The fullerene graphs with a perfect star packing[*]

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Abstract

Fullerene graph G is a connected plane cubic graph with only pentagonal and hexagonal faces, which is the molecular graph of carbon fullerene. A spanning subgraph of G is called a perfect star packing in G if its each component is isomorphic to $K_{1,3}$. For an independent set $D \subseteq V(G)$, if each vertex in $V(G) \setminus D$ has exactly one neighbor in D, then D is called an efficient dominating set of G. In this paper we show that the number of vertices of a fullerene graph admitting a perfect star packing must be divisible by 8. This answers an open problem asked by Došlić et al. and also shows that a fullerene graph with an efficient dominating set has $8n$ vertices. In addition, we find some counterexamples for the necessity of Theorem 14 of paper of Došlić et al. from 2020 and list some subgraphs that preclude the existence of a perfect star packing of type $P0$.

Keywords: Fullerene graph, perfect star packing, efficient dominating set.

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Fulerenski grafi s popolnim pakiranjem zvezd[*]

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Povzetek

Fulerenski graf G je povezan ravninski kubični graf s samimi peterokotnimi in šesterokotnimi lici, ki predstavlja molekularni graf ogljikovega fulerena. Vpeti podgraf grafa G se imenuje popolno zvezdno pakiranje v grafu G, če je vsaka njegova komponenta izomorfna $K_{1,3}$. Neodvisna množica $D \subseteq V(G)$, v kateri ima vsako vozlišče iz $V(G) \setminus D$ natanko enega soseda v D, se imenuje učinkovita dominantna množica grafa G. V tem članku pokažemo, da mora biti število vozlišč fulerenskega grafa, ki dopušča popolno zvezdno pakiranje, deljivo z 8. To odgovarja na odprt problem, ki so ga zastavili Došlić in dr. in kaže, da ima fulerenski graf z učinkovito dominantno množico $8n$ vozlišč. Pokažemo tudi nekaj protiprimerov za nujnost izreka 14 v članku Došlić in dr. iz leta 2020 in prikažemo nekatere podgrafe, ki izključujejo obstoj popolnega zvezdnega pakiranja tipa $P0$.

Ključne besede: Fulerenski graf, popolno zvezdno pakiranje, učinkovita dominantna množica.

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