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The total weak discrepancy of a partially ordered set

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Abstract

We define the total weak discrepancy of a poset P as the minimum nonnegative integer k for whichthere exists a function $f : V \rightarrow \mathbf{Z}$ satisfying (i) if a \prec b then $f(a) + 1 \leq f(b)$ and (ii) $\Sigma|f(a) - f(b)| \leq k$, where the sum is taken over all unordered pairs $\{a, b\}$ of incomparable elements. If we allow k and f to take real values, we call the minimum k the fractional total weak discrepancy of P. These concepts are related to the notions of weak and fractional weak discrepancy, where (ii) must hold not for the sum but for each individual pair of incomparable elements of P. We prove that, unlike the latter, the total weak and fractional total weak discrepancy of P are always the same, and we give a polynomial-time algorithm to find their common value. We use linear programming duality and complementary slackness to obtain this result.

Keywords: Posets, weak discrepancy, fractional weak discrepancy, total linear discrepancy.

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Popolno šibko neskladje delno urejene množice

Povzetek

Popolno šibko neskladje delno urejene množice P definiramo kot minimalno nenegativno celo število k, za katerega obstaja funkcija $f : V \to \mathbf{Z}$, ki zadošča pogojema: (i) če je a \prec b potem je $f(a) + 1 \leq f(b)$ in (ii) $\Sigma|f(a) - f(b)| \leq k$, kjer seštevamo po vseh neurejenih parih $\{a, b\}$ neprimerljivih elementov. Če smeta k in f zavzeti realne vrednosti, pravimo najmanjšemu številu k *frakcionalno popolno šibko neskladje* delno urejene množice P. Ta dva koncepta sta sorodna pojmoma šibko in racionalno šibko neskladje, kjer mora (ii) veljati ne za vsoto, pač pa za vsak posamični par neprimerljivih elementov množice P. Dokažemo, da sta, za razliko od zadnjega od omenjenih pojmov, popolnoma šibko in racionalno popolnoma šibko neskladje delno urejene množice P vedno enaka, in podamo polinomski algoritem za iskanje njune vrednosti.

Ključne besede: delno urejene množice, šibko neskladje, frakcionalno šibko neskladje, popolno linearno neskladje.