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A note on domination and independence-domination numbers of graphs*

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Abstract

Vizing's conjecture is true for graphs G satisfying $\gamma^i(G) = \gamma(G)$, where $\gamma(G)$ is the *domination number* of a graph G and $\gamma^i(G)$ is the *independence-domination number* of G, that is, the maximum, over all independent sets I in G, of the minimum number of vertices needed to dominate I. The equality $\gamma^i(G) = \gamma(G)$ is known to hold for all chordal graphs and for chordless cycles of length $0 \pmod 3$. We prove some results related to graphs for which the above equality holds. More specifically, we show that the problems of determining whether $\gamma^i(G) = \gamma(G) = 2$ and of verifying whether $\gamma^i(G) \geq 2$ are NP-complete, even if G is weakly chordal. We also initiate the study of the equality $\gamma^i = \gamma$ in the context of hereditary graph classes and exhibit two infinite families of graphs for which $\gamma^i < \gamma$.

Keywords: Vizing's conjecture, domination number, independence-domination number, weakly chordal graph, NP*-completeness, hereditary graph class,* IDD*-perfect graph.*

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1 Introduction

The closed neighborhood $N_G[v]$ of a vertex in a (finite, simple, undirected) graph G is the set consisting of v itself and its neighbors in the graph. A set A of vertices is said to *dominate* a set B if $B \subseteq \cup\{N_G[a] : a \in A\}$. The minimum size of a set of vertices dominating a set A is denoted by $\gamma_G(A)$. A *dominating set* in a graph G is a set D of vertices that dominates $V(G)$. We write $\gamma(G)$ for $\gamma_G(V(G))$. The concept of domination in graphs has been extensively studied, both in structural and algorithmic graph theory, because of its numerous applications to a variety of areas. Domination naturally arises in facility location

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problems, in problems involving finding sets of representatives, in monitoring communication or electrical networks, and in land surveying. The two books [14, 15] discuss the main results and applications of domination in graphs. Many variants of the basic concepts of domination have appeared in the literature. Again, we refer to [14, 15] for a survey of the area, and to [4, 10, 11, 13, 16, 18, 19, 21, 22] for some recent papers on domination and variants thereof.

The *Cartesian product* of two graphs G and H is the graph $G\Box H$ with vertex set $V(G) \times V(H)$ and edge set $\{(u,x)(v,y) : (u,x),(v,y) \in V(G) \times V(H), u = v$ and $xy \in E(H)$, or $x = y$ and $uv \in E(G)\}$. In 1968 Vizing made the following conjecture, according to Brešar et al. [8] "arguably the main open problem in the area of domination theory":

Conjecture 1. For every two graphs G and H, it holds that $\gamma(G\Box H) \geq \gamma(G)\gamma(H)$.

The conjecture is still open and was verified for several specific classes of graphs; see, e.g., [8].

An *independent set* in a graph is a set of pairwise non-adjacent vertices. The *independence-domination number* $\gamma^i(G)$ is the maximum of $\gamma_G(I)$ over all independent sets I in G. The independence-domination number has arisen in the context of matching theory, see, e.g., [2, 20], and was first introduced in the context of domination by Aharoni and Szabó in 2009 [3]. Obviously, $\gamma^i(G) \leq \gamma(G)$, and in general the gap between the two may be large [3]. However, equality holds for:

- *cycles of length* $0 \pmod 3$, and more generally, for graphs that have a set of $\gamma(G)$ vertices with pairwise disjoint closed neighborhoods [17];
- *chordal graphs*, as proved by Aharoni, Berger and Ziv [1] in a result on width and matching width of families of trees.

Recall that a graph G is called *chordal* if it does not contain any induced cycle of length at least 4, and *weakly chordal* if it does not contain any induced cycles of length at least 5 or their complements.

Theorem 2 ([1]). For every chordal graph G, it holds that $\gamma^i(G) = \gamma(G)$.

The independence-domination number is related to Vizing's conjecture via the following result proved by Aharoni and Szabó [3]:

Theorem 3 ([3]). For every two graphs G and H, it holds that $\gamma(G\Box H) \geq \gamma^i(G)\gamma(H)$.

In particular, Vizing's conjecture is true for chordal graphs. More generally, if G is a graph with $\gamma^i(G) = \gamma(G)$ then $\gamma(G\Box H) \geq \gamma(G)\gamma(H)$ for every graph H. In a recent survey paper on Vizing's conjecture [8], Brešar et al. asked what other classes of graphs can be found for which $\gamma^i(G) = \gamma(G)$ for every G in the class.

In this note, we prove some results related to graphs for which the independence-domination number coincides with the domination number. First, using a relationship between the independence-domination number and the notion of a dominating clique, we prove that determining whether $\gamma^i(G) = \gamma(G)$ is NP-hard. More specifically, we show that it is NP-complete to determine whether $\gamma^i(G) \geq 2$, as well as to determine whether $\gamma^i(G) = \gamma(G) = 2$. These results, obtained in Section 2, remain valid for weakly chordal graphs.

In Section 3, we turn our attention to graphs in which the equality $\gamma^i = \gamma$ holds in the hereditary sense. We show that this class, which properly contains the class of chordal graphs, is properly contained in the class of graphs in which all induced cycles are of length $0 \pmod 3$. We do this by constructing an infinite family of graphs in which all induced cycles are of length $0 \pmod 3$ but where the independence-domination number is strictly smaller than the domination number. In conclusion, we propose three related problems.

2 The complexity of computing γ^i and testing $\gamma^i = \gamma$

In this section, we study some computational complexity aspects of computing the indepedence-domination number and comparing it to the domination number. We first recall some notions needed in our proofs. For a graph $G = (V, E)$, we denote by $\overline{G}$ its *complement*, that is, the graph with the same vertex set as G, in which two vertices are adjacent if and only if they are not adjacent in G. A *clique* in a graph is a subset of pairwise adjacent vertices. A dominating set that is also a clique is called a *dominating clique*. We assume familiarity with basic notions of computational complexity (see, e.g., [12]).

Theorem 4. Given a weakly chordal graph G, it is NP-complete to determine whether $\gamma^i(G) \geq 2$.

Proof. To show membership in NP, observe that a short certificate for the fact that $\gamma^i(G) \geq 2$ is any independent set I such that for every vertex $v \in V(G)$, it holds that $I \not\subseteq N_G[v]$.

To show hardness, we make a reduction from the problem of determining whether a given weakly chordal graph contains a dominating clique. This is an NP-complete problem, see, e.g., [6]. Clearly, the problem remains NP-complete if we assume that the input graph G does not have a dominating vertex.

Suppose that we are given a weakly chordal graph G without dominating vertices. We compute its complementary graph $H = \overline{G}$. Since H is also weakly chordal, the theorem follows immediately from the claim below.

Claim: G has a dominating clique if and only if $\gamma^i(H) \geq 2$.

For the forward implication, suppose that G has a dominating clique K. We will show that $\gamma^i(H) \geq 2$ by showing that $\gamma_H(K) \geq 2$. Suppose for a contradiction that $\gamma_H(K) = 1$. Then, there exists a vertex $v \in V(H) = V(G)$ such that $K \subseteq N_H[v]$. In particular, v must belong to K, since otherwise in G, vertex v would not have any neighbors in K, contrary to the assumption that K is dominating in G. Since K is independent in H, that facts that $v \in K$ and $K \subseteq N_K[v]$ imply that $K = \{v\}$, that is, v is a dominating vertex in G, which is impossible since we assumed that G has no dominating vertices. Hence, it holds that $\gamma_H(K) \geq 2$ and consequently $\gamma^i(H) \geq 2$.

For the converse implication, suppose that $\gamma^i(H) \geq 2$, and let I be an independent set in H such that $\gamma_H(I) \geq 2$. Clearly, I is a clique in G, and, in fact, a dominating clique: If this were not the case, then there would exist a vertex $v \in V(G) \setminus I$ such that in G, vertex v is not adjacent to any vertex from I. Equivalently, for every $u \in I$, $uv \in E(H)$. But then $\{v\}$ would dominate I in H, contrary to the assumption that $\gamma_H(I) \geq 2$. $\square$

Corollary 5. Given a (weakly chordal) graph G and an integer k, it is NP-hard to determine whether $\gamma^i(G) \geq k$.

Corollary 6. Given a (weakly chordal) graph G and an integer k, it is NP-hard to determine whether $\gamma^i(G) \leq k$.

How difficult it is to determine whether the values of γ^i and γ coincide? Since $\gamma^i(G) \leq \gamma(G)$ holds for every graph G, in order to show that

$$\gamma^i(G) = \gamma(G) = k\,, \tag{2.1}$$

it suffices to argue that $\gamma^i(G) \geq k$ and $\gamma(G) \leq k$. Clearly, for $k = 1$, whether (2.1) holds can be determined in polynomial time: a necessary and sufficient condition for $\gamma^i(G) = \gamma(G) = 1$ is that G has a dominating vertex.

We now show that already for $k = 2$, the problem becomes NP-complete, even for weakly chordal graphs. The proof will also imply intractability of the problem of verifying whether $\gamma^i = \gamma$.

Theorem 7. Given a weakly chordal graph G, it is NP-complete to determine whether $\gamma^i(G) = \gamma(G) = 2$.

Proof. Membership in NP follows from the fact that a short certificate for $\gamma^i(G) = \gamma(G) = 2$ is given by a pair (I, D) where I is an independent set not dominated by any vertex (proving $\gamma^i(G) \geq 2$) and D is a dominating set of size two (proving $\gamma(G) \leq 2$).

To show hardness, we make a reduction from 3-SAT [12]. The reduction is an adaptation of the reduction by Brandstädt and Kratsch [6] used to prove that the dominating clique problem is NP-complete for weakly chordal graphs.

Suppose that we are given an instance to 3-SAT, that is, a Boolean formula φ over variables $x_1, \ldots, x_n$, consisting of m clauses of length 3, say $C_i = x_{i_1}^{\alpha_{i_1}} \vee x_{i_2}^{\alpha_{i_2}} \vee x_{i_3}^{\alpha_{i_3}}$ for $i = 1, \ldots, m$, where $\alpha_{i_j} \in \{0, 1\}$, with the usual interpretation that $x_i^1 = x_i$ and $x_i^0 = \overline{x_i}$. Without loss of generality, we may assume the following properties of the formula:

Property 1: No clause contains both a literal and its negation. (This is because clauses containing both a literal and its negation can be discarded as they will always be satisfied.)

Property 2: There exist two clauses, say C_1 and C_2, that have no literals in common. (If the given formula φ does not have this property, we simply add to it a new clause consisting of three new variables. If necessary, we relabel the clauses.)

Consider the graph H defined as follows:

$$\begin{aligned} V(H) &= \{x_1, \overline{x_1}, \ldots, x_n, \overline{x_n}\} \cup \{C_1, \ldots, C_m\}\,, \\ E(H) &= \{x_i^{\alpha_1} x_j^{\alpha_2} \,|\, 1 \leq i, j \leq n,\ i \neq j,\ \alpha_1, \alpha_2 \in \{0, 1\}\} \cup \\ &\quad\ \{x_i^{\alpha} C_j \,|\, 1 \leq i \leq n,\ 1 \leq j \leq m,\ \alpha \in \{0, 1\},\ x_i^{\alpha} \text{ is a literal in } C_j\}\,. \end{aligned}$$

We complete the reduction by computing the complementary graph $G = \overline{H}$.

Using Property 1, it is easy to verify that neither H nor G contain an induced cycle of length at least 5, that is, G is weakly chordal. Moreover, the following properties are equivalent:

(i) φ is satisfiable.

(ii) H has a dominating clique.

(iii) $\gamma^i(G) = \gamma(G) = 2$.

(iv) $\gamma^i(G) = \gamma(G)$.

The equivalence between (i) and (ii) has been established in [6].

(ii) implies (iii): Suppose that H has a dominating clique. Since H has no dominating vertex, similar arguments as in the proof of Theorem 4 allow us to conclude that $\gamma^i(G) \geq 2$. Furthermore, by Property 1 and by construction of H, vertices C_1 and C_2 have no common neighbors in H. This implies that $\{C_1, C_2\}$ is a dominating set in G. Therefore $\gamma(G) \leq 2$, and the conclusion follows since $2 \leq \gamma^i(G) \leq \gamma(G) \leq 2$.

Trivially, (iii) implies (iv).

(iv) implies (ii): Suppose that $\gamma^i(G) = \gamma(G)$. Since H has no isolated vertices, G has no dominating vertices. Therefore $\gamma^i(G) = \gamma(G) \geq 2$, and it can be shown, similarly as in the proof of Theorem 4, that H has a dominating clique.

This completes the proof. □

Theorem 8. Given a weakly chordal graph G, it is NP-hard to determine whether $\gamma^i(G) = \gamma(G)$.

Proof. Perform the same reduction as in the proof of Theorem 7 and use the fact that the formula is satisfiable if and only if $\gamma^i(G) = \gamma(G)$. □

3 A hereditary view on $\gamma^i = \gamma$

In this section, we initiate the study of the equality between the domination and independence-domination number of graphs in the context of hereditary graph classes. A graph class is said to be *hereditary* if it is closed under vertex deletions. The family of hereditary graph classes is of particular interest, first of all, since many natural graph properties are hereditary, and second, since hereditary (and only hereditary) classes admit a uniform description in terms of forbidden induced subgraphs. For a set $\mathcal{F}$ of graphs, we say that a graph G is $\mathcal{F}$*-free* if it does not contain an induced subgraph isomorphic to a member of $\mathcal{F}$. The set of all $\mathcal{F}$-free graphs will be denoted by $\mathit{Free}(\mathcal{F})$. Notice that for two sets $\mathcal{F}_1$ and $\mathcal{F}_2$ of graphs, it holds that $\mathit{Free}(\mathcal{F}_1 \cup \mathcal{F}_2) = \mathit{Free}(\mathcal{F}_1) \cap \mathit{Free}(\mathcal{F}_2)$.

Given a hereditary class $\mathcal{G}$, denote by $\mathcal{F}$ the set of all graphs G with the property that $G \notin \mathcal{G}$ but $H \in \mathcal{G}$ for every proper induced subgraph H of G. The set $\mathcal{F}$ is said to be the set of (minimal) forbidden induced subgraphs for $\mathcal{G}$, and $\mathcal{G}$ is precisely the class of $\mathcal{F}$*-free* graphs. The set $\mathcal{F}$ can be either finite or infinite, and many interesting classes of graphs can be characterized as being $\mathcal{F}$-free for some family $\mathcal{F}$. Such characterizations can be useful for establishing inclusion relations among hereditary graph classes, and were obtained for numerous graph classes (see, e.g. [7]). The most famous such class is probably the class of *perfect graphs*, for which the forbidden induced subgraph characterization is given by the Strong Perfect Graph Theorem conjectured by Berge in 1961 [5] and proved by Chudnovsky, Robertson, Seymour and Thomas in 2006 [9].

Since Vizing's conjecture holds for graphs G such that $\gamma^i(G) = \gamma(G)$, it would be interesting to determine the largest hereditary class of graphs with this property. Moreover, since recognizing graphs with $\gamma^i = \gamma$ is NP-hard, it would also be interesting to determine whether graphs in which the property $\gamma^i = \gamma$ holds in the hereditary sense can be recognized efficiently. With this motivation in mind, we introduce the class of *independence-domination-domination-perfect graphs*, or shortly, IDD*-perfect graphs*, that is, graphs for which the above equality holds in the hereditary sense:

$$\text{IDD-}\textit{perfect graphs} = \{G \,:\, \gamma^i(H) = \gamma(H) \text{ for every induced subgraph } H \text{ of } G\}\,.$$

We now provide some partial results towards a characterization of IDD-perfect graphs. By Theorem 2, we can immediately relate the class of IDD-perfect graphs to a well studied hereditary subclass of perfect graphs, the class of chordal graphs:

Theorem 9.

$$\textit{Chordal graphs} \subset \textsc{idd}\textit{-perfect graphs}\,.$$

Proof. Since every induced subgraph of a chordal graph is chordal, Theorem 2 implies that the class of IDD-perfect graphs contains the class of chordal graphs. This inclusion is proper since chordless cycles of length congruent to $0 \pmod 3$ are IDD-perfect [17] (but not chordal). □

In the rest of this section, we bound the class of IDD-perfect graphs from above, by exhibiting two infinite families of graphs that do not belong to class of IDD-perfect graphs: the chordless cycles of length not congruent to $0 \pmod 3$ and another graph family, which we describe now. For positive integers $k_1, k_2, k_3 > 1$, let F_{k_1,k_2,k_3} denote the graph obtained from the disjoint union of three cycles C_1, C_2 and C_3 where $|V(C_j)| = 3k_j$ as follows: denoting by $(v_1^j, \ldots, v_{3k_j}^j)$ a cyclic order of vertices of C_j, we identify vertex v_1^2 with vertex $v_{3k_1}^1$, vertex v_1^3 with vertex $v_{3k_2}^2$, and vertex v_1^1 with vertex $v_{3k_3}^3$. See Fig. 1 for an example.

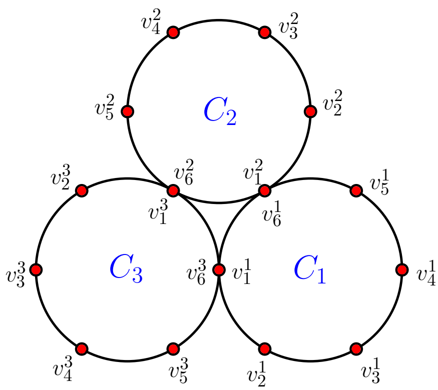

Figure 1: The graph $F_{2,2,2}$

Theorem 10.

$$\textsc{idd}\textit{-perfect graphs} \subseteq \textit{Free}\Bigg(\bigcup_{k\ge 1}\{C_{3k+1}, C_{3k+2}\} \cup \bigcup_{k_1,k_2,k_3>1}\{F_{k_1,k_2,k_3}\}\Bigg)\,.$$

Proof. First, we establish the inclusion IDD-*perfect graphs* $\subseteq$ *Free*$\big(\bigcup_{k\ge1}\{C_{3k+1}, C_{3k+2}\}\big)$. To this end, we show that for every chordless cycle C of order $n = 3k + 1$ or $n = 3k + 2$ (where k is a positive integer), it holds that $\gamma^i(C) = k$ and $\gamma(C) = k + 1$. Let $(v_1, \ldots, v_n)$ be a cyclic order of the vertices of such a cycle C. Observe that for every set $S \subseteq V(C)$ with $|S| \le k$, it holds that

$$|\cup_{v\in S} N_C(v)| \le \sum_{v\in S} |N_C[v]| = 3|S| < n\,.$$

Thus, we immediately have $\gamma(C) \ge k + 1$. On the other hand, the set

$$\{v_{3j-2} : 1 \le j \le k + 1\}$$

is dominating, proving $\gamma(C) = k+1$. Suppose now that I is an independent set in C. We may assume w.l.og. that $v_1 \notin I$. In case that $n = 3k+2$, we may also assume that $v_n \notin I$. In either case, the set $\{v_{3j} : 1 \leq j \leq k\}$ is a set of size k dominating I. This shows that $\gamma^i(C) \leq k$. Conversely, since the set $I = \{v_{3j} : 1 \leq j \leq k\}$ is a set of k vertices with pairwise disjoint closed neighborhoods, we have $\gamma^i(C) \geq \gamma^C(I) = |I| = k$. Thus $k = \gamma^i(C) < \gamma(C) = k+1$ and hence no IDD-perfect graph can contain C as an induced subgraph.

It remains to show that IDD-*perfect graphs* $\subseteq$ *Free*$\left(\bigcup_{k_1,k_2,k_3>1}\{F_{k_1,k_2,k_3}\}\right)$. Equivalently, we must show that for every three integers $k_1, k_2, k_3 > 1$, it holds that $\gamma^i(F_{k_1,k_2,k_3}) < \gamma(F_{k_1,k_2,k_3})$. We will show this in two steps, by computing the exact values of $\gamma^i(F_{k_1,k_2,k_3})$ and $\gamma(F_{k_1,k_2,k_3})$.

Let $F = F_{k_1,k_2,k_3}$ for some $k_1, k_2, k_3 > 1$. First, we show that $\gamma(F) = k_1+k_2+k_3-1$. Consider the set

$$D = \{v^1_{3j-2} : 1 \leq j \leq k_1\} \cup \{v^2_{3j-1} : 1 \leq j \leq k_2\} \cup \{v^3_{3j} : 1 \leq j \leq k_3-1\}\,.$$

Then, D is a dominating set of size $k_1+k_2+k_3-1$, showing that $\gamma(F) \leq k_1+k_2+k_3-1$. Now, we show that $\gamma(F) \geq k_1+k_2+k_3-1$. Suppose for a contradiction that D is a dominating set in F with $|D| \leq k_1+k_2+k_3-2$. Clearly, for every $p \in \{1,2,3\}$, we have that $|D \cap V(C_p)| \geq k_p$. Moreover, D must contain at least $k_p - 1$ vertices from C_p other than v^p_1 and $v^p_{3k_p}$ since otherwise not all vertices in the set $\{v^1_{3p-2} : 2 \leq j \leq k_p\}$ can be dominated by D. This implies that $|D \cap \{v^1_1, v^2_1, v^3_1\}| = 1$. We may assume without loss of generality that $D \cap \{v^1_1, v^2_1, v^3_1\} = \{v^1_1\}$. But this implies that $|D \cap V(C_2)| = k_2 - 1$, a contradiction. Hence $\gamma(F) = k_1+k_2+k_3-1$.

In the rest of the proof, we show that $\gamma^i(F) = k_1+k_2+k_3-2$. Consider the set

$$I = \{v^1_{3j} : 1 \leq j \leq k_1\} \cup \{v^2_{3j-2} : 1 \leq j \leq k_2\} \cup \{v^3_{3j} : 1 \leq j \leq k_3-1\}\,.$$

This is is a set of $k_1+k_2+k_3-2$ vertices with pairwise disjoint closed neighborhoods. Therefore $\gamma^i(F) \geq |I| = k_1+k_2+k_3-2$. To see that $\gamma^i(F) \leq k_1+k_2+k_3-2$, we will verify that $\gamma_F(I) \leq k_1+k_2+k_3-2$ for every independent set I in F. Up to symmetry, it is sufficient to consider the following two cases:

- *Case 1:* $v^1_2 \notin I$.
  In this case, the set
  $$D = \{v^1_{3j-2} : 2 \leq j \leq k_1\} \cup \{v^2_{3j} : 1 \leq j \leq k_2\} \cup \{v^3_{3j-2} : 2 \leq j \leq k_3\}$$
  is a set of size $k_1+k_2+k_3-2$ dominating I.
- *Case 2:* $\{v^1_2, v^1_{3k_1-1}, v^2_2, v^2_{3k_2-1}, v^3_2, v^3_{3k_3-1}\} \subseteq I$.
  In this case, the set
  $$D = \{v^1_1, v^2_1, v^3_1\} \cup \{v^1_{3j-1} : 2 \leq j \leq k_1-1\} \cup \{v^2_{3j-1} : 2 \leq j \leq k_2-1\} \cup \{v^3_{3j-1} : 2 \leq j \leq k_3-1\}$$
  is a set of size $k_1+k_2+k_3-3$ dominating I.

This shows that $k_1+k_2+k_3-2 = \gamma^i(F) < \gamma(F) = k_1+k_2+k_3-1$ and hence no IDD-perfect graph in can contain $F = F_{k_1,k_2,k_3}$ as an induced subgraph.

This completes the proof. □

Remark. Theorem 10 shows that the class of IDD-perfect graphs is not comparable with the class of perfect graphs. On the one hand, the 9-cycle is an IDD-perfect graph that is not perfect. On the other hand, the 4-cycle is a (bipartite, hence) perfect graph that is not IDD-perfect.

4 Conclusion

We conclude this note with three problems related to results from Section 3.

Problem 1. Determine whether every graph of the form F_{k_1,k_2,k_3} is a *minimal* forbidden induced subgraph for the class of IDD-perfect graphs.

Problem 2. Determine the set of minimal forbidden induced subgraphs for the class of IDD-perfect graphs.

Problem 3. Determine the computational complexity of recognizing IDD-perfect graphs.

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