

# IMPROVED HOLT-WINTERS METHOD: A CASE OF OVERNIGHT STAYS OF TOURISTS IN REPUBLIC OF SLOVENIA

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**ABSTRACT:** *Exponential smoothing methods are very commonly used for forecasting demand because they are simple, fast and inexpensive. The Holt-Winters (HW) methods estimate three smoothing parameters, associated with level, trend and seasonal factors. The seasonal variation can be of either an additive or multiplicative form. The multiplicative version is used more widely and on average works better than the additive, but if a data series contains some values equal to zero, the multiplicative HW method may not be used. In this paper we propose an improved additive HW method and we treat the initial values for the level, trend and seasonal components as well as three smoothing constants as decision variables. Through our results we demonstrate that a considerable reduction in forecast error (mean square error) can be achieved. The presented new method is applied to the case of overnight stays of tourists in Republic of Slovenia and comparisons with other methods are made on this case study data.*

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**Keywords:** Demand forecasting, Holt-Winters method, Optimization

**JEL Classification:** C53, C61

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## 1. INTRODUCTION

Exponential smoothing is used frequently throughout the world, because the method is simple, fast and inexpensive. It is particularly suitable for production planning and stock control, where forecasts are made with large numbers of variables (stock accuracy forecasts are particularly important, because excessive forecasts lead to over-stocks and insufficient forecasts lead to stock shortages) (Holt, 2004).

Exponential smoothing methods are a class of methods that produce forecasts with simple formulae, taking into account trend and seasonal effects of the data. These procedures are widely used as forecasting techniques in inventory management and sales forecasting. Some papers (Koehler, Snyder & Ord, 2001; Ord, Koehler & Snyder, 1997) have stimulated renewed interest in the technique, putting exponential smoothing procedures on sound theoretical ground by identifying and examining the underlying statistical models.

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The HW method estimates three smoothing parameters, associated with level, trend and seasonal factors. The seasonal variation can be of either an additive or multiplicative form. The multiplicative version is used more widely and on average works better than the additive (Bermúdez, Segura & Vercher, 2006); of course, if a data series contains some values equal to zero, the multiplicative HW method may not be used. A problem which affects all exponential smoothing methods is the selection of smoothing parameters and initial values, so that forecasts better accord with time series data ("Author", 2010). We estimate smoothing (and initial) parameters in HW methods by minimising the mean square error (MSE). The minimising problem is solved by using Solver (Microsoft Excel 2007).

The aim of this paper is to introduce an improved forecasting method that will provide as good results as the multiplicative HW method and at the same time can be used for a time series containing zero values. In this paper we present an improved HW method and we show that this new method contributes to reduction in the MSE. From the results obtained from real data we prove that the proposed method is more efficient than the ordinary (additive and multiplicative) HW method and consequently we show that the new, improved HW method does not only achieve multiplicative HW method results but can give even better results in the measuring of the MSE.

The remainder of the paper is organized as follows. We begin with the description of a dataset included in our study (see Section 2). In Section 3 we describe Holt-Winters forecasting procedure and we present an improved Holt-Winters procedure. In Section 4, the presented new method is applied to the case of overnight stays of tourists in Republic of Slovenia and comparisons with other methods are made on this case study data. Finally, in conclusions we suggest some further steps of research.

## 2. EMPIRICAL DATA

The improved HW method was first tested on some examples, taken from the monograph *Forecasting: Methods and Applications* (Makridakis, Wheelwright & Hyndman, 1998). If we used the improved HW method instead of additive or multiplicative HW method, the MSE was reduced for all selected examples; especially, for time series with high variations in data.

For research we used quarterly data of overnight stays of domestic (D) and foreign (F) tourists in the Republic of Slovenia between 2000 and 2009, produced by the Statistical Office of the Republic of Slovenia (SI-STAT Data Portal – Economy – Tourism). We selected only those communities that represent time series with seasonality and/or trend and high variations in data. We dealt with 10 non zero time series and 6 intermittent time series for chosen Slovenian communities but in this chapter we will present only one (Medvode-F) in detail.

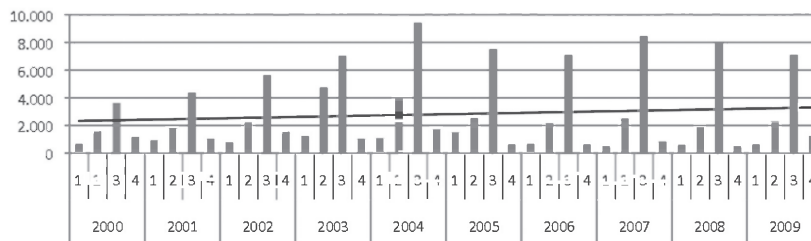
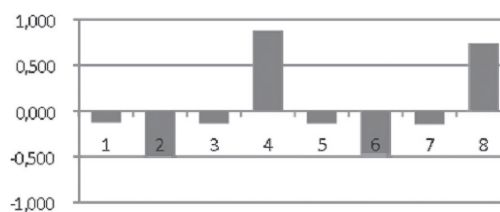
Figure 1: *Overnight stays of foreign guests (Medvode-F) with a trend line.*

Figure 1 represents the original data – the number of overnight stays of foreign guests in the community of Medvode between 2000 and 2009 – and the trend line that is slightly increasing.

Analysed time series exhibits clear seasonal effects. In the period 2000-2009, the demand reached its annual peak in the third quarter (in months of July, August and September) when it was 145% higher than quarterly average (seasonal factor=245). All other quarters were below the quarterly average. The bottom was touched in the first quarter (in months January, February and March) with demand being 69% below the quarterly average (seasonal factor=31). In the fourth quarter demand was 64% below the quarterly average (seasonal factor=36) and in the second quarter the demand was 11% below the quarterly average (seasonal factor=89).

Seasonal effects are also confirmed (see Figure 2) by a positive significant autocorrelation coefficient reaching maximum (0.878) at the lag of 4 quarters and by a negative significant autocorrelation coefficient reaching minimum (-0.505) at the lag of 2 quarters.

Figure 2: *Graph of autocorrelation coefficients.*

From Table 1 we can observe that analysed data represents a time series with relatively high variations in data. Coefficient of variation for original data equals 0.94, for seasonal adjusted series 0.35 and for trend-cycle it is still more than 0.26, which means that the time series includes large noise.

Table 1: Descriptive Statistics for Overnight stays of foreign guests (Medvode-F).

| Time Series                 | N  | Minimum  | Maximum  | Mean     | Std. Deviation | Variance     | Coefficient of variation |
|-----------------------------|----|----------|----------|----------|----------------|--------------|--------------------------|
| Overnight stays (Medvode-F) | 40 | 447.00   | 9,416.00 | 2,811.28 | 2,642.60       | 6,983,312.46 | 0.94                     |
| Seasonal adjusted series    | 40 | 1,245.24 | 5,323.49 | 2,817.80 | 981.97         | 964,273.56   | 0.35                     |
| Trend-cycle                 | 40 | 1,603.95 | 4,348.79 | 2,807.34 | 732.39         | 536,398.74   | 0.26                     |

As a result of such characteristics of the dataset the additive, multiplicative and the improved HW method for forecasting demand were used in our further analysis.

### 3. THE HOLT-WINTERS AND IMPROVED METHOD

The HW method of exponential smoothing involves trend and seasonality and is based on three smoothing equations: for level, for trend and for seasonality. The decision regarding which method to use depends on time series characteristics: the additive method is used when the seasonal component is constant, the multiplicative method is used when the size of the seasonal component is proportional to trend level (Chatfield, 1978).

#### Multiplicative HW method (MHW)

Fundamental equations for level, trend, seasonal factors and forecast are (Makridakis, Wheelwright & Hyndman, 1998):

$$(1) \quad L_t = \alpha \frac{Y_t}{S_{t-s}} + (1 - \alpha)(L_{t-1} + b_{t-1})$$

$$(2) \quad b_t = \beta(L_t - L_{t-1}) + (1 - \beta)b_{t-1}$$

$$(3) \quad S_t = \gamma \frac{Y_t}{L_t} + (1 - \gamma)S_{t-s}$$

$$(4) \quad F_{t+m} = (L_t + b_t m) S_{t-s+m}$$

where  $\alpha$ ,  $\beta$ ,  $\gamma$  are smoothing parameters (which must lie in the interval  $[0,1]$ ),  $m$  is the number of forecast ahead,  $s$  is the length of seasonality (e.g., number of months or quarters in a year) and  $Y_t$  is the observed data at time point  $t$ .

Empirical study shows that the method used to designate the initial vector has very little effect on the accuracy of the predictions obtained (Bermúdez, Segura & Vercher, 2006). To initialize the level, we set  $L_s = (Y_1 + Y_2 + \dots + Y_s) / s$  (in our case  $s = 12$  (months)); to initialize the trend, we use  $b_s = (Y_s + 1 - Y_1) / s$ ; and for initial seasonal indices we calculate  $S_p = Y_p / L_s$ ,  $p = 1, 2, \dots, s$ .

### Additive HW method (AHW)

Fundamental equations for level, trend, seasonal factors and forecast are:

$$(5) \quad L_t = \alpha(Y_t - S_{t-s}) + (1 - \alpha)(L_{t-1} + b_{t-1})$$

$$(6) \quad b_t = \beta(L_t - L_{t-1}) + (1 - \beta)b_{t-1}$$

$$(7) \quad S_t = \gamma(Y_t - L_t) + (1 - \gamma)S_{t-s}$$

$$(8) \quad F_{t+m} = L_t + b_t m + S_{t-s+m}$$

Equation (6) is identical to equation (2). The only differences in the other equations are that the seasonal indices are now added and subtracted instead of taking products and ratios. The initial values for level and trend are identical to those for the multiplicative method. To initialize the seasonal indices we use  $S_p = Y_p - L_p$ ,  $p = 1, 2, \dots, s$ .

### Improved HW method (IHW)

The only difference between the AHW method and the improved additive HW method is in the equation (5) for level, which changes to equation (9), while the equations (6), (7) and (8) - for trend, seasonal factors and forecast - remain unchanged:

$$(9) \quad L_t = \alpha Y_t - S_{t-s} + (1 - \alpha)(L_{t-1} + b_{t-1})$$

For the improved HW method, in contrast to the AHW method, the smoothing parameter  $\alpha$  occurs only at observed data  $Y_t$  and not at seasonal factor  $S_{t-s}$ . This is done to de-seasonalise (eliminate seasonal fluctuations from) the smoothing value of  $Y_t$ . When  $\alpha Y_t > S_{t-s}$  (the smoothed value in period  $t$  is greater than the average in its seasonality in period  $t-s$ ) the level increases in comparison with the level in the earlier period; the opposite adjustment occurs when  $\alpha Y_t < S_{t-s}$ .

The initial values for level, trend and seasonal indices are identical to those for the AHW method.

## 4. FORECAST CALCULATIONS AND RESULTS

We calculate forecasts by using AHW, MHW and the improved HW method and compare results with each other. Regarding the Ferbar Tratar (2010) study we also calculate forecasts for all three methods with additional optimization – smoothing and initial parameters are estimated by minimising the MSE (we use the notation *init*).

In the tables (2-7) we use the following notations:  $s = 4$ ,  $E^2 = (F_t - Y_t)^2$ ,  $MSE = \frac{1}{36} \sum_{t=5}^{40} E^2$ .

We use the first year (first four quarters) for initialization, the following nine years (periods from 5 to 40) represent test series, which is used for minimization of the MSE.

Tables 2 and 3 show calculations of forecasts for overnight stays of foreign guests in Medvode. In Table 2 forecasts are calculated using AHW method, where we estimated (only) smoothing parameters by minimising the MSE. In Table 3 forecasts are calculated using the AHW-init method, where smoothing and initial parameters are estimated by minimising the MSE.

Table 2: *Forecasts calculated with AHW method (Medvode-F).*

| Year               | T   | $Y_t$ | $L_t$    | $b_t$ | $S_t$     | $F_t$    | $E^2$             |
|--------------------|-----|-------|----------|-------|-----------|----------|-------------------|
| 2000               | 1   | 656   |          |       | -1,101.50 |          |                   |
|                    | 2   | 1,569 |          |       | -188.50   |          |                   |
|                    | 3   | 3,628 |          |       | 1,870.50  |          |                   |
|                    | 4   | 1,177 | 1,757.50 | 66.25 | -580.50   |          |                   |
| 2001               | 5   | 908   | 1,849.00 | 66.25 | -958.18   | 722.25   | 34,503.06         |
|                    | 6   | 1,795 | 1,924.53 | 66.25 | -135.84   | 1,726.75 | 4,657.76          |
|                    | 7   | 4,367 | 2,059.53 | 66.25 | 2,260.70  | 3,861.28 | 255,752.36        |
|                    | 8   | 1,020 | 2,054.37 | 66.25 | -985.79   | 1,545.28 | 275,920.83        |
| ...                | ... | ...   | ...      | ...   | ...       | ...      | ...               |
| 2009               | 37  | 616   | 3,481.17 | 66.25 | -2,860.99 | 661.20   | 2,043.41          |
|                    | 38  | 2,297 | 3,572.38 | 66.25 | -1,292.36 | 2,113.40 | 33,708.75         |
|                    | 39  | 7,093 | 3,482.52 | 66.25 | 3,716.68  | 8,241.37 | 1,318,742.51      |
|                    | 40  | 1,248 | 3,634.73 | 66.25 | -2,933.07 | 615.70   | 399,806.21        |
| <b>alpha =</b>     |     |       |          |       |           |          | <b>0.136</b>      |
| <b>beta =</b>      |     |       |          |       |           |          | <b>0.000</b>      |
| <b>gamma =</b>     |     |       |          |       |           |          | <b>0.893</b>      |
| <b>MSE (5-40)=</b> |     |       |          |       |           |          | <b>818,375.50</b> |

Table 3: *Forecasts calculated with AHW-init method (Medvode-F).*

| Year               | T   | $Y_t$ | $L_t$    | $b_t$ | $S_t$     | $F_t$    | $E^2$             |
|--------------------|-----|-------|----------|-------|-----------|----------|-------------------|
| 2000               | 1   | 656   |          |       | -1,187.07 |          |                   |
|                    | 2   | 1,569 |          |       | 25.56     |          |                   |
|                    | 3   | 3,628 |          |       | 2,664.43  |          |                   |
|                    | 4   | 1,177 | 1,942.07 | 24.41 | -1,318.35 |          |                   |
| 2001               | 5   | 908   | 1,986.14 | 24.41 | -1,106.78 | 779.41   | 16,535.31         |
|                    | 6   | 1,795 | 1,973.68 | 24.41 | -124.97   | 2,036.10 | 58,130.20         |
|                    | 7   | 4,367 | 1,952.91 | 24.41 | 2,479.93  | 4,662.52 | 87,332.14         |
|                    | 8   | 1,020 | 2,032.51 | 24.41 | -1,092.94 | 658.96   | 130,346.79        |
| ...                | ... | ...   | ...      | ...   | ...       | ...      | ...               |
| 2009               | 37  | 616   | 2,755.84 | 24.41 | -2,150.62 | 567.61   | 2,341.88          |
|                    | 38  | 2,297 | 2,811.26 | 24.41 | -559.45   | 2,094.15 | 41,149.72         |
|                    | 39  | 7,093 | 2,682.52 | 24.41 | 4,633.65  | 8,094.74 | 1,003,487.05      |
|                    | 40  | 1,248 | 2,820.77 | 24.41 | -2,203.56 | 503.36   | 554,481.96        |
| <b>alpha =</b>     |     |       |          |       |           |          | <b>0.153</b>      |
| <b>beta =</b>      |     |       |          |       |           |          | <b>0.000</b>      |
| <b>gamma =</b>     |     |       |          |       |           |          | <b>0.737</b>      |
| <b>MSE (5-40)=</b> |     |       |          |       |           |          | <b>770,745.12</b> |

As is evident from the results (Tables 2 and 3), better results are gained with the AHW-init method in comparison to the AHW method: the MSE for AHW-init method equals 770,745.12; for the AHW method 818,375.50. This means that the MSE is reduced by almost 6% (see also Table 8).

In Table 4 and 5 we present the forecasts, calculated with the MHW method and MHW-init method respectively.

Table 4: Forecasts calculated with MHW-method (Medvode-F).

| Year                | t   | $Y_t$ | $L_t$    | $b_t$  | $S_t$ | $F_t$    | $E^2$             |
|---------------------|-----|-------|----------|--------|-------|----------|-------------------|
| 2000                | 1   | 656   |          |        | 0.37  |          |                   |
|                     | 2   | 1,569 |          |        | 0.89  |          |                   |
|                     | 3   | 3,628 |          |        | 2.06  |          |                   |
|                     | 4   | 1,177 | 1,757.50 | 66.25  | 0.67  |          |                   |
| 2001                | 5   | 908   | 1,989.28 | 80.30  | 0.39  | 680.73   | 51,652.42         |
|                     | 6   | 1,795 | 2,053.56 | 78.94  | 0.89  | 1,847.60 | 2,766.97          |
|                     | 7   | 4,367 | 2,127.87 | 78.55  | 2.06  | 4,402.10 | 1,231.90          |
|                     | 8   | 1,020 | 2,020.65 | 62.78  | 0.63  | 1,477.64 | 209,435.19        |
| ...                 | ... | ...   | ...      | ...    | ...   | ...      | ...               |
| 2009                | 37  | 616   | 1,970.04 | -40.78 | 0.32  | 941.33   | 105,839.51        |
|                     | 38  | 2,297 | 2,042.17 | -31.20 | 1.02  | 2,911.95 | 378,168.13        |
|                     | 39  | 7,093 | 2,158.83 | -18.65 | 2.90  | 8,251.83 | 1,342,886.10      |
|                     | 40  | 1,248 | 2,533.41 | 14.73  | 0.38  | 1,034.21 | 45,708.23         |
| <b>alpha =</b>      |     |       |          |        |       |          | <b>0.272</b>      |
| <b>beta =</b>       |     |       |          |        |       |          | <b>0.085</b>      |
| <b>gamma =</b>      |     |       |          |        |       |          | <b>0.251</b>      |
| <b>MSE (5-40) =</b> |     |       |          |        |       |          | <b>741,763.48</b> |

Table 5: Forecasts calculated with MHW-init method (Medvode-F).

| Year                | T   | $Y_t$ | $L_t$    | $b_t$ | $S_t$ | $F_t$    | $E^2$             |
|---------------------|-----|-------|----------|-------|-------|----------|-------------------|
| 2000                | 1   | 656   |          |       | 0.41  |          |                   |
|                     | 2   | 1,569 |          |       | 0.89  |          |                   |
|                     | 3   | 3,628 |          |       | 2.32  |          |                   |
|                     | 4   | 1,177 | 1,880.78 | -7.32 | 0.34  |          |                   |
| 2001                | 5   | 908   | 1,948.98 | -7.32 | 0.41  | 768.12   | 19,566.57         |
|                     | 6   | 1,795 | 1,958.23 | -7.32 | 0.89  | 1,728.36 | 4,441.29          |
|                     | 7   | 4,367 | 1,935.69 | -7.32 | 2.32  | 4,526.64 | 25,484.75         |
|                     | 8   | 1,020 | 2,157.58 | -7.32 | 0.34  | 663.63   | 126,998.80        |
| ...                 | ... | ...   | ...      | ...   | ...   | ...      | ...               |
| 2009                | 37  | 616   | 2,084.40 | -7.32 | 0.41  | 1,246.87 | 397,991.74        |
|                     | 38  | 2,297 | 2,188.51 | -7.32 | 0.89  | 2,707.05 | 168,139.04        |
|                     | 39  | 7,093 | 2,375.04 | -7.32 | 2.32  | 7,056.24 | 1,351.49          |
|                     | 40  | 1,248 | 2,646.32 | -7.32 | 0.34  | 1,046.58 | 40,570.42         |
| <b>alpha =</b>      |     |       |          |       |       |          | <b>0.221</b>      |
| <b>beta =</b>       |     |       |          |       |       |          | <b>0.000</b>      |
| <b>gamma =</b>      |     |       |          |       |       |          | <b>0.000</b>      |
| <b>MSE (5-40) =</b> |     |       |          |       |       |          | <b>547,269.28</b> |

From Table 4 and 5 we can observe that better results are gained by using the MHW-init method compared to the MHW method: the MSE for MHW-init method equals 547,269.28; for MHW method 741,763.48. This means that the MSE is reduced by more than 26% (see also Table 8).

In Table 6 and 7 we present the forecasts, calculated with the improved HW method (IHW) and with IHW-init method respectively.

Table 6: Forecasts calculated with IHW method (Medvode-F).

| Year               | t   | $Y_t$ | $L_t$    | $b_t$ | $S_t$     | $F_t$    | $E^2$             |
|--------------------|-----|-------|----------|-------|-----------|----------|-------------------|
| 2000               | 1   | 656   |          |       | -1,101.50 |          |                   |
|                    | 2   | 1,569 |          |       | -188.50   |          |                   |
|                    | 3   | 3,628 |          |       | 1,870.50  |          |                   |
|                    | 4   | 1,177 | 1,757.50 | 66.25 | -580.50   |          |                   |
| 2001               | 5   | 908   | 2,663.22 | 66.25 | -1,227.35 | 722.25   | 34,503.06         |
|                    | 6   | 1,795 | 2,650.59 | 66.25 | -316.92   | 2,540.97 | 556,477.80        |
|                    | 7   | 4,367 | 1,318.50 | 66.25 | 2,097.27  | 4,587.34 | 48,550.22         |
|                    | 8   | 1,020 | 1,860.89 | 66.25 | -630.63   | 804.25   | 46,545.99         |
| ...                | ... | ...   | ...      | ...   | ...       | ...      | ...               |
| 2009               | 37  | 616   | 4,938.68 | 66.25 | -3,209.82 | -167.19  | 613,393.95        |
|                    | 38  | 2,297 | 5,758.00 | 66.25 | -1,900.03 | 1,249.42 | 1,097,421.23      |
|                    | 39  | 7,093 | 991.56   | 66.25 | 5,370.08  | 7,973.04 | 774,469.50        |
|                    | 40  | 1,248 | 2,683.90 | 66.25 | -1,545.54 | 1,205.65 | 1,793.88          |
| <b>alpha =</b>     |     |       |          |       |           |          | <b>0.286</b>      |
| <b>beta =</b>      |     |       |          |       |           |          | <b>0.000</b>      |
| <b>gamma =</b>     |     |       |          |       |           |          | <b>0.193</b>      |
| <b>MSE (5-40)=</b> |     |       |          |       |           |          | <b>621,079.87</b> |

Table 7: Forecasts calculated with IHW-init method (Medvode-F).

| Year               | T   | $Y_t$ | $L_t$    | $b_t$ | $S_t$     | $F_t$    | $E^2$             |
|--------------------|-----|-------|----------|-------|-----------|----------|-------------------|
| 2000               | 1   | 656   |          |       | -1,145.02 |          |                   |
|                    | 2   | 1,569 |          |       | -214.40   |          |                   |
|                    | 3   | 3,628 |          |       | 2,590.31  |          |                   |
|                    | 4   | 1,177 | 1,684.60 | 89.10 | -939.83   |          |                   |
| 2001               | 5   | 908   | 2,492.87 | 89.10 | -1,235.64 | 628.68   | 78,021.67         |
|                    | 6   | 1,795 | 2,409.25 | 89.10 | -296.78   | 2,367.57 | 327,831.12        |
|                    | 7   | 4,367 | 827.25   | 89.10 | 2,785.93  | 5,088.66 | 520,793.34        |
|                    | 8   | 1,020 | 1,907.17 | 89.10 | -928.98   | -23.47   | 1,088,836.52      |
| ...                | ... | ...   | ...      | ...   | ...       | ...      | ...               |
| 2009               | 37  | 616   | 3,964.71 | 89.10 | -2,708.77 | 131.33   | 234,908.30        |
|                    | 38  | 2,297 | 4,059.52 | 89.10 | -1,053.82 | 1,804.12 | 242,933.87        |
|                    | 39  | 7,093 | -29.80   | 89.10 | 5,935.04  | 8,300.85 | 1,458,893.37      |
|                    | 40  | 1,248 | 2,197.27 | 89.10 | -1,428.79 | 1,120.81 | 16,177.24         |
| <b>alpha =</b>     |     |       |          |       |           |          | <b>0.492</b>      |
| <b>beta =</b>      |     |       |          |       |           |          | <b>0.000</b>      |
| <b>gamma =</b>     |     |       |          |       |           |          | <b>0.206</b>      |
| <b>MSE (5-40)=</b> |     |       |          |       |           |          | <b>535,270.19</b> |



From Table 6 and 7 we can observe that better results are gained with the IHW-init method compared to the IHW method: the MSE for the IHW-init method equals 535,270.19; for the IHW method 621,079.87. This means that the MSE is reduced by almost 14% (see also Table 8).

If we compare the improved HW method with the AHW (MHW) method, MSE is reduced by more than 24% (16%). So, if we use the improved HW method with initial optimization (IHW-init) instead of the AHW (MHW) method, the MSE can be reduced by more than 34% (27%) (see also Table 8).

Table 8: Review of results for different communities (none zero time series).

| Community               | MSE          | Improvement (in %) |                                |                              |                     |
|-------------------------|--------------|--------------------|--------------------------------|------------------------------|---------------------|
|                         |              | IHW/<br>A(M)HW     | A(M,I)<br>HW-init/<br>A(M,I)HW | IHW-init/<br>A(M)HW<br>-init | IHW-init/<br>A(M)HW |
| Brda-AHW                | 35,676.22    | -15.77%            |                                |                              | 36.28%              |
| Brda-AHW-init           | 25,676.77    |                    | 28.03%                         | 11.46%                       |                     |
| Brda-MHW                | 50,925.71    | 16.83%             |                                |                              | 55.36%              |
| Brda-MHW-init           | 22,782.78    |                    | 55.26%                         | 0.22%                        |                     |
| Brda-IHW                | 42,355.00    |                    |                                |                              |                     |
| Brda-IHW-init           | 22,733.75    |                    | 46.33%                         |                              |                     |
| Dobrna-AHW              | 6,116,327.74 | 8.64%              |                                |                              | 49.09%              |
| Dobrna-AHW-init         | 3,955,478.85 |                    | 35.33%                         | 21.28%                       |                     |
| Dobrna-MHW              | 4,497,040.04 | -19.52%            |                                |                              | 30.76%              |
| Dobrna-MHW-init         | 3,146,752.41 |                    | 30.03%                         | 1.05%                        |                     |
| Dobrna-IHW              | 5,587,653.21 |                    |                                |                              |                     |
| Dobrna-IHW-init         | 3,113,754.61 |                    | 44.27%                         |                              |                     |
| Hrpelje-Kozina-AHW      | 575,116.75   | 8.22%              |                                |                              | 30.56%              |
| Hrpelje-Kozina-AHW-init | 420,002.10   |                    | 26.97%                         | 4.91%                        |                     |
| Hrpelje-Kozina-MHW      | 530,708.52   | 0.54%              |                                |                              | 24.75%              |
| Hrpelje-Kozina-MHW-init | 404,106.56   |                    | 23.86%                         | 1.17%                        |                     |
| Hrpelje-Kozina-IHW      | 527,836.82   |                    |                                |                              |                     |
| Hrpelje-Kozina-IHW-init | 399,371.55   |                    | 24.34%                         |                              |                     |
| Komen-AHW               | 6,197.67     | 14.93%             |                                |                              | 35.65%              |
| Komen-AHW-init          | 4,037.17     |                    | 34.86%                         | 1.21%                        |                     |
| Komen-MHW               | 8,573.13     | 38.50%             |                                |                              | 53.48%              |
| Komen-MHW-init          | 4,286.47     |                    | 50.00%                         | 6.95%                        |                     |
| Komen-IHW               | 5,272.29     |                    |                                |                              |                     |
| Komen-IHW-init          | 3,988.42     |                    | 24.35%                         |                              |                     |
| Kranj-AHW               | 306,279.15   | -15.82%            |                                |                              | 38.65%              |
| Kranj-AHW-init          | 198,239.33   |                    | 35.27%                         | 5.21%                        |                     |
| Kranj-MHW               | 278,940.20   | -23.34%            |                                |                              | 32.64%              |
| Kranj-MHW-init          | 195,502.81   |                    | 29.91%                         | 3.89%                        |                     |
| Kranj-IHW               | 363,852.42   |                    |                                |                              |                     |
| Kranj-IHW-init          | 187,903.88   |                    | 48.36%                         |                              |                     |

| Community             | MSE            | Improvement (in %) |                                |                              |                     |
|-----------------------|----------------|--------------------|--------------------------------|------------------------------|---------------------|
|                       |                | IHW/<br>A(M)HW     | A(M,I)<br>HW-init/<br>A(M,I)HW | IHW-init/<br>A(M)HW<br>-init | IHW-init/<br>A(M)HW |
| Litija-TUJ-AHW        | 8,697.44       | 5.17%              |                                |                              | 38.28%              |
| Litija-TUJ-AHW-init   | 5,658.42       |                    | 34.94%                         | 5.14%                        |                     |
| Litija-TUJ-MHW        | 8,385.01       | 1.64%              |                                |                              | 35.98%              |
| Litija-TUJ-MHW-init   | 5,815.28       |                    | 30.65%                         | 7.70%                        |                     |
| Litija-TUJ-IHW        | 8,247.62       |                    |                                |                              |                     |
| Litija-TUJ-IHW-init   | 5,367.79       |                    | 34.92%                         |                              |                     |
| Ljutomer-TUJ-AHW      | 192,437.24     | 5.40%              |                                |                              | 43.68%              |
| Ljutomer-TUJ-AHW-init | 165,449.84     |                    | 14.02%                         | 34.49%                       |                     |
| Ljutomer-TUJ-MHW      | 195,915.75     | 7.08%              |                                |                              | 44.68%              |
| Ljutomer-TUJ-MHW-init | 129,297.45     |                    | 34.00%                         | 16.18%                       |                     |
| Ljutomer-TUJ-IHW      | 182,037.82     |                    |                                |                              |                     |
| Ljutomer-TUJ-IHW-init | 108,380.96     |                    | 40.46%                         |                              |                     |
| Lukovica-TUJ-AHW      | 68,916.47      | -27.11%            |                                |                              | 21.55%              |
| Lukovica-TUJ-AHW-init | 56,963.49      |                    | 17.34%                         | 5.09%                        |                     |
| Lukovica-TUJ-MHW      | 74,524.53      | -21.17%            |                                |                              | 27.45%              |
| Lukovica-TUJ-MHW-init | 57,141.52      |                    | 23.33%                         | 5.38%                        |                     |
| Lukovica-TUJ-IHW      | 94,544.19      |                    |                                |                              |                     |
| Lukovica-TUJ-IHW-init | 54,064.82      |                    | 42.82%                         |                              |                     |
| Maribor-AHW           | 1,262,288.67   | 7.55%              |                                |                              | 30.25%              |
| Maribor-AHW-init      | 937,201.44     |                    | 25.75%                         | 6.05%                        |                     |
| Maribor-MHW           | 1,246,297.47   | 6.36%              |                                |                              | 29.35%              |
| Maribor-MHW-init      | 1,075,392.58   |                    | 13.71%                         | 18.13%                       |                     |
| Maribor-IHW           | 1,166,998.83   |                    |                                |                              |                     |
| Maribor-IHW-init      | 880,456.62     |                    | 24.55%                         |                              |                     |
| Medvode-TUJ-AHW       | 818,375.50     | 24.11%             |                                |                              | 34.59%              |
| Medvode-TUJ-AHW-init  | 770,745.12     |                    | 5.82%                          | 30.55%                       |                     |
| Medvode-TUJ-MHW       | 741,763.48     | 16.27%             |                                |                              | 27.84%              |
| Medvode-TUJ-MHW-init  | 547,269.28     |                    | 26.22%                         | 2.19%                        |                     |
| Medvode-TUJ-IHW       | 621,079.87     |                    |                                |                              |                     |
| Medvode-TUJ-IHW-init  | 535,270.19     |                    | 13.82%                         |                              |                     |
|                       | <b>Average</b> | <b>1.53%</b>       | <b>25.83%</b>                  | <b>12.54%</b>                | <b>35.86%</b>       |
|                       |                | <b>2.32%</b>       | <b>31.70%</b>                  | <b>6.29%</b>                 | <b>36.23%</b>       |
|                       |                |                    | <b>34.42%</b>                  |                              |                     |

From Table 8 we observe the following:

- AHW-init can reduce MSE on average by almost 26% in comparison with AHW
- MHW-init can reduce MSE on average by more than 31% in comparison with MHW
- IHW-init can reduce MSE on average by more than 34% in comparison with IHW
- IHW-init can reduce MSE on average by more than 12% in comparison with AHW-init

- IHW-init can reduce MSE on average by more than 6% in comparison with MHW-init
- IHW-init can reduce MSE on average by 35.86% in comparison with AHW and
- IHW-init can reduce MSE on average by 36.23% in comparison with MHW

In some cases (Brda, Dobrna, Kranj, Lukovica) we can see that the IHW method yields worse results than the AHW and/or the MHW method but when we use the IHW-init method on these time series the results are better than results obtained with the AHW and the MHW method. Even more, if we use the IHW-init method the results are better than results obtained with the AHW-init and the MHW-init method. So, we can conclude: if we use the IHW-init method instead of AHW or MHW method, the MSE is reduced for all cases.

Our goal was to find an improved method that will provide as good results as the multiplicative method and at the same time can be used for time series containing zero values. We showed in cases that the improved HW method does not only achieve multiplicative methods results but in the sense of measuring forecasting errors (MSE) can provide even better results: the IHW-init provides better results than the MHW (MHW-init) on average by more than 36% (6%).

In Table 9 we present results of forecasting for 6 intermittent time series. The table shows the percentage of improvement of the MSE, calculated by using the improved HW (init) method compared to the AHW (init) method (forecasting with the MHW method is not possible, because these time series contain zero values). It is obvious that if we treat the initial values for the level, trend and seasonal components as well as three smoothing constants as decision variables with the improved HW method a considerable reduction in MSE can be achieved.

Table 9: Review of results for different communities (intermittent time series).

| Community                     | MSE          | Improvement (%) |                  |                  |                       |                  |
|-------------------------------|--------------|-----------------|------------------|------------------|-----------------------|------------------|
|                               |              | IHW/<br>AHW     | AHW-init/<br>AHW | IHW-init/<br>IHW | IHW-init/<br>AHW-init | IHW-init/<br>AHW |
| Komenda-D-AHW                 | 5,699.55     |                 |                  |                  |                       |                  |
| Komenda-D-AHW-init            | 3,605.81     |                 | 36.74%           |                  |                       |                  |
| Komenda-D-IHW                 | 6,255.00     | -8.88%          |                  |                  |                       |                  |
| Komenda-D-IHW-init            | 3,455.36     |                 |                  | 44.76%           | 4.17%                 | 39.37%           |
| Komenda-F-AHW                 | 42,076.54    |                 |                  |                  |                       |                  |
| Komenda-F-AHW-init            | 41,395.41    |                 | 1.62%            |                  |                       |                  |
| Komenda-F-IHW                 | 42,138.34    | -0.15%          |                  |                  |                       |                  |
| Komenda-F-IHW-init            | 40,730.58    |                 |                  | 3.34%            | 1.61%                 | 3.20%            |
| Logatec-F-AHW                 | 1,020,874.68 |                 |                  |                  |                       |                  |
| Logatec-F-AHW-init            | 987,429.14   |                 | 3.28%            |                  |                       |                  |
| Logatec-F-IHW                 | 943,746.05   | 7.56%           |                  |                  |                       |                  |
| Logatec-F-IHW-init            | 823,322.34   |                 |                  | 12.76%           | 16.62%                | 19.35%           |
| Lovrenc na Pohorju-D-AHW      | 106.30       |                 |                  |                  |                       |                  |
| Lovrenc na Pohorju-D-AHW-init | 64.99        |                 | 38.86%           |                  |                       |                  |
| Lovrenc na Pohorju-D-IHW      | 109.93       | -3.30%          |                  |                  |                       |                  |

| Community                     | MSE       | Improvement (%) |                  |                  |                       |                  |
|-------------------------------|-----------|-----------------|------------------|------------------|-----------------------|------------------|
|                               |           | IHW/<br>AHW     | AHW-init/<br>AHW | IHW-init/<br>IHW | IHW-init/<br>AHW-init | IHW-init/<br>AHW |
| Lovrenc na Pohorju-D-IHW-init | 57.37     |                 |                  | 47.82%           | 11.73%                | 46.04%           |
| Miren-Kostanjevica-D-AHW      | 11,731.89 |                 |                  |                  |                       |                  |
| Miren-Kostanjevica-D-AHW-init | 10,200.77 |                 | 13.05%           |                  |                       |                  |
| Miren-Kostanjevica-D-IHW      | 11,158.74 | 4.89%           |                  |                  |                       |                  |
| Miren-Kostanjevica-D-IHW-init | 9,335.60  |                 |                  | 16.34%           | 8.48%                 | 20.43%           |
| Miren-Kostanjevica-F-AHW      | 14,913.19 |                 |                  |                  |                       |                  |
| Miren-Kostanjevica-F-AHW-init | 10,952.78 |                 | 26.56%           |                  |                       |                  |
| Miren-Kostanjevica-F-IHW      | 13,645.80 | 8.50%           |                  |                  |                       |                  |
| Miren-Kostanjevica-F-IHW-init | 10,605.29 |                 |                  | 22.28%           | 3.17%                 | 28.89%           |
|                               |           | <b>Average</b>  | <b>20.02%</b>    | <b>24.55%</b>    | <b>7.63%</b>          | <b>26.21%</b>    |

From Table 9 (results for the intermittent demand) we observe the following:

- AHW-init can reduce MSE on average by 20% in comparison with AHW
- IHW-init can reduce MSE on average by more than 24% in comparison with IHW
- IHW-init can reduce MSE on average by 7.63% in comparison with AHW-init
- IHW-init can reduce MSE on average by more than 26% in comparison with AHW

## 5. CONCLUSION AND FURTHER RESEARCH

Demand forecasting is used throughout the world more often because of proper source management and a rising need to plan. Which method is going to be used depends on multiple factors: demanded comprehension of forecasts, further use of forecasts, and available data and price. One of the most commonly used forecasting techniques is exponential smoothing, which is relatively inexpensive, fast and simple and does not require special software. There has been a lot of attention given to the Holt-Winters forecasting procedure in recent years. Researchers have discovered new ways to improve the method itself, especially in dealing with more seasonal cycles and forecasting intervals.

The aim of this paper was to find an improved forecasting method that will provide as good results as the multiplicative method and at the same time may be used for time series containing zero values. We proposed an improved HW method and we showed that a reduction in the forecast error (MSE) can be achieved. We dealt with 10 non zero time series and 6 intermittent time series for chosen Slovenian communities. We calculated forecasts by using AHW, MHW and the improved HW method and compare results with each other. We also calculate forecasts for all three methods with additional optimization – smoothing and initial parameters were estimated by minimising the MSE. We showed in cases that the improved HW method does not only achieve MHW method results but in the sense of measuring MSE can provide even better results. From the results obtained from the real data we proved that the proposed IHW-init method is more efficient than the AHW (MHW) method, on average by more than 35% (36%) for nonzero time series and more than 26% for time series containing zero values.

As is obvious from the given cases, the improved HW method yields good results for data with large fluctuations, therefore it would make sense to examine new methods for time series with multiplicative seasonal fluctuations and/or multiplicative trend. Further research includes a detailed study of the improved HW method for demand patterns with multiplicative trend and seasonality.

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