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18 Could Experimental Anomalies Reflect Non-perturbative Effects?

H.B. Nielsen[1] and C.D. Froggatt[2]

[1]Niels Bohr Institute, University of Copenhagen, Blegdamsvej 17,
Copenhagen Ø, Denmark
[2] Glasgow University, Department of Physics and Astronomy
Glasgow University, Glasgow G12 8QQ, Scotland, UK

Abstract. We investigate whether some of the rather few anomalies, in the sense of deviations from the Standard Model, could be explained as due to non-perturbative effects caused by the top-Yukawa-coupling being of order unity (in a sense to be discussed briefly in this article). The main achievement of our non-perturbative rule or model is to relate the deviations of ratios between B-meson decay rates for flavour universality violation for neutral currents to the deviations for the charged current flavour universality violations. In fact the anomaly in the ratio $R(D^*)$ for a charged current with τ and its neutrino relative to the rate with the μ and its neutrino is being related in our model for non-perturbative effects to an analogous effect in a neutral current B-meson decay. It is suggested that the ratio of the anomalous amplitudes contributing to these two combinations of decay processes are to very first approximation given by the squared mass ratio of the heaviest lepton involved in the two ratios, which by their deviation from the Standard Model prediction signal lack of flavour universality.

The muon $g-2$ anomaly also fits well in our non-perturbative model. But we have to mutilate the model somewhat in order to avoid a far too large anomaly prediction for, say B_s-$\bar{B}_s$, particle - antiparticle mixing.

Povzetek. Avtorja v prispevku raziskujeta, ali lahko odstopanja od napovedi Standardnega modela pojasnita z neperturbativnimi efekti, ki se pojavijo, ker so Yukawine sklopitve za top kvark reda ena (v smislu razloženem v prispevku). Povežeta odstopanja med dosedanjimi napovedmi razmerij razpadnih stanj B mezonov za kršitve univerzalnosti tokov za nevtralne in za nabite tokove in rezultati meritev. Odstopanje v razmerju $R(D^*)$ za nabite tokove za delec τ in njegov nevtrino in za delec μ in njegov nevtrino je povezano z analognimi odstopanji v primeru razpadov nevtralnih mezonov B. Predlagata, da je razmerje anomalnih amplitud, ki prispevajo k tem dvem kombinacijam razpadnih procesov, v prvem približku dano s kvadratom razmerij mas najtežjih leptonov v teh razpadih. Odstopanje od napovedi Standardnega modela nakazuje odvisnost od okusa (flavor).

Model je uporabljiv tudi za odstopanja med poskusi in računi za vrednost $g-2$ za mione, denimo za mešanje B_s-$\bar{B}_s$, če model popačita in se tako izogneta velikim odstopanjem.

Keywords: Decay rate anomalies, non-perturbative effects, flavor universality

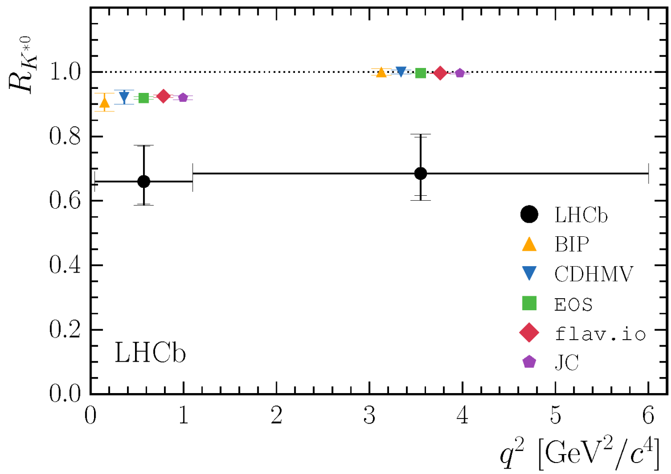
18.1 Introduction

Are the Tensions in LHCb etc data due to Non-perturbative Effects in the Pure Standard Model ?

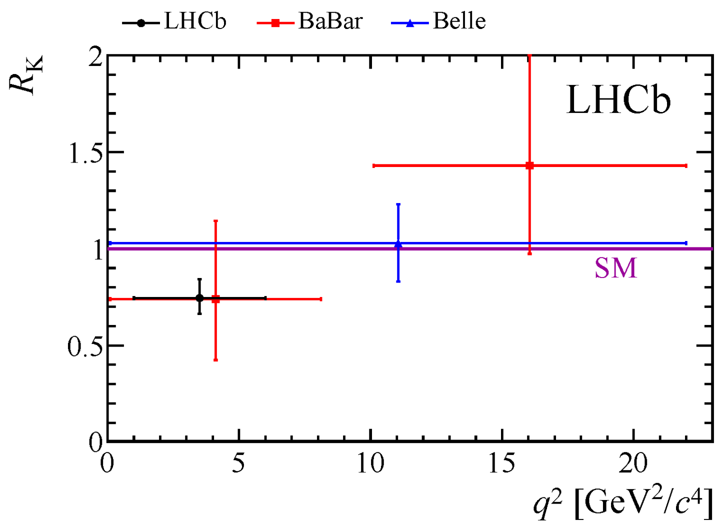
The Standard Model works surprisingly well for LHC physics: Almost no new physics, and at least nothing truly statistically significant! However there is a small number of tensions in the data with a few standard deviations significance: Small lepton universality violating deviations [1–3] , say.

The present proposal is that even these small tensions are not due to genuine new physics, but rather to effects forgotten because of the systematic use of perturbation theory except for the QCD-sector; i.e. the tensions should be non-perturbative effects.

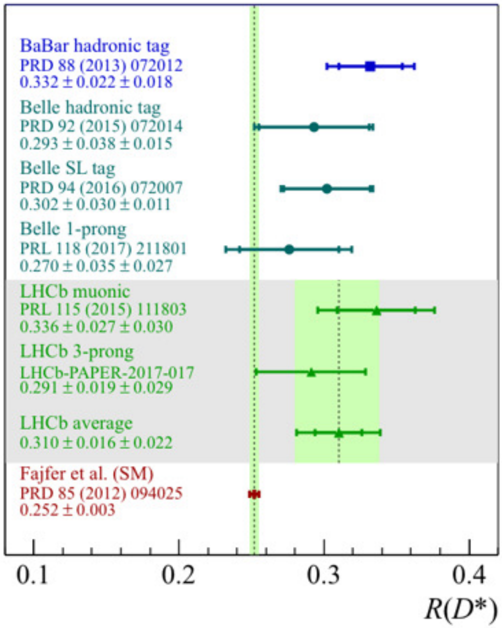
Ratio R_{K*} of $\mu\mu$ versus ee for $B \to K^* l\bar{l}$, anomalous.



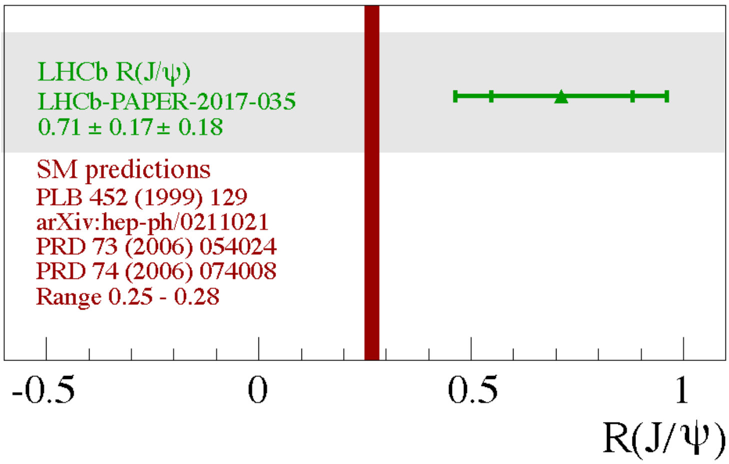
Ratio R_K of $\mu\bar{\mu}$ to $e\bar{e}$ Ratio for $B^+ \to K^+ l\bar{l}$ decay, anomalous for separate q^2?.



Ratio $\tau\nu_\tau$ versus $\mu\nu_\mu$ for $B \to D^*\nu + \text{lepton}$, an anomaly



Ratio $R(J/\Psi)$ of $\tau\nu_\tau$ versus $\mu\nu_\mu$ also in $B \to J/\Psi + \nu + \text{lepton}$ an anomaly.



The two Deviations from SM at LHCb:

In the following table we summarize the two deviations from the Standard Model at LHCb and compare our prediction for the ratio of the corresponding anomaly amplitudes with the data.

Channel	Branch. fraction	"R" Ratio	Deviation relative	Anomaly-amplitude
$B \to K^*\mu^+\mu^-$ neutral c current	10^{-6}	exp. 0.66 SM 1.00	-34 %	$-0.34\sqrt{10^{-6}}/2$ $= -1.7 * 10^{-4}$ $= -1.7 * 10^{-3}\sqrt{\%}$
$B \to D^*\tau\nu_\tau$ charged current	2%	exp. 0.31 SM 0.25	+24 %	$0.24\sqrt{0.02}/2$ =0.017 $=0.17\sqrt{\%}$
Ratio	$2 * 10^4$			-10^2
Pred. ratio				$\sim 0.4 * (\frac{m_\tau}{m_\mu})^2$ $=\sim 115$

In the table we perform a very crude estimate of the ratio of the anomalous contributions to the amplitude of the two decay processes $B \to D^*\tau\nu_\tau$ (which is a charged current process) relative to the anomalous contribution for $B \to K^*\mu^+\mu^-$ (which is a neutral current one). It is based on a few very crude but we think reasonable assumptions in our model:

- Since our non-perturbative anomalous prediction is strongly increasing with the mass of the charged lepton involved, we of course blame practically the whole anomaly on the decay rate for the process involved in the ratio revealing the deviation from lepton universality, which has the biggest mass. In $R(D^*)$ for instance it is the τ channel that has the anomaly, while for $R(K^*)$ which is a ratio between a μ and an e channel it is the μ channel that carries the anomaly.
- We make the approximation that the channels all have the same phase space - which means ignoring the differences between the masses of the particles in the final state of the decays (compared roughly to the B-meson mass). This also implies that, in this approximation, we can simply talk about the amplitude for going into the single final state for each of the considered channels of decay. This allows us to use the normalization of simply writing the amplitude of a decay measured in square roots of %, and simply in this notation have the decay fraction to a channel be the square numerically of the added up amplitudes.

The columns of the table denote the following

- The first column represents the decay channel, corresponding to the two different ratios revealing the violation of flavour universality (for leptons), which has the heavier lepton in the decay. These decay channels are thus, according to our assumption, the ones that are (most) anomalous in our model. We shall neglect the anomaly in the other decay channels in the ratios.
- The next column gives the branching fraction of these two channels thought to be carrying an anomaly.
- The next - 3rd - column now gives both the experimental ratio and the Standard Model predictions for the ratio associated with the channels lined up in column

1. That is to say for the first row, or rather the one associated with $B \to K^*\mu^+\mu^-$, we talk about the ratio $R(K^*)$ being the ratio of this decay rate to the corresponding one with the muons replaced by electrons. Similarly the second of the genuine rows refers to the ratio of the decay listed in first column divided by the corresponding one with the lepton replaced by the lighter lepton, in this case thus $B \to D^*\mu\nu_\mu$.

- The relative deviation between experiment and Standard Model is calculated in the next - the 4th - column. In our philosophy this also gives the relative deviation between the size of the decay in column 1 experimentally relative to the Standard Model. Thus the anomalous probability contribution is the product of this relative percentage and the rate as in column 2.
- Finally in the last column we identify the deviation corresponding to the anomaly with 2 times the amplitude - meaning the square root - of the rate (from column 2) multiplied with the "anomalous amplitude". It is then the latter that is presented in the last column.

Finally the result of interest is that we estimate the ratio of the anomalous amplitudes for the anomalous parts of the decay amplitudes of the two "rows". It is this ratio we have a chance to predict, because as a ratio it means that our parameter K gets divided out.

The calculation in the table, which we can at the moment hope to confront with our model, is an *order of magnitude one* meaning that neither factors 2 or π etc nor even the sign are our under control so far.

It might seem that just substituting a mu-coupling by a tau-coupling would only change the anomalous amplitude by a very well-defined real positive ratio given by the masses actually very precisely. However, in our comparison, we have it interfere with the Standard Model amplitude for two very different processes from the Standard Model point of view. So to get even the sign one would need the relative sign of these Standard Model amplitudes, something that would be quite a complicated task. We hope to come back to this exercise of calculating the relative sign of the Standard Model amplitudes, so as to make possible a sign prediction for our model about the sign of the ratio of the two anomalies which we studied.

0.4 **some order of unity number in the last entry in the table.**

In fact the order unity factor 0.4 in our predicted ratio is given in our non-perturbative model by

$$\frac{V_{tb}V_{ts}g_2^2}{V_{bc}g_t^2} = 0.4. \tag{18.1}$$

The numerically more significant factor is the ratio

$$\frac{g_\tau^2}{g_\mu^2} = \frac{m_\tau^2}{m_\mu^2} = \frac{1777^2}{105.7^2} = 283. \tag{18.2}$$

The numerical coincidence, that should suggest the truth of our non-perturbative effect idea, is:

$$\frac{(R(D^*)|_{exp}/R(D^*)|_{SM}-1)\sqrt{B(B\to D^*\tau\nu_\tau)}}{(R(K^*)|_{exp}/R(K^*)|_{SM}-1)\sqrt{B(B\to K^*\mu\bar{\mu})}}\approx\frac{m_\tau^2}{m_\mu^2}. \quad (18.3)$$

Here the "R″ ratios are defined as:

$$R(K^*)=\frac{B(B\to K^*\mu\bar{\mu})}{B(B\to K^*e\bar{e})}; \quad (18.4)$$

$$R(D^*)=\frac{B(B\to D^*\tau\bar{\nu_\tau})}{B(B\to D^*\mu\bar{\nu_\mu})}. \quad (18.5)$$

Note that these "R″ ratios test the lepton universality, the numerator and the denominator only deviating by the flavour of the lepton pair produced. But in $R(D^*)$it is the ratio τ-pair over μ-pair, while $R(K^*)$ is for μ-pair over e-pair.

Decays into channels only deviating by "hadronic details" support such models as e.g. our "non-perturbative" model.

That is to say the approximate equalities

$$\frac{R(K)|_{exp}}{R(K)|_{SM}}=0.75\approx 0.66=\frac{R(K^*)_{exp}}{R(K^*)_{SM}}, \quad (18.6)$$

$$\frac{R(J/\psi)|_{exp}}{R(J/\psi)|_{SM}}=2.3\approx 1.24=\frac{R(D^*)_{exp}}{R(D^*)|_{SM}} \quad (18.7)$$

confirm that the anomaly is approximately the same for different hadronic developments with the same underlying weak process behind, thus supporting an e.g. non-perturbative effect, or a new physics at the weak scale.

Have now to build arguments that the lepton pair needs to couple twice with its Higgs Yukawa coupling to the strongly interacting particles/sector.

We imagine there is some coupling g_t which is so strong that very complicated diagrams involving it become relevant. But somehow we hope to argue that the leptons only get interacting with the bunch of "new strong" interaction particles via two Higgs couplings in the processes we looked at with the anomalies.

Also an agreement for the anomaly in the anomalous magnetic moment for the muon, $a_\mu=(g-2)/2|_\mu$.

We get a correction to the anomalous magnetic moment for the muon in our non-perturbative model, using an overall fitting constant K for the non-perturbative effects (to be explained later):

$$(a_\mu|_{full}-a_\mu|_{perturbative})*\frac{e}{m_\mu}\approx K*<\phi_{Higgs}>(\frac{g_\mu}{g_t})^3. \quad (18.8)$$

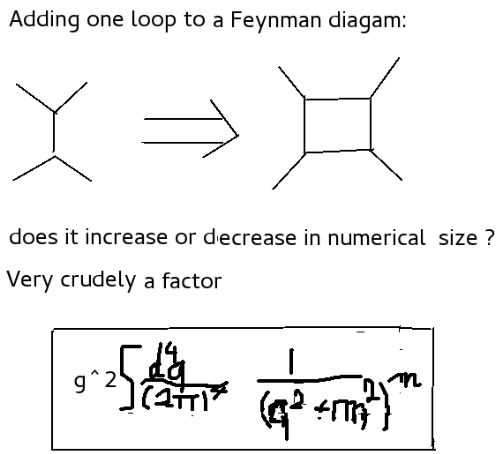
With our fitted value $K\sim\frac{1}{5GeV^2}$, we get

$$a_\mu|_{full}-a_\mu|_{perturbative}\approx\frac{246GeV*0.105GeV}{5GeV^2*1700^3}=1*10^{-9}$$

to be compared with the anomaly found experimentally $2.7*10^{-9}$.

18.2 Strong Coupling

Except for α_S the strongest coupling in Standard Model is the Top Yukawa Coupling g_t.



The Coupling on the Border between Weak and Strong Interactions for Particle with Only One Component is $g \sim 4\pi$.

Taking very crudely by a "dimensional argument"

$$\int \frac{d|q|}{|q|} \sim 1 \text{ (by dimensional argument)}$$

and the borderline coupling g_{border} to
have the increase factor by adding a loop of

$$g^2 \int \frac{d^4q}{(2\pi)^4 |q|^4} \approx 1 \text{ (ignoring the mass squares}$$

in the propagators) we get

$$g_{\text{border}} \approx \sqrt{\frac{(2\pi)^4}{\pi^2}} = 4\pi. \quad (18.9)$$

Another crude estimate of the border coupling corresponds to taking the Rydberg constant

$$R_\infty = \frac{\alpha^2 m_e c}{4\pi\hbar}$$

to be of the order of the mass-energy $m_e c^2$:

$$R_\infty = m_e c^2 \quad (18.10)$$

implying (18.11)

$$\alpha^2 = 4\pi \text{ for } c = \hbar = 1. \quad (18.12)$$

meaning

$$e = \sqrt[4]{(4\pi)^3} \approx 6 \quad (18.13)$$

Size of Borderline Coupling and Number of "Components"

If there were e.g. a color quantum number taking N values for the particle type encircling the loop, then there would be N various loops for each one. According to our philosophy of the increase factor by inserting a loop

$$g_{\mathrm{border}}^2 N \int \frac{d^4 q}{(2\pi)^4 |q|^4} \approx 1 \tag{18.14}$$

then the N-dependence of the borderline coupling between perturbative and non-perturbative regimes would be

$$g_{\mathrm{border}} \propto \sqrt{\frac{1}{N}}. \tag{18.15}$$

For say 16 "Components" Borderline Coupling ~ 1.5 to 3

Very crudely counting particle and antiparticle also as different "components" and counting together both the Higgs with its 4 real components and the top with its 3*2*2=12 components, we get in total for the particles interacting via the top Yukawa coupling g_t 12+4 = 16 components. Thus the borderline value for g_t becomes

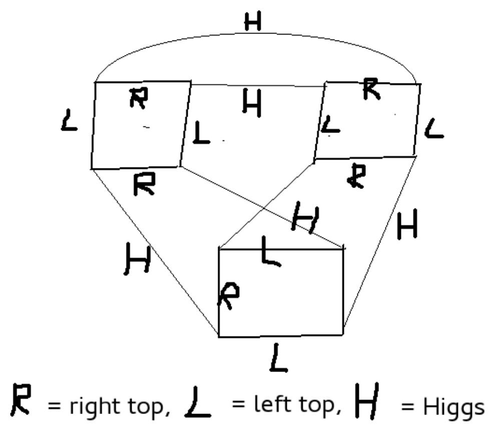
$$g_{t\ \mathrm{border}} \approx (6 \text{ to } 4\pi)/\sqrt{16} = 1.5 \text{ to } 3. \tag{18.16}$$

Experimentally

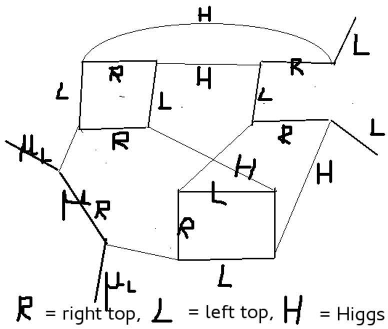
$$g_{t\ \mathrm{exp}} = 0.93_5 \tag{18.17}$$

18.3 Procedure

Very High Order Diagrams Likely to be Important



Diagrams with Almost Only Top-Yukawa Couplings of High Order Could be Significant and give the Anomalies about to be Statistically Significant "Tensions".



L	can be both left top and left bottom, strange, d
R	right can be only top.
H	can be both eaten Higgs and the "radial" observed Higgs

Suggested Procedure of Model

We imagine a lot of Feynman diagrams - that shall be summed up of course - each with almost only the top-Yukawa coupling g_t in it, and only a few external lines/propagators of other types (like muon say). Then the rules/assumptions of our non-perturbative model are as follows:

- The sum over the many diagrams with only g_t (from which we modify a bit by putting external lines on) is supposed to give just one **overall factor K, which we must fit.**
- When we use an external L line as a left bottom, strange or d quark line, we include a V_{tb}, V_{ts}, or V_{td} **mixing angle factor**
- Other couplings than g_t needed must give rise to the extra factors being these couplings, compared to the g_t they replace.
- Propagators for W, Higgs, top,... are similar order of magnitudewise, and we ignore the differences in our crude rule.

From the Physics involving Rather Heavy Particles the Result of the Non-perturbative Effects should be Effective Lagrangian Terms of Unrenormalizable Dimensionality.

The rather high mass of the particles, like the top quark and Higgs partticle, involved in the diagrams developing non-perturbative effects suggests these effects at the relatively low energies involved, in B-meson decay say, should be described by an effective field theory. The effective terms which have an operator dimension like in renormalizable theory are already present in the Standard Model. Thus such non-perturbative effects contributing to terms with dimension less than or equal to $[\mathrm{GeV}^4]$ would just be absorbed into these terms already present in the Standard Model.

We can only realistically hope to measure terms not of this renormalizable type, because otherwise we would need some knowledge about the bare couplings not coming from the usual measurements:

Denoting say leptons and quark fields by ψ_q and ψ_l and the bosons as W_μ, Z_μ and ϕ, effective field theory terms that might result from non-perturbative effects could have e.g. the forms (P_L is left handed γ_5 projector)

$$\bar{\psi_t}\phi\psi_t: \text{ of renormalizable theory dimension } [GeV^4]$$
$$\bar{\psi_b}\gamma_\nu P_L\psi_s\bar{\psi_\mu}\gamma^\nu P_L\psi_\mu: \text{ Dimension } [GeV^6], \text{ so not renormalizable.}$$

Example of an Effective Lagrangian Density Coefficient Estimated in Our Non-perturbative Scheme:

Say we want the coefficient to the term of the form

$$\bar{\psi}_b(x)\gamma_\nu\psi_s(x)\bar{\psi}_\mu(x)\gamma^\nu\psi_\mu(x),$$

which can represent that a bottom quark b described by $\psi_b(x)$ becomes a strange quark s described by ψ_s by a "neutral current exchange" and the production of a muon antimuon pair produced by the operator

$$\bar{\psi}_\mu(x)\gamma^\nu\psi_\mu(x).$$

Then we need a non-perturbative diagram with the four external particles corresponding to b→s, μ and $\bar{\mu}$. In fact it shall be a series of diagrams with an arbitrary number of g_t vertices and associated with t_L, t_R and Higgs, but as few as possible other - and therefore smaller - couplings (except we might include the strong QCD couplings).

If the b and the s are taken to be of the left handed helicity, b_L and s_L, we are really interested in the coefficient to the effective term

$$\bar{\psi}_b(x)\gamma_\nu P_L\psi_s(x)\bar{\psi}_\mu(x)\gamma^\nu\psi_\mu(x). \tag{18.18}$$

We can interpret it, that the weak $SU(2)$ partners of the left handed top-components t_L, which are also allowed in the bulk of our diagrams, are already present with amplitudes V_{tb} and V_{ts} respectively for the left handed b_L and s_L. So they do not "cost" extra coupling factors except for these CKM matrix elements, V_{tb} and V_{ts}.

Ignoring the propagators and thereby the masses, we have in the bulk diagram perfect formal conservation of weak charge $SU(2)$, and thus the two left handed quarks b and s being doublets cannot couple to only one Higgs. We must have **two** external Higgs bosons coupling to the muon-antimuon pair.

The muon cannot be interpreted as being already there in the bulk diagram and must instead be coupled, as we already argued, to two Higgs-bosons. This causes the applicable type of diagram to include a factor g_μ^2 - or if we want to consider it a replacement of g_t couplings by analogous g_μ's, it must include a factor $\left(\frac{g_\mu}{g_t}\right)^2$. So the coefficient to the b → s,$\bar{\mu}$, μ transition operator (18.18) becomes

$$\text{“coefficient to c}\rightarrow s\bar{\mu}\ \mu\text{''} = K * V_{tb}V_{ts}\left(\frac{g_\mu}{g_t}\right)^2. \tag{18.19}$$

Here K is an overall constant depending on the non-perturbative part of the calculation, which we cannot do. Thus we must fit via this overall factor K, while g_t, and g_μ are the Yukawa couplings to the Higgs of the top quark and the muon respectively. V_{tb} and V_{ts} are the mixing matrix elements.

Another Example: b → c, $\bar{\tau}$, ν_τ; Charged Current Process

The coefficient to the "non-renormalizable" charged current simulating effective field theory term

$$\bar{\psi}_b \gamma_\nu P_L \psi_c \bar{\psi}_\tau \gamma^\nu P_L \psi_{\nu_\tau} \tag{18.20}$$

becomes similarly

$$K * V_{tb}(V_{tb}V_{bc} \text{``}+\text{''}\, V_{ts}V_{sc} \text{``}+\text{''}\, V_{td}V_{dc}) \left(\frac{g_2}{g_t}\frac{g_\tau}{g_t}\right)^2. \tag{18.21}$$

Here g_2 is the weak $SU(2)$ gauge theory coupling, and as before: K is the overall non-perturbative constant, $V_{qq'}$ the mixing matrix elements, and g_t, g_τ the respective Yukawa Higgs couplings. Order of magnitudewise we only care for the dominant one of the three mixing matrix element products.

Fitting our overall constant K**:**

With the notation

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \frac{e^2}{16\pi} \sum (C_i O_i + C_i' O_i') + h.c. \tag{18.22}$$

and

$$O_9^{(\prime)} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{l}\gamma^\mu l), \tag{18.23}$$

the fit of the "new physics" NP in the coefficient C_9 to the effective term O_9, which we considered is about

$$C_9 \approx -1.3. \tag{18.24}$$

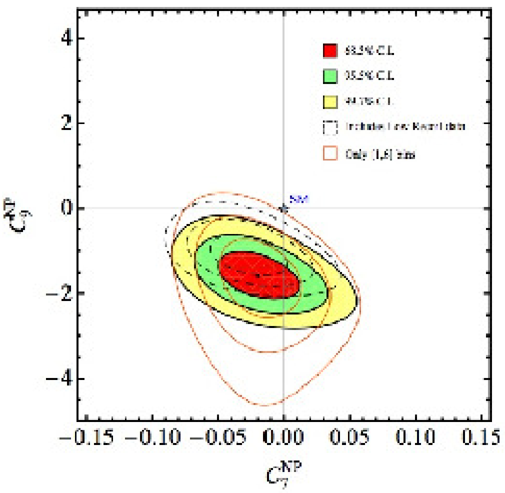


FIG. 1: Fit to (C_7^{NP}, C_9^{NP}), using the three large-recoil bins for $B \to K^*\mu^+\mu^-$ observables, together with $B \to X_s\gamma$, $B \to X_s\mu^+\mu^-$, $B \to K^*\gamma$ and $B_s \to \mu^+\mu^-$. The dashed contours include both large- and low-recoil bins, whereas the orange (solid) ones use only the 1-6 GeV^2 bin for $B \to K^*\mu^+\mu^-$ observables. The origin $C_{7,9}^{NP} = (0,0)$ corresponds to the SM values for the Wilson coefficients $C_{7\text{eff},9}^{SM} = (-0.29, 4.07)$ at $\mu_b = 4.8$ GeV.

The conventional $V_{tb}V_{ts}^*$ factors in

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \frac{e^2}{16\pi} \sum (C_i O_i + C_i' O_i') + h.c.$$

are just the same as in our formula for the non-perturbative effect coefficient

$$\text{“coefficient to } c{\to}s\bar{\mu}\,\mu\text{''} = K * V_{tb}V_{ts} \left(\frac{g_\mu}{g_t}\right)^2.$$

Thus we should fit to

$$\begin{aligned} K * \left(\frac{g_\mu}{g_t}\right)^2 &= -\frac{4G_F}{\sqrt{2}} \frac{e^2}{16\pi} * C_9 = -\frac{G_F}{\sqrt{2}} \alpha * C_9 \\ &= 1.1663787(6) \times 10^{-5} GeV^{-2} / (\sqrt{2} * 137.037) * (-1.3) \\ &= -6.01847886 * 10^{-8} GeV^{-2} * (-1.3). \end{aligned}$$

Since $\left(\frac{g_\mu}{g_t}\right)^2 = (0.1056583745/172.44)^2 = 3.77 * 10^{-7}$, we get from fitting the O_9 coefficient

$$K = \frac{6.018 * 10^{-8} GeV^{-2}}{3.77 * 10^{-7}} * 1.3 \tag{18.25}$$

$$= 0.21\ GeV^{-2} \tag{18.26}$$

$$= \frac{1}{4 \text{ to } 5\ GeV^2} \tag{18.27}$$

Embarrassingly Huge Overall Constant $K \sim \frac{1}{4\ GeV^2}$ **for the Non-perturbative Effect.**

Imagine that the non-perturbative effect in reality is the effect of some loop with, or just the effect of, a bound state formed from the top-quarks and the Higgs. If consisting, as we usually speculate, of 6 top + 6 anti-top quarks its constituent mass would be $12m_t = 2.1$ TeV. So, even if we did not count suppression from there being a loop say, an order of magnitude $K \sim \frac{1}{4\ TeV^2}$ would have been rather expected.

But now, if we have about 12 constituents in the bound state, a top-quark or a Higgs would couple to such a bound state with a total coupling of the order of $12g_t$. Very optimistically a diagram with four external lines would have four such factors and the resulting K would be enhanced by a factor $(12g_t)^4 \approx 20000$ which would bring $K \sim \frac{1}{4\ TeV^2}$ up to $K \sim \frac{1}{200\ GeV^2}$.

If the bound state mass were say 750 GeV rather than 2.1 TeV, a reduction by a factor $(2.1/.75)^2$ of the above speculated value $\frac{1}{200 GeV^2}$ would be argued for. Then we might say that we could understand if K were of order of magnitude $\frac{1}{20\ GeV^2}$, but the fitted value $K \sim \frac{1}{4\ GeV^2}$ still seems to be a bit - a factor 5 - bigger than we would even speculate optimistically.

But of course the point is that it is too hard to compute or even speculate on the overall strength K, so that we must rather trust a fit to the data.

Our Prediction for the Ratio of the anomalous Charged Current B$\to X_c\tau\nu_\tau$ **to the anomalous Neutral Current B**$\to X_s\bar{\mu}\mu$ **amplitudes**

The ratio of the experimentally found quite separate anomalies measured in their rates/branching ratios is

$$\frac{\text{“Anomalous rate } B \to X_c \tau \nu_\tau\text{”}}{\text{“Anomalous rate } B \to X_s \mu \nu_\mu\text{”}} = (-)1 * 10^4$$

while the ratio of the normal rates is:

$$\frac{BR(B \to X_c \tau \nu_\tau)}{BR(B \to X_s \mu \nu_\mu)} = \frac{2\%}{2 * 10^{-6}} = 1 * 10^4$$

corresponding to an amplitude ratio:

$$\frac{A(B \to X_c \tau \nu_\tau)}{A(B \to X_s \mu \nu_\mu)} = \sqrt{\frac{2\%}{2 * 10^{-6}}} = 1 * 10^2.$$

By accident it does not matter whether the anomalies come by interference - as we think they do - or by just adding to the rate. In any case it is needed experimentally that the ratio of the two anomalous parts of the amplitude must be ~ 100:

$$\frac{A_{an}(B \to X_c \tau \nu_\tau)}{A_{an}(B \to X_s \bar{\mu}\mu)} = 100. \tag{18.28}$$

Is that then what our model predicts? Our prediction for the ratio of the anomalous parts of the amplitudes is:

$$\begin{aligned} \frac{A_{an}(B \to X_c \tau \nu_\tau)}{A_{an}(B \to X_s \bar{\mu}\mu)} &= \frac{K * V_{tb}(V_{tb}V_{bc} \text{“}+\text{”} V_{ts}V_{sc} \text{“}+\text{”} V_{td}V_{dc}) \left(\frac{g_2}{g_t}\frac{g_\tau}{g_t}\right)^2}{K * V_{tb}V_{ts}\left(\frac{g_\mu}{g_t}\right)^2} \\ &\approx \frac{V_{tb}V_{bc}}{V_{ts}} * \frac{g_2^2 g_\tau^2}{g_\mu^2 g_t^2} \\ &\approx 1 * 0.4 * \frac{m_\tau^2}{m_\mu^2} \\ &= 0.4 * \frac{1777^2}{105^2} = 115. \end{aligned} \tag{18.29}$$

Very good agreement with experiment!

Dominant Anomaly in $B^+ \to K^+\tau^+\tau^-$

Our prediction for the branching ratio for $B^+ \to K^+\tau^+\tau^-$:

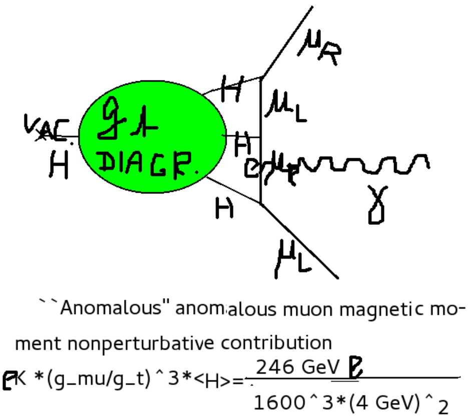
The anomaly amplitude is enhanced by the factor m_τ^2/m_μ^2 compared to the $B^+ \to K^+\mu^+\mu^-$ anomaly amplitude and therefore **dominates the usual SM** amplitude.

So the branching ratio value for $B^+ \to K^+\tau^+\tau^-$is:

	Branching ratio
For SM	$\sim 2 \times 10^{-7}$
For our anomaly	$\sim 3 \times 10^{-4}$
Experiment.	$< 2.25 \times 10^{-3}$

18.4 g minus 2

There is a small deviation from experiment in the perturbative Standard Model prediction for the anomalous magnetic moment for the muon. The non-perturbative contribution of our model is illustrated in the following diagram, which is followed by a list of comments on it.



- The muon anomalous magnetic moment term in the effective Lagrangian density

$$a_\mu \bar{\psi}_\mu(x) F_{\nu\rho}(x)\gamma^\nu\gamma^\rho\psi_\mu(x) = a_\mu \bar{\mu}(x)F_{\nu\rho}(x)\gamma^\nu\gamma^\rho\mu(x) \qquad (18.30)$$

  makes a transition between the chirality left to right or opposite. (Contrary to simple electromagnetic coupling making it left to left or right to right.)
- Thus we need to couple the muon line series an **odd** number of times to **Higgs** in our non-perturbative contribution.
- Only **one Higgs exchanged would just give a renormalization of the Higgs propagator**, and would thus already be included in the Standard Model calculation and not count as an anomalous term for the anomalous magnetic moment.
- This contribution must then, because we ignore the propagator masses in it, have a Higgs-line **couple to vacuum via the expectation value** $< H >=$ 246 GeV, so that it **conserves weak isospin**.
- These remarks give the factor $\left(\frac{g_\mu}{g_t}\right)^3 < H >$.
- When we use our speculated non-perturbative effect, we have the "overall" factor $K \sim \frac{1}{4\ GeV^2}$.
- Finally we get a non-perturbative contribution to $a_\mu = (g-2)/2|_\mu$ for the muon.

$$a_\mu|_{full} - a_\mu|_{perturbative} \approx \frac{246 GeV * 0.105 GeV}{4 GeV^2 * 1700^3} = 1.3 * 10^{-9}. \qquad (18.31)$$

  This is to be compared with the anomaly found experimentally $2.7 * 10^{-9}$.

18.5 Mixing

The mixing of mesons and their antiparticles such as B_s mixing with $\bar{B}_s$ is a problem, as was pointed out by a member of the audience when HBN gave a talk about this work in Tallinn. The problem is that at first sight it looks as though we have, according to our rule above, just a few mixing angles suppressing the transition from say B_s to $\bar{B}_s$. This is very analogous to the way we got the b to s transition, using that both s and b can for the left handed case be considered to be in the doublet with the left handed top and thus indeed participating significantly in the diagrams supposed to be of very high order and still important. However this is not quite true, because the quarks that have to be converted in the mixing process for pseudoscalar mesons - which are w.r.t. strong interactions stable ones, so that mixing experiments can be practically performed - are both right handed and left handed.

18.5.1 Formal

If we take completely formally our rules as set up, including the rule of neglecting propagators and thereby especially the masses of the quarks and leptons in the strong diagram, then a right handed quark of electric charge 2/3 (like the top) can, by interaction with a Higgs-doublet, only be converted into the left handed one of the same flavour or the weak isodoublet partner of this left handed one of the same flavour. This weak isodoublet partner is a superposition of all three flavours of the quark with the other electric charge than the starting right quark. This superposition carries in principle the signal of the flavour of the starting right handed quark. If we ignore the masses and only have it interact via the Higgses in the supposed to dominate diagrams, this superposition can only go back to the right handed quark of just the same flavour as from the start. In this way the "right flavour" has become formally a conserved quantum number, as long as we exclude other interactions than in our rule.

Only if there is transition into a right handed quark of the other charge, i.e. charge -1/3, will another set of Yukawa-couplings (namely the -1/3 charge ones) come into the game and more complicated flavour changes become possible.

The value of $K = \frac{1}{(4 \text{ to } 5) GeV^2}$ we found, by fitting flavour universality violations, would give us a non-renormalizable Lagrangian term for say top-quark scattering, which would not be suppressed,

$$\sim \frac{1}{5GeV^2} \bar{t}(x)\gamma^{\mu}t(x) * \bar{t}(x)\gamma_{\mu}t(x). \tag{18.32}$$

This is quite absurd, if you think of using it up to a cut-off scale of say the order of $\Lambda \sim 0.5$ TeV or a "lattice scale" of the order $a \sim \frac{1}{0.5\ TeV}$.

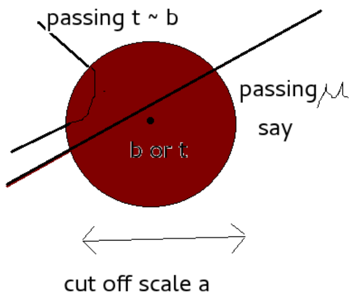
We would in fact like to argue *that you cannot use perturbation theory for such a coupling unless for*

$$K * \bar{t}(x)\gamma^{\mu}t(x) * \bar{t}(x)\gamma_{\mu}t(x) \tag{18.33}$$

one has

$$K/a^2 \leq 1. \tag{18.34}$$

Too Strong (Effective) Coupling Term gets Absurd/not Perturbatively Applicable, when $K/a^2 > 1$ for dim =6



This figure is supposed to make clear the absurdity in the too strong coupling regime, which does not at least crudely obey $K/a^2 < 1$. The figure is based on the assumption that inside the interacting particles (in our example top quarks) we have some structure or fields with which they interact with the other particle, and now illustrates how one particle passes into the field or matter belonging to the other one.

Then the idea is to estimate the phase rotation of the amplitude of the scattering, i.e. after the passage. For the case that the particles did indeed pass within the distance a, we can argue dimensionally that the phase rotation δ must be of the order

$$\delta \approx K/a^2 \tag{18.35}$$

or if there is some suppression factor such as e.g. "suppression"$^{-1} = \frac{g_\mu^2}{g_t^2}$:

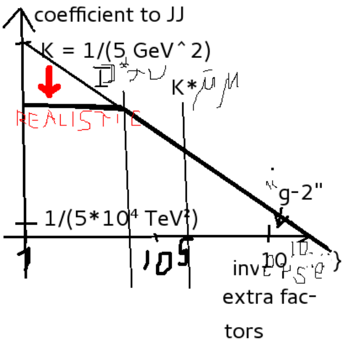
$$\delta \approx \frac{K}{\text{“suppression”}a^2}. \tag{18.36}$$

Now the important point is that such a phase rotation δ only makes sense modulo 2π. So it cannot be expected to give any sensible result when it becomes very big compared to 2π. First the point is that you simply cannot "see" the difference in various sizes once the 2π is past. Realistically, we would physically rather imagine that interference between slightly different passage ways of the one particle through the field or matter around the other one would get relative to 2π rather big phase differences, so that strong (destructive) interference would take place. Spoiled by such interference it seems unavoidable that, seen from outside, the end result would be an effective coupling looking much smaller than the a priori one K/"suppression". Therefore we would like to conclude that the very strong coupling, not obeying our requirement $K/(\text{“suppression”}a^2) < 1$ is not realistic in practice.

Basically the strong interaction would cause further interactions or make different details in the interaction come out of phase. Thus the effective resulting interaction would be brought back to a size obeying the upper limit, which we suggest.

A slightly different way to think of this "strong couplings killing themselves down" to only of order unity, would be to notice that passing a region with too strong interactions would cause reflection. So the particle would never come through but rather get reflected on the surface. In this way the interaction would be reduced to a size compatible with only the surface regions being used in the effective interaction as seen form outside. This is illustrated in the figure by the track of a particle turning around and going out again.

If there is not a correction factor reducing the K to be sensible, we cannot take it seriously, but must correct it down:



On this figure we now illustrate what we shall effectively do in our model, so as to take into account that the absurdly strong couplings cannot be taken seriously. From the rule of our non-perturbative model one starts from our fitted constant K and then has to put various factors such as g_μ/g_t to some powers etc. so that one at the end divide by a "suppression" - a suppression factor.

In the figure to give an idea of what we shall do, this suppression factor "suppression" is plotted as the abscissa. As the ordinate is plotted the effective field theory term coupling coefficient. If we did not modify our model this effective field theory coupling would of course just be K/"suppression" and that is represented by the skew straight line, simply with slope -1 in the logarithmic plot. If the suppression factor is sufficiently big, perturbation theory on top of our non-perturbative effect is still o.k. and we can take the result seriously. If, however, the suppression factor for some effective field theory interaction, we look for, turns out so small that the effective coupling becomes bigger than the limit, we should cut the coupling down to agree with the limit. This is indicated by the red arrow on the figure.

So in reality we shall use the kinky curve given on this figure which for small "suppression" is flat, but for large enough "suppression" kinks into the -1 slope straight curve piece.

In order that we can claim the success of our main result on the ratio of the anomalous amplitudes fort the two B-meson anomalies, it is crucial that they both fall in the region with the skew part of the curve. I.e. that suppression is enough.

18.5.2 Conservations

In order to put forward a little better the problems with making contributions to meson anti-meson mixing in our scheme, we shall think of a certain truncated Standard Model:

In the region of our new strong interaction it is only right and left top quarks and the Higgs doublet, which are present. We must though consider the left top to also include a certain superposition of bottom, strange and down left quarks, namely the one that is in a doublet with the left top.

At least for pedagogical reasons, but also really logically, we are allowed to use as a strictly speaking more accurate model a restriction of the Standard Model which also includes the three important particles for the new strong sector: the right top, the Higgs doublet and the doublet containing the left top.

Let us indeed for our study, pedagogically or logically, choose the model with all the quarks and for that matter also the leptons, both right and left, and the Higgs doublet. However, we do not let into this restricted model the gauge bosons, so there is no transverse W nor transverse Z. (Only the longitudinal components in the form of eaten Higgses are let in).

This sub-model contains all the components that are crucial for the non-perturbative effects. So it is in principle "better" than the only new strong interaction approximation.

Now let us contemplate the conserved quantities of this "better" restriction of the Standard Model, and let us in the spirit of our proposed rule of ignoring the propagators or at least their masses, take all the quarks and leptons to be massless except for vacuum expectation values for the Higgs. But the Higgs vacuum expectation is assumed to be small on the mass scale we have in mind, so we indeed ignore the masses in the propagators, even for the Higgs, which has a mass of a similar order of magnitude.

In this our "better" restricted Standard Model the weak isospin is only a **global** $SU(2)$ symmetry, as is also the electric charge. We can without problems use a different flavour basis for the $T_3 = 1/2$ and the $T_3 = -1/2$ quarks, as one in fact does in practice. In such a notation then all the flavours get totally conserved. Roughly speaking: We switched off the weak interactions and then the flavours are conserved. It should though be borne in mind that our restricted sub-model of the Standard Model only had the transverse weak gauge bosons switched off, while the longitudinal components in the form of eaten Higgs components are still included.

But this is then at first very promising for the mixing of the various pseudoscalar mesons with their antiparticles in our model. Namely in first approximation, in which we could claim that we only need the just constructed restricted Standard Model, we can say that flavour changing is totally forbidden. Without flavour changing we can have no meson anti-meson mixing and thus our non-perturbative sector cannot produce any contribution to the mixing in this first approximation.

18.5.3 Problem

However, there still seems to be a problem: The Standard Model contribution to meson anti meson mixing already has in amplitude two W-exchanges - as are needed for the flavour violation. Now the experimental method of measuring mixing is very sensitive and we cannot rely on the anomalous contribution from our non-perturbative model being negligible even if decorated with two W-propagators.

We could therefore expect a non-negligible anomalous contribution basically simulating the Standard model term, but letting the two top quark propagators present in the Standard Model main term for the mixing interact via our non-perturbative effect. This would mean crudely some usual top-propagators, being of the order $1/m_t$ each, if counted as fermion propagators, would in our anomalous term be replaced - following dimensionality rules - by a top-quark scattering effective coupling proportional to our K parameter with associated suppression factors. However, for top quark scattering we have in our model no further suppression and thus we simply get a K replacing the factor $1/m_t^2$ from the Standard Model perturbatively. Our estimating of the correction factor to the full contribution from the Standard Model would then be of the order $m_t^2 K = \frac{173^2}{5} \approx 5000$. This prediction would of course be catastrophic for the hope that our model could be right. There is certainly no place for an extra mixing even of the same order as the Standard Model mixing, let alone 5000 times as much.

Now, however, although formally correct according to our rules, such an estimate is physically rather crazy. We must realistically expect that the effectively "new physics", due to the non-perturbative effects, has to do with say some bound state or some little clump of a new vacuum or whatever, which only truly comes into play when the interacting particles come sufficiently close to each other that the bound state or a couple of them say could be exchanged between them. Such bound state would presumably already have been observed if it were not of mass of the order of say the by now disappearing $F(750)$ digamma.

Let as say that, since no such bound state or replacement for it has been seen, a mass of the order of 1 TeV at least should be estimated.

We would then say that we have an effective field theory and may take the scale μ for it to be of the order of 1 TeV.

18.5.4 Coupling's Maximum

Now we then want to argue that when we consider an effective field theory at a scale $\mu = a^{-1}$, where a is the typical length for the scale of phenomena considered, there must be an upper bound of what the effective field theory coupling G on some vertex such as $G\bar{\psi}_1\bar{\psi}_2 \ldots \psi_3\psi_4$ can physically be. Here the ... just stands for some γ-matrices or the like. In fact we want to argue that order of magnitudewise we must have

$$\mu^2 G = G/a^2 < O(1). \tag{18.37}$$

This condition is of course the same as that given in eq. (18.34) and discussed above.

It is very natural, when we have our bound state ideas, to think of the particles for the purpose of estimating what goes on as having extensions of the order $a = \mu^{-1}$. Then one particle passing another one will get a phase rotation of its wave function as it goes by given by G, in such a way that when it has passed through it is by dimensional arguments rotated by $\mu^2 G = G/a^2$. But if this dimensionless quantity is big compared to unity (or 2π) there will not result a particle with a phase as estimated, but rather some superposition of particles with many somewhat different phases for their amplitudes, and they may typically interfere out to much less. So we cannot really expect a coupling not obeying our suggested bound to have any chance to survive in practice.

This means that unless we have enough suppression factors, such as the g_μ/g_t to some power, to bring the $\mu^2 G$ a priori equal to $\mu^2 K$ down to under 1, we are not allowed to take our model seriously. But now with our suggested number of $\mu = 1$ TeV and our fit $K = 1/(5\ \text{GeV}^2)$, we have $\mu^2 K = 200000$. So unless our suppression factors for the interacting particles - the Yukawa coulings needed etc. - make a suppression of a factor 200000, we cannot take our model seriously. We must then claim that, for the physical reason of the particles being able to pass through each other, we must anyway suppress the non-perturbative effect by the rest of this needed factor 200000.

The idea now is that this suppression by the full factor 200000 is needed in the least suppressed case of top on top interaction as we use in the mixing. This should help to reduce our discrepancy w.r.t. mixing predictions.

In the cases of the anomalies, which we fitted as our main point, even in the least suppressed of the two cases we had a suppression factor $m_\tau^2/m_t^2 \approx 1/10000$. This is only barely enough suppression to avoid further suppression in order to get down by 200000. However the factor 200000 was really somewhat arbitrary, and we could fit the μ to be a bit smaller by a square root of 20. But our problem with mixing getting predicted too strong of course gets worse by such a choice.

18.6 Review

We have worked for a long time on the speculation that non-perturbative effects in the Standard Model produce a very strongly bound state of 6 top + 6 anti-top quarks [4–6], and a new vacuum with a condensate of such bound states. This idea leads to a model of dark matter[7–10] without any physics beyond the Standard Model:

- Dark matter consists of bubbles of a new phase of the vacuum filled with atoms.
- These dark matter "pearls" with mass ~ 500000t made 6400 volcanoes of the Kimberlite pipe type found on earth (and probably many more not found).

Some Successful Numbers Fitted/Predicted by Our Non-perturbative Standard Model Based Model for Dark Matter:

Quantity	Predicted	"experiment"	from
Weak scale	$\sim$ 30GeV	$\sim$ 100GeV	"Tunguska"
3.5 keV line	4.5 keV	3.5 keV	"homolumo-gap"
"Life time, 3.5 keV"	10^{29}s?	10^{28}s	pearl collisions
Double supernova burst	14 hours	5 hours	neutron-eating

18.7 Conclusion

- We proposed, that two (small) tensions found in respectively neutral current ($c \to s$) and charged current ($b \to c$) transitions in B-decay are due to non-perturbative effects inside the Standard Model.
- The observed ratio between the anomalous amplitudes for the two processes/decays of B-mesons $B \to X_s \bar{\mu}\mu$ and $B \to X_c \tau\nu_\tau$ seems to be $\sim \frac{1}{100}$. This is in agreement with the prediction resulting from our "practical procedure" for calculating this ratio of amplitudes from our assumption that they result from non-perturbative effects, due to the top-Yukawa coupling g_t being of order unity.
- So the Standard Model could be perfectly correct even with these anomalies/tensions being true physical effects.
- In the neutral current decay $B \to K\tau^+\tau^-$ we PREdict the anomaly to dominate.
- We have earlier used this non-perturbative effect for a model for dark matter, thus completely inside the Standard Model.

Acknowledgement

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