

A note on girth-diameter cages*

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Abstract

In this paper we introduce a problem closely related to the Cage Problem and the Degree Diameter Problem. For integers $k \geq 2$, $g \geq 3$ and $d \geq 1$, we define a $(k; g, d)$ -graph to be a

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k -regular graph with girth g and diameter d . We denote by $n_0(k; g, d)$ the smallest possible order of such a graph, and, if such a graph exists, we call it a $(k; g, d)$ -cage. In particular, we focus on $(k; 5, 4)$ -graphs. We show that $n_0(k; 5, 4) \geq k^2 + k + 2$ for all k , and report on the determination of all $(k; 5, 4)$ -cages for $k = 3, 4$ and 5 and of examples with $k = 6$, and describe some examples of $(k; 5, 4)$ -graphs which prove that $n_0(k; 5, 4) \leq 2k^2$ for infinitely many k .

Keywords: Cages, girth, degree-diameter problem.

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Opomba o kletkah z dano ožino in premerom*

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Povzetek

V tem članku predstavimo problem, ki je tesno povezan s problemom kletke in problemom premera in stopnje. Za cela števila $k \geq 2$, $g \geq 3$ in $d \geq 1$ definiramo $(k; g, d)$ -graf kot k -regularen graf z ožino g in premerom d . Z $n_0(k; g, d)$ označimo najmanjši možni

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red takega grafa; če tak graf obstaja, ga imenujemo $(k; g, d)$ -kletka. Še posebej se osredotočamo na $(k; 5, 4)$ -grafe. Pokažemo, da je $n_0(k; 5, 4) \geq k^2 + k + 2$ za vse k , in poročamo o določitvi vseh $(k; 5, 4)$ -kletek za $k = 3, 4$ in 5 ter o primerih s $k = 6$; opišemo tudi nekaj primerov $(k; 5, 4)$ -grafov, ki dokazujejo, da je $n_0(k; 5, 4) \leq 2k^2$ za neskončno mnogo k .

Ključne besede: Kletke, ožina, problem stopnje in premera.

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