

Subgraphs of Kneser graphs with large girth and large chromatic number

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Received 13 October 2021, accepted 18 October 2022, published online 9 December 2022

Abstract

It is well known that for any integers k and g , there is a graph with chromatic number at least k and girth at least g . In 1960s, Erdős and Hajnal conjectured that for any k and g , there exists a number $h(k, g)$, such that every graph with chromatic number at least $h(k, g)$ contains a subgraph with chromatic number at least k and girth at least g . The Erdős-Hajnal Conjecture has been recently proposed in the setting of the fractional chromatic number. In support of the original and the modified conjecture, we prove that every Kneser graph, whose (fractional) chromatic number is sufficiently large, contains a subgraph of large girth, whose (fractional) chromatic number is large.

Keywords: Kneser graph, graph coloring, chromatic number, fractional chromatic number.

Math. Subj. Class.: 05C15

*Corresponding author. Supported in part by the NSERC Discovery Grant R611450 (Canada), by the Canada Research Chairs program, and by the Research Project J1-2452 of ARRS (Slovenia). On leave from IMFM, Jadranska 19, 1000 Ljubljana.

†Part of this work was done while the author was a PIMS Postdoctoral Fellow at the Department of Mathematics, Simon Fraser University, Burnaby, B.C.

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Podgrafi Kneserjevih grafov z veliko ožino in velikim kromatskim številom

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Prejeto 13. oktobra 2021, sprejeto 18. oktobra 2022, objavljeno na spletu 9. decembra 2022

Povzetek

Dobro znano je, da za poljubni celi števili k in g obstaja graf s kromatskim številom najmanj k in ožino najmanj g . V 1960-ih sta Erdős in Hajnal postavila domnevo, da za poljubni števili k in g obstaja tako število $h(k, g)$, da vsak graf, katerega kromatsko število je najmanj $h(k, g)$, vsebuje podgraf, katerega kromatsko število je najmanj k , ožina pa najmanj g . Erdős-Hajnalova domneva je bila nedavno na novo formulirana s pojmi frakcijskega kromatskega števila. V podporo izvirni in modificirani domnevi dokažemo, da vsak Kneserjev graf, katerega (frakcijsko) kromatsko število je dovolj veliko, vsebuje podgraf velike ožine, katerega (frakcijsko) kromatsko število je veliko.

Ključne besede: Kneserjev graf, barvanje grafov, kromatsko število, frakcijsko kromatsko število.

Math. Subj. Class.: 05C15

*Kontaktni avtor. Delno podprt s strani raziskovalne štipendije NSERC R611450 (Kanada), s strani Canada Research Chairs programa, in v okviru raziskovalnega projekta J1-2452 agencije ARRS (Slovenija). IMFM, Jadranska 19, 1000 Ljubljana.

†Del te raziskave je bil opravljen, ko je bil avtor podoktorski študent PIMS na Department of Mathematics, Simon Fraser University, Burnaby, B.C.

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