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Baryon transition form factors from space-like into time-like regions

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Pion electroproduction is the main source for investigations of the transition form factors of the nucleon to excited N^* and Δ baryons. After early measurements of the G_M^* form factor of the $N\Delta$ transition, in the 1990s a large program was running at Mainz, Bonn, Bates Brookhaven and JLab in order to measure the E/M ratio of the $N\Delta$ transition and the Q^2 dependence of the E/M and S/M ratios in order to get information on the internal quadrupole deformations of the nucleon and the Δ. Only at JLab both the energy and the photon virtuality were available to measure transition form factors for a set of nucleon resonances up to $Q^2 \approx 5$ GeV^2. Two review articles on the electromagnetic excitation of nucleon resonances, which give a very good overview over experiment and theory, were published a few years ago [1,2].

In the spirit of the dynamical approach to pion photo- and electroproduction, the t-matrix of the unitary isobar model MAID is set up by the ansatz [1]

$$t_{\gamma\pi}(W) = t^B_{\gamma\pi}(W) + t^R_{\gamma\pi}(W)\,, \tag{1}$$

with a background and a resonance t-matrix, each of them constructed in a unitary way. Of course, this ansatz is not unique. However, it is a very important prerequisite to clearly separate resonance and background amplitudes within a Breit-Wigner concept also for higher and overlapping resonances. For a specific partial wave $\alpha = \{j, l, \ldots\}$, the background t-matrix is set up by a potential multiplied by the pion-nucleon scattering amplitude in accordance with the K-matrix approximation,

$$t^{B,\alpha}_{\gamma\pi}(W, Q^2) = v^{B,\alpha}_{\gamma\pi}(W, Q^2)\,[1 + i t^{\alpha}_{\pi N}(W)]\,, \tag{2}$$

where the on-shell part of pion-nucleon rescattering is maintained in the non-resonant background and the off-shell part from pion-loop contributions is absorbed in the phenomenological renormalized (dressed) resonance contribution. In the latest version MAID2007 [3], the S, P, D, and F waves of the background contributions are unitarized as explained above, with the pion-nucleon elastic scattering amplitudes $t^{\alpha}_{\pi N}$ described by phase shifts and inelasticities taken from the GWU/SAID analysis [4].

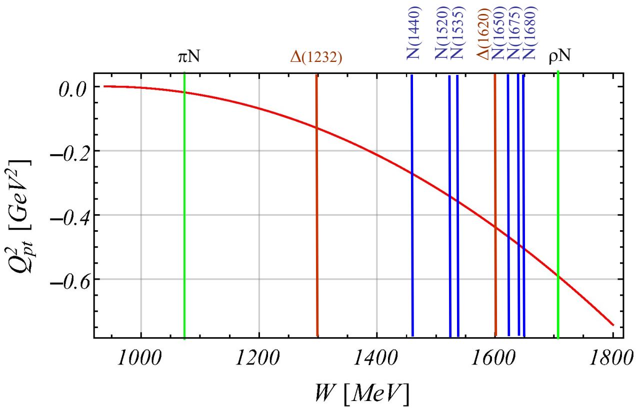

Fig. 1. The W dependence of the pseudo-threshold, where the Siegert theorem strictly holds and which also limits the physical region, where time-like form factors can be obtained from Dalitz decays $N\pi \to Ne^+e^-$. At πN threshold, the pseudo-threshold value is $Q^2_{pt} = -m^2_\pi = -0.018$ GeV^2, at $W = 1535$ MeV, $Q^2_{pt} = -0.356$ GeV^2. The vertical lines denote the pion threshold and nucleon resonance positions, where space-like transition form factors have been analyzed from electroproduction experiments.

For the resonance contributions we follow Ref. [3] and assume Breit-Wigner forms for the resonance shape,

$$t^{R,\alpha}_{\gamma\pi}(W, Q^2) = \bar{\mathcal{A}}^R_\alpha(W, Q^2)\, \frac{f_{\gamma N}(W)\Gamma_{tot}(W)\, M_R\, f_{\pi N}(W)}{M_R^2 - W^2 - iM_R\,\Gamma_{tot}(W)}\, e^{i\phi_R(W)}\,, \tag{3}$$

where $f_{\pi N}(W)$ is a Breit-Wigner factor describing the decay of a resonance with total width $\Gamma_{tot}(W)$. The energy dependence of the partial widths and of the γNN^* vertex can be found in Ref. [3]. The phase $\phi_R(W)$ in Eq. (3) is introduced to adjust the total phase such that the Fermi-Watson theorem is fulfilled below two-pion threshold.

In most cases, the resonance couplings $\bar{\mathcal{A}}^R_\alpha(W, Q^2)$ are assumed to be independent of the total energy and a simple Q^2 dependence is assumed for $\bar{\mathcal{A}}_\alpha(Q^2)$. Generally, these resonance couplings, taken at the Breit-Wigner mass $W = M_R$ are called transition form factors $\bar{\mathcal{A}}_\alpha(Q^2)$. In the literature, baryon transition form factors are defined in three different ways as helicity form factors $A_{1/2}(Q^2)$, $A_{3/2}(Q^2)$, $S_{1/2}(Q^2)$, Dirac form factors $F_1(Q^2)$, $F_2(Q^2)$, $F_3(Q^2)$ and Sachs form factors $G_E(Q^2)$, $G_M(Q^2)$, $G_C(Q^2)$. For detailed relations among them see Ref. [1]. In MAID they are parameterized in an ansatz with polynomials and exponentials, where the free parameters are determined in a fit to the world data of pion photo- and electroproduction.

In the case of the $N\Delta$ transition, the form factors are usually discussed in the Sachs definition and are denoted by $G^*_E(Q^2)$, $G^*_M(Q^2)$, $G^*_C(Q^2)$. While the G^*_M form factor by far dominates the $N \to \Delta$ transition, the electric and Coulomb transitions are usually presented as E/M and S/M ratios. In pion electroproduction they are defined as the ratios of the multipoles. Within our ansatz they can be expressed in terms of the $N\Delta$ transition form factors by

$$R_{EM}(Q^2) = -\frac{G_E^\star(Q^2)}{G_M^\star(Q^2)}, \tag{4}$$

$$R_{SM}(Q^2) = -\frac{k(M_\Delta, Q^2)}{2M_\Delta}\frac{G_C^\star(Q^2)}{G_M^\star(Q^2)}, \tag{5}$$

with the virtual photon 3-momentum

$$k(W, Q^2) = \sqrt{((W - M_N)^2 + Q^2)(W + M_N)^2 + Q^2)}/(2W).$$

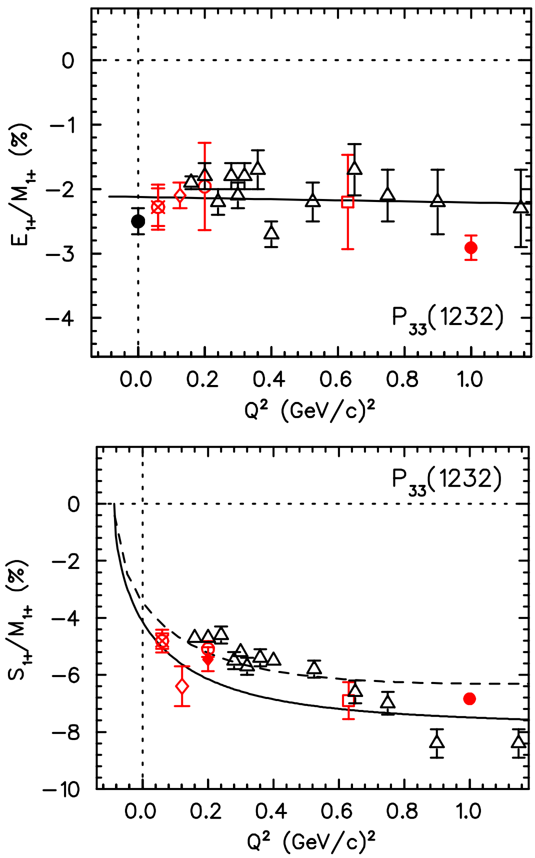

Fig. 2. The Q^2 dependence of the E/M and S/M ratios of the $\Delta(1232)$ excitation for low Q^2. The data are from Mainz, Bonn, Bates and JLab. For details see Ref. [1]. The behavior of the S/M ratio at low Q^2 and in particular for $Q^2 < 0$ in the unphysical region is a consequence of the Siegert theorem.

Whereas in photo- and electroproduction, data are only available for space-like momentum transfers, $Q^2 = -k_\mu k^\mu \geq 0$, the inelastic form factors can be extended into the time-like region, $Q^2 \leq 0$, down to the so-called pseudo-threshold, Q^2_{pt}, which is defined as the momentum transfer, where the 3-momentum of the virtual photon vanishes,

$$k(W, Q^2_{pt}) = 0 \quad \rightarrow \quad Q^2_{pt} = -(W - M_N)^2 . \tag{6}$$

This time-like region is called the Dalitz decay region. The energy dependence of this region is shown in Fig. 1. At pion threshold, the Dalitz decay region is very small and extends only down to $Q^2_{pt} = -0.018$ GeV^2, for transitions to the $\Delta(1232)$ resonance down to -0.086 GeV^2 and to the Roper resonance $N(1440)$ down to -0.252 GeV^2.

In Fig. 2 we have extended our parametrization of the E/M and S/M ratios for $N \rightarrow \Delta$ from space-like to time-like regions and show a comparison to the data obtained from photo- and electroproduction [1,2].

In general, the extrapolation of the transverse form factors G_E and G_M into the time-like region is more reliable than the extrapolation of the longitudinal form factor G_C, which can not be measured at $Q^2 = 0$ with photoproduction. For longitudinal transitions, the photon point is only reachable asymptotically, and in practise, only at MAMI-A1 in Mainz, momentum transfers as low as $Q^2 \simeq 0.05$ GeV^2 are accessible. Therefore, the longitudinal form factors become already quite uncertain in the real-photon limit $Q^2 = 0$.

Because of this practical limitation, the Siegert Theorem, already derived in the 1930s, give a powerful constraint for longitudinal form factors. In the long-wavelength limit, where $k \rightarrow 0$, all three components of the e.m. current become identical, $J_x = J_y = J_z$, because of rotational symmetry. As a consequence, excitations as $N \rightarrow \Delta(1232)3/2^+$ or $N \rightarrow N(1535)1/2^-$ obtain charge form factors that are proportional to the electric form factors. For the $N \rightarrow N(1440)1/2^+$ transition, where no electric form factor exists, still a minimal constraint remains, namely

$$S_{1/2}(Q^2) \sim k(Q^2) , \tag{7}$$

forcing the longitudinal helicity form factor to vanish at the pseudo-threshold. This is a requirement for all $S_{1/2}$ transition form factors to any nucleon resonance. In Fig. 3 the longitudinal transition form factor for the Roper resonance transition is shown. Different model predictions are compared to previous data of the JLab-CLAS analysis and a new data point measured at MAMI-A1 for $Q^2 = 0.1$ GeV^2 [5]. Only the MAID prediction comes close to the new measurement because of the build-in constraint from the Siegert theorem.

The study of baryon resonances is still an exciting field in hadron physics. With the partial wave analyses from MAID and the JLab group of electroproduction data a series of transition form factors has been obtained in the space-like region. We have shown that with the help of the long-wavelength limit (Siegert Theorem) extrapolations to the time-like region can be obtained satisfying minimal constraints at the pseudo-threshold. In this time-like region, Dalitz decays in the process $N\pi \rightarrow N^*/\Delta \rightarrow Ne^+e^-$ can be measured and time-like form fac-

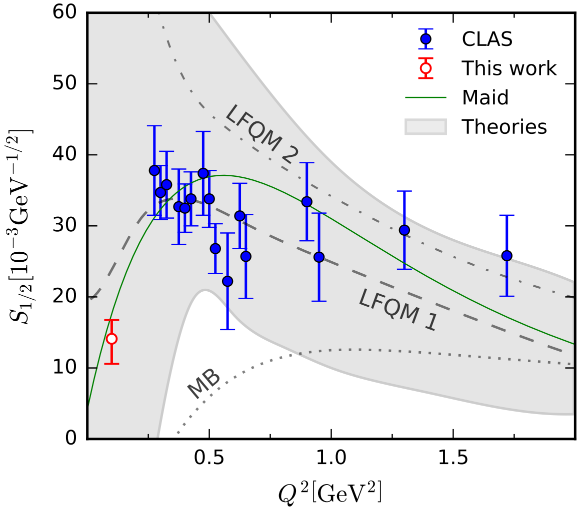

Fig. 3. Longitudinal transition form factor $S_{1/2}(Q^2)$ for the transition from the proton to the Roper resonance. The figure and the red exp. data point at $Q^2 = 0.1$ GeV^2 are from Štajner et al. [5], the blue data points are from the CLAS collaboration. The MAID model prediction which satisfies the Siegert's Theorem in the time-like region is in very good agreement with the new data point. For further details, see Ref. [5].

tors can be analyzed experimentally. Such experiments are already in progress at HADES@GSI and are also planned with the new FAIR facility at GSI.

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