) in each of the classes 1 , 2^P , $2^P ex$, 3 , 4^P and 5^P , there exists a self-dual edge-transitive map, (b) there exists an edge-transitive map with simple underlying graph on an orientable surface of genus g for every

integer $g \geq 0$, and (c) there exists an orientable surface that carries an edge-transitive map of each of the 14 classes, and indeed that these three things still hold when we insist that both the map and its dual have simple underlying graph. Also there is at least one group that occurs as the automorphism group of some edge-transitive map in each of the 14 classes.

Every 2-closed group of degree qp^2 has a semiregular element

Ted Dobson

University of Primorska

In 1981, Marušič asked if the automorphism group of every vertex-transitive graph contains permutation which is a product of p -cycles and is without fixed points, where p is prime. Such an element is called a **semiregular element** of order p . The same problem was later posed by Jordan in 1988. A more general form of this conjecture was made by Klin in 1987; he conjectured that every 2-closed group contains a semiregular element of order p . We show that if q and p are distinct primes, then every 2-closed group of degree qp^2 contains a semiregular element of prime order.

Some non-Beauville groups: Why you should always pay attention to what is said at wine receptions

Ben Fairbairn

Birkbeck, University of London

A Beauville surface is a complex surface obtained by considering a well-chosen pair of regular dessins defined by the same group. These have numerous nice properties and they are unusually easy to work with since doing so boils down to finite group theory. Many groups, however, cannot be used in such a construction for various reasons. One such obstruction, that it is easy to find examples of, is that a group cannot be made to act on a well-chosen pair of dessins but can be made to act on a well-chosen set of 4 dessins giving higher dimensional analogues of Beauville surfaces with equally nice properties. What examples are there? What about 3 dessins? What about 5 dessins?

This is joint work with Ludovico Carta.

Arc-transitive bicirculants

Michael Giudici

The University of Western Australia

A graph on $2n$ vertices is a bicirculant if it admits an automorphism that is a permutation with two cycles of length n . For example, the Petersen and Heawood graphs. Arc-transitive bicirculants of valencies three, four and five have previously been classified by various authors. In this talk I will discuss recent joint work with Alice Devillers and Wei Jin that characterises all arc-transitive bicirculants and provides a framework for their complete classification.

Generalized Cayley maps

Robert Jajcay

Comenius University, Bratislava and University of Primorska, Koper

In the case of general (i.e., both orientable and unorientable) maps, a map \mathcal{M} is called *regular* if its full automorphism group (which in the case of an orientable map includes both orientation-preserving and orientation-reversing map automorphisms) acts regularly on the set of flags of \mathcal{M} .

One important limitation to the use of Cayley maps in the general theory of regular maps is the fact that Cayley maps are by definition orientable and the subgroups of their automorphism groups acting regularly on their vertices are always orientation preserving.

Thus, it is only natural to try to generalize the concept to groups acting regularly on the sets of the vertices but not necessarily consisting of orientation preserving automorphisms as well as to introduce a parallel concept within the theory of non-orientable maps.

We present some preliminary results concerning generalized Cayley maps, and summarize related work of Kwak and Kwon.

This is joint work with Yan Wang and Jozef Širáň.

Realisation of groups as automorphism groups of maps and hypermaps

Gareth A. Jones

University of Southampton, UK

It is shown that in various categories, including many consisting of maps or hypermaps, oriented or unoriented, of a given hyperbolic type, or of coverings of a suitable topological space, every countable group A is isomorphic to the automorphism group of uncountably many non-isomorphic objects, infinitely many of which are finite if A is finite. In particular, the latter applies to dessins d'enfants, regarded as finite oriented hypermaps. The objects realising A are obtained as regular coverings by A of certain basic objects with primitive monodromy groups, corresponding to maximal subgroups of triangle groups. A preprint is available at <https://arxiv.org/abs/1807.00547>.

Symmetry breaking in claw-free graphs of small maximum degree

Rafał Kalinowski

AGH University, Krakow, Poland

A colouring c of a graph G *breaks* an automorphism φ of G if φ does not preserve c . The *distinguishing number* $D(G)$ is the least number of colours in a (not necessarily proper) vertex colouring of G that breaks all nontrivial automorphisms of G .

Perhaps the most intriguing open problem in this area is the Infinite Motion Conjecture of Tucker: if every nontrivial automorphism of a connected, locally finite graph G moves infinitely many vertices, then $D(G) \leq 2$. This conjecture has been confirmed for several classes of graphs. In particular, Lehner confirmed it for line graphs.

The well-known theorem of Beineke states that the claw $K_{1,3}$ is one of the nine forbidden subgraphs of line graphs. We prove that, with the exception of three finite graphs, $D(G) \leq 2$ whenever G is a countable, connected, claw-free graph with $\Delta(G) \leq 4$ whose every nontrivial

automorphism moves more than two vertices.

This is joint work with Wilfried Imrich, Monika Pilśniak and Mariusz Woźniak.

Edge-primitive 3-arc-transitive graphs

Carlisle King

University of Western Australia

Let Γ be a finite graph with $G = \text{Aut}(\Gamma)$. We say that Γ is edge-primitive if G acts primitively on the edges of Γ . An s -arc in Γ is an ordered path of length s . We say that Γ is s -arc-transitive if G is transitive on the set of s -arcs of Γ .

In 1981, Weiss proved that there exists no finite s -arc-transitive graph of valency at least 3 for $s \geq 8$. Since then, there has been considerable effort to characterise s -arc-transitive graphs for $s \leq 7$. One interesting family of graphs is that of edge-primitive graphs. Many famous graphs are edge-primitive, such as the Heawood graph and the Higman-Sims graph. In 2011, Li and Zhang classified finite edge-primitive s -arc-transitive graphs for $s \geq 4$. We study the problem of classifying finite edge-primitive 3-arc-transitive graphs. This is joint work with Michael Giudici.

Groups all of whose Haar graphs are Cayley graphs

István Kovács

University of Primorska

A Cayley graph of a group H is a graph Γ such that $\text{Aut}(\Gamma)$ contains a subgroup isomorphic to H acting regularly on $V(\Gamma)$, while a Haar graph of H is a bipartite graph Σ such that $\text{Aut}(\Sigma)$ contains a subgroup isomorphic to H acting semiregularly on $V(\Sigma)$ and the H -orbits are equal to the bipartite sets of Σ . It is well known that Haar graphs of abelian groups are also Cayley graphs. I. Estélyi and T. Pisanski asked which are the finite non-abelian groups all of whose Haar graphs are Cayley graphs (Electron. J. Combin. 23 (2016)). In the talk, I will discuss this question. This is based on joint work with Yan-Quan Feng and Da-Wei Yang.

Square roots of automorphisms of cyclic groups

Young Soo Kwon

Yeungnam University

A skew morphism of a finite group A is a permutation φ on A fixing the identity element of A and for which there exists an integer-valued function π on A such that $\varphi(ab) = \varphi(a)\varphi^{\pi(a)}(b)$ for all $a, b \in A$. If $\pi(a) = 1$ for all $a \in A$, then φ is an automorphism of A . Thus, skew morphisms can be viewed as a generalization of group automorphisms. If φ is not an automorphism but φ^2 is an automorphism, then φ is called a proper square root of automorphism. In this talk, we classify proper square roots of automorphism of the cyclic groups.

Tilings of the Sphere by Almost Equilateral Pentagons

Hoi Ping Luk

The Hong Kong University of Science & Technology

The classification of edge-to-edge tilings of the sphere by congruent pentagons can be divided into three cases in terms of edge: variable edge lengths, equilateral, and almost equilateral. The first two cases have been largely settled by Min Yan and his collaborators. The almost equilateral case (four edges of the same length and the fifth different) is the most difficult one, and earlier techniques are insufficient. We have introduced decision-making algorithms in wxMaxima and new geometric constraints to handle the challenge. We have obtained full classification for almost equilateral pentagons with three distinct angles and partial results for those with five distinct angles. We will discuss the findings which include fundamental objects, Earth Map Tilings, and some special tilings which are not seen in the other pentagon cases. We will also discuss the linkage between Earth Map Tilings and geometric realisation of the duals of certain spherical maps. This talk is based on the joint work with Min Yan of Hong Kong University of Science & Technology and Yohji Akama of Tohoku University.

On external symmetries of Wilson maps

Martin Mačaj

Comenius University, Bratislava, Slovakia

Regular maps are embeddings of graphs or multigraphs on closed surfaces (which may be orientable or non-orientable), in which the automorphism group of the embedding acts regularly on flags. Such maps may admit external symmetries that are not automorphisms of the embedding, but correspond to combinations of well known operators that may transform the map into an isomorphic copy: duality, Petrie duality, and the ‘hole operators’. The group generated by the external symmetries admitted by a regular map is the external symmetry group of the map.

Kaleidoscopic regular maps with trinity symmetry (KRTs) are regular maps which admit both the above dualities and all feasible hole operators. Existence of finite kaleidoscopic regular maps was conjectured for every even valency by Wilson (1976), and proved by Archdeacon, Conder and Širáň (2010). Further, Conder Kwon and Širáň (2013) studied the groups of external symmetries of these maps.

We give more detailed description of groups external symmetries of Wilson’s maps, both as abstract groups and as subgroups of groups of ‘algebraic external symmetries’.

This is a joint work with K. Hriňáková, V. Hucíková and J. Širáň.

Covers of digraphs

Aleksander Malnič

University of Ljubljana and University of Primorska

So far, covers of digraphs have been considered mostly in connection with their spectra, much less, if at all, in connection with studying their symmetry properties. In order to develop a unified theory of graph and digraph covers dealing with isomorphism, equivalence, and lifting groups of automorphisms, one has to work with digraphs containing directed and undirected edges, directed and undirected loops, and semiedges.

But then, covers of such digraphs can behave very differently from covers of graphs or covers of “pure” digraphs (where no arc has its inverse). For instance, a covering projection of digraphs might not induce a covering projection of their underlying graphs; combinatorial description in terms of voltages might not exist; homotopy might not lift (and one has to rethink the concept of homotopy in digraphs, which has not been done so far); this also means that one has to work with the fundamental monoid of all closed walks at a given base vertex and not with the fundamental group – which triggers further questions related to monoid actions (in semigroup theory, it appears, the relevant and rather basic questions that emerge along these lines have not been considered thus far).

In the talk I will present some basic facts related to questions mentioned above.

Patterns of edge-transitive maps

Adnan Melekoğlu

Aydın Adnan Menderes University

A map \mathcal{M} whose automorphism group $\text{Aut}\mathcal{M}$ acts transitively on the set of flags is called regular. Each reflection of \mathcal{M} fixes a number of geodesics on the underlying surface, which are called *mirrors*. Then every mirror passes through some geometric points of \mathcal{M} and these geometric points form a periodic sequence, which is called the *pattern* of the mirror. By geometric points we mean the vertices, edge-centers and face-centers of \mathcal{M} .

Graver and Watkins partitioned edge-transitive maps into fourteen classes, and ten of these classes contain reflexible maps including class 1, which consists of regular maps. In this talk, we determine the patterns of edge-transitive maps.

Infinite vertex-transitive graphs and their arc-types

Rögnvaldur G. Möller

University of Iceland

Let X be a locally finite vertex transitive graph of degree d . and let G be the full automorphism group of X . The *arc-type* of X is defined in terms of the sizes of the orbits of the stabilizer G_v of a given vertex v in X on the set of neighbouring vertices of v . The orbit of w under G_v is *paired* with the orbit of w' if there is an element $g \in G$ such that $gw = v$ and $gv = w'$. The arc-type of X is a partition of d as a sum

$$n_1 + n_2 + \dots + n_t + (m_1 + m'_1) + (m_2 + m'_2) + \dots + (m_s + m'_s)$$

where n_1, n_2, \dots, n_t are the sizes of self-paired orbits and $m_1, m'_1, m_2, m'_2, \dots, m_s, m'_s$ are the sizes of non-self-paired orbits, such that an orbit with size m_i is paired with an orbit with size m'_i . If the graph is finite then paired orbits of G_v have the same size so $m_i = m'_i$, but that need not hold for infinite graphs.

In Kranjska Gora four years ago Arjana Žitnik spoke about her joint work with Marston Conder and Tomaž Pisanski on arc-types of finite vertex-transitive graphs. They showed that every arc-type with $m_i = m'_i$ for all i , except the arc-types $1 + 1$ and $(1 + 1)$, occurs as the arc-type of some finite vertex-transitive graph. Determining all the possible arc-types of an infinite locally finite graph is more complicated since the arc-type $(1 + m)$ for $m \geq 1$ is not realizable in an infinite graph. In this talk, the general problem of finding out which arc-types are realizable for an infinite vertex transitive locally finite graph will be discussed briefly. Then the focus will shift to the arc-type $k + (1 + m)$ for $k, m \geq 2$. This arc-type is realizable and the struggle to

construct examples led to an interesting problem about finite graphs (solved by Luke Morgan, Primož Potočnik and Gabriel Verret) and also led to a natural definition of a free product with amalgamation for graphs.

Joint work with Norbert Seifert, Wolfgang Woess and Sara Zemljič.

The distinguishing number of 2-arc-transitive graphs and permutation groups

Luke Morgan

University of Primorska

The distinguishing number of a permutation group G acting on a set X is the smallest size of a partition of X such that only the identity of G fixes each part of the partition. The symmetric and alternating groups of degree n have distinguishing number n and $n - 1$ respectively. On the other hand, most primitive groups one meets in the wild have distinguishing number two. In fact, a result of Cameron, Neumann and Saxl shows that apart from the symmetric and alternating groups, the distinguishing number of a primitive group is bounded by an absolute constant. In joint work with Alice Devillers and Scott Harper, we looked at a larger class of finite permutation groups, the quasiprimitive and semiprimitive groups to see if this behaviour is generic. As an application of our results, we have considered the distinguishing number of 2-arc-transitive graphs.

Reductions of maps preserving the isomorphism relation I

Roman Nedela

University of West Bohemia

Theorems by Steinitz (1916), Whitney (1933) and Mani (1971) show that the isomorphism problems for polyhedra, for 3-connected planar graphs, and for the spherical maps are closely related. In 1974, Hopcroft and Wong investigated the complexity of the graph isomorphism problem for polyhedral graphs. They proved that the problem can be solved in linear time. We describe a modified linear-time algorithm solving the isomorphism problem for spherical maps based on the approach by Hopcroft and Wong. The algorithm employs a set of operations defined on maps preserving the isomorphism relation. The operations apply to any map, not just to planar ones. With some effort one can generalize the algorithm so that it works for any map on a fixed closed surface. This way we have an algorithm testing the map isomorphism for maps on a fixed surface in linear time. The procedure assigns to any map a locally homogeneous map. In the final step the algorithm compares the two irreducible maps assigned to the input maps. In the planar case the irreducible maps are the Archimedean and Platonic maps, cycles and their duals.

Joint work with Ken-Ichi Kawarabayashi, P. Klavik, B. Mohar and P. Zeman.

Distinguishing colorings of maps

Monika Pilśniak

AGH University, Krakow, Poland

The distinguishing number of a group A acting on a finite set Ω , denoted by $D(A, \Omega)$ is the least k such that there is a k -coloring of Ω which is preserved only by elements of A fixing all points in Ω . For a map M , also called a cellular graph embedding or a ribbon graph, the action of $\text{Aut}(M)$ on the vertex set V gives the distinguishing number $D(M)$. It is known that $D(M) \leq 2$ whenever $|V| > 10$.

The action of $\text{Aut}(M)$ on the edge set E gives the distinguishing index $D'(M)$, which has not been studied before. It is shown that the only maps M with $D'(M) > 2$ are the following: the tetrahedron; the maps in the sphere with underlying graphs C_n , or $K_{1,n}$ for $n = 3, 4, 5$; a map in the projective plane with underlying graph C_4 ; two one-vertex maps with 4 or 5 edges; one two-vertex map with 4 edges; or any map obtained from these maps using duality or Petrie duality. There are 39 maps in all.

This is joint work with Thomas Tucker.

Duality and Chiral-Duality

Daniel Pinto

University of Coimbra

There are two duality operations on oriented hypermaps $\mathcal{H} = (G, x, y)$: one that interchanges the two generators x and y ; and another, that we can call chiral-duality, that also reverses the orientation by sending x to y^{-1} and y to x^{-1} . Since the automorphisms of Δ^+ which induce them are conjugate in $\text{Aut}(\Delta^+)$, both dualities have the same general properties. However, their effect on a specific hypermap might be distinct. This also means that the duality index of a hypermap might be different than the chiral-duality index of that same hypermap. We study the effect of those two operations in some families of hypermaps, and how some results might hold in one case but not in another.

Generalised voltage graphs

Primož Potočnik

University of Ljubljana

If Γ is a graph admitting a group of automorphism G such that the vertex-stabiliser G_v is trivial for every vertex v (that is, G acts semiregularly on the vertex-set of Γ), then it is well known that the quotient projection $\Gamma \rightarrow \Gamma/G$ is a covering projection and the graph Γ can be reconstructed as the covering graph of Γ/G with respect to an appropriate voltage assignment ζ , assigning an element of G to every dart of Γ/G . In such a situation various properties of the graph Γ can often be encoded and studied in a possibly much smaller graph Γ/G endowed with the voltage assignment ζ , and the theory of graph coverings defined via voltage assignments has become one of the central tools in the problems pertaining to symmetries of graphs.

The purpose of this talk is to introduce a similar theory for the case where G is an arbitrary (not necessarily semiregular) group of automorphisms. We show how one can generalise the notion of the voltage assignment on Γ/G in order to encode the complete information of Γ . We also discuss some generalisation of the classical results of the theory of graph covers to this

more general situation and show some application of the this new approach.

The original research presented in this talk is a joint work with Micael Toledo.

Normal quotients of four-valent G -oriented graphs

Nemanja Poznanovic

University of Melbourne

A finite graph Γ is said to be G -oriented (or G -half-arc-transitive) with respect to some group $G \leq \text{Aut}(\Gamma)$ if G acts transitively on the vertices and edges of Γ but does not act transitively on the arcs (ordered pairs of adjacent vertices). These graphs have been studied for several decades in various contexts, with the four-valent case receiving special attention.

Recently, a framework for analysing the four-valent G -oriented graphs using a normal quotient reduction has been developed. Among other things, this approach lets us apply several results from the theory of permutation groups, and in particular, lets us exploit the classification of finite simple groups in order to study these graphs. I will describe this approach and discuss some recent results.

Joint work with Cheryl Praeger.

LDPC codes constructed from cubic symmetric graphs

Marina Šimac

University of Rijeka, Croatia

Low-density parity-check (LDPC) codes were first presented by Gallager in 1962. and since then they have been the subject of much interest and study. In this talk we focus on the construction of LDPC codes using adjacency matrices of cubic symmetric graphs. We will discuss some of the properties of the constructed codes and present bounds for the code parameters. Moreover, information on the constructed codes will be presented. Based on joint work with Dean Crnković and Sanja Rukavina.

On loosely attached tetravalent half-arc-transitive graphs

Primož Šparl

University of Ljubljana and University of Primorska

Half-arc-transitive (vertex- and edge- but not arc-transitive) graphs and graphs admitting half-arc-transitive group actions have been extensively studied in the last three decades. Most of the work was done for the tetravalent graphs which remain to be of primary interest.

One of the reasons for this is probably that in these graphs the so-called alternating cycles and their intersections give a useful insight into the local structure of such graphs. This viewpoint has led to some very important results. For instance, a result by Marušič and Praeger from 1999 and a recent improvement by Ramos Rivera and Šparl shows that all such graphs are either members of the already classified family of so-called tightly attached graphs or are particularly nice cyclic covers of the so-called *loosely attached* graphs in which two non-disjoint alternating cycles meet in a single vertex. This shows that the loosely attached graphs are of extreme importance for a better understanding of all tetravalent half-arc-transitive graphs.

Unfortunately, not much is known about them so far.

In this talk we present some recent results on loosely attached tetravalent half-arc-transitive graphs. We also exhibit, for (almost) each $r \geq 7$ and $s \geq 2$, an infinite family of tetravalent half-arc-transitive loosely attached graphs with radius r and vertex-stabilizers of order 2^s .

Based on joint work with Štefko Miklavič and Steve Wilson.

Cubic graphs with long orbits

Micael Toledo

IMFM and University of Primorska

Let Γ be a graph of order n and let φ be an automorphism of Γ . For a fixed integer k , we say that φ has a long orbit if at least $\frac{n}{k}$ vertices lie in the same orbit under the action of the cyclic group $\langle \varphi \rangle$. We give a general overview of cubic vertex-transitive graphs that admit an automorphism with a long orbit and we completely classify such graphs when $k = 3$.

Joint work with Primož Potočnik.

Surface Symmetry: Kulkarni Revisited

Thomas Tucker

Colgate University

Which finite groups G act on which closed topological surfaces S ? One can either fix G or fix S . One can also prescribe the action on S (for example, preserving orientation or having no reflections). If one fixes G , Kulkarni's classic 1987 paper proves that there is a number $d(G)$ such that if G acts preserving orientation on the surface of genus g , then $g \equiv 1 \pmod{d(G)}$ and that G acts preserving orientation on all but finitely many such surfaces. A corollary is that G acts preserving orientation on all but finitely many closed surfaces if and only if G is almost Sylow-cyclic with Sylow 2-subgroup G_2 cyclic, dihedral, semi-dihedral, or generalized quaternion. We generalize this result to non-orientable surfaces, with or without reflections. We also consider which surfaces S have the property that if G acts on S , then G acts on all but finitely many surfaces. Finally, we show that the dihedral groups D_4, D_6 and their subgroups are the only groups that act on all surfaces.

Reductions of maps preserving the isomorphism relation II

Peter Zeman

Charles University, Prague

This talk is a follow-up to the talk of Roman Nedela. In this talk we describe an algorithm which tests isomorphism of maps on a fixed (possibly non-orientable) surface S in linear time. Precisely, the complexity of the algorithm is $f(g)$ times n , where $f(g)$ is some function of the genus of S and n is the size of the map. We mainly focus on the differences in the algorithm for the spherical case, described in the previous talk, and the algorithm for an arbitrary surface. We also discuss several details which are necessary for the algorithm to run in linear time.

Joint work with Ken-Ichi Kawarabayashi, B. Mohar and R. Nedela.

SPEAKER INDEX

- Abiad, Aida, 138
Aboulker, Pierre, 148
Abreu, Marién, 96
Adriaensen, Sam, 78
Alon, Noga, 22
Ambat, Vijayakumar, 138
Andelic, Milica, 140
Ando, Kiyoshi, 96
Anholcer, Marcin, 118
Antunović, Suzana, 97
- Bachraty, Martin, 156
Bacsó, Gábor, 97
Bailey, Robert F., 68
Ball, Simeon, 78
Ban, Sara, 54
Barát, János, 118
Becerra López, Fernando I., 97
Beisegel, Jesse, 148
Belardo, Francesco, 140
Berman, Leah Wrenn, 48
Blau, Harvey, 28
Blázsik, Zoltán L., 79
Bockting-Conrad, Sarah, 68
Bokal, Drago, 62
Bokowski, Jürgen, 48
Bonvicini, Simona, 98
Bracho, Javier, 132
Breda d’Azevedo, Antonio, 156
Brešar, Boštjan, 72
Brezovnik, Simon, 40
Brunetti, Maurizio, 140
Bujtás, Csilla, 88
Buratti, Marco, 22
- Cabello, Sergio, 62
Catalano, Domenico, 156
Che, Zhongyuan, 40
Chen, Haiyan, 40
Cichacz, Sylwia, 98
Colin de Verdière, Éric, 62
Conder, Marston, 132, 156
Cooper, Jacob, 99
Costa, Simone, 54
- Crnković, Dean, 55
Csajbók, Bence, 80
- Dalfo, Cristina, 141
Dankelmann, Peter, 41
Dantas, Simone, 88
DasGupta, Bhaskar, 124
De Boeck, Maarten, 80
Dębski, Michał, 99
Denaux, Lins, 81
D’haeseleer, Jozefien, 81
Dobson, Ted, 157
Doerr, Daniel, 34
Dorbec, Paul, 72
Došlić, Tomislav, 41
Drgas-Burchardt, Ewa, 118
- Egan, Ronan, 55
Ellingham, Mark, 99
- Fabrici, Igor, 100
Fairbairn, Ben, 157
Farrugia, Alexander, 141
Ferme, Jasmina, 119
Fernandes, Maria Elisa, 132
Fernández, Blas, 68
Fijavž, Gašper, 100
French, Christopher, 28
Fujita, André, 34
Furmańczyk, Hanna, 101
Furtula, Boris, 41
- Garus, Jerzy, 101
Gavoille, Cyril, 63
Gavrilyuk, Alexander, 141
Giudici, Michael, 157
Glasby, Stephen, 82
Goedgebeur, Jan, 101
Gologranc, Tanja, 72
Grech, Mariusz, 102
Gropp, Harald, 49
Grytczuk, Jarosław, 89
Gévay, Gábor, 48
- Hawtin, Dan, 103

- Hecht, Michael, 103, 148
 Hellmuth, Marc, 35
 Henning, Michael A., 73
 Herbst, Lina, 35
 Herman, Allen, 28
 Hernandez Lucas, Lisa, 73
 Hliněný, Petr, 63
 Hoersch, Florian, 103
 Horvát, Szabolcs, 104
 Hubard, Isabel, 132
 Humphries, Stephen, 29
 Huynh, Tony, 148

 Ihringer, Ferdinand, 82
 Imrich, Wilfried, 142
 Iršič, Vesna, 89
 Izquierdo, Milagros, 49

 Jajcay, Robert, 157
 Jakovac, Marko, 73
 James, Tijo, 89
 Janssen, Remie, 36
 Jin, Xian'an, 42
 Jones, Gareth A., 22, 158
 Junnila, Ville, 124

 Kabanov, Vladislav, 55
 Kalinowski, Rafał, 158
 Kang, Cong X., 125
 Katona, Gyula Y., 104
 Kelenc, Aleksander, 125
 Kempner, Yulia, 63
 Kim, Eunjung, 149
 King, Carlisle, 159
 Kiss, György, 83
 Klavžar, Sandi, 74
 Knor, Martin, 42
 Korchmáros, Gábor, 23
 Kos, Tim, 74
 Kovič, Jurij, 49
 Kovács, István, 159
 Kraner Šumenjak, Tadeja, 74
 Krnc, Matjaž, 150
 Král', Daniel, 23
 Krész, Miklós, 149
 Kubicka, Ewa, 119
 Kubicki, Grzegorz, 104
 Kuziak, Dorota, 125
 Kwiatkowski, Mariusz, 105
 Kwon, Young Soo, 159
 Kühn, Daniela, 24

 Labbate, Domenico, 105

 Lafond, Manuel, 36
 Laihonen, Tero, 126
 Lakshmanan S., Aparna, 74
 Lando, Sergei, 24
 Lavrauw, Michel, 83
 Leemans, Dimitri, 133
 Lehner, Florian, 105
 Li, Xueliang, 43, 119
 Lindorfer, Christian, 106
 Liotta, Giuseppe, 64
 Lochet, William, 150
 Luk, Hoi Ping, 160
 Lužar, Borut, 120

 Maceková, Mária, 106
 Mačaj, Martin, 160
 Majstorović, Snježana, 107
 Malnič, Aleksander, 160
 Marc, Tilen, 133
 Marino, Giuseppe, 84
 Martínez-Barona, Berenice, 75
 Mattheus, Sam, 85
 Mattiolo, Davide, 107
 Mazzuocolo, Giuseppe, 107
 Melekoğlu, Adnan, 161
 Merola, Francesca, 56
 Mihailović, Bojana, 142
 Miklavič, Štefko, 68
 Milanič, Martin, 151
 Mochan, Elias, 133
 Mohar, Bojan, 142
 Molinari, Maria Chiara, 108
 Montejano, Luis, 134
 Montero, Antonio, 134
 Mora, Mercè, 126
 Morgan, Luke, 162
 Munemasa, Akihiro, 29
 Muzychuk, Misha, 29
 Möller, Rögnvaldur G., 161

 Nakamoto, Atsuhiko, 108
 Napolitano, Vito, 49
 Nedela, Roman, 162
 Neumaier, Arnold, 69
 Newman, Alantha, 151
 Nøjgaard, Nikolai, 37
 Nyul, Gábor, 109

 Olmez, Oktay, 56
 Onn, Shmuel, 152
 Osthus, Deryk, 109

 Pach, János, 24

- Pagani, Silvia, 109
 Pallozzi Lavorante, Vincenzo, 110
 Pankov, Mark, 110
 Pasotti, Anita, 57
 Patkós, Balázs, 110
 Patra, Kamal Lochan, 142
 Pavčević, Mario Osvin, 57
 Pavlíková, Soňa, 143
 Pellicer, Daniel, 134
 Penjić, Safet, 69
 Pepe, Valentina, 85
 Peterin, Iztok, 75
 Piedade, Claudio Alexandre, 134
 Pilśniak, Monika, 163
 Pinheiro, Sofia J., 111
 Pinto, Daniel, 91, 163
 Pisanski, Tomaž, 49
 Pivač, Nevena, 152
 Pizaña, Miguel, 111, 152
 Pongrácz, András, 91
 Ponomarenko, Ilia, 30
 Potočnik, Primož, 163
 Pott, Alexander, 58
 Poznanovic, Nemanja, 164
 Praeger, Cheryl E., 24
 Przybyło, Jakub, 120
 Puertas, María Luz, 127
 Puleo, Gregory J., 91
 Pálvölgyi, Dömötör, 64
- Rácz, Gabriella, 112
 Rall, Douglas, 120
 Rama, Paula, 143
 Ramírez-Cruz, Yuniór, 127
 Raney, Michael, 50
 Rosenfeld, Moshe, 135
 Ryabov, Grigory, 30
- Sahin, Murat, 112
 Sahoo, Binod Kumar, 112
 Saniga, Metod, 50
 Saumell, Maria, 65
 Schiermeyer, Ingo, 121
 Schmitt, John R., 58
 Scholz, Guillaume, 37
 Sciriha, Irene, 144
 Sedlar, Jelena, 43
 Seemann, Carsten R., 37
 Simanjuntak, Rinovia, 129
 Sintuari, Ni Luh Dewi, 153
 Somlai, Gábor, 30
 Sopena, Éric, 92
- Stanić, Zoran, 144
 Stokes, Klara, 50
 Suda, Sho, 31
 Suzuki, Hiroshi, 70, 113
 Szőnyi, Tamás, 85
- Śleszyńska-Nowak, Małgorzata, 121
- Šimac, Marina, 164
 Škrekovski, Riste, 121
 Šparl, Primož, 164
 Štesl, Daša, 92
 Švob, Andrea, 58
- Taniguchi, Tetsuji, 144
 Tararenko, Andrej, 129
 Tepeh, Aleksandra, 76
 Terwilliger, Paul, 70
 Tiwary, Hans Raj, 65
 Toledo, Micael, 165
 Traetta, Tommaso, 59
 Tratnik, Niko, 43
 Tucker, Thomas, 165
 Tuza, Zsolt, 25
 Tyc, Adam, 113
- Vasil'ev, Andrey, 31
 Vegi Kalamar, Alen, 65
 Veldsman, Stefan, 113
 Vesel, Aleksander, 114
 Vidali, Janoš, 32
 Vizer, Mate, 114
 Vučićić, Tanja, 59
- Wang, Jianfeng, 44, 145
 Wassermann, Alfred, 59
 Weiss, Asia Ivić, 135
 West, Douglas B., 92
 Wicke, Kristina, 38
 Williams, Gordon, 135
 Wilson, Steve, 136
 Wissing, Pepijn, 145
- Xiao, Chuanqi, 114
- Ye, Dong, 114
 Yero, Ismael G., 129
 Yi, Eunjeong, 130
- Zeman, Peter, 165
 Zerafa, Jean Paul, 115
 Zhang, Heping, 44
 Zhu, Xuding, 25
 Zieschang, Paul-Hermann, 32

Žak, Bogdan, 115

Žigert Pleteršek , Petra, 45

Žitnik, Arjana, 51



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