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Resonances in the Nambu–Jona-Lasinio model

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Abstract. We have designed a soluble model similar to the Nambu–Jona-Lasino model, regularized in a box with periodic boundary conditions, in order to explore the properties of resonances when only discrete eigenvalues are available. The study might give a lesson to similar problems in Lattice QCD.

1 The quasispin NJL-like model

It is very instructive to understand the key features of a simplified model containing the spontaneous chiral symmetry breaking. Some time ago we have constructed a soluble version of the Nambu–Jona-Lasino model [1,2]. Now we explore what it tells about the sigma meson.

We make the following simplifications:

1. We assume a sharp 3-momentum cutoff $0 \leq |\mathbf{p}_i| \leq \Lambda$;
2. The space is restricted to a box of volume $\mathcal{V}$ with periodic boundary conditions. This gives a finite number of discrete momentum states, $\mathcal{N} = N_h N_c N_f \mathcal{V}\Lambda^3/6\pi^2$ occupied by N quarks. (N_h, N_c and N_f are the number of quark helicities, colours and flavours.)
3. We take an average value of kinetic energy for all momentum states: $|\mathbf{p}_i| \to P = \frac{3}{4}\Lambda$.
4. While in the NJL model the interaction conserves the sum of momenta of both quarks we assume that each quark conserves its momentum and only switches from the Dirac level to Fermi level.
5. Temporarily, we restrict to one flavour of quarks, $N_f = 1$.

Let us repeat the "Quasispin Hamiltonian" [1,2].

$$H = \sum_{k=1}^{N} \Big(\gamma_5(k) h(k) P + m_0 \beta(k) \Big) + \\ - \frac{g}{2} \left(\sum_{k=1}^{N} \beta(k) \sum_{l=1}^{N} \beta(l) + \sum_{k=1}^{N} i\beta(k)\gamma_5(k) \sum_{l=1}^{N} i\beta(l)\gamma_5(l) \right) \quad .$$

Here γ_5 and β are Dirac matrices, m_0 is the bare quark mass and $g = 4G/\mathcal{V}$ where G is the interaction strength in the original (continuum) NJL.

We introduce the quasispin operators which obey the spin commutation relations

$$j_x = \frac{1}{2}\beta\,, \quad j_y = \frac{1}{2}i\beta\gamma_5\,, \quad j_z = \frac{1}{2}\gamma_5\,,$$

$$R_\alpha = \sum_{k=1}^{N} \frac{1+h(k)}{2} j_\alpha(k)\,,\ L_\alpha = \sum_{k=1}^{N} \frac{1-h(k)}{2} j_\alpha(k)\,,\ J_\alpha = R_\alpha + L_\alpha = \sum_{k=1}^{N} j_\alpha(k)\,.$$

The model Hamiltonian can then be written as

$$H = 2P(R_z - L_z) + 2m_0 J_x - 2g(J_x^2 + J_y^2) \quad .$$

The three model parameters

$$\Lambda = 648\,\text{MeV}, \quad G = 40.6\,\text{MeV fm}^3, \quad m_0 = 4.58\,\text{MeV}$$

have been fitted (in a Hartree-Fock + RPA approximation) to the observables

$$M = \sqrt{\Big(E_g(N) - E_g(N-1)\Big)^2 - P^2} = 335\,\text{MeV}$$

$$Q = \langle\, g|\bar{\psi}\psi|g\rangle = \frac{1}{\mathcal{V}}\langle\, g|\sum_i \beta(i)|g\,\rangle = \frac{1}{\mathcal{V}}\langle\, g|J_x|g\,\rangle = 250^3\,\text{MeV}^3$$

$$m_\pi = E_1(N) - E_g(N) = 138\,\text{MeV}\,.$$

The values of our model parameters are very close to those of the full Nambu-Jona Lasinio model used by the Coimbra group [3] and by Buballa [4].

2 The spectrum of 0^- and 0^+ excitations – Emergence of the σ meson

It is easy to evaluate the matrix elements of the quasispin Hamiltonian using the angular momentum algebra. If N is not too large the corresponding sparse matrix can be diagonalized using *Mathematica*.

Excited levels of the ground state band (R=L=N/4) in Fig. 1 are almost equidistant and are suggestive of n-pion states (in s-state). The level spacings ΔE are slightly decreasing with the assumed number of pions n_π due to the attractive interaction between pions. Inbetween appear also "intruder states" which can be interpreted as sigma excitations. The interpretation as σ meson is further supported by the large value of the matrix element of J_x between the ground state and the "intruder state". (Odd "multipion states" have zero value and even ones have a rather small value.)

n_π	parity	E [MeV]	ΔE [MeV]	Intruder
8	+	866	63	
	-	816		$\sigma(667)+\pi(136)+13$ MeV
7	-	803	93	
6	+	710	99	
	+	667		$\sigma(667)$
5	-	611	108	
4	+	503	115	
3	-	388	123	
2	+	265	129	
1	-	136	136	
0	+	0	0	

Fig. 1. Levels of the ground state band (R=L=N/4), level spacing between opposite parity states, and the assumed number of pions n_π pions

3 The width of the σ meson

In the attempt to describe resonances when only discrete eigenvalues are available we get a discrete sigma resonance energy, but not its width. We are trying to get the complex pole. For that purpose, we explore the method of analytic continuation from the bound state [5]. For this purpose, we vary one of the model parameters, the bare quark mass m from the region where the σ meson would be bound ($E_\sigma < E_{2\pi}$) down to the physical value of $m \to m_0$ (where $E_\sigma \gg E_{2\pi}$).

At $m > 64$ MeV there are two positive parity states between the first and second negative parity states (the one-pion and three-pion excitations); the lower one is the intruder (σ meson) and the upper one is the correlated two-pion state. At $m = 64$ MeV both positive parity states coincide – the threshold for $\sigma \to 2\pi$. When we decrease m further, the energy of the σ meson decreases slower than the 2π energy and it appears at higher multipion states. For the physical value $m = m_0 = 4.58$ MeV σ is already the sixth excited state, next to the six-pion state. It is obviously in the continuum, prompt to decay into 2π, in a more complete choice of interaction.

The method consists of the following steps:

- Determine the threshold value m_{th} and calculate $\epsilon = E_\sigma - E_{2\pi}$ as a function of m for $m > m_{th}$.
- Introduce a variable $x = \sqrt{m - m_{th}}$; calculate $k(x) = i\sqrt{-\epsilon}$ in the bound state region (Fig. 2).
- Fit $k(x)$ by a polynomial $k(x) = i(c_0 + c_1 x + c_2 x^2 + \ldots + c_{2M} x^{2M})$.

- Construct a Padé approximant:

$$k(x) = i\,\frac{a_0 + a_1 x + \ldots + a_M x^M}{1 + b_1 x + \ldots + b_M x^M}\ .$$

- Analytically continue $k(x)$ to the region $m < m_{th}$ (i.e. to imaginary x) where $k(x)$ becomes complex.
- Determine the position and the width of the resonance as analytic continuation in m (Fig. 3 and Fig. 4):

$$E_{res} = \mathrm{Re}\,(\mathrm{cont}_{m\to m_0} k^2)\,, \qquad \Gamma = -2\,\mathrm{Im}\,(\mathrm{cont}_{m\to m_0} k^2)\ .$$

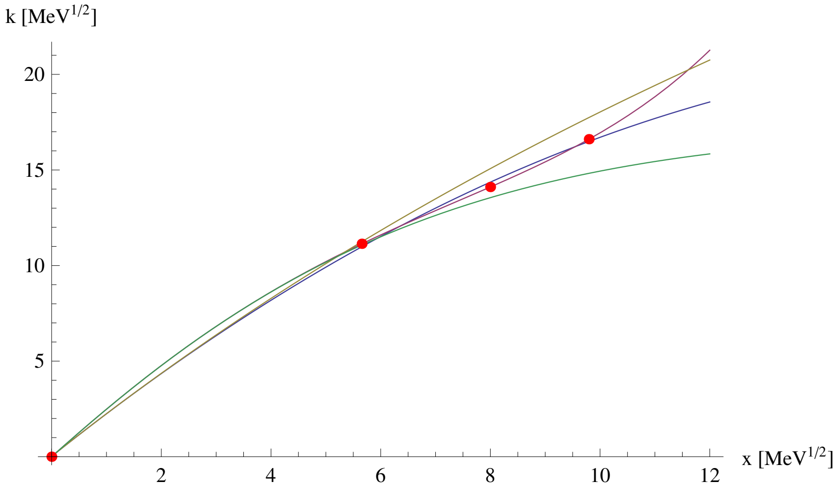

Fig. 2. The fit of $k(x)$ with quadratic(lower middle) and quartic polynomial (upper middle) and with Padé approximants of order 1 (below) and 2 (above)

We notice that the results for E_{res} and Γ in Fig. 3 and 4 deviate strongly for first and second order Padé approximants. This is due to the large stretch for the analytic continuation so that convergence at higher orders cannot be expected. Nevertheless, it is rewarding that the physical values for E_{res} and Γ lie somewhere in the middle between both curves.

To conclude, the method of analytic continuation in this case is just a game, but it is instructive. Intentionally, we have plotted the energy and width of the σ meson as a function of the corresponding pion mass rather than as a function of the model parameter m. This is reminiscent of the extrapolation of pion mass from about 500 Mev towards its physical value the way tha lattice people have to struggle.

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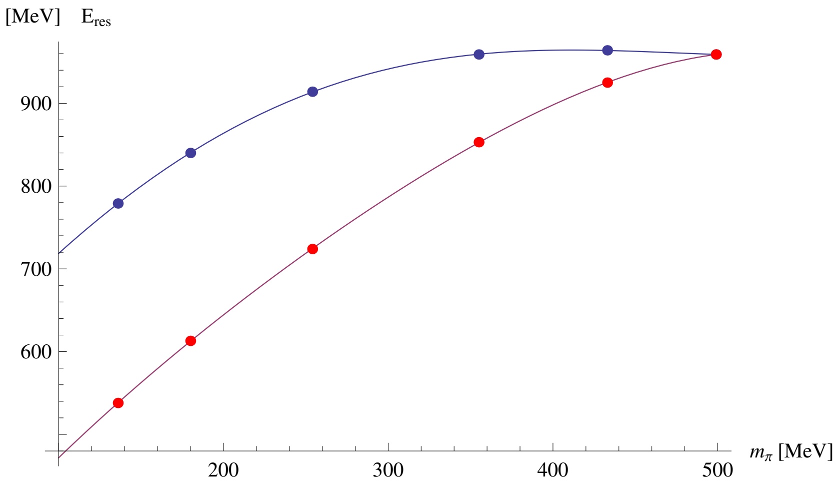

Fig. 3. The resonance energy E_{res} of the σ meson as a function of the pion mass – extrapolation using Padé approximants of order 1 (below) and 2 (above)

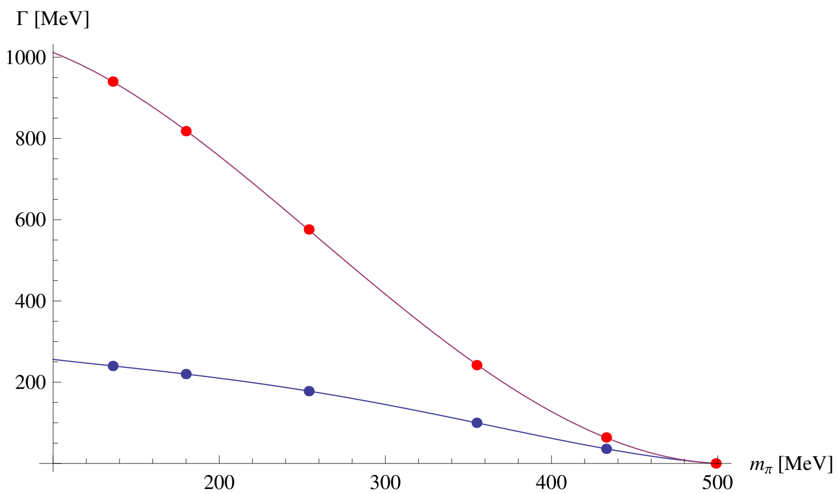

Fig. 4. The width Γ of the σ meson as a function of the pion mass – extrapolation using Padé approximants of order 1 (below) and 2 (above)

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Vektorske in skalarne resonance čarmonija v kromodinamiki na mreži

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Proučujemo sipanje mezonov $\bar{D}$ na D s kromodinamiko na mreži, da bi določili mase in razpadne širine vektorskih in skalarnih resonanc čarmonija nad pragom za razpad v čarobne mezone. V vektorskem kanalu dobimo znano resonanco $\psi(3770)$. Simulacija pri vrednosti pionove mase $m_\pi = 156$ MeV da maso in razpadno širino resonance, ki se ujema z eksperimentalnimi podatki znotraj velike statistične negotovosti. V skalarnem kanalu proučujemo prvo vzbujeno stanje $\chi_{c0}(1P)$, za katero ni zaenkrat nobenega sprejetega kandidata. Za sipanje $\bar{D}$ na D v s-valu raziskujemo razne scenarije. Simulacija nakazuje še neopaženo ozko resonanco z maso malo pod 4 GeV. Potrebne so nadaljnje raziskave, da bi osvetlili uganke pri skalarnih vzbujenih stanjih čarmonija.

Resonance v modelu Nambuja in Jona-Lasinia

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Pred leti smo sestavili rešljivo verzijo modela Nambuja in Jona-Lasinia, ki še vedno ustrezno opiše spontani zlom kiralne simetrije in pojav mezonov pi. Njeni značilnosti sta regularizacija polja v škatli s periodičnimi robnimi pogoji ter poenostavljena kinetična energija in interakcija. Sedaj nas pa zanima opis resonanc, kadar so na voljo le diskretne laste vrednosti energije. Kot zgled navajamo mezon σ. Raziskava je lahko poučna za podobne probleme pri kromodinamiki na mreži.

Roperjeva resonanca — ignoramus ignorabimus?

S. Širca

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V tem prispevku ponudimo kratek pregled nekaterih zadnjih dosežkov na področju raziskav Roperjeve resonance. Naštejemo nekaj najbolj razburljivih eksperimentalnih rezultatov iz centrov MAMI in Jefferson Lab ter drugih laboratorijev; osvetlimo nekaj poskusov, da bi razložili naravo te zagonetne strukture v okviru modelov s kvarkovskimi in mezonskimi ali barionskimi in mezonskimi prostostnimi stopnjami; in odpremo vpogled v znaten napredek, ki so ga v zadnjih letih naredili kromodinamski računi na mreži.