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## A note on domination and independence-domination numbers of graphs

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### Abstract

Vizing's conjecture is true for graphs  $G$  satisfying  $\gamma^i(G) = \gamma(G)$ , where  $\gamma(G)$  is the *domination number* of a graph  $G$  and  $\gamma_i(G)$  is the *independence-domination number* of  $G$ , that is, the maximum, over all independent sets  $I$  in  $G$ , of the minimum number of vertices needed to dominate  $I$ . The equality  $\gamma^i(G) = \gamma(G)$  is known to hold for all chordal graphs and for chordless cycles of length  $0 \pmod{3}$ . We prove some results related to graphs for which the above equality holds. More specifically, we show that the problems of determining whether  $\gamma^i(G) = \gamma(G) = 2$  and of verifying whether  $\gamma^i(G) \geq 2$  are NP-complete, even if  $G$  is weakly chordal. We also initiate the study of the equality  $\gamma^i = \gamma$  in the context of hereditary graph classes and exhibit two infinite families of graphs for which  $\gamma^i < \gamma$ .

**Keywords:** Vizing's conjecture, domination number, independence-domination number, weakly chordal graph, NP-completeness, hereditary graph class, IDD-perfect graph.

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# Opomba o dominacijskem številu in neodvisnostno-dominacijskem številu grafov

## Povzetek

Vizingova domneva je pravilna za grafe  $G$ , ki zadoščajo pogoju  $\gamma^i(G) = \gamma(G)$ , kjer je  $\gamma(G)$  *dominacijsko število* grafa  $G$  in  $\gamma^i(G)$  *neodvisnostno-dominacijsko število* grafa  $G$ , to je maksimum po vseh neodvisnih množicah  $I$  v  $G$ , ki vsebujejo minimalno število vozlišč, potrebnih za dominacijo  $I$ . Znano je, da enakost  $\gamma^i(G) = \gamma(G)$  velja za vse strunske grafe in za brezstrunske cikle dolžine  $0 \pmod{3}$ . Dokažemo nekatere rezultate, povezane z grafi, za katere velja  $\gamma^i(G) = \gamma(G)$ . Natančneje, pokažemo, da sta problema ugotavljanja, ali je  $\gamma^i(G) = \gamma(G) = 2$  in verificiranja, ali je  $\gamma^i(G) \geq 2$ , NP-polna, celo če je  $G$  šibko strunski graf. Poleg tega vpeljemo študij enakosti  $\gamma^i = \gamma$  v kontekstu razredov hereditarnih grafov in poiščemo dve neskončni družini grafov, za katere velja  $\gamma^i < \gamma$ .

**Ključne besede:** Vizingova domneva, dominacijsko število, neodvisnostno-dominacijsko število, šibko strunski graf, NP-polnost, hereditaren razred grafov, IDD-popoln graf.