MODIFICIRANA METODA DEFORMACIJSKE ANALIZE PO POSTOPKU MÜNCHEN

MODIFIED METHOD OF DEFORMATION ANALYSIS ACCORDING TO THE MUNICH APPROACH

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IZVLEČEK

Kljub tehnološkemu napredku v stroki ostaja spremljanje stabilnosti gradbeno-inženirskih objektov ena zahtevnejših nalog v inženirski geodeziji. V članku obravnavamo deformacijsko analizo po postopku München, ki jo je predlagal W. M. Welsch. Metoda obravnava testiranje skladnosti geodetske mreže, testiranje preoblikovanja izbranih trikotnikov in testiranje kinematičnih parametrov v 2D-geodetskih mrežah. Deformacijsko analizo po postopku München lahko izvedemo z metodo X, ki temelji na primerjavi koordinat identičnih točk v geodetski mreži med dvema terminskima izmerama, ki so odvisne od geodetskega datuma, ali z metodo L, s katero se ugotavljajo spremembe vrednosti dolžin in kotov, ki so od geodetskega datuma neodvisne količine. V modificirani metodi predlagamo ugotavljanje skladnosti vseh dolžin in vseh kotov v mreži med dvema terminskima izmerama ter ugotavljanje skladnosti vseh trikotnikov med točkami v mreži, ne le izbranih, kot je običaj pri klasičnem pristopu deformacijske analize po postopku München. Predlagane izboljšave testiramo na primeru znanega šestkotnika in izdelamo analizo uspešnosti odkrivanja stabilnih točk v mreži glede na klasični pristop metode München. Rezultati predlaganega postopka se na obravnavanem primeru niso razlikovali od rezultatov drugih postopkov deformacijske analize.

ABSTRACT

Despite the technical progress monitoring the stability of engineering structures remains one of the most difficult tasks in engineering geodesy. This article presents the deformation analysis according to the Munich approach of W. M. Welsch. The method deals with the testing of the geodetic network's congruence, the affinity of selected triangles, and the testing of other kinematic parameters in 2D geodetic networks. Deformation analysis using the Munich approach can be performed using the X-method, which is based on the comparison of the coordinates of identical points in the geodetic network between two sets of measurements that depend on the geodetic datum, or using the L-method, which determines changes in the values of lengths and angles which are quantities independent of the geodetic datum. In the modified method, we propose to determine the congruence of all lengths and all angles in the network between two sets of measurements and to determine the congruence of all triangles between points in the network, not only selected ones, as it is common in the classical approach of deformation analysis based on the Munich approach. The proposed improvements are tested on the example of a known hexagon, and an analysis is made of the success of detecting stable points in the network using the classical Munich approach. In the present case, the results of the proposed method did not differ from the results of other deformation analysis methods.

KLJUČNE BESEDE

deformacijska analiza, postopek München, modificirana metoda, metoda X, metoda L, testiranje skladnosti, testiranje preoblikovanja trikotnikov

KEY WORDS

deformation analysis, Munich approach, modified method, X-method, L-method, testing of congruence, strain analysis of triangles

1 INTRODUCTION

Monitoring the stability of engineering structures or natural objects is the systematic measurement and tracking of changes in the shape and/or dimensions of the object in question as a result of loads. Continuous monitoring of deformations and recording the measured values is a prerequisite for deformation analysis, based on which we propose timely maintenance or renovation of the object in question. Deformation monitoring primarily belongs to the field of engineering geodesy, using various measuring devices or sensors. Various methods have been developed for adequate strain analysis (Chrzanowski et al., 1986; Ašanin, 1986; Mihailović and Aleksić, 1994).

This paper deals with the deformation analysis according to the Munich approach which was developed by W. Welsch who worked at the Institute of Geodesy at the Military Academy in Munich. The method is based on the analysis of the deformations of triangles whose vertices are formed by geodetic points in the network and whose coordinate changes are calculated based on the angle and length changes in the network. Deformation analysis according to the Munich approach can be performed in two ways (Welsch, 1983; Welsch and Zhang, 1983; Soldo and Ambrožič, 2018):

- with the **X-method**, which is based on the comparison of the coordinates of identical points of the geodetic network of two sets of measurements. Since the coordinates of the points depend on the geodetic datum, the results of the free network adjustment must refer to the same geodetic datum. If the number of points in the network is different for two epochs, the coordinate unknowns of the non-identical points are eliminated using the S-transformation (van Mierlo, 1987),
- with the **L-method**, which is based on comparing quantities independent of the geodetic datum, i.e. bearings and lengths.

The classical approach to deformation analysis using the Munich approach requires free network adjustment for each epoch by providing an identical geodetic datum and testing the homogeneity of the accuracy of the measurements of the dimensions in question. In the following, we perform testing of the congruence of a geodetic network, trying to determine whether displacements and deformations of the whole object have occurred. If we find statistically significant deformations of the object, we proceed to the testing of affinity of the geodesic network by dividing the network into selected triangles and determining the change in the shape of each triangle. We calculate the basic kinematic parameters in each triangle and then other parameters such as normal and shear deformations. Finally, we examine the changes in the datum-independent lengths in the network, as suggested by the author of the W. M. Welsch method, and in this way discover the points that have moved in a statistically significant way (Welsch and Zhang, 1983; Ašanin, 1986).

The modified method we propose in this paper is based on forming all possible triangles in the network rather than just the selected ones, and in addition to testing changes in the datum-independent lengths, also testing independent angles in all the triangles formed. In this way, we capture much more information about the changes in the network and find with greater certainty the points that have moved significantly, which is a considerable improvement on the classical approach to deformation analysis using the Munich approach.

According to the two methods of deformation analysis based on the Munich approach, i.e. according to the X-method and according to the L-method, exactly the same results are obtained in the phase of testing of congruence of the geodetic network and the phase of testing of affinity of the geodesic network (Welsch, 1983; Welsch and Zhang, 1983; Soldo and Ambrožič, 2018), therefore, in the proposed modified method, we will use only the X-method for testing of congruence and testing of affinity of the geodetic network.

2 DEFORMATION ANALYSIS ACCORDING TO THE MUNICH APPROACH

2.1 Prerequisites for the performance of deformation analyses

To perform deformation analysis, it is important to ensure the quality and consistency of measurement accuracy between two sets of measurements. This is done by checking the homogeneity of the accuracy of the measurements with the null hypothesis H_0: $E(s_1^2) = E(s_2^2)$, which states that the accuracy of the measurements between two sets of measurements is the same, where s_i are a posteriori reference standard deviation of the unit weights for each set of measurements. Gross errors must be excluded from the measurements according to established procedures such as Baard's, Pope's, Danish, or other appropriate methods (Caspary, 1988; Grigillo and Stopar, 2003; Vrce, 2011).

Methods of deformation analysis require adjustment of an individual set of measurements as a free network with a minimal trace of the cofactor matrix of unknown (Ambrožič, 2001; Sušić et al., 2017). It is important to consider only the coordinate unknowns, so the orientation unknowns are eliminated by reducing the unknowns in the observation equations. If we consider only length measurements, we must also reduce the unknowns due to the scaling factor in the network (Van Mierlo, 1978). The deformation analysis can only be performed on identical points in the network, so the S-transformation eliminates coordinate unknowns from non-identical points in the considered set of measurements, if necessary (Van Mierlo, 1978; Caspary 1988; Marjetič and Stopar, 2007; Sušić et al, 2015a; Marković et al, 2019). The results of the adjustment are the estimated vectors of the adjusted coordinates of the points $\hat{\mathbf{x}}_i$ with the corresponding cofactor matrices of the coordinate unknowns $\mathbf{Q}_{\hat{\mathbf{x}}_i\hat{\mathbf{x}}_i}$ and the a posteriori reference variance of the weight unit s_i^2 for each set of measurement.

After providing an identical geodetic datum and verifying the homogeneity of the accuracy of the two sets of measurements discussed, we compute the reference variance a posteriori using the expression

$$s^2 = \frac{\mathbf{v}_1^{\mathrm{T}}\mathbf{P}_{\mathbf{ll}_1}\mathbf{v}_1 + \mathbf{v}_2^{\mathrm{T}}\mathbf{P}_{\mathbf{ll}_2}\mathbf{v}_2}{f_1 + f_2} = \frac{f_1 s_1^2 + f_2 s_2^2}{f}, \tag{1}$$

where $\mathbf{v}_1$ and $\mathbf{v}_2$ are the vectors of residuals of the measurements, $\mathbf{P}_{\mathbf{ll}_1}$ and $\mathbf{P}_{\mathbf{ll}_2}$ are the weight matrices of the measurements, f_1 and f_2 are the number of degrees of freedom and we can write the relation $f_1 + f_2 = f$, s_1^2 and s_2^2 are the a posteriori reference variance after adjustment the 1st and 2nd sets of measurements at t_1 and t_2.

If statistically significant deformations are found when testing the conformity of the geodetic network, we continue with the classical approach of testing of affinity of the geodetic network by analysing individual triangles and determining whether the triangle has significantly changed the shape between two sets of measurements.

2.2 Testing of congruence of the geodetic network

The decision of whether displacements and deformations have occurred in the geodetic network is made using the testing of congruence. We formulate the null and alternative hypotheses (Welsch, 1983):

$H_0 : E(\hat{\mathbf{x}}_1) = E(\hat{\mathbf{x}}_2)$ or $E(\mathbf{u}) = \mathbf{0}$... the coordinates of all points of the network have not changed between two epochs;

$H_a : E(\hat{\mathbf{x}}_1) \neq E(\hat{\mathbf{x}}_2)$ or $E(\mathbf{u}) \neq \mathbf{0}$... the coordinates of at least one point of the network have changed between two epochs.

When testing the consistency of the X-method with a global stability test for all points in the network, the variance of the point coordinate differences $s_{\mathbf{u}}^2$ is compared with a posteriori reference variance estimate s^2.

Let us compile the test statistics:

$$T_1^2 = \frac{s_{\mathbf{u}}^2}{s^2} = \frac{\mathbf{u}^{\mathrm{T}} \mathbf{Q}_{\mathbf{u}}^{-1} \mathbf{u}}{f_{\mathbf{u}} \cdot s^2}, \tag{2}$$

The vector of displacements of identical points is calculated using the following expression:

$$\mathbf{u} = \hat{\mathbf{x}}_2 - \hat{\mathbf{x}}_1, \tag{3}$$

where $\hat{\mathbf{x}}_1$ and $\hat{\mathbf{x}}_2$ are the vector of the adjusted coordinates of all points after the adjusting 1st and 2nd sets of measurements t_1 and t_2.

The cofactors matrix of the coordinate differences is written as

$$\mathbf{Q}_{\mathbf{u}} = \mathbf{Q}_{\hat{\mathbf{x}}_1\hat{\mathbf{x}}_1} + \mathbf{Q}_{\hat{\mathbf{x}}_2\hat{\mathbf{x}}_2}, \tag{4}$$

where $\mathbf{Q}_{\hat{\mathbf{x}}_1\hat{\mathbf{x}}_1}$ and $\mathbf{Q}_{\hat{\mathbf{x}}_2\hat{\mathbf{x}}_2}$ are the cofactor matrices of the coordinate unknowns after smoothing, which are calculated with the expressions $\mathbf{Q}_{\hat{\mathbf{x}}_1\hat{\mathbf{x}}_1} = (\mathbf{B}_1^{\mathrm{T}} \mathbf{P}_{\mathbf{ll}_1} \mathbf{B}_1 + \mathbf{G}^{\mathrm{T}}\mathbf{G})^{-1}$ and $\mathbf{Q}_{\hat{\mathbf{x}}_2\hat{\mathbf{x}}_2} = (\mathbf{B}_2^{\mathrm{T}} \mathbf{P}_{\mathbf{ll}_2} \mathbf{B}_2 + \mathbf{G}^{\mathrm{T}}\mathbf{G})^{-1}$. $\mathbf{B}_1$ and $\mathbf{B}_2$ are the design matrices. $\mathbf{P}_{\mathbf{ll}_1}$ and $\mathbf{P}_{\mathbf{ll}_2}$ are the weight matrices of the 1st and 2nd sets of measurements at moments t_1 and t_2 (Kuang, 1996).

Datum matrix $\mathbf{G}^{\mathrm{T}}$ in 2D geodetic triangulation network has the form (Welsch, 1986):

$$\mathbf{G}^{\mathrm{T}} = \begin{bmatrix} 1 & 0 & 1 & 0 & \dots & 1 & 0 \\ 0 & 1 & 0 & 1 & \dots & 0 & 1 \\ x_i^0 & -y_i^0 & x_j^0 & -y_j^0 & \dots & x_m^0 & -y_m^0 \\ y_i^0 & x_i^0 & y_j^0 & x_j^0 & \dots & y_m^0 & x_m^0 \end{bmatrix}, \tag{5}$$

where $y_i^0, x_i^0, y_j^0, x_j^0, y_m^0, x_m^0$ are the approximate coordinates of m points of geodetic network.

The number of degrees of freedom is:

$$f_{\mathbf{u}} = u - d. \tag{6}$$

We denote the number of all points in the geodetic network by m. In equation (6) $u = 2m$ represents the number of unknown coordinates in the 2D network and d the datum defect of the geodetic network (Welsch, 1983; Welsch and Zhang, 1983; Soldo and Ambrožič, 2018).

The test statistic (2) is distributed according to the Fisher distribution $F_{f_{\mathbf{u}},f}$:

- $T_1^2 \leq F_{f_{\mathbf{u}},f}$: the null hypothesis H_0 cannot be rejected and with significance level α we cannot claim that deformations have occurred in the network. In other words, the deformations are not statistically significant.
- $T_1^2 > F_{f_{\mathbf{u}},f}$: we reject the null hypothesis H_0 and claim with a significance level smaller than α that deformation has occurred in the network. In other words, the deformations are statistically significant.

If compliance testing of the geodetic network confirms the presence of deformations in the network, we proceed to a more detailed consideration and localization of the displacements of points in the geodetic network.

2.3 Testing of affinity of the geodetic network

The classical approach to testing of affinity of the geodetic network with the X-method is based on the formation of selected triangles and the calculation of the kinematic parameters (vector **p**) in these triangles. As a rule, such triangles are not overlapping. The choice of the vertices of the triangles has a decisive influence on the calculation of the kinematic parameters. If the points in the selected triangle move in any direction and the movements are large enough, we calculate large values of the kinematic parameters. However, if the points in the selected triangle all move in the same direction and their displacements are similar in magnitude, we usually calculate small values of the kinematic parameters and the change in the shape and size of the triangle is smaller. In the classical approach, we usually choose a limited number of triangles.

From the adjusted coordinates of the 1st and 2nd sets of measurement epoch t_1 and t_2, we calculate the change in the measurement value which can also be written as a function of point movements (Welsch and Zhang, 1983; Soldo and Ambrožič, 2018):

$$d\mathbf{l} = \mathbf{l}_2 - \mathbf{l}_1 = \mathbf{L} \cdot \mathbf{u}, \tag{7}$$

with the corresponding cofactor matrix of the change in measured value

$$\mathbf{Q}_{d\mathbf{l}} = \mathbf{L}\mathbf{Q}_{\mathbf{u}}\,\mathbf{L}^{\mathrm{T}}. \tag{8}$$

It is important to emphasize that the matrix of partial derivates of measurements by coordinate unknowns $\mathbf{L} = \left[\frac{\partial \mathbf{l}}{\partial \hat{\mathbf{x}}}\right]$ depends on the nature of the measurements, i.e., whether we consider only lengths, only angles, or both.

In our modified method we create all possible triangles in the network to obtain more information about changes in the network. We may miss important changes due to the inappropriate selection of triangles. It is natural to calculate lengths and angles in a triangle, therefore in our proposal we first test the change of lengths, then the change of angles and only finally the change of kinematic parameters in the triangle. Since calculating all possible combinations is not a computational problem anymore, it makes sense to gather all information about changes in the network.

2.3.1 Testing lengths in the geodetic network

The test is performed by testing changes in the datum independent lengths connecting a single point to other points. We write down the null and alternative hypothesis (Welsch, 1982):

H_0: $E(dl_{dD_{ij}}) = 0$… the length difference between the points P_i and P_j has not changed between two epochs;

H_a: $E(dl_{dD_{ij}}) \neq 0$… the length difference between the points P_i and P_j has changed between two epochs.

Let us calculate the test statistic for the length difference between points P_i and P_j (Welsch, 1982):

$$T^2_{2D_{ij}} = \frac{dl_{dD_{ij}} Q^{-1}_{dD_{ij}} dl_{dD_{ij}}}{n_D \cdot s^2}, \tag{9}$$

where $n_D = 1$ is the number of degrees of freedom (Welsch, 1982; Soldo and Ambrožič, 2018).

The length difference is calculated with the following equation

$$dl_{dD_{ij}} = D_{ij_2} - D_{ij_1}, \tag{10}$$

where the element $dl_{dD_{ij}}$ refers to the length $D_{ij_1} = \sqrt{(\hat{y}_{j_1} - \hat{y}_{i_1})^2 + (\hat{x}_{j_1} - \hat{x}_{i_1})^2}$ and $D_{ij_2} = \sqrt{(\hat{y}_{j_2} - \hat{y}_{i_2})^2 + (\hat{x}_{j_2} - \hat{x}_{i_2})^2}$, calculated from the adjusted coordinates between the points P_i and P_j of the 1st and 2nd sets of measurements.

The element of the cofactor matrix of the change in the value of length in equation (9) is calculated by equation (8) $Q_{dD_{ij}} = \mathbf{L}_{dD_{ij}} \mathbf{Q}_{\mathbf{u}dD_{ij}} \mathbf{L}^{\mathrm{T}}{}_{dD_{ij}}$, where the vector of partial derivatives of the measurements $\mathbf{L}_{dD_{ij}}$ is written as follows (Welsch and Zhang, 1983; Soldo and Ambrožič, 2018):

$$\mathbf{L}_{dD_{ij}} = \left[\frac{\partial \mathbf{l}_{dD_{ij}}}{\partial \hat{\mathbf{x}}}\right] = \left[-\sin\nu_{ij} \quad -\cos\nu_{ij} \quad \sin\nu_{ij} \quad \cos\nu_{ij}\right], \tag{11}$$

where $\nu_{ij} = (\nu_{ij_1} + \nu_{ij_2})/2$ is the average of the bearings ν_{ij_1} and ν_{ij_2}, calculated from the adjusted coordinates between points P_i and P_j, $\nu_{ij_1} = \arctan\frac{\hat{y}_{j_1} - \hat{y}_{i_1}}{\hat{x}_{j_1} - \hat{x}_{i_1}}$ and $\nu_{ij2} = \arctan\frac{\hat{y}_{j_2} - \hat{y}_{i_2}}{\hat{x}_{j_2} - \hat{x}_{i_2}}$ in the 1st and 2nd sets of measurements.

In the submatrix $\mathbf{Q}_{\mathbf{u}dD_{ij}}$ are the corresponding elements of the matrix $\mathbf{Q}_{\mathbf{u}}$, which refer to points P_i and P_j.

The test statistic (9) is distributed according to the Fisher distribution $F_{1,f}$:

- $T^2_{2D_{ij}} \leq F_{1,f}$: the null hypothesis H_0 cannot be rejected and with significance level α we cannot claim that the length between considered points has changed,
- $T^2_{2D_{ij}} > F_{1,f}$: we reject the null hypothesis H_0 and claim with a significance level smaller than α that the length between the considered points has changed.

2.3.2 Testing angles in the geodetic network

The test is performed by testing changes in datum independent angles connecting a single point to other points. We write down the null and the alternative hypothesis:

H_0: $E(dl_{d\alpha_{ijk}}) = 0$... the angle difference between the points P_i, P_j and P_k has not changed between two epochs;

H_a: $E(dl_{d\alpha_{ijk}}) \neq 0$... the angle difference between the points P_i, P_j and P_k has changed between two epochs.

We create the test statistic for the angle difference between the points P_i, P_j and P_k:

$$T^2_{2\alpha_{ijk}} = \frac{dl_{d\alpha_{ijk}} Q^{-1}_{d\alpha_{ijk}} dl_{d\alpha_{ijk}}}{n_\alpha \cdot s^2}, \tag{12}$$

where $n_\alpha = 1$ is the number of degrees of freedom.

The angle difference is calculated with following expression

$$dl_{d\alpha_{ijk}} = \alpha_{ijk_2} - \alpha_{ijk_1}, \tag{13}$$

where the element $dl_{d\alpha_{ijk}}$ refers to the angles $\alpha_{ijk_1} = v_{ik_1} - v_{ij_1}$ and $\alpha_{ijk_2} = v_{ik_2} - v_{ij_2}$ with the vertex at the point P_i against the points P_j and P_k in the 1st and 2nd sets of measurements, where v_{ij_1} and v_{ik_1}, v_{ij_2} and v_{ik_2} are the bearings from the offset coordinates between points P_i and P_j, P_i and P_k in the 1st and 2nd sets of measurement.

The element of the cofactor matrix of the change in the value of angular in equation (12) is calculated by equation (8) $Q_{d\alpha_{ijk}} = \mathbf{L}_{d\alpha_{ijk}} \mathbf{Q}_{\mathbf{u}d\alpha_{ijk}} \mathbf{L}^{\mathrm{T}}_{d\alpha_{ijk}}$, where the vector of the partial derivatives of the measurements $\mathbf{L}_{d\alpha_{ijk}}$ is written as

$$\mathbf{L}_{d\alpha_{ijk}} = \left[\frac{\partial \mathbf{l}_{d\alpha_{ijk}}}{\partial \hat{\mathbf{x}}}\right] = \left[\left(-\frac{\cos v_{ik}}{D_{ik}} + \frac{\cos v_{ij}}{D_{ij}}\right)\left(\frac{\sin v_{ik}}{D_{ik}} + \frac{\sin v_{ij}}{D_{ij}}\right)\left(-\frac{\cos v_{ij}}{D_{ij}}\right)\left(\frac{\sin v_{ij}}{D_{ij}}\right)\left(\frac{\cos v_{ik}}{D_{ik}}\right)\left(-\frac{\sin v_{ik}}{D_{ik}}\right)\right], \tag{14}$$

where $v_{ij} = (v_{ij_1} + v_{ij_2})/2$ and $v_{ik} = (v_{ik_1} + v_{ik_2})/2$ are the averages of bearings v_{ij_1} and v_{ik_1}, v_{ij_2} and v_{ik_2}. The lengths $D_{ij} = (D_{ij_1} + D_{ij_2})/2$ and $D_{ik} = (D_{ik_1} + D_{ik_2})/2$ represent the average values of the lengths obtained from the adjusted coordinates between points P_i and P_j, P_i and P_k in the 1st and 2nd sets of measurements.

In the submatrix $\mathbf{Q}_{\mathbf{u}d\alpha_{ijk}}$ are the corresponding elements of the matrix $\mathbf{Q}_\mathbf{u}$ referring to the points P_i, P_j and P_k.

The test statistic (12) is distributed according to the Fisher distribution $F_{1,f}$:

- $T^2_{2\alpha_{ijk}} \leq F_{1,f}$: the null hypothesis H_0 cannot be rejected and with significance level α we cannot claim that the angle between considered points has changed,
- $T^2_{2\alpha_{ijk}} > F_{1,f}$: we reject the null hypothesis H_0 and claim with a significance level smaller than α that the angle between the considered points has changed.

2.3.3 Testing triangles in a geodetic network

We divide the network in question into triangles and determine whether a single triangle has changed shape between the two sets of measurements and in this way explain the coordinate differences in the network. We note the null and alternative hypotheses (Welsch, 1983):

$H_0 : E(\hat{\mathbf{x}}_1) = E(\hat{\mathbf{x}}_2)$ or $E(\mathbf{u}_{\Delta_{ijk}}) = \mathbf{0}$... the coordinates of the points of the triangle have not changed between two epochs;

$H_a : E(\hat{\mathbf{x}}_1) \neq E(\hat{\mathbf{x}}_2)$ or $E(\mathbf{u}_{\Delta_{ijk}}) \neq \mathbf{0}$... the coordinates of at least one point in the triangle have changed between two epochs.

Let us calculate a test statistic for the vertices P_i, P_j and P_k of the triangle:

$$T^2_{2\Delta_{ijk}} = \frac{\mathbf{u}^{\mathrm{T}}_{\Delta_{ijk}} \mathbf{Q}_{\mathbf{u}}^{-1} \mathbf{u}_{\Delta_{ijk}}}{n_\Delta \cdot s^2}, \tag{15}$$

where $n_\Delta = 3$ is the number of degrees of freedom (Welsch, 1983; Welsch and Zhang, 1983; Soldo and Ambrožič, 2018).

We calculate the vector of displacements of the vertices in the triangle $\mathbf{u}_{\Delta_{ijk}}$ by the equation (3), where $\hat{\mathbf{x}}_1$ and $\hat{\mathbf{x}}_2$ are the vectors of the adjusted coordinates of vertices P_i, P_j and P_k in the triangle after the 1st and 2nd sets of measurements have been adjusted.

The cofactor matrix of the coordinate differences $\mathbf{Q}_{\mathbf{u}}$ is calculated by equation (4), where $\mathbf{Q}_{\hat{\mathbf{x}}_1\hat{\mathbf{x}}_1}$ and $\mathbf{Q}_{\hat{\mathbf{x}}_2\hat{\mathbf{x}}_2}$ are the cofactor matrices of the unknown coordinates of the vertices P_i, P_j and P_k in the triangle after the adjustment of the geodetic network.

The test statistic (15) is distributed according to the Fisher distribution $F_{1,f}$:

- $T^2_{2\Delta_{ijk}} \le F_{3,f}$: the null hypothesis H_0 cannot be rejected and with the significance level α we cannot claim that the coordinates in the triangle with vertices P_i, P_j and P_k have changed,
- $T^2_{2\Delta_{ijk}} > F_{3,f}$: we reject the null hypothesis H_0 and claim with a significance level smaller than α that the coordinates in the triangle with vertices P_i, P_j and P_k have changed.

In the following, we can calculate the kinematic parameters in each triangle:

$$\mathbf{p} = \mathbf{H}_{\mathbf{u}}^{-1} \cdot \mathbf{u}, \tag{16}$$

where $\mathbf{H}_{\mathbf{u}}$ is the deformation model matrix that relates the kinematic parameters to the displacements of the points in the triangle

$$\mathbf{H}_{\mathbf{u}} = \begin{bmatrix} \hat{x}_i & \hat{y}_i & 0 & -\hat{y}_i & 1 & 0 \\ 0 & \hat{x}_i & \hat{y}_i & \hat{x}_i & 0 & 1 \\ \hat{x}_j & \hat{y}_j & 0 & -\hat{y}_j & 1 & 0 \\ 0 & \hat{x}_j & \hat{y}_j & \hat{x}_j & 0 & 1 \\ \hat{x}_k & \hat{y}_k & 0 & -\hat{y}_k & 1 & 0 \\ 0 & \hat{x}_k & \hat{y}_k & \hat{x}_k & 0 & 1 \end{bmatrix}, \tag{17}$$

$$\mathbf{P}^{\mathrm{T}} = \begin{bmatrix} e_{xx} & e_{xy} & e_{yy} & \omega & t_x & t_y \end{bmatrix} \tag{18}$$

and $\mathbf{p}$ is a vector of kinematic parameters, where e_{xx} and e_{yy} are the normal deformations in the direction of the coordinate axes x and y, $e_{xy} = e_{yx}$ is the shear deformation (only e_{xx}, e_{yy} and e_{xy} are the deformation parameters), ω represents rotation, t_x and t_y are the translations in the direction of the coordinate axes x and y.

Based on the basic parameters, we calculate other kinematic parameters (Welsch, 1983; Mihailović and Aleksić, 1994; Ašanin, 1986; Acar, 2010; Labant et al., 2014; Sušić et al., 2015b; Boresi and Sidebottom, 1985):

$\Delta = e_{xx} + e_{yy}$ represents the change of area,

$e_1 = ½\,(e_{xx} + e_{yy} + ee)$ is the principal (the largest) normal strain,

$e_2 = ½\,(e_{xx} + e_{yy} - ee)$ is the principal (the smallest) normal strain,

where $ee^2 = (e_{xx} - e_{yy})^2 + 4e_{xy}^2$,

$e_1 = \dfrac{e_1 - e_2}{2}$ is the principal shear strain and

$\gamma = 2e_{xy}$ is the engineering shear strain and represents the change of the right angle between x and y directions.

The bearings of the principal normal strains are calculated with the expression $\tan 2\vartheta = \dfrac{2e_{xy}}{e_{xx} - e_{yy}}$ and the bearing of the principal shear strain is calculated with the expression $\Psi = \vartheta + 45°$.

3 CALCULATION EXAMPLE

For a simple and direct comparison of the discussed modified method of deformation analysis according to the Munich approach with other methods of deformation analysis, we choose the example of a simulated geodetic network, calculated with other methods of deformation analysis (Ambrožič, 2001; Ambrožič, 2004; Marjetič et al., 2012; Vrečko and Ambrožič, 2013; Soldo and Ambrožič, 2018; Ambrožič et al., 2019; Hamza et al., 2020; Batilović et al., 2022). In the cited literature, we can see the sketch of the network and obtain all the necessary data for adjustment. Briefly summarizing the results of free network adjustment by reducing the orientation unknowns of the 1st set of measurements, we obtain $s_1 = 0.9699$, $f_1 = 30$ and the 2nd set of measurements $s_2 = 1.1562$ and $f_2 = 30$. Since we confirm the identity of the geodetic datum and the homogeneity of the accuracy in the considered term measurements, we calculate the total reference variance a posteriori $s^2 = 1.1387$ and the total number of degrees of freedom is $f = 60$ (equation 1). In addition, a significance level $\alpha = 5\ \%$ is chosen for all.

The test of congruence of the geodetic network by the classical approach and by the proposed modified method of deformation analysis is exactly the same, so we write only that $T_1^2 = 141.29$ (equation 2), which exceeds the critical value at the chosen test characteristic $F_{11,60} = 1.95$. Therefore, we reject the null hypothesis and claim with a significance level smaller than $\alpha = 5\ \%$, that deformations occurred in the network, and continue the deformation analysis by testing of affinity of the geodetic network.

The deformation analysis according to the proposed modified method is performed by testing of affinity of the geodesic network by first considering all the lengths in the geodetic network, of which there are 21. For each calculated length we create a test statistic $T_{2\,D_{ij}}^2$ (equation 9) and compare it with the critical value at the chosen level of test characteristics $F_{1,60} = 4.00$, the results are given in Table 1. We calculate the actual risk α_T for rejecting the null hypothesis, which we compare with the significance level $\alpha = 5\ \%$. In the cases where the actual risk is lower than the chosen one, we must reject the null hypothesis and claim that the deformations are statistically significant. However, in the opposite cases, we can conclude that the lengths between stationary points have not changed statistically significantly, since the actual risk is greater than 5 % in all cases.

Table 1: Analysis of testing all lengths between the points.

Length	Between the points	$T^2_{2D_{ij}}$	H_0	α_T [%]	Length	Between the points	$T^2_{2D_{ij}}$	H_0	α_T [%]
1	1-2	19.89	reject	0.00*	12	3-4	124.81	reject	0.00*
2	1-3	49.38	reject	0.00*	13	3-5	62.37	reject	0.00*
3	1-4	87.04	reject	0.00*	14	3-6	10.41	reject	0.20
4	1-5	64.84	reject	0.00*	15	3-7	9.20	reject	0.36
5	1-6	10.05	reject	0.24	16	4-5	0.08	not rej.	77.85
6	1-7	689.26	reject	0.00*	17	4-6	0.01	not rej.	91.54
7	2-3	109.75	reject	0.00*	18	4-7	186.39	reject	0.00*
8	2-4	84.02	reject	0.00*	19	5-6	0.63	not rej.	42.87
9	2-5	163.39	reject	0.00*	20	5-7	83.12	reject	0.00*
10	2-6	113.96	reject	0.00*	21	6-7	77.68	reject	0.00*
11	2-7	113.61	reject	0.00*					

* actual risk is smaller than 0.005 %

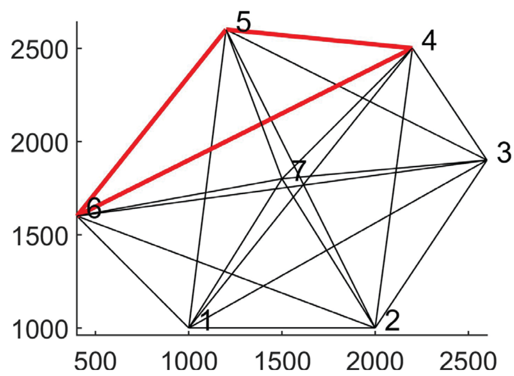

Figure 1: Analysis of the lengths between the points: the lengths where the null hypothesis cannot be rejected are marked in red and the lengths where the null hypothesis is rejected are marked in black.

Table 1 and Figure 1 show that only the lengths between points 4-5, 4-6 and 5-6 did not change statistically significantly.

Then we continue the deformation analysis using the proposed modified method by testing of affinity of the geodetic network considering all the angles in the geodetic network, of which there are 105. For each calculated angle, we create the test statistic $T^2_{2\,\alpha_{ijk}}$ (equation 12) and compare it with the critical value at the chosen level of test characteristics $F_{1,60} = 4.00$, the results are shown in Table 2. We calculate the actual risk α_T for rejecting the null hypothesis, which we compare with the significance level $\alpha = 5$ %. When testing angles, it is not possible to unambiguously determine which angle does not change its value, since we also test angles between two fixed and one moving points, or even between one fixed and two moving points. From Table 2, we can conclude that the actual risk is greater than 5 % for all combinations of angles where all three stationary points occur, such as angles 4-5-6 ($\alpha_T = 33$ %), 5-4-6 ($\alpha_T = 44$ %), and 6-4-5 ($\alpha_T = 74$ %).

Table 2: Analysis of testing all angles between the points.

Angle	Between the points	$T^2_{2\alpha_{ijk}}$	H_0	α_T [%]	Angle	Between the points	$T^2_{2\alpha_{ijk}}$	H_0	α_T [%]
1	1-2-3	765.25	reject	0.00*	36	3-2-4	172.51	reject	0.00*
2	1-2-4	382.51	reject	0.00*	37	3-2-5	256.93	reject	0.00*
3	1-2-5	377.49	reject	0.00*	38	3-2-6	322.97	reject	0.00*
4	1-2-6	409.63	reject	0.00*	39	3-2-7	15.28	reject	0.02
5	1-2-7	304.97	reject	0.00*	40	3-4-5	2.50	not rej.	11.88
6	1-3-4	55.84	reject	0.00*	41	3-4-6	4.70	reject	3.42
7	1-3-5	2.08	not rej.	15.41	42	3-4-7	183.15	reject	0.00*
8	1-3-6	38.50	reject	0.00*	43	3-5-6	2.24	not rej.	14.01
9	1-3-7	32.56	reject	0.00*	44	3-5-7	368.79	reject	0.00*
10	1-4-5	24.42	reject	0.00*	45	3-6-7	342.61	reject	0.00*
11	1-4-6	104.38	reject	0.00*	46	4-1-2	153.77	reject	0.00*
12	1-4-7	0.33	not rej.	56.77	47	4-1-3	7.99	reject	0.64
13	1-5-6	78.35	reject	0.00*	48	4-1-5	0.05	not rej.	83.07
14	1-5-7	20.96	reject	0.00*	49	4-1-6	3.88	not rej.	5.36
15	1-6-7	101.10	reject	0.00*	50	4-1-7	33.76	reject	0.00*
16	2-1-3	844.94	reject	0.00*	51	4-2-3	17.86	reject	0.01
17	2-1-4	474.80	reject	0.00*	52	4-2-5	42.39	reject	0.00*
18	2-1-5	448.55	reject	0.00*	53	4-2-6	69.33	reject	0.00*
19	2-1-6	406.03	reject	0.00*	54	4-2-7	22.08	reject	0.00*
20	2-1-7	916.07	reject	0.00*	55	4-3-5	4.18	reject	4.52
21	2-3-4	361.72	reject	0.00*	56	4-3-6	3.20	not rej.	7.89
22	2-3-5	426.83	reject	0.00*	57	4-3-7	0.28	not rej.	59.82
23	2-3-6	456.29	reject	0.00*	58	4-5-6	0.98	not rej.	32.59
24	2-3-7	76.78	reject	0.00*	59	4-5-7	14.57	reject	0.03
25	2-4-5	69.08	reject	0.00*	60	4-6-7	15.39	reject	0.02
26	2-4-6	132.49	reject	0.00*	61	5-1-2	1.17	not rej.	28.29
27	2-4-7	37.28	reject	0.00*	62	5-1-3	4.23	reject	4.40
28	2-5-6	68.82	reject	0.00*	63	5-1-4	5.51	reject	2.23
29	2-5-7	230.59	reject	0.00*	64	5-1-6	5.41	reject	2.34
30	2-6-7	365.41	reject	0.00*	65	5-1-7	230.42	reject	0.00*
31	3-1-2	561.33	reject	0.00*	66	5-2-3	15.02	reject	0.03
32	3-1-4	0.39	not rej.	53.44	67	5-2-4	4.47	reject	3.86
33	3-1-5	1.06	not rej.	30.63	68	5-2-6	1.15	not rej.	28.79
34	3-1-6	11.15	reject	0.14	69	5-2-7	232.58	reject	0.00*
35	3-1-7	468.25	reject	0.00*	70	5-3-4	38.75	reject	0.00*

Angle	Between the points	$T^2_{2\alpha ijk}$	H_0	α_T [%]	Angle	Between the points	$T^2_{2\alpha ijk}$	H_0	α_T [%]
71	5-3-6	9.61	reject	0.30	89	6-4-7	127.84	reject	0.00*
72	5-3-7	240.57	reject	0.00*	90	6-5-7	90.46	reject	0.00*
73	5-4-6	0.60	not rej.	44.04	91	7-1-2	132.04	reject	0.00*
74	5-4-7	73.79	reject	0.00*	92	7-1-3	199.15	reject	0.00*
75	5-6-7	106.42	reject	0.00*	93	7-1-4	9.90	reject	0.26
76	6-1-2	296.34	reject	0.00*	94	7-1-5	41.02	reject	0.00*
77	6-1-3	35.57	reject	0.00*	95	7-1-6	33.70	reject	0.00*
78	6-1-4	98.75	reject	0.00*	96	7-2-3	27.83	reject	0.00*
79	6-1-5	69.53	reject	0.00*	97	7-2-4	44.72	reject	0.00*
80	6-1-7	314.80	reject	0.00*	98	7-2-5	233.68	reject	0.00*
81	6-2-3	258.54	reject	0.00*	99	7-2-6	207.06	reject	0.00*
82	6-2-4	38.20	reject	0.00*	100	7-3-4	215.43	reject	0.00*
83	6-2-5	22.19	reject	0.00*	101	7-3-5	432.37	reject	0.00*
84	6-2-7	20.63	reject	0.00*	102	7-3-6	332.44	reject	0.00*
85	6-3-4	97.78	reject	0.00*	103	7-4-5	129.96	reject	0.00*
86	6-3-5	31.15	reject	0.00*	104	7-4-6	57.94	reject	0.00*
87	6-3-7	321.56	reject	0.00*	105	7-5-6	5.96	reject	1.76
88	6-4-5	0.11	not rej.	74.42					

* actual risk is smaller than 0.005 %

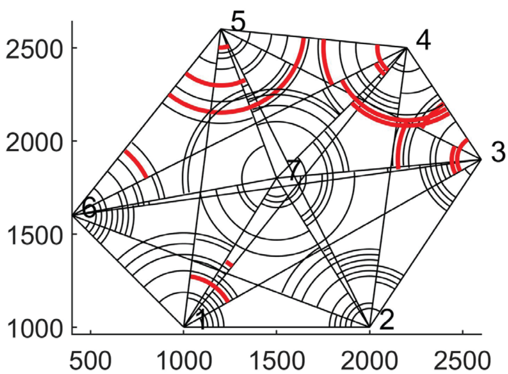

Figure 2: Analysis of angles between the points: angles are marked in red where the null hypothesis cannot be rejected and the angles are marked in black where the null hypothesis is rejected.

Table 2 and Figure 2 show that some angles between points did not change statistically significantly.

The deformation analysis is continued according to the proposed modified method by testing of affinity of the geodesic network by considering the consistency of all triangles in the geodetic network, of which there are 35. For each calculated triangle, we create a test statistic $T^2_{2\Delta_{ijk}}$ (equation 15) and compare it with the critical value at the chosen level characteristics of the test $F_{3,60} = 2.76$, the results are given in

Table 3. We calculate the actual risk α_T for rejecting the null hypothesis, which we compare with the significance level α = 5 %. We can conclude that the triangle between stationary points 4-5-6 did not change statistically significantly, since the actual risk is much higher than 5 % (α_T = 77 %).

Table 3: Analysis of the triangle congruence test between the points.

Triangle	Between the points	$T^2_{2\Delta ijk}$	H_0	α_T [%]	Triangle	Between the points	$T^2_{2\Delta ijk}$	H_0	α_T [%]
1	1-2-3	286.17	reject	0.00*	19	2-3-7	62.03	reject	0.00*
2	1-2-4	159.09	reject	0.00*	20	2-4-5	60.86	reject	0.00*
3	1-2-5	157.59	reject	0.00*	21	2-4-6	68.60	reject	0.00*
4	1-2-6	166.60	reject	0.00*	22	2-4-7	110.69	reject	0.00*
5	1-2-7	336.48	reject	0.00*	23	2-5-6	67.44	reject	0.00*
6	1-3-4	62.88	reject	0.00*	24	2-5-7	147.62	reject	0.00*
7	1-3-5	42.33	reject	0.00*	25	2-6-7	133.31	reject	0.00*
8	1-3-6	24.16	reject	0.00*	26	3-4-5	42.87	reject	0.00*
9	1-3-7	278.24	reject	0.00*	27	3-4-6	41.92	reject	0.00*
10	1-4-5	35.00	reject	0.00*	28	3-4-7	116.79	reject	0.00*
11	1-4-6	46.79	reject	0.00*	29	3-5-6	20.86	reject	0.00*
12	1-4-7	271.23	reject	0.00*	30	3-5-7	162.34	reject	0.00*
13	1-5-6	33.96	reject	0.00*	31	3-6-7	140.22	reject	0.00*
14	1-5-7	249.07	reject	0.00*	32	4-5-6	0.37	not rej.	77.30
15	1-6-7	229.98	reject	0.00*	33	4-5-7	95.96	reject	0.00*
16	2-3-4	136.82	reject	0.00*	34	4-6-7	98.21	reject	0.00*
17	2-3-5	158.79	reject	0.00*	35	5-6-7	56.68	reject	0.00*
18	2-3-6	163.70	reject	0.00*					

* actual risk is smaller than 0.005 %

We do not show the image of the triangles between the points because the figure becomes unlegible due to a large number of triangles.

Due to testing a large number of all lengths between points in the network, an even larger number of all angles between points in the network, and also a large number of testing of congruence of all possible triangles in the network, which is the essence of the proposed modified method of deformation analysis, the analysis is difficult to represent. We can simplify the analysis and make it more legible by performing all three testing of quantities per unit, i.e. per individual triangle. Table 4 shows the results of testing all three quantities in a single triangle.

Table 4: Analysis of testing all three quantities in triangles.

Triangle	Between the points *i-j-k*	H_{0-D} *i-j* *j-k* *i-k*	$H_{0-\alpha}$ *i-j-k* *j-i-k* *k-i-j*	$H_{0-\Delta}$ *i-j-k*
1	1-2-3	reject reject reject	reject reject reject	reject
2	1-2-4	reject reject reject	reject reject reject	reject
3	1-2-5	reject reject reject	reject reject not reject	reject

Triangle	Between the points	$H_{0\text{-}D}$			$H_{0\text{-}\alpha}$			$H_{0\text{-}\Delta}$
	i-j-k	*i-j*	*j-k*	*i-k*	*i-j-k*	*j-i-k*	*k-i-j*	*i-j-k*
4	1-2-6	reject	reject	reject	reject	reject	reject	reject
5	1-2-7	reject	reject	reject	reject	reject	reject	reject
6	1-3-4	reject	reject	reject	reject	not reject	reject	reject
7	1-3-5	reject	reject	reject	not rej.	not rej.	reject	reject
8	1-3-6	reject	reject	reject	reject	reject	reject	reject
9	1-3-7	reject	reject	reject	reject	reject	reject	reject
10	1-4-5	reject	not reject	reject	reject	not reject	reject	reject
11	1-4-6	reject	not reject	reject	reject	not reject	reject	reject
12	1-4-7	reject	reject	reject	not reject	reject	reject	reject
13	1-5-6	reject	not reject	reject	reject	reject	reject	reject
14	1-5-7	reject	reject	reject	reject	reject	reject	reject
15	1-6-7	reject	reject	reject	reject	reject	reject	reject
16	2-3-4	reject	reject	reject	reject	reject	reject	reject
17	2-3-5	reject	reject	reject	reject	reject	reject	reject
18	2-3-6	reject	reject	reject	reject	reject	reject	reject
19	2-3-7	reject	reject	reject	reject	reject	reject	reject
20	2-4-5	reject	not reject	reject	reject	reject	reject	reject
21	2-4-6	reject	not reject	reject	reject	reject	reject	reject
22	2-4-7	reject	reject	reject	reject	reject	reject	reject
23	2-5-6	reject	not reject	reject	reject	not reject	reject	reject
24	2-5-7	reject	reject	reject	reject	reject	reject	reject
25	2-6-7	reject	reject	reject	reject	reject	reject	reject
26	3-4-5	reject	not reject	reject	not reject	reject	reject	reject
27	3-4-6	reject	not reject	reject	reject	not reject	reject	reject
28	3-4-7	reject	reject	reject	reject	not reject	reject	reject
29	3-5-6	reject	not reject	reject	not reject	reject	reject	reject
30	3-5-7	reject	reject	reject	reject	reject	reject	reject
31	3-6-7	reject	reject	reject	reject	reject	reject	reject
32	4-5-6	not rej.	not rej.	not rej.	not rej.	not rej.	not rej.	not rej.
33	4-5-7	not reject	reject	reject	reject	reject	reject	reject
34	4-6-7	not reject	reject	reject	reject	reject	reject	reject
35	5-6-7	not reject	reject	reject	reject	reject	reject	reject

Figure 3 does not show the kinematic parameters for each triangle, but the differences of the adjusted coordinates between two epochs, which is one of the results of the deformation analysis according to the Munich method (Welsch and Zhang, 1983; Welsch, 1983; Soldo and Ambrožič, 2018; Sušić et al., 2015b; Sušić et al., 2017; Mihailović and Aleksić, 1994). There are too many triangles, and some of them are very elongated, so the image would be very illegible.

Based on the test results presented in Table 4, it can be quickly observed that all lengths, all angles and congruence in triangle 32 did not change statistically significantly, which is shown in Figure 3 by plotting the displacements. With standard significance level $\alpha = 5$ %, we claim that the points of this triangle have not shifted significantly, since we tested the triangle both in terms of changes in the lengths and angles in the triangle, and in the coherence of the entire triangle.

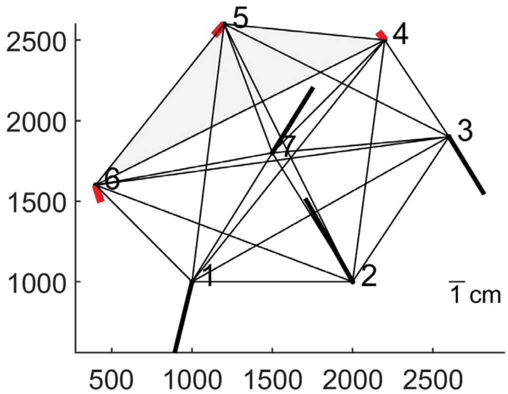

Figure 3: Illustration of point displacements: the movements at points 4, 5 and 6 are marked in red if the null hypothesis cannot be rejected and if the null hypothesis is rejected, they are marked in black.

According to equation (18) we calculate the basic kinematic parameters and show them in Table 5. Other deformation parameters are not calculated and displayed, as they can be easily calculated with the above equations.

Table 5: Calculated kinematic parameters in all triangles.

Triangle	Between the points *i-j-k*	e_{xx} [μs]	e_{xy} [μs]	e_{yy} [μs]	ω ['']	t_x [mm]	t_y [mm]
1	1-2-3	-161.27	82.79	-18.70	-2.7	0.021	-0.062
2	1-2-4	-43.92	57.70	-18.70	-7.9	-0.096	-0.012
3	1-2-5	11.14	50.49	-18.70	-9.4	-0.151	0.003
4	1-2-6	151.73	50.35	-18.70	-9.4	-0.292	0.003
5	1-2-7	46.19	76.48	-18.70	-4.0	-0.186	-0.049
6	1-3-4	52.09	-22.84	32.64	0.3	-0.072	-0.022
7	1-3-5	24.18	-4.11	20.39	0.9	-0.060	-0.031
8	1-3-6	39.05	5.69	4.66	4.6	-0.066	-0.044
9	1-3-7	158.66	-16.92	-7.75	13.8	-0.119	-0.053
10	1-4-5	21.33	8.38	2.64	-1.2	-0.080	-0.016
11	1-4-6	43.04	0.08	-10.54	2.7	-0.074	-0.013
12	1-4-7	368.00	-113.88	-233.38	102.9	0.007	0.031
13	1-5-6	26.76	-12.01	-18.44	3.5	-0.042	0.003
14	1-5-7	2.38	79.05	85.25	17.9	-0.212	-0.088
15	1-6-7	86.78	33.95	13.46	0.6	-0.162	-0.061
16	2-3-4	-14.58	-58.59	75.42	13.5	0.315	-0.187
17	2-3-5	-62.75	-9.55	36.67	8.7	0.218	-0.136

Triangle	Between the points *i-j-k*	e_{xx} [μs]	e_{xy} [μs]	e_{yy} [μs]	ω ['']	t_x [mm]	t_y [mm]
18	2-3-6	-98.67	29.36	0.77	5.6	0.146	-0.088
19	2-3-7	-54.19	-0.77	-8.92	13.2	0.235	-0.075
20	2-4-5	-32.33	12.71	4.05	0.8	0.066	-0.054
21	2-4-6	-34.60	22.41	-13.45	-0.7	0.034	-0.021
22	2-4-7	-28.08	1.59	-68.24	5.1	0.126	0.081
23	2-5-6	-21.31	18.08	-18.57	-2.7	0.011	0.002
24	2-5-7	-129.06	-193.66	-434.50	-1.9	0.550	1.042
25	2-6-7	13.88	58.63	6.90	3.0	-0.050	-0.116
26	3-4-5	81.75	-12.17	-2.07	-6.7	-0.244	0.112
27	3-4-6	55.49	-26.77	13.21	-1.6	-0.092	0.053
28	3-4-7	20.50	-57.18	0.71	3.0	0.111	0.102
29	3-5-6	14.21	-15.96	11.03	-0.5	-0.028	0.029
30	3-5-7	-83.15	-50.90	0.43	2.3	0.284	0.096
31	3-6-7	1279.93	35.50	-26.54	45.7	-1.984	-0.397
32	4-5-6	-5.80	0.43	1.32	-2.3	-0.006	0.020
33	4-5-7	-57.45	-17.02	-1.65	-4.8	0.135	0.101
34	4-6-7	-119.44	-3.87	34.03	14.9	0.153	0.112
35	5-6-7	-39.65	11.85	25.68	-8.7	0.031	0.041

4 CONCLUSIONS

In previous research, we have demonstrated that the results are very similar regardless of which method of deformation analysis is used for the test case (Ambrožič, 2001; Ambrožič, 2004; Marjetič et al., 2012; Vrečko and Ambrožič, 2013; Soldo and Ambrožič, 2018; Ambrožič et al., 2019; Hamza et al., 2020; Batilović et al., 2022). In our research, we focused on important improvements that would allow a more reliable determination of the deformations of the object under consideration or the measured surface. In the proposed modified method of deformation analysis according to the Munich approach, we propose that before the phase of testing of affinity of the geodetic network, therefore before the formation of triangles in the network, in addition to testing changes in independent lengths, we also perform testing of independent angles between all points in the network. In the next phase, we form all possible triangles between the points of the network and not only the selected ones, because in this way we obtain more reliable information about possible changes in the shape of the triangle and, consequently, about the points that have shifted in a statistically significant way.

When analyzing the test of all lengths in the network, with the proposed method, we managed to find the lengths between points 4-5, 4-6 and 5-6, that did not change statistically significantly, so the null hypothesis cannot be rejected (Table 1 and Figure 1). This is consistent with the simulated network movements when points 4, 5 and 6 are presumed stable. When analyzing the test of all angles in the network using the proposed method, we managed to find all combinations of angles between points 4-5-6, 5-4-6 and 6-4-5 that did not change statistically significantly, so the null hypothesis cannot be rejected (Table 2 and

Figure 2). Analyzing the consistency test of all triangles in the network, we managed to find one triangle with vertices in points 4, 5 and 6 that did not change statistically significantly, so the null hypothesis cannot be rejected (Table 3). If we write down the results of the tests more transparently, we can find that all lengths, all angles and the congruence in triangle 32, i.e. between points 4-5-6, did not change statistically significantly (Table 4). We conclude that with the significance level $\alpha = 5$ % we cannot claim that the points of this triangle moved, which is consistent with the simulated movements in the network.

The advantages of the proposed modified method are that the user of the deformation analysis usually forms:

- triangles between adjacent points, but we propose between all points and thus obtain information about the deformations between all points, which can be overlooked if we form the triangles themselves.
- "nice" shapes of triangles, which in turn causes us to lose information about the deformations of such triangle in the case of a very elongated triangle or a triangle where one side is much shorter than the other two.

The disadvantage of the proposed modified method is in the case of a geodetic network when only two points are stationary. In this case, we cannot form a triangle (with three points) if all three tested quantities (lengths, angles and congruence of the triangle) would not change statistically significantly. According to the proposed method, we could conclude that there are too few stable points.

We estimate that the proposed modified method contains important improvements that help us to find more reliably stable points or statistically characteristic deformations of the considered object, which is the essence of any deformation analysis. Considering the advanced capabilities of computational operations and storage of computational results, we conclude that the spatial and temporal complexity of the proposed algorithm is not problematic.

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Literature and references

Acar, M. (2010). Determination of strain accumulation in landslide areas with GPS measurements. Scientific Research and Essays, 5 (8), 763-768.

Ambrožič, T., Mulahusić, A., Tuno, N., Topoljak, J., Hajdar, A., Kogoj, D. (2019). Deformacijska analiza v geodetskih mrežah z robustnimi metodami. Deformation analysis with robust methods in geodetic nets. Geodetski vestnik, 63 (2), 163-178. DOI: 10.15292/geodetski-vestnik.2019.02.163-178.

Ambrožič, T. (2001). Deformacijska analiza po postopku Hannover. Geodetski vestnik, 45 (1&2), 38-53. http://www.geodetski-vestnik.com/45/gv45-12.pdf.

Ambrožič, T. (2004). Deformacijska analiza po postopku Karlsruhe. Geodetski vestnik, 48 (3), 315-331. https://www.geodetski-vestnik.com/arhiv/48/3/gv48-3_315-331.pdf.

Ašanin, S. (1986). Prilog obradi i analizi geodetskih merenja za odredjivanje pomeranja i deformacija objekta i tla. Doktorska disertacija. Beograd: Univerzitet u Beogradu, Gradjevinski fakultet, Institut za geodeziju.

Batilović M., Kanović Ž., Sušić Z., Marković M.Z., Bulatović V. (2022). Deformation analysis: the modified GREDOD method. Geodetski vestnik, 66 (1), 60-75. DOI: 10.15292/geodetski-vestnik.2022.01.60-75.

Boresi, A. P., Sidebottom, O. M. (1985). Advanced Mechanics of Materials. New York: John Wiley&Sons, Inc.

Caspary, W. F. (1988). Concepts of Network and Deformation Analysis. Kensington: The University of New South Wales, School of Surveying.

Chrzanowski, A., Chen, Y.Q., Secord, J.M. (1986). Geometrical analysis of deformation surveys. V Bock, Y. (ur.), Proceedings of the Deformation Measurements Workshop, 31 October-1 November, Institute of Technology, Boston, Massachusetts, pp. 170-206.

Grigillo, D., Stopar, B. (2003). Metode odkrivanja grobih pogreškov v geodetskih opazovanjih. Methods of Gross Error Detection in Geodetic Observations. Geodetski vestnik, 47 (4), 387-403. http://www.geodetski-vestnik.com/47/4/gv47-4_387-403.pdf.

Hamza, V., Stopar, B., Ambrožič, T. (2020). Deformacijska analiza po postopku Caspary. Deformation analysis: the Caspary approach. Geodetski vestnik, 64 (1), 68-88. DOI: 10.15292/geodetski-vestnik.2020.01.68-88.

Kuang, S. L. 1996. Geodetic Network Analysis and Optimal Design: Concepts and Applications. Ann Arbor Press, Chelsea, Michigan.

Labant, S., Weiss, G., Zuzik, J., Baran, M. (2014). Graphical interpretation deformation analysis of stability area using of strain analysis. Acta Montanistica Slovaca, 19 (1), 31-40.

Marjetič, A., Stopar, B. (2007). Geodetski datum in S-transformacija. Geodetic datum and S-transformation. Geodetski vestnik, 51 (3), 549-564. http://www.geodetski-vestnik.com/51/3/gv51-3_549-564.pdf.

Marjetič, A., Zemljak, M., Ambrožič, T. (2012). Deformacijska analiza po postopku DELFT. Deformation analysis: the DELFT approach. Geodetski vestnik, 56 (1), 9-26. DOI: 10.15292/geodetski-vestnik.2012.01.009-026.

Marković, M., Bajić, J., Batilović, M., Sušić, Z., Joža, A., Stojanović, G. (2019). Comparative Analysis of Deformation Determination by Applying Fiber-optic 2D Deflection Sensors and Geodetic Measurements. Sensors, 19 (4), 844, 1-4. DOI: https://doi.org/10.3390/s19040844.

Van Mierlo, J. 1978. Testing Procedure for Analyzing Geodetic Deformation Measurements. V Hallermann, L. (ur.), Proceedings of the 2nd International Symposium on Deformation Measurements by Geodetic Methods, Konrad Wittwer, Stuttgart, pp. 321-353.

Mihailović, K., Aleksić, I. (1994). Deformaciona analiza geodetskih mreža. Beograd: Univerzitet u Beogradu, Građevinski fakultet, Institut za geodeziju.

Soldo, J., Ambrožič, T. (2018). Deformacijska analiza po postopku München. Deformation analysis: the München approach. Geodetski vestnik, 62 (3), 392-414. DOI: http://dx.doi.org/10.15292/geodetski-vestnik.2018.03.392-414.

Sušić, Z., Batilović, M., Ninkov, T., Aleksić, I., Bulatović, V. (2015a). Identification of movements using different geodetic methods of deformation analysis. Identifikacija premikov z uporabo različnih geodetskih metod deformacijske analize. Geodetski vestnik, 59 (3), 537-553. DOI: 10.15292/geodetski-vestnik.2015.03.537-553.

Sušić, Z., Batilović, M., Ninkov, T., Bulatović, V., Aleksić, I., Nikolić, G. (2017). Geometric Deformation Analysis in Free Geodetic Networks: Case Study for Fruska Gora in Serbia. Acta Geodynamica et Geomaterialia, 4 (3), 341-355. DOI: 10.13168/AGG.2017.0017.

Sušić, Z., Toljić, M., Bulatović, V., Ninkov, T., Stojadinović, U. (2015b). Present-day Horizontal Mobility in the Serbian Part of the Pannonian Basin; Inferences from the Geometric Analysis of Deformations. Acta Geophysica, 64 (5), 1626-1654. DOI: 10.1515/acgeo-2016-0074.

Vrce, E. (2011). Deformacijska analiza mikrotriangulacijske mreže. Deformation analysis of microtriangulation network. Geodetski glasnik, 40, 14-27. http://www.suggsbih.ba/GEODETSKI%20GLASNIK/GEODETSKI%20GLASNIK%2040/Vrce_Clanak_Geodetski_glasnik_B45_P.pdf, pridobljeno 24. 2. 2023.

Vrečko, A., Ambrožič, T. (2013). Deformacijska analiza po postopku Fredericton. Deformation analysis: the Fredericton approach. Geodetski vestnik, 57 (3), 479-497. DOI: 10.15292/geodetski-vestnik.2013.03.479-497.

Welsch, W. (1982). Einige Erweiterungen der Deformationsermitlung in geodätischen Netzen durch Methoden der Strainanlyse. V Joó, I. (ur.), Detreköi, A. (ur.), Proceedings of the 3rd International Symposium on Deformation Measurements by Geodetic Methods, Budapest, Akademiai Kiadó, pp. 83-97.

Welsch, W. 1983. Finite Element Analysis of Strain Patterns from Geodetic Observations Across a Plate Margin. Tectonophysics, 97, 57-71.

Welsch, W., Zhang, Y. (1983). Einige Methoden zur Untersuchung kongruenter und affiner Beziehungen in geodätischen Überwachungsnetzen zur Ermittlung von Deformationen. V Welsch, W. (ur.), Deformationsanalysen '83, Geometrische Analyse und Interpretation von Deformationen Geodätischer Netze. Beiträge zum Geodätischen Seminar. Schriftenreihe des Wissenschaftlichen Studiengangs Vermessungswesen der Hochschule der Bundeswehr, Heft 9, München, pp. 299-328.

Welsch, W.M. (1986). Some aspects of the analysis of geodetic strain observations in kinematic models. Tectonophysics, 39 (1-4), 437-458. DOI: https://doi.org/10.1016/0040-1951(86)90132-0.

Savšek S., Ambrožič T. (2023). Modified method of deformation analysis according to the Munich approach.
Geodetski vestnik, 67 (2), 131-164.
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MODIFICIRANA METODA DEFORMACIJSKE ANALIZE PO POSTOPKU MÜNCHEN

OSNOVNE INFORMACIJE O ČLANKU:

GLEJ STRAN 131

1 UVOD

Spremljanje stabilnosti gradbeno-inženirskih ali naravnih objektov, tako imenovani monitoring, je sistematično merjenje in sledenje spremembam oblike in/ali dimenzije obravnavanega objekta kot posledica napetosti, ki jih povzročajo obremenitve. Kontinuirano spremljanje deformacij z beleženjem izmerjenih vrednosti je osnovni pogoj za analizo deformacij, na podlagi katere pravočasno predlagamo vzdrževanje ali sanacijo obravnavanega objekta. Spremljanje premikanja je primarno povezano s področjem inženirske geodezije, kjer lahko uporabimo različne merilne naprave ali senzorje. Za ustrezno analizo deformacij so bili razviti različni postopki deformacijske analize (Chrzanowski et al., 1986; Ašanin, 1986; Mihailović in Aleksić, 1994).

V prispevku obravnavamo deformacijsko analizo po postopku München. Razvil jo je W. M. Welsch, ki je deloval na Inštitutu za geodezijo v okviru Visoke vojaške šole v Münchnu. Metoda temelji na analizi deformacij trikotnikov, katerih oglišča tvorijo geodetske točke v mreži in katerih spremembe koordinat izračunamo na osnovi spremembe kotov in dolžin v mreži. Deformacijsko analizo po postopku München lahko izvedemo na dva načina (Welsch, 1983; Welsch in Zhang, 1983; Soldo in Ambrožič, 2018):

- z **metodo X**, ki temelji na primerjavi koordinat identičnih točk geodetske mreže dveh terminskih izmer. Ker so koordinate točk odvisne od geodetskega datuma, je pomembno, da se rezultati izravnave proste mreže nanašajo na isti geodetski datum. Če se število točk v mreži v terminskih izmerah razlikuje, izločimo koordinatne neznanke neidentičnih točk s transformacijo S (van Mierlo, 1987);
- z **metodo L**, ki temelji na primerjavi količin, neodvisnih od geodetskega datuma, to so koti in dolžine.

Klasični pristop deformacijske analize po postopku München zahteva izravnavo geodetske mreže kot proste mreže za vsako terminsko izmero z zagotovitvijo identičnega geodetskega datuma ter testiranje homogenosti natančnosti meritev obravnavanih izmer. V nadaljevanju se izvede testiranje skladnosti geodetske mreže, s katerim skušamo ugotoviti, ali je prišlo do premikov in deformacij celotnega objekta. Če ugotovimo statistično značilne deformacije objekta, nadaljujemo s testiranjem preoblikovanja geodetske mreže, pri čemer mrežo razdelimo na izbrane trikotnike in ugotavljamo spremembo oblike posameznega trikotnika. Izračunamo osnovne kinematične parametre v posameznem trikotniku in nato še druge parametre, kot so normalne in strižne deformacije. Nazadnje izvedemo še testiranje sprememb datumsko neodvisnih dolžin v mreži kot predlaga avtor postopka W. M. Welsch, in tako odkrijemo točke, ki so se statistično značilno premaknile (Welsch in Zhang, 1983; Ašanin, 1986).

Modificiran pristop, ki ga predlagamo v prispevku, temelji na tvorjenju vseh možnih trikotnikov v mreži, ne le izbranih, ter poleg testiranja sprememb datumsko neodvisnih dolžin vključuje tudi testiranje neodvisnih kotov v vseh trikotnikih, ki jih tvorimo. Tako zajamemo bistveno več informacij o spremembah v mreži in z večjo gotovostjo najdemo točke, ki so se značilno premaknile, kar je pomembna izboljšava klasičnega pristopa deformacijske analize po postopku München.

Z obema načinoma deformacijske analize po postopku München, torej po metodi X in po metodi L, dobimo v fazi testiranja skladnosti geodetske mreže in v fazi testiranja preoblikovanja geodetske mreže popolnoma enake rezultate (Welsch, 1983; Welsch in Zhang, 1983; Soldo in Ambrožič, 2018), zato bomo za testiranje skladnosti in testiranje preoblikovanja geodetske mreže v predlaganem modificiranem pristopu uporabili le metodo X.

2 DEFORMACIJSKA ANALIZA PO POSTOPKU MÜNCHEN

2.1 Osnovni pogoji za izvedbo deformacijske analize

Za izvedbo deformacijske analize je pomembno, da zagotovimo kakovost in skladnost natančnosti meritev v terminskih izmerah. To izvedemo tako, da preverimo homogenost natančnosti meritev v terminskih izmerah z ničelno domnevo H_0: $E(s_1^2) = E(s_2^2)$, ki pravi, da sta natančnosti meritev v terminskih izmerah enaki, kjer sta s_i a posteriori referenčni standardni deviaciji enote uteži za posamezno terminsko izmero. Iz meritev moramo izločiti grobo pogrešene meritve po uveljavljenih postopkih, kot so Baardova, Popeova, danska ali ustrezna druga metoda (Caspary, 1988; Grigillo in Stopar, 2003; Vrce, 2011).

Metode deformacijske analize zahtevajo izravnavo meritev posamezne terminske izmere kot prosto mrežo z minimalno sledjo matrike kofaktorjev neznank (Ambrožič, 2001; Sušić et al., 2017). Pomembno je, da obravnavamo le koordinatne neznanke, zato orientacijske neznanke izločimo z redukcijo neznank v enačbah popravkov. Če obravnavamo le dolžinske meritve, moramo reducirati tudi neznanko zaradi faktorja merila v mreži (Van Mierlo, 1978). Deformacijsko analizo lahko izvedemo le na identičnih točkah v mreži, zato s transformacijo S po potrebi izločimo koordinatne neznanke neidentičnih točk v obravnavanih terminskih izmerah (Van Mierlo, 1978; Caspary 1988; Marjetič in Stopar, 2007; Sušić et al., 2015a; Marković et al., 2019). Rezultat izravnave sta ocenjena vektorja izravnanih koordinat točk $\hat{\mathbf{x}}_i$ s pripadajočima matrikama kofaktorjev koordinatnih neznank $\mathbf{Q}_{\hat{\mathbf{x}}_i\hat{\mathbf{x}}_i}$ ter referenčni varianci a posteriori enote uteži s_i^2 za posamezno terminsko izmero.

Ko zagotovimo identični geodetski datum in preverimo homogenost natančnosti v obravnavanih terminskih izmerah, izračunamo referenčno varianco a posteriori z izrazom

$$s^2 = \frac{\mathbf{v}_1^{\mathbf{T}} \mathbf{P}_{\mathbf{ll}_1} \mathbf{v}_1 + \mathbf{v}_2^{\mathbf{T}} \mathbf{P}_{\mathbf{ll}_2} \mathbf{v}_2}{f_1 + f_2} = \frac{f_1 s_1^2 + f_2 s_2^2}{f}, \tag{1}$$

kjer sta $\mathbf{v}_1$ in $\mathbf{v}_2$ vektorja popravkov meritev, $\mathbf{P}_{\mathbf{ll}_1}$ in $\mathbf{P}_{\mathbf{ll}_2}$ matriki uteži meritev, f_1 in f_2 števili nadštevilnih meritev in velja $f_1 + f_2 = f$, s_1^2 in s_2^2 referenčni varianci a posteriori po izravnavi predhodne in tekoče terminske izmere t_1 in t_2.

Če v postopku testiranja skladnosti geodetske mreže ugotovimo statistično značilne deformacije, nadaljujemo s klasičnim pristopom testiranja preoblikovanja geodetske mreže, tako da mrežo delimo na trikotnike in ugotavljamo, ali je posamezen trikotnik značilno spremenil obliko med dvema terminskima izmerama.

2.2 Testiranje skladnosti geodetske mreže

Odločitev, ali so se v geodetski mreži pojavili premiki in deformacije, opravimo s testiranjem skladnosti.

Sestavimo ničelno in alternativno hipotezo (Welsch, 1983):

$H_0 : E(\hat{\mathbf{x}}_1) = E(\hat{\mathbf{x}}_2)$ oz. $E(\mathbf{u}) = \mathbf{0}$ … koordinate vseh točk v mreži se med dvema terminskima izmerama niso spremenile;

$H_a : E(\hat{\mathbf{x}}_1) \neq E(\hat{\mathbf{x}}_2)$ oz. $E(\mathbf{u}) \neq \mathbf{0}$ … koordinate vsaj ene točke v mreži so se med dvema terminskima izmerama spremenile.

Pri testiranju skladnosti z metodo X z globalnim testom stabilnosti vseh točk mreže primerjamo varianco razlik koordinat točk $s_{\mathbf{u}}^2$ z oceno referenčne variance a posteriori s^2.

Sestavimo testno statistiko:

$$T_1^2 = \frac{s_{\mathbf{u}}^2}{s^2} = \frac{\mathbf{u}^{\mathrm{T}}\mathbf{Q}_{\mathbf{u}}^{-1}\mathbf{u}}{f_{\mathbf{u}} \cdot s^2}, \tag{2}$$

Vektor premikov identičnih točk izračunamo z naslednjim izrazom:

$$\mathbf{u} = \hat{\mathbf{x}}_2 - \hat{\mathbf{x}}_1, \tag{3}$$

kjer sta $\hat{\mathbf{x}}_1$ in $\hat{\mathbf{x}}_2$ vektorja izravnanih koordinat vseh točk po izravnavi predhodne in tekoče terminske izmere t_1 in t_2.

Matriko kofaktorjev koordinatnih razlik zapišemo kot

$$\mathbf{Q}_{\mathbf{u}} = \mathbf{Q}_{\hat{\mathbf{x}}_1\hat{\mathbf{x}}_1} + \mathbf{Q}_{\hat{\mathbf{x}}_2\hat{\mathbf{x}}_2}, \tag{4}$$

kjer sta $\mathbf{Q}_{\hat{\mathbf{x}}_1\hat{\mathbf{x}}_1}$ in $\mathbf{Q}_{\hat{\mathbf{x}}_2\hat{\mathbf{x}}_2}$ matriki kofaktorjev koordinatnih neznank po izravnavi, ki ju izračunamo z izrazoma $\mathbf{Q}_{\hat{\mathbf{x}}_1\hat{\mathbf{x}}_1} = (\mathbf{B}_1^{\mathrm{T}}\mathbf{P}_{\mathbf{ll}_1}\mathbf{B}_1 + \mathbf{G}^{\mathrm{T}}\mathbf{G})^{-1}$ in $\mathbf{Q}_{\hat{\mathbf{x}}_2\hat{\mathbf{x}}_2} = (\mathbf{B}_2^{\mathrm{T}}\mathbf{P}_{\mathbf{ll}_2}\mathbf{B}_2 + \mathbf{G}^{\mathrm{T}}\mathbf{G})^{-1}$. $\mathbf{B}_1$ in $\mathbf{B}_2$ sta matriki koeficientov enačb popravkov, $\mathbf{P}_{\mathbf{ll}_1}$ in $\mathbf{P}_{\mathbf{ll}_2}$ pa sta matriki uteži predhodne in tekoče terminske izmere v trenutkih t_1 in t_2 (Kuang, 1996).

Datumska matrika $\mathbf{G}^{\mathrm{T}}$ ima v 2D-geodetski triangulacijski mreži obliko (Welsch, 1986):

$$\mathbf{G}^{\mathrm{T}} = \begin{bmatrix} 1 & 0 & 1 & 0 & \dots & 1 & 0 \\ 0 & 1 & 0 & 1 & \dots & 0 & 1 \\ x_i^0 & -y_i^0 & x_j^0 & -y_j^0 & \dots & x_m^0 & -y_m^0 \\ y_i^0 & x_i^0 & y_j^0 & x_j^0 & \dots & y_m^0 & x_m^0 \end{bmatrix}, \tag{5}$$

kjer so y_i^0, x_i^0, y_j^0, x_j^0, y_m^0, x_m^0 približne koordinate m točk geodetske mreže.

Število prostostnih stopenj je:

$$f_{\mathbf{u}} = u - d. \tag{6}$$

Z m označimo število vseh točk v geodetski mreži. V enačbi (6) $u = 2m$ predstavlja število koordinatnih neznank v 2D-mreži in d defekt datuma geodetske mreže (Welsch, 1983; Welsch in Zhang, 1983; Soldo in Ambrožič, 2018).

Testna statistika (2) se porazdeljuje po Fisherjevi porazdelitvi $F_{f_{\mathbf{u}},f}$:

- $T_1^2 \leq F_{f_{\mathbf{u}},f}$: ničelne hipoteze H_0 ne moremo zavrniti in s tveganjem α ne moremo trditi, da so se v mreži pojavile deformacije. Z drugimi besedami, deformacije niso statistično značilne.
- $T_1^2 > F_{f_{\mathbf{u}},f}$: ničelno hipotezo H_0 zavrnemo in s tveganjem, manjšim od α, trdimo, da so se v mreži pojavile deformacije. Z drugimi besedami, deformacije so statistično značilne.

Če s testiranjem skladnosti geodetske mreže potrdimo prisotnost deformacij v mreži, nadaljujemo s podrobnejšo obravnavo in lociranjem premikov točk v geodetski mreži.

2.3 Testiranje preoblikovanja geodetske mreže

Klasični pristop testiranja preoblikovanja mreže z metodo X sloni na tvorjenju izbranih trikotnikov in računanju kinematičnih parametrov (vektor **p**) v teh trikotnikih. Navadno izberemo trikotnike, ki se med seboj ne prekrivajo. Izbira oglišč trikotnikov pa ključno vpliva na izračun kinematičnih parametrov. Če se točke v izbranem trikotniku premaknejo v poljubno smer in so premiki dovolj veliki, izračunamo velike vrednosti kinematičnih parametrov. Če pa se točke v izbranem trikotniku premaknejo vse v isto smer in je njihov premik podobno velik, navadno izračunamo male vrednosti kinematičnih parametrov in je sprememba oblike in velikosti trikotnika manjša. Pri klasičnem pristopu navadno izberemo omejeno število trikotnikov.

Iz izravnanih koordinat točk predhodne in tekoče terminske izmere t_1 in t_2 izračunamo spremembo vrednosti meritev, ki jo lahko zapišemo tudi kot funkcijo premikov točk (Welsch in Zhang, 1983; Soldo in Ambrožič, 2018):

$$d\mathbf{l} = \mathbf{l}_2 - \mathbf{l}_1 = \mathbf{L} \cdot \mathbf{u}, \tag{7}$$

s pripadajočo matriko kofaktorjev spremembe vrednosti meritev

$$\mathbf{Q}_{d\mathbf{l}} = \mathbf{L}\mathbf{Q}_{\mathbf{u}}\, \mathbf{L}^{\mathrm{T}}. \tag{8}$$

Pomembno je poudariti, da je matrika parcialnih odvodov meritev po koordinatnih neznankah $\mathbf{L} = \left[\frac{\partial \mathbf{l}}{\partial \hat{\mathbf{x}}}\right]$ odvisna od vrste meritev, bodisi obravnavamo samo dolžine, samo kote ali oboje.

V naši modificirani metodi tvorimo vse možne trikotnike v mreži in tako dobimo bistveno več informacij o spremembah v njej. Ob ponesrečenem izboru trikotnikov lahko spregledamo pomembne spremembe. Naravno je, da v trikotniku izračunamo dolžine in kote, zato tudi v našem predlogu najprej testiramo spremembo dolžin, nato spremembo kotov in šele na koncu spremembo kinematičnih parametrov v trikotniku. Ker izračun vseh mogočih kombinacij računsko ni več težava, je smiselno, da zajamemo vse informacije o spremembah v mreži.

2.3.1 Testiranje dolžin v geodetski mreži

Testiranje izvedemo s testiranjem sprememb datumsko neodvisnih dolžin, ki povezujejo posamezno točko z drugimi točkami. Zapišemo ničelno in alternativno hipotezo (Welsch, 1982):

H_0: $E(dl_{dD_{ij}}) = 0$ … dolžina med točkama P_i in P_j se med dvema terminskima izmerama ni spremenila;

H_a: $E(dl_{dD_{ij}}) \neq 0$ … dolžina med točkama P_i in P_j se je med dvema terminskima izmerama spremenila.

Sestavimo testno statistiko za razliko dolžin med točkama P_i in P_j v dveh izmerah (Welsch, 1982):

$$T^2_{2D_{ij}} = \frac{dl_{dD_{ij}} Q^{-1}_{dD_{ij}} dl_{dD_{ij}}}{n_D \cdot s^2}, \tag{9}$$

kjer je $n_D = 1$ število prostostnih stopenj (Welsch, 1982; Soldo in Ambrožič, 2018).

Spremembo dolžine izračunamo z izrazom

$$dl_{dD_{ij}} = D_{ij_2} - D_{ij_1}, \tag{10}$$

kjer se element $dl_{dD_{ij}}$ nanaša na dolžini $D_{ij_1} = \sqrt{(\hat{y}_{j_1} - \hat{y}_{i_1})^2 + (\hat{x}_{j_1} - \hat{x}_{i_1})^2}$ in $D_{ij_2} = \sqrt{(\hat{y}_{j_2} - \hat{y}_{i_2})^2 + (\hat{x}_{j_2} - \hat{x}_{i_2})^2}$, ki ju izračunamo iz izravnanih koordinat med točkama P_i in P_j v predhodni in tekoči terminski izmeri.

Element matrike kofaktorjev spremembe vrednosti dolžine v enačbi (9) izračunamo z enačbo (8) $Q_{dD_{ij}} = \mathbf{L}_{dD_{ij}} \mathbf{Q}_{\mathbf{u}dD_{ij}} \mathbf{L}^{\mathrm{T}}_{dD_{ij}}$, kjer vektor parcialnih odvodov meritev $\mathbf{L}_{dD_{ij}}$ zapišemo kot (Welsch in Zhang, 1983; Soldo in Ambrožič, 2018):

$$\mathbf{L}_{dD_{ij}} = \left[\frac{\partial \mathbf{l}_{dD_{ij}}}{\partial \hat{\mathbf{x}}}\right] = \left[-\sin\nu_{ij} \quad -\cos\nu_{ij} \quad \sin\nu_{ij} \quad \cos\nu_{ij}\right], \tag{11}$$

kjer je $\nu_{ij} = (\nu_{ij_1} + \nu_{ij_2})/2$ srednja vrednost smernih kotov ν_{ij_1} in ν_{ij_2}, ki ju izračunamo iz izravnanih koordinat med točkama P_i in P_j, $\nu_{ij_1} = \arctan\frac{\hat{y}_{j_1} - \hat{y}_{i_1}}{\hat{x}_{j_1} - \hat{x}_{i_1}}$ in $\nu_{ij_2} = \arctan\frac{\hat{y}_{j_2} - \hat{y}_{i_2}}{\hat{x}_{j_2} - \hat{x}_{i_2}}$ v predhodni in tekoči terminski izmeri.

V podmatriki $\mathbf{Q}_{\mathbf{u}dD_{ij}}$ so pripadajoči elementi matrike $\mathbf{Q}_{\mathbf{u}}$, ki se nanašajo na točki P_i in P_j.

Testna statistika (9) se porazdeljuje po Fisherjevi porazdelitvi $F_{1,f}$:

- $T^2_{2D_{ij}} \leq F_{1,f}$: ničelne hipoteze H_0 ne moremo zavrniti in s tveganjem α ne moremo trditi, da se je dolžina med obravnavanima točkama spremenila,
- $T^2_{2D_{ij}} > F_{1,f}$: ničelno hipotezo H_0 zavrnemo in s tveganjem, manjšim od α, trdimo, da se je dolžina med obravnavanima točkama spremenila.

2.3.2 Testiranje kotov v geodetski mreži

Testiranje izvedemo s testiranjem sprememb datumsko neodvisnih kotov, ki povezujejo posamezno točko z drugimi točkami. Zapišemo ničelno in alternativno hipotezo:

H_0: $E(dl_{d\alpha_{ijk}}) = 0$ … kot med točkami P_i, P_j in P_k se med dvema terminskima izmerama ni spremenil;

H_a: $E(dl_{d\alpha_{ijk}}) \neq 0$ … kot med točkami P_i, P_j in P_k se je med dvema terminskima izmerama spremenil.

Sestavimo testno statistiko za kot med točkami P_i, P_j in P_k:

$$T^2_{2\alpha_{ijk}} = \frac{dl_{d\alpha_{ijk}} Q^{-1}_{d\alpha_{ijk}} dl_{d\alpha_{ijk}}}{n_\alpha \cdot s^2}, \tag{12}$$

kjer je $n_\alpha = 1$ število prostostnih stopenj.

Spremembo kota izračunamo z izrazom

$$dl_{d\alpha_{ijk}} = \alpha_{ijk_2} - \alpha_{ijk_1}, \tag{13}$$

kjer se element $dl_{d\alpha_{ijk}}$ nanaša na kota $\alpha_{ijk_1} = \nu_{ik_1} - \nu_{ij_1}$ in $\alpha_{ijk_2} = \nu_{ik_2} - \nu_{ij_2}$ z vrhom v točki P_i proti točkam P_j in P_k v predhodni in tekoči terminski izmeri, pri čemer so ν_{ij_1} in ν_{ik_1}, ν_{ij_2} ter ν_{ik_2} smerni koti iz izravnanih koordinat med točkama P_i in P_j ter P_i in P_k v predhodni in tekoči terminski izmeri.

Element matrike kofaktorjev spremembe vrednosti kota v enačbi (12) izračunamo z enačbo (8) $Q_{d\alpha_{ijk}} = \mathbf{L}_{d\alpha_{ijk}} \mathbf{Q}_{\mathbf{u}d\alpha_{ijk}} \mathbf{L}^{\mathrm{T}}_{d\alpha_{ijk}}$, kjer vektor parcialnih odvodov meritev $\mathbf{L}_{d\alpha_{ijk}}$ zapišemo kot

$$\mathbf{L}_{d\alpha_{ijk}} = \left[\frac{\partial \mathbf{l}_{d\alpha_{ijk}}}{\partial \hat{\mathbf{x}}}\right] = \left[\left(-\frac{\cos\nu_{ik}}{D_{ik}} + \frac{\cos\nu_{ij}}{D_{ij}}\right)\left(\frac{\sin\nu_{ik}}{D_{ik}} + \frac{\sin\nu_{ij}}{D_{ij}}\right)\left(-\frac{\cos\nu_{ij}}{D_{ij}}\right)\left(\frac{\sin\nu_{ij}}{D_{ij}}\right)\left(\frac{\cos\nu_{ik}}{D_{ik}}\right)\left(-\frac{\sin\nu_{ik}}{D_{ik}}\right)\right], \tag{14}$$

kjer sta $\nu_{ij} = (\nu_{ij_1} + \nu_{ij_2})/2$ in $\nu_{ik} = (\nu_{ik_1} + \nu_{ik_2})/2$ srednji vrednosti smernih kotov ν_{ij_1} in ν_{ik_1}, ν_{ij_2} in ν_{ik_2}. Dolžini $D_{ij} = (D_{ij_1} + D_{ij_2})/2$ in $D_{ik} = (D_{ik_1} + D_{ik_2})/2$ predstavljata srednji vrednosti dolžin, ki ju izračunamo iz izravnanih koordinat med točkama P_i in P_j ter P_i in P_k v predhodni in tekoči terminski izmeri t_1 in t_2.

V podmatriki $\mathbf{Q}_{\mathbf{u}d\alpha_{ijk}}$ so pripadajoči elementi matrike $\mathbf{Q}_{\mathbf{u}}$, ki se nanašajo na točke P_i, P_j in P_k.

Testna statistika (12) se porazdeljuje po Fisherjevi porazdelitvi $F_{1,f}$:

- $T^2_{2\alpha_{ijk}} \leq F_{1,f}$: ničelne hipoteze H_0 ne moremo zavrniti in s tveganjem α ne moremo trditi, da se je kot med obravnavanimi točkami spremenil,
- $T^2_{2\alpha_{ijk}} > F_{1,f}$: ničelno hipotezo H_0 zavrnemo in s tveganjem, manjšim od α, trdimo, da se je kot med obravnavanimi točkami spremenil.

2.3.3 Testiranje trikotnikov v geodetski mreži

Obravnavano mrežo razdelimo na trikotnike in ugotavljamo, ali je posamezni trikotnik spremenil obliko med terminskima izmerama, s čimer pojasnimo koordinatne razlike v mreži. Zapišemo ničelno in alternativno hipotezo (Welsch, 1983):

$H_0 : E(\hat{\mathbf{x}}_1) = E(\hat{\mathbf{x}}_2)$ oz. $E(\mathbf{u}_{\Delta_{ijk}}) = \mathbf{0}$ … koordinate točk v trikotniku se med dvema terminskima izmerama niso spremenile;

$H_a : E(\hat{\mathbf{x}}_1) \neq E(\hat{\mathbf{x}}_2)$ oz. $E(\mathbf{u}_{\Delta_{ijk}}) \neq \mathbf{0}$ … koordinate vsaj ene točke v trikotniku se so med dvema terminskima izmerama spremenile.

Sestavimo testno statistiko za oglišča P_i, P_j in P_k v trikotniku:

$$T^2_{2\Delta_{ijk}} = \frac{\mathbf{u}^{\mathrm{T}}_{\Delta_{ijk}} \mathbf{Q}^{-1}_{\mathbf{u}} \mathbf{u}_{\Delta_{ijk}}}{n_\Delta \cdot s^2}, \tag{15}$$

kjer je $n_\Delta = 3$ število prostostnih stopenj (Welsch, 1983; Welsch in Zhang, 1983; Soldo in Ambrožič, 2018).

Izračunamo vektor premikov oglišč v trikotniku $\mathbf{u}_{\Delta_{ijk}}$ po enačbi (3), kjer sta $\hat{\mathbf{x}}_1$ in $\hat{\mathbf{x}}_2$ vektorja izravnanih koordinat oglišč P_i, P_j in P_k v trikotniku po izravnavi predhodne in tekoče terminske izmere t_1 in t_2.

Matriko kofaktorjev koordinatnih razlik $\mathbf{Q_u}$ izračunamo po enačbi (4), kjer sta $\mathbf{Q}_{\hat{\mathbf{x}}_1\hat{\mathbf{x}}_1}$ in $\mathbf{Q}_{\hat{\mathbf{x}}_2\hat{\mathbf{x}}_2}$ matriki kofaktorjev koordinatnih neznank oglišč P_i, P_j in P_k v trikotniku po izravnavi geodetske mreže.

Testna statistika (15) se porazdeljuje po Fisherjevi porazdelitvi $F_{1,f}$:

- $T^2_{2\Delta_{ijk}} \leq F_{3,f}$: ničelne hipoteze H_0 ne moremo zavrniti in s tveganjem α ne moremo trditi, da so se koordinate v trikotniku z oglišči P_i, P_j in P_k spremenile,
- $T^2_{2\Delta_{ijk}} > F_{3,f}$: ničelno hipotezo H_0 zavrnemo in s tveganjem, manjšim od α, trdimo, da so se koordinate v trikotniku z oglišči P_i, P_j in P_k spremenile.

V nadaljevanju lahko izračunamo kinematične parametre v posameznem trikotniku:

$$\mathbf{p} = \mathbf{H_u^{-1}} \cdot \mathbf{u}, \tag{16}$$

kjer je $\mathbf{H_u}$ matrika deformacijskega modela, ki povezuje kinematične parametre s premiki točk v trikotniku

$$\mathbf{H_u} = \begin{bmatrix} \hat{x}_i & \hat{y}_i & 0 & -\hat{y}_i & 1 & 0 \\ 0 & \hat{x}_i & \hat{y}_i & \hat{x}_i & 0 & 1 \\ \hat{x}_j & \hat{y}_j & 0 & -\hat{y}_j & 1 & 0 \\ 0 & \hat{x}_j & \hat{y}_j & \hat{x}_j & 0 & 1 \\ \hat{x}_k & \hat{y}_k & 0 & -\hat{y}_k & 1 & 0 \\ 0 & \hat{x}_k & \hat{y}_k & \hat{x}_k & 0 & 1 \end{bmatrix}, \tag{17}$$

$$\mathbf{P}^{\mathrm{T}} = \begin{bmatrix} e_{xx} & e_{xy} & e_{yy} & \omega & t_x & t_y \end{bmatrix}, \tag{18}$$

$\mathbf{p}$ pa je vektor kinematičnih parametrov, kjer so e_{xx} in e_{yy} normalni deformaciji v smeri koordinatnih osi x in y, $e_{xy} = e_{yx}$ strižna deformacija (samo e_{xx}, e_{yy} in e_{xy} so deformacijski parametri), ω predstavlja rotacijo, t_x in t_y pa translaciji v smeri koordinatnih osi x in y.

Na podlagi osnovnih parametrov izračunamo še druge kinematične parametre (Welsch, 1983; Mihailović in Aleksić, 1994; Ašanin, 1986; Acar, 2010; Labant et al., 2014; Sušić et al., 2015b; Boresi in Sidebottom, 1985):

$\Delta = e_{xx} + e_{yy}$ predstavlja spremembo ploščine,

$e_1 = ½\,(e_{xx} + e_{yy} + ee)$ je glavna (največja) normalna deformacija,

$e_2 = ½\,(e_{xx} + e_{yy} - ee)$ je glavna (najmanjša) normalna deformacija,

kjer je $ee^2 = (e_{xx} - e_{yy})^2 + 4e_{xy}^2$,

$e_{\mathrm{I}} = \dfrac{e_1 - e_2}{2}$ je glavna strižna deformacija in

$\gamma = 2e_{xy}$ je inženirska strižna deformacija in predstavlja spremembo pravega kota med x in y smerjo.

Smerna kota glavnih normalnih deformacij izračunamo z izrazom $\tan 2\vartheta = \dfrac{2e_{xy}}{e_{xx} - e_{yy}}$, smerni kot glavne strižne deformacije pa z izrazom $\Psi = \vartheta + 45°$.

3 RAČUNSKI PRIMER

Zaradi enostavne in neposredne primerjave obravnavane modificirane metode deformacijske analize po postopku München z drugimi metodami deformacijske analize izberemo primer simulirane geodetske mreže, ki smo ga obravnavali z drugimi metodami deformacijske analize (Ambrožič, 2001; Ambrožič, 2004; Marjetič et al., 2012; Vrečko in Ambrožič, 2013; Soldo in Ambrožič, 2018; Ambrožič et al., 2019; Hamza et al., 2020; Batilović et al., 2022). V navedeni literaturi si lahko ogledamo skico mreže in dobimo vse potrebne podatke za izravnavo. Če na kratko povzamemo rezultate izravnave proste mreže z redukcijo orientacijskih neznank predhodne izmere, dobimo $s_1 = 0{,}9699$ in $f_1 = 30$, tekoče izmere pa $s_2 = 1{,}1562$ in $f_2 = 30$. Ker ne moremo zavrniti identičnosti geodetskega datuma in homogenosti natančnosti v obravnavanih terminskih izmerah, izračunamo skupno referenčno varianco a posteriori $s^2 = 1{,}1387$ in skupno število nadštevilnih meritev $f = 60$ (enačba 1). Povejmo še, da pri vseh uporabljenih testih v nadaljevanju izberemo tveganje $\alpha = 5$ %.

Testiranje skladnosti geodetske mreže je po klasičnem pristopu in po predlagani modificirani metodi deformacijske analize popolnoma enako, zato zapišimo le, da je $T_1^2 = 141{,}29$ (enačba 2), kar presega kritično vrednost pri izbrani stopnji značilnosti testa $F_{11,60} = 1{,}95$. Tako zavrnemo ničelno hipotezo in s tveganjem, manjšim od $\alpha = 5$ %, trdimo, da so se v mreži pojavile deformacije, in deformacijsko analizo nadaljujemo s testiranjem preoblikovanja geodetske mreže.

Deformacijsko analizo po predlagani modificirani metodi nadaljujemo s testiranjem preoblikovanja geodetske mreže, tako da najprej obravnavamo vse dolžine v geodetski mreži, teh je 21. Za vsako izračunano dolžino sestavimo testno statistiko $T^2_{2D_{ij}}$ (enačba 9) in jo primerjamo s kritično vrednostjo pri izbrani stopnji značilnosti testa $F_{1,60} = 4{,}00$, rezultate podajamo v preglednici 1. Izračunamo dejansko tveganje α_T za zavrnitev ničelne hipoteze, ki ga primerjamo z izbranim tveganjem $\alpha = 5$ %. Kjer je dejansko tveganje manjše od izbranega, moramo ničelno hipotezo zavrniti in trdimo, da so pomiki statistično značilni. V nasprotnih primerih pa lahko zaključimo, da se dolžine med mirujočimi točkami niso statistično značilno spremenile, saj je dejansko tveganje večje od 5 %.

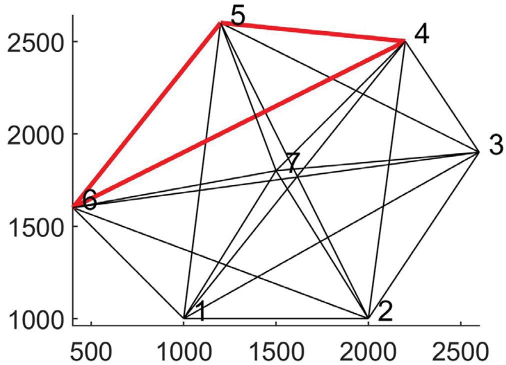
Preglednica 1: Analiza testiranja vseh dolžin med točkami

Dolžina	**Med točkama**	$T^2_{2D_{ij}}$	H_0	α_T [%]	**Dolžina**	**Med točkama**	$T^2_{2D_{ij}}$	H_0	α_T [%]
1	1-2	19.89	zavr.	0.00*	12	3-4	124.81	zavr.	0.00*
2	1-3	49.38	zavr.	0.00*	13	3-5	62.37	zavr.	0.00*
3	1-4	87.04	zavr.	0.00*	14	3-6	10.41	zavr.	0.20
4	1-5	64.84	zavr.	0.00*	15	3-7	9.20	zavr.	0.36
5	1-6	10.05	zavr.	0.24	16	4-5	0.08	ne z.	77.85
6	1-7	689.26	zavr.	0.00*	17	4-6	0.01	ne z.	91.54
7	2-3	109.75	zavr.	0.00*	18	4-7	186.39	zavr.	0.00*
8	2-4	84.02	zavr.	0.00*	19	5-6	0.63	ne z.	42.87
9	2-5	163.39	zavr.	0.00*	20	5-7	83.12	zavr.	0.00*
10	2-6	113.96	zavr.	0.00*	21	6-7	77.68	zavr.	0.00*
11	2-7	113.61	zavr.	0.00*					

* Dejansko tveganje je manjše od 0,005 %.

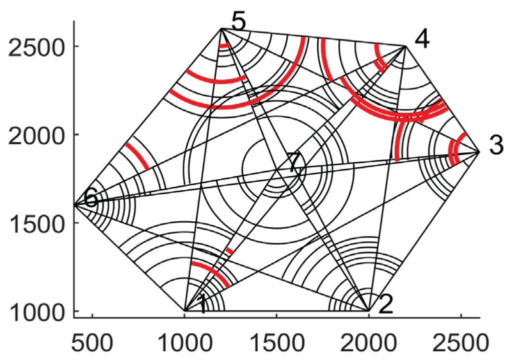
Iz preglednice 1 in slike 1 vidimo, da se le dolžine med točkami 4-5, 4-6 in 5-6 niso statistično značilno spremenile.

Nato nadaljujemo deformacijsko analizo po predlagani modificirani metodi s testiranjem preoblikovanja geodetske mreže, tako da obravnavamo vse kote v geodetski mreži, teh je 105. Za vsak izračunan kot sestavimo testno statistiko $T^2_{2\,\alpha_{ijk}}$ (enačba 12) in jo primerjamo s kritično vrednostjo pri izbrani stopnji značilnosti testa $F_{1,60} = 4{,}00$, rezultate podajamo v preglednici 2. Izračunamo dejansko tveganje α_T za zavrnitev ničelne hipoteze in ga primerjamo z izbranim dopustnim tveganjem $\alpha = 5$ %. Pri testiranju kotov ni mogoče enolično ugotoviti, kateri kot ne spremeni svoje vrednosti, saj testiramo tudi kote med dvema mirujočima in eno premično točko ali celo med eno mirujočo in dvema premičnima točkama. Iz preglednice 2 lahko zaključimo, da je dejansko tveganje večje od 5 % pri vseh kombinacijah kotov, kjer nastopajo vse tri mirujoče točke, kot na primer v kotu 4-5-6 ($\alpha_T = 33$ %), 5-4-6 ($\alpha_T = 44$ %) in 6-4-5 ($\alpha_T = 74$ %).



Slika 1: Analiza dolžin med točkami: z rdečo barvo so označene dolžine, ko ničelne hipoteze ne moremo zavrniti, s črno pa dolžine, ko ničelno hipotezo zavrnemo.

Iz preglednice 2 in slike 2 vidimo, da se kar nekaj kotov med točkami ni statistično značilno spremenilo.



Slika 2: Analiza kotov med točkami: z rdečo barvo so označeni koti, ko ničelne hipoteze ne moremo zavrniti, s črno pa koti, ko ničelno hipotezo zavrnemo.

Preglednica 2: Analiza testiranja vseh kotov med točkami

Kot	Med točkami	$T^2_{2\alpha ijk}$	H_0	α_T [%]	Kot	Med točkami	$T^2_{2\alpha ijk}$	H_0	α_T [%]
1	1-2-3	765.25	zavr.	0.00*	36	3-2-4	172.51	zavr.	0.00*
2	1-2-4	382.51	zavr.	0.00*	37	3-2-5	256.93	zavr.	0.00*
3	1-2-5	377.49	zavr.	0.00*	38	3-2-6	322.97	zavr.	0.00*
4	1-2-6	409.63	zavr.	0.00*	39	3-2-7	15.28	zavr.	0.02
5	1-2-7	304.97	zavr.	0.00*	40	3-4-5	2.50	ne z.	11.88
6	1-3-4	55.84	zavr.	0.00*	41	3-4-6	4.70	zavr.	3.42
7	1-3-5	2.08	ne z.	15.41	42	3-4-7	183.15	zavr.	0.00*
8	1-3-6	38.50	zavr.	0.00*	43	3-5-6	2.24	ne z.	14.01
9	1-3-7	32.56	zavr.	0.00*	44	3-5-7	368.79	zavr.	0.00*
10	1-4-5	24.42	zavr.	0.00*	45	3-6-7	342.61	zavr.	0.00*
11	1-4-6	104.38	zavr.	0.00*	46	4-1-2	153.77	zavr.	0.00*
12	1-4-7	0.33	ne z.	56.77	47	4-1-3	7.99	zavr.	0.64
13	1-5-6	78.35	zavr.	0.00*	48	4-1-5	0.05	ne z.	83.07
14	1-5-7	20.96	zavr.	0.00*	49	4-1-6	3.88	ne z.	5.36
15	1-6-7	101.10	zavr.	0.00*	50	4-1-7	33.76	zavr.	0.00*
16	2-1-3	844.94	zavr.	0.00*	51	4-2-3	17.86	zavr.	0.01
17	2-1-4	474.80	zavr.	0.00*	52	4-2-5	42.39	zavr.	0.00*
18	2-1-5	448.55	zavr.	0.00*	53	4-2-6	69.33	zavr.	0.00*
19	2-1-6	406.03	zavr.	0.00*	54	4-2-7	22.08	zavr.	0.00*
20	2-1-7	916.07	zavr.	0.00*	55	4-3-5	4.18	zavr.	4.52
21	2-3-4	361.72	zavr.	0.00*	56	4-3-6	3.20	ne z.	7.89
22	2-3-5	426.83	zavr.	0.00*	57	4-3-7	0.28	ne z.	59.82
23	2-3-6	456.29	zavr.	0.00*	58	4-5-6	0.98	ne z.	32.59
24	2-3-7	76.78	zavr.	0.00*	59	4-5-7	14.57	zavr.	0.03
25	2-4-5	69.08	zavr.	0.00*	60	4-6-7	15.39	zavr.	0.02
26	2-4-6	132.49	zavr.	0.00*	61	5-1-2	1.17	ne z.	28.29
27	2-4-7	37.28	zavr.	0.00*	62	5-1-3	4.23	zavr.	4.40
28	2-5-6	68.82	zavr.	0.00*	63	5-1-4	5.51	zavr.	2.23
29	2-5-7	230.59	zavr.	0.00*	64	5-1-6	5.41	zavr.	2.34
30	2-6-7	365.41	zavr.	0.00*	65	5-1-7	230.42	zavr.	0.00*
31	3-1-2	561.33	zavr.	0.00*	66	5-2-3	15.02	zavr.	0.03
32	3-1-4	0.39	ne z.	53.44	67	5-2-4	4.47	zavr.	3.86
33	3-1-5	1.06	ne z.	30.63	68	5-2-6	1.15	ne z.	28.79
34	3-1-6	11.15	zavr.	0.14	69	5-2-7	232.58	zavr.	0.00*
35	3-1-7	468.25	zavr.	0.00*	70	5-3-4	38.75	zavr.	0.00*

Kot	Med točkami	$T^2_{2\alpha ijk}$	H_0	α_T [%]	Kot	Med točkami	$T^2_{2\alpha ijk}$	H_0	α_T [%]
71	5-3-6	9.61	zavr.	0.30	89	6-4-7	127.84	zavr.	0.00*
72	5-3-7	240.57	zavr.	0.00*	90	6-5-7	90.46	zavr.	0.00*
73	5-4-6	0.60	ne z.	44.04	91	7-1-2	132.04	zavr.	0.00*
74	5-4-7	73.79	zavr.	0.00*	92	7-1-3	199.15	zavr.	0.00*
75	5-6-7	106.42	zavr.	0.00*	93	7-1-4	9.90	zavr.	0.26
76	6-1-2	296.34	zavr.	0.00*	94	7-1-5	41.02	zavr.	0.00*
77	6-1-3	35.57	zavr.	0.00*	95	7-1-6	33.70	zavr.	0.00*
78	6-1-4	98.75	zavr.	0.00*	96	7-2-3	27.83	zavr.	0.00*
79	6-1-5	69.53	zavr.	0.00*	97	7-2-4	44.72	zavr.	0.00*
80	6-1-7	314.80	zavr.	0.00*	98	7-2-5	233.68	zavr.	0.00*
81	6-2-3	258.54	zavr.	0.00*	99	7-2-6	207.06	zavr.	0.00*
82	6-2-4	38.20	zavr.	0.00*	100	7-3-4	215.43	zavr.	0.00*
83	6-2-5	22.19	zavr.	0.00*	101	7-3-5	432.37	zavr.	0.00*
84	6-2-7	20.63	zavr.	0.00*	102	7-3-6	332.44	zavr.	0.00*
85	6-3-4	97.78	zavr.	0.00*	103	7-4-5	129.96	zavr.	0.00*
86	6-3-5	31.15	zavr.	0.00*	104	7-4-6	57.94	zavr.	0.00*
87	6-3-7	321.56	zavr.	0.00*	105	7-5-6	5.96	zavr.	1.76
88	6-4-5	0.11	ne z.	74.42					

* Dejansko tveganje je manjše od 0,005 %.

Deformacijsko analizo nadaljujemo po predlagani modificirani metodi še s testiranjem preoblikovanja geodetske mreže, tako da obravnavamo skladnost vseh trikotnikov v geodetski mreži, teh je 35. Za vsak izračunan trikotnik sestavimo testno statistiko $T^2_{2\Delta ijk}$ (enačba 15) in jo primerjamo s kritično vrednostjo pri izbrani stopnji značilnosti testa $F_{3,60} = 2,76$, rezultate podajamo v preglednici 3. Izračunamo dejansko tveganje α_T za zavrnitev ničelne hipoteze, ki ga primerjamo z izbranim dopustnim tveganjem $\alpha = 5$ %. Ugotovimo lahko, da se trikotnik med mirujočimi točkami 4-5-6 ni statistično značilno spremenil, saj je dejansko tveganje bistveno večje od 5 % in znaša $\alpha_T = 77$ %.

Preglednica 3: Analiza testiranja skladnosti trikotnikov med točkami

Trikotnik	Med točkami	$T^2_{2\Delta ijk}$	H_0	α_T [%]	Trikotnik	Med točkami	$T^2_{2\Delta ijk}$	H_0	α_T [%]
1	1-2-3	286.17	zavr.	0.00*	10	1-4-5	35.00	zavr.	0.00*
2	1-2-4	159.09	zavr.	0.00*	11	1-4-6	46.79	zavr.	0.00*
3	1-2-5	157.59	zavr.	0.00*	12	1-4-7	271.23	zavr.	0.00*
4	1-2-6	166.60	zavr.	0.00*	13	1-5-6	33.96	zavr.	0.00*
5	1-2-7	336.48	zavr.	0.00*	14	1-5-7	249.07	zavr.	0.00*
6	1-3-4	62.88	zavr.	0.00*	15	1-6-7	229.98	zavr.	0.00*
7	1-3-5	42.33	zavr.	0.00*	16	2-3-4	136.82	zavr.	0.00*
8	1-3-6	24.16	zavr.	0.00*	17	2-3-5	158.79	zavr.	0.00*
9	1-3-7	278.24	zavr.	0.00*	18	2-3-6	163.70	zavr.	0.00*

Trikotnik	Med točkami	$T^2_{2\Delta ijk}$	H_0	α_T [%]	Trikotnik	Med točkami	$T^2_{2\Delta ijk}$	H_0	α_T [%]
19	2-3-7	62.03	zavr.	0.00*	28	3-4-7	116.79	zavr.	0.00*
20	2-4-5	60.86	zavr.	0.00*	29	3-5-6	20.86	zavr.	0.00*
21	2-4-6	68.60	zavr.	0.00*	30	3-5-7	162.34	zavr.	0.00*
22	2-4-7	110.69	zavr.	0.00*	31	3-6-7	140.22	zavr.	0.00*
23	2-5-6	67.44	zavr.	0.00*	32	4-5-6	0.37	ne z.	77.30
24	2-5-7	147.62	zavr.	0.00*	33	4-5-7	95.96	zavr.	0.00*
25	2-6-7	133.31	zavr.	0.00*	34	4-6-7	98.21	zavr.	0.00*
26	3-4-5	42.87	zavr.	0.00*	35	5-6-7	56.68	zavr.	0.00*
27	3-4-6	41.92	zavr.	0.00*					

* Dejansko tveganje je manjše od 0,005 %.

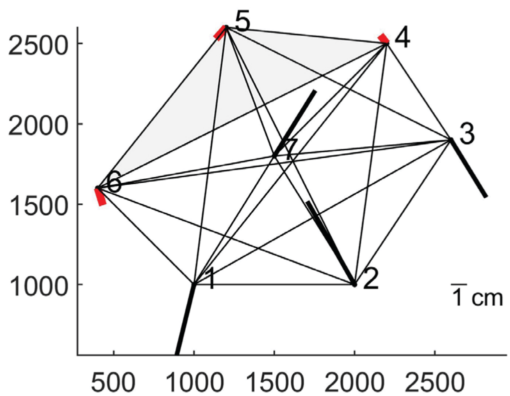
Slike trikotnikov med točkami ne prikažemo, saj je zaradi velikega števila trikotnikov nepregledna.

Zaradi testiranja velikega števila vseh dolžin med točkama v mreži, še večjega števila vseh kotov med točkami v mreži in tudi velikega števila testiranja skladnosti vseh mogočih trikotnikov v mreži, kar je bistvo predlagane modificirane metode deformacijske analize po postopku München, je analiza težavna in nepregledna. Lahko pa jo poenostavimo in njeno preglednost povečamo tako, da vsa tri testiranja količin naredimo na enoto, to je na posamezen trikotnik. V preglednici 4 prikazujemo rezultate testiranja vseh treh količin v posameznem trikotniku.

Preglednica 4: Analiza testiranja vseh treh količin v trikotnikih

Trikotnik	Med točkami	$H_{0\text{-}D}$			$H_{0\text{-}\alpha}$			$H_{0\text{-}\Delta}$
	i-j-k	*i-j*	*j-k*	*i-k*	*i-j-k*	*j-i-k*	*k-i-j*	*i-j-k*
1	1-2-3	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.
2	1-2-4	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.
3	1-2-5	zavr.	zavr.	zavr.	zavr.	zavr.	ne z.	zavr.
4	1-2-6	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.
5	1-2-7	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.
6	1-3-4	zavr.	zavr.	zavr.	zavr.	ne z.	zavr.	zavr.
7	1-3-5	zavr.	zavr.	zavr.	ne z.	ne z.	zavr.	zavr.
8	1-3-6	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.
9	1-3-7	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.
10	1-4-5	zavr.	ne z.	zavr.	zavr.	ne z.	zavr.	zavr.
11	1-4-6	zavr.	ne z.	zavr.	zavr.	ne z.	zavr.	zavr.
12	1-4-7	zavr.	zavr.	zavr.	ne z.	zavr.	zavr.	zavr.
13	1-5-6	zavr.	ne z.	zavr.	zavr.	zavr.	zavr.	zavr.
14	1-5-7	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.
15	1-6-7	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.
16	2-3-4	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.
17	2-3-5	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.
18	2-3-6	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.
19	2-3-7	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.
20	2-4-5	zavr.	ne z.	zavr.	zavr.	zavr.	zavr.	zavr.
21	2-4-6	zavr.	ne z.	zavr.	zavr.	zavr.	zavr.	zavr.
22	2-4-7	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.
23	2-5-6	zavr.	ne z.	zavr.	zavr.	ne z.	zavr.	zavr.

Trikotnik	Med točkami	H_{0-D}			$H_{0-\alpha}$			$H_{0-\Delta}$
	i-j-k	*i-j*	*j-k*	*i-k*	*i- j-k*	*j-i-k*	*k-i-j*	*i-j-k*
24	2-5-7	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.
25	2-6-7	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.
26	3-4-5	zavr.	ne z.	zavr.	ne z.	zavr.	zavr.	zavr.
27	3-4-6	zavr.	ne z.	zavr.	zavr.	ne z.	zavr.	zavr.
28	3-4-7	zavr.	zavr.	zavr.	zavr.	ne z.	zavr.	zavr.
29	3-5-6	zavr.	ne z.	zavr.	ne z.	zavr.	zavr.	zavr.
30	3-5-7	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.
31	3-6-7	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.
32	4-5-6	ne z.	ne z.	ne z.	ne z.	ne z.	ne z.	ne z.
33	4-5-7	ne z.	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.
34	4-6-7	ne z.	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.
35	5-6-7	ne z.	zavr.	zavr.	zavr.	zavr.	zavr.	zavr.



Slika 3: Izris premikov točk: z rdečo barvo so označeni premiki na točkah 4, 5 in 6, ko ničelne hipoteze ne moremo zavrniti, s črno pa premiki, ko ničelno hipotezo zavrnemo.

Na sliki 3 prikazujemo razlike izravnanih koordinat obeh terminskih izmer in ne deformacijskih parametrov za posamezni trikotnik, kar je eden od rezultatov deformacijske analize po postopku München (Welsch in Zhang, 1983; Welsch, 1983; Soldo in Ambrožič, 2018; Sušić et al., 2015b; Sušić et al., 2017; Mihailović in Aleksić, 1994). Trikotnikov je preveč, nekaj jih je zelo iztegnjenih, zato bi bila slika zelo nepregledna.

Na podlagi tako zapisanih rezultatov testiranja v preglednici 4 pa hitro ugotovimo, da se vse dolžine, vsi koti in skladnost v trikotniku 32 niso statistično značilno spremenili, kar z izrisom premikov prikažemo na sliki 3. Z izbranim dopustnim tveganjem $\alpha = 5$ % trdimo, da se tudi točke tega trikotnika niso statistično značilno premaknile, saj smo trikotnik testirali tako glede spremembe dolžin in kotov v trikotniku kakor tudi skladnosti celotnega trikotnika.

Po enačbi (18) izračunamo osnovne kinematične parametre in jih prikažemo v preglednici 5. Drugih deformacijskih parametrov ne izračunamo in prikazujemo, saj si jih lahko po zgoraj zapisanih enačbah enostavno izračunamo.

Preglednica 5: Izračunani kinematični parametri v vseh trikotnikih

Trikotnik	Med točkami *i -j-k*	e_{xx} [μs]	e_{xy} [μs]	e_{yy} [μs]	ω ['']	t_x [mm]	t_y [mm]
1	1-2-3	-161.27	82.79	-18.70	-2.7	0.021	-0.062
2	1-2-4	-43.92	57.70	-18.70	-7.9	-0.096	-0.012
3	1-2-5	11.14	50.49	-18.70	-9.4	-0.151	0.003
4	1-2-6	151.73	50.35	-18.70	-9.4	-0.292	0.003
5	1-2-7	46.19	76.48	-18.70	-4.0	-0.186	-0.049
6	1-3-4	52.09	-22.84	32.64	0.3	-0.072	-0.022
7	1-3-5	24.18	-4.11	20.39	0.9	-0.060	-0.031
8	1-3-6	39.05	5.69	4.66	4.6	-0.066	-0.044
9	1-3-7	158.66	-16.92	-7.75	13.8	-0.119	-0.053
10	1-4-5	21.33	8.38	2.64	-1.2	-0.080	-0.016
11	1-4-6	43.04	0.08	-10.54	2.7	-0.074	-0.013
12	1-4-7	368.00	-113.88	-233.38	102.9	0.007	0.031
13	1-5-6	26.76	-12.01	-18.44	3.5	-0.042	0.003
14	1-5-7	2.38	79.05	85.25	17.9	-0.212	-0.088
15	1-6-7	86.78	33.95	13.46	0.6	-0.162	-0.061
16	2-3-4	-14.58	-58.59	75.42	13.5	0.315	-0.187
17	2-3-5	-62.75	-9.55	36.67	8.7	0.218	-0.136
18	2-3-6	-98.67	29.36	0.77	5.6	0.146	-0.088
19	2-3-7	-54.19	-0.77	-8.92	13.2	0.235	-0.075
20	2-4-5	-32.33	12.71	4.05	0.8	0.066	-0.054
21	2-4-6	-34.60	22.41	-13.45	-0.7	0.034	-0.021
22	2-4-7	-28.08	1.59	-68.24	5.1	0.126	0.081
23	2-5-6	-21.31	18.08	-18.57	-2.7	0.011	0.002
24	2-5-7	-129.06	-193.66	-434.50	-1.9	0.550	1.042
25	2-6-7	13.88	58.63	6.90	3.0	-0.050	-0.116
26	3-4-5	81.75	-12.17	-2.07	-6.7	-0.244	0.112
27	3-4-6	55.49	-26.77	13.21	-1.6	-0.092	0.053
28	3-4-7	20.50	-57.18	0.71	3.0	0.111	0.102
29	3-5-6	14.21	-15.96	11.03	-0.5	-0.028	0.029
30	3-5-7	-83.15	-50.90	0.43	2.3	0.284	0.096
31	3-6-7	1279.93	35.50	-26.54	45.7	-1.984	-0.397
32	4-5-6	-5.80	0.43	1.32	-2.3	-0.006	0.020
33	4-5-7	-57.45	-17.02	-1.65	-4.8	0.135	0.101
34	4-6-7	-119.44	-3.87	34.03	14.9	0.153	0.112
35	5-6-7	-39.65	11.85	25.68	-8.7	0.031	0.041

4 SKLEP

V preteklih raziskavah smo dokazali, da so rezultati zelo podobni ne glede na to, katero metodo deformacijske analize uporabimo na testnem primeru (Ambrožič, 2001; Ambrožič, 2004; Marjetič et al., 2012; Vrečko in Ambrožič, 2013; Soldo in Ambrožič, 2018; Ambrožič et al., 2019; Hamza et al., 2020; Batilović et al., 2022). V naši raziskavi smo se osredotočili na pomembne izboljšave, ki bi omogočile zanesljivejše ugotavljanje deformacij obravnavanega objekta oziroma merjenega območja. V modificirani metodi deformacijske analize po postopku München pred fazo preoblikovanja geodetske mreže, torej pred tvorjenjem trikotnikov v mreži, predlagamo, da poleg testiranja sprememb neodvisnih dolžin izvedemo tudi testiranje neodvisnih kotov med vsemi točkami v mreži. V naslednji fazi pa tvorimo vse možne trikotnike med točkami v mreži in ne le izbranih, saj tako pridobimo zanesljivejšo informacijo o morebitnih spremembah oblike trikotnikov in posledično o točkah, ki so se statistično značilno premaknile.

Pri analizi testiranja vseh dolžin v mreži nam je s predlagano metodo uspelo najti dolžine med točkami 4-5, 4-6 in 5-6, ki se niso statistično značilno spremenile, in zato ničelne hipoteze ne moremo zavrniti (preglednica 1 in slika 1). To je skladno s simuliranimi premiki v mreži, ko so točke 4, 5 in 6 obravnavane kot mirujoče. Pri analizi testiranja vseh kotov v mreži nam je s predlagano metodo uspelo najti vse kombinacije kotov med točkami 4-5-6, 5-4-6 in 6-4-5, ki se niso statistično značilno spremenili, in zato ničelne hipoteze ne moremo zavrniti (preglednica 2 in slika 2). Pri analizi testiranja skladnosti vseh trikotnikov v mreži nam je s predlagano metodo uspelo najti trikotnik z oglišči v točkah 4, 5 in 6, ki se ni statistično značilno spremenil, in zato ničelne hipoteze ne moremo zavrniti (preglednica 3). Če rezultate testiranj zapišemo bolj pregledno, lahko ugotovimo, da se vse dolžine, vsi koti in skladnost v trikotniku 32, torej med točkami 4-5-6, niso statistično značilno spremenili (preglednica 4). Sklepamo, da z dopustnim tveganjem $\alpha = 5$ % ne moremo trditi, da so se točke tega trikotnika premaknile, kar je skladno s simuliranimi premiki v mreži.

Prednosti predlagane modificirane metode so, da uporabnik deformacijske analize navadno tvori:

– trikotnike med sosednjimi točkami, mi pa predlagamo tvorjenje trikotnikov med vsemi točkami, s čimer dobimo informacijo o deformacijah med vsemi točkami, ki jo lahko spregledamo, če trikotnike tvorimo sami;
– »lepe« oblike trikotnikov, kar spet vodi do tega, da v zelo iztegnjenem trikotniku ali v trikotniku, kjer je ena stranica veliko krajša od drugih dveh, izgubimo informacijo o deformacijah takega trikotnika.

Slabost predlagane modificirane metode se pokaže pri geodetski mreži, ko mirujeta samo dve točki. V tem primeru ne moremo tvoriti trikotnika (s tremi točkami), ko se vse tri testirane količine (dolžine, koti in skladnost trikotnika) ne bi statistično značilno spremenile. Glede na predlagano metodo lahko ugotovimo, da je mirujočih točk premalo.

Ocenjujemo, da predlagana modificirana metoda vključuje pomembne izboljšave, ki nam pomagajo bolj zanesljivo najti stabilne točke oziroma statistično značilne deformacije obravnavanega objekta, kar je bistvo vsake deformacijske analize. Glede na napredne možnosti računskih operacij in shranjevanja rezultatov izračunov sklepamo, da prostorska in časovna zahtevnost predlaganega algoritma ni problematična.

Zahvala

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Literatura in viri

Glej stran 147.

Savšek S., Ambrožič T. (2023). Modificirana metoda deformacijske analize po postopku München.
Geodetski vestnik, 67 (2), 131-164.
DOI: https://doi.org/10.15292/geodetski-vestnik.2023.02.131-164

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