



Limits on hadron spectrum from bulk medium properties^{*}

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Abstract. We bring up the fact that the bulk thermal properties of the hadron gas, as measured on the lattice, preclude a very fast rising of the number of resonance states in the QCD spectrum, as assumed by the Hagedorn hypothesis, unless a substantial repulsion between hadronic resonances is present. If the Hagedorn growth continued above masses ~ 1.8 GeV, then the thermodynamic functions would noticeably depart from the measured lattice values at temperatures above 140 MeV, just below the transition temperature to quark-gluon plasma.

In this talk we point out the sensitivity of thermal bulk medium properties (energy density, entropy, sound velocity...) to the spectrum of the hadron resonance gas. In particular, we explore the effects of the high-lying part of the spectrum, above ~ 1.8 GeV, where it is poorly known, on the thermal properties still below the cross-over transition to the quark-gluon plasma phase. Such investigations were carried out in the past by various authors, see [1–5] and references therein, where the reader may find more details and results.

The presently established QCD spectrum reaches about 2 GeV, and it is a priori not clear what happens above. Does the growth continue, or is saturated? As is evident from Fig. 1, the Hagedorn hypothesis [7] works very well up to about 1.8 GeV [8]. In the following, we explore two models: 1) hadron resonance gas with the Breit-Wigner width, HRG(Γ), which takes into account all states listed in the Particle Data Group tables [6] with mass below 1.8 GeV, and 2) this model amended with the states above 1.8 GeV, modeled with the Hagedorn formula fitted to the spectrum at lower masses (see Fig. 1). In short, model 1) includes the up-to-now established resonances, and model 2) extends them according to the Hagedorn hypothesis.

First, we recall the fact that the inclusion of widths of resonances [11], as listed in the Particle Data Group tables, affects the results noticeably and in fact improves them. This is shown in Fig. 2, where the hadron resonance gas calculation for the QCD trace anomaly, $\epsilon - 3p$ divided by T^4 . Here ϵ stands for the energy density, p for the pressure, and T for the temperature. In the calculation,

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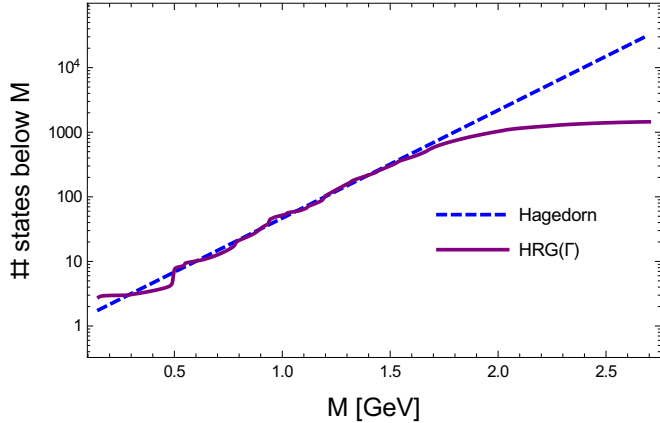


Fig. 1. Solid line, labeled HRG(Γ): Number of QCD states (mesons, baryons, and antibaryons combined) with mass below M . All stable particles and resonances from the Particle Data Group tables [6] are included and their Breit-Wigner width is taken into account. Dashed line: the fit with the Hagedorn formula for the density of states, $\rho(m) = \Lambda \exp(m/T_H)$, with $T_H = 260$ MeV.

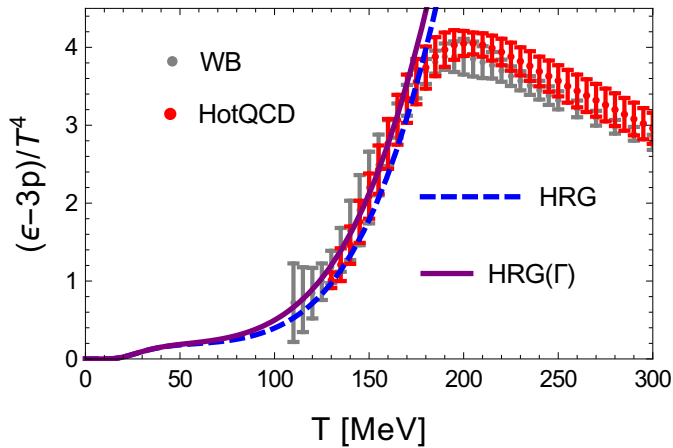


Fig. 2. The QCD trace anomaly (divided by T^4) plotted as a function of temperature T . The inclusion of width of resonances to the hadron resonance gas improves the agreement with the lattice data from the Wuppertal-Budapest (WB) [9] and Hot QCD [10] collaborations.

the hadrons are treated as components of an ideal gas of fermions and bosons. We note that the overall agreement with of the hadron resonance gas model HRG(Γ) with the lattice measurement is remarkable.

The virial expansion of Kamerlingh Onnes yields $p/T = \rho + B_2(T)\rho^2 + B_3(T)\rho^3 + \dots$. Correspondingly, for the partition function of a thermodynamic system including the $1 \rightarrow 1$, $2 \rightarrow 2$, etc., processes one has

$$\ln Z = \ln Z^{(1)} + \ln Z^{(2)} + \dots \quad (1)$$

The non-interacting term

$$\ln Z^{(1)} = \sum_k \ln Z_k^{\text{stable}} = \sum_k f_k V \int \frac{d^3 p}{(2\pi)^3} \ln \left[1 \pm e^{-E_p/T} \right]^{\pm 1} \quad (2)$$

includes the sum over all stable particles, whereas the second-order virial term involves the sum over pairs of stable particles denoted as K ,

$$\ln Z^{(2)} = \sum_K f_K V \int_0^\infty \frac{d\delta_K(M)}{\pi dM} dM \int \frac{d^3 P}{(2\pi)^3} \ln \left[1 \pm e^{-E_P/T} \right]^{\pm 1}, \quad (3)$$

where $\delta_K(M)$ stands for the phase shift in the channel K . For narrow resonances the correction to the density of two-body states $d\delta_K(M)/(\pi dM)$ [12] can be accurately approximated with the Breit-Wigner form, which is a basis of the hadron resonance gas model.

In Fig. 3 we show the result of extending the Hagedorn hypothesis above the present experimental limit on the QCD spectrum. We note that the inclusion of extra (non-interacting) states above $M = 1.8$ GeV has a quite dramatic effect on the trace anomaly $\theta_{\mu}^{\mu} = \epsilon - 3p$, placing it way above the lattice data at $T > 140$ MeV (the model calculation is credible below $T \simeq 170$ MeV, where a cross-over to the quark-gluon plasma occurs). A similar conclusion is drawn for other thermodynamic quantities, such as the entropy (cf. Fig. 4) or the sound velocity (cf. Fig. 5).

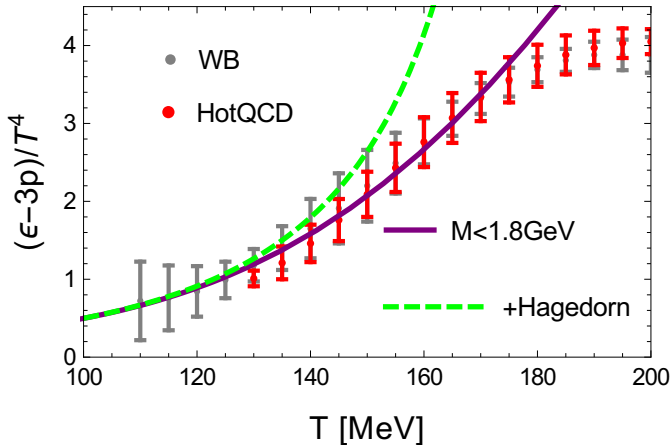


Fig. 3. Same as in Fig. 2 but with the lines denoting the hadron resonance gas model, $\text{HRG}(\Gamma)$, up to $M = 1.8$ GeV, and this model amended with the Hagedorn spectrum above $M = 1.8$ GeV.

Therefore, if the hadron resonances were non-interacting, there would be no room for extra states above 1.8 GeV in the QCD spectrum. This conclusion may be affected by repulsion between the states (e.g., the excluded volume corrections), which decreases the contribution to the partition function. The issue is

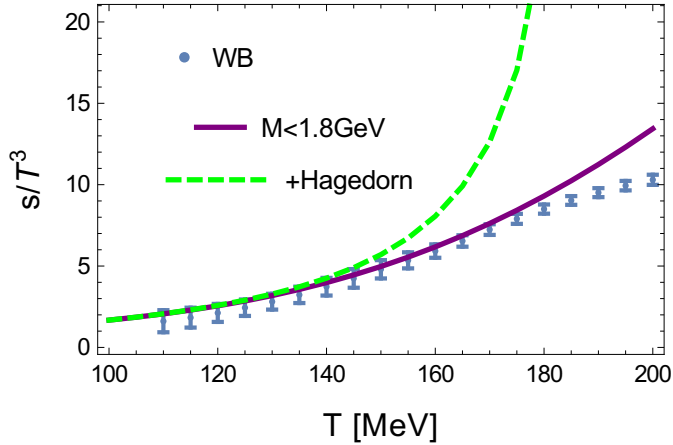


Fig. 4. Same as in Fig. 3 but for the entropy density divided by T^3 .

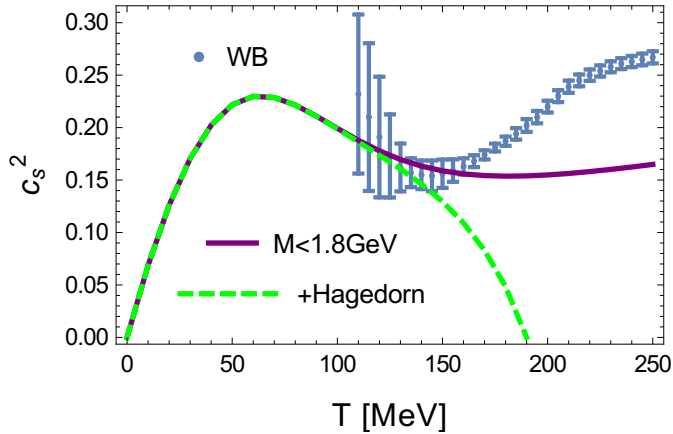


Fig. 5. Same as in Fig. 3 but for the square of the sound velocity.

discussed quantitatively in [3, 4], where a reduction of contributions to the thermodynamic quantities is assessed. The excluded volume reduces the contribution of resonances, and this makes them possible to appear in the spectrum in an “innocuous” way. The effect is explicit in Eq. (3), as repulsion leads to a decrease of the phase shift with M , or a negative correction to the density of states $d\delta_\kappa(M)/(\pi dM)$.

An important example of such an explicit cancellation occurs in the case of the σ meson, whose contribution to one-body observables is canceled by the isospin-2 channel [12]. The case of the trace anomaly is shown in Fig. 6. Note that the phase shift taken into account in this analysis automatically includes the short-distance repulsion in specific channel, hence there is no need to model it separately. The cancellation experienced by the σ state may occur also for other states with higher mass.

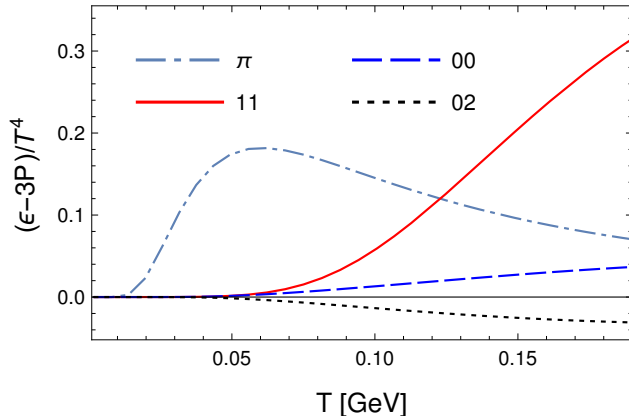


Fig. 6. Contributions to the trace anomaly from the pions, ρ mesons, σ meson, and the isospin-2 component of the pion-pion interaction. We note an almost perfect cancellation of the σ and isospin-2 channels.

In conclusion, the thermodynamic quantities offered by the modern lattice QCD calculations allow to place limits on the high-lying spectrum on the QCD resonances, but the interactions between the states, such as the short-range repulsion, must be properly taken into account, as the two effects: increasing the number of states and introducing repulsion works in the opposite way.

References

1. F. Karsch, K. Redlich, and A. Tawfik, *Eur. Phys. J.* **C29**, 549 (2003), arXiv:hep-ph/0303108 [hep-ph].
2. P. Huovinen and P. Petreczky, *Nucl. Phys.* **A837**, 26 (2010), arXiv:0912.2541 [hep-ph].
3. A. Andronic, P. Braun-Munzinger, J. Stachel, and M. Winn, *Phys. Lett.* **B718**, 80 (2012), arXiv:1201.0693 [nucl-th].
4. E. Ruiz Arriola, L. L. Salcedo, and E. Megias, *54th Cracow School of Theoretical Physics: QCD meets experiment: Zakopane, Poland, June 12-20, 2014*, *Acta Phys. Polon.* **B45**, 2407 (2014), arXiv:1410.3869 [hep-ph].
5. P. M. Lo, M. Marzecenko, K. Redlich, and C. Sasaki, *Phys. Rev.* **C92**, 055206 (2015), arXiv:1507.06398 [nucl-th].
6. K. A. Olive, *Chin. Phys.* **C40**, 100001 (2016).
7. R. Hagedorn, *Nuovo Cim. Suppl.* **3**, 147 (1965).
8. W. Broniowski, W. Florkowski, and L. Ya. Glozman, *Phys. Rev.* **D70**, 117503 (2004), arXiv:hep-ph/0407290 [hep-ph].
9. S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, and K. K. Szabo, *Phys. Lett.* **B730**, 99 (2014), arXiv:1309.5258 [hep-lat].
10. A. Bazavov *et al.* (HotQCD), *Phys. Rev.* **D90**, 094503 (2014), arXiv:1407.6387 [hep-lat].
11. E. Ruiz Arriola, W. Broniowski, and P. Masjuan, *Proceedings, Conference on Modern approaches to nonperturbative gauge theories and their applications (Light Cone 2012): Cracow, Poland, July 8-13, 2012*, *Acta Phys. Polon. Supp.* **6**, 95 (2013), arXiv:1210.7153 [hep-ph].
12. W. Broniowski, F. Giacosa, and V. Begun, *Phys. Rev.* **C92**, 034905 (2015), arXiv:1506.01260 [nucl-th].