UNIVERSITY OF LJUBLJANA
FACULTY OF ELECTRICAL ENGINEERING

Martin Povh

STOCHASTIC MODELLING OF LONG-TERM ELECTRICITY FORWARD PRICES

Doctoral dissertation

ADVISER: Assoc. Prof. Dr. Robert Golob
COADVISER: Prof. Dr. Stein-Erik Fleten

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SOMENTOR: prof. dr. Stein-Erik Fleten

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Abstract

In contrast to forwards and futures on storable commodities, prices of long-term electricity forwards exhibit a dynamics different to that of short-term and mid-term prices. Since electricity cannot be stored, the supply and demand shocks in the spot prices are not transferred to the long-term forward prices, as it is usual in the case of storable commodities. While forward prices of storable commodities can be modelled sufficiently with spot prices and storage costs, other factors are needed to explain the dynamics of long-term electricity forward prices.

We design a model for long-term electricity forward prices dependent on variables that influence the supply and demand for electricity and the risk premium. A specific model for long-term electricity forwards from Nord Pool is conditioned on information that includes long-term forward prices of crude oil, coal, natural gas, emission allowances, imported electricity and aluminium. Risk premium is modelled as a function of time-to-maturity. All variables have weekly resolution.

Based on the variables in the information set we set up a vector autoregressive model. The variables are non-stationary and we find that all of them are cointegrated, except for natural gas price. This indicates that although these prices may wander away from each other in the short-run due to the impact of random forces, in the long-run, they are drawn together due to the effect of long-run equilibrium forces. The Nord Pool price increases approximately 1:1 with oil price, a bit more than 1:1 with aluminium price, and a bit less than 1:1 with coal price. On the other hand, the Nord Pool price falls a bit more than 1:1 when EEX price increase, and falls by 0.15% if the emission allowance price rises by 1%. A presence of cointegration could be considered as evidence against the semi-strong form of market efficiency. However, cointegration could also be a result of a volatile risk premium. We also find that crude oil, coal, natural gas and aluminium prices influence the prices of emission allowances and both electricity prices, but not vice versa.

We find strong causality of all variables on emission allowance price and both electricity prices. Most of this is captured by the long-run relationship. Residual covariances show that there is also a strong contemporaneous relationship between variables. The system is therefore governed by the dynamics shorter than one week and by long-run dynamics in which the steady state after the shock is achieved in approximately 5 weeks. The structural model reveals that most of the permanent price components come from the crude oil price, which is by far the most dominant variable in the system. We find that time-to-maturity is significant only in emission allowance prices and EEX forward price. This indicates that during the analysed period, the risk premium dynamics in the long-term electricity forward prices from Nord Pool can be considered as constant with respect to time-to-maturity.

Key words: electricity prices, electricity markets, long-term forward prices, fundamental models, vector autoregressive models, cointegration, structural VAR.

Povzetek

Cene kratkoročnih in dolgoročnih terminskih pogodb dobrin, ki jih lahko shranjujemo, se obnašajo zelo podobno. Te lastnosti ne opazimo pri cenah električne energije, saj se cene dolgoročnih terminskih pogodb električne energije obnašajo drugače, kot cene kratkoročnih ali srednjeročnih terminskih pogodb. Ker električne energije ni mogoče shraniti, se vplivi šokov v ponudbi in povpraševanju na promptno ceno ne morejo prenesti na terminske cene, kot pri dobrinah, ki jih lahko shranjujemo. Medtem ko lahko terminske cene dobrin, ki se lahko shranjujejo, dovolj dobro modeliramo s promptno ceno in stroški skladiščenja, je potrebno pri modeliranju dolgoročnih terminskih cen električne energije upoštevati druge faktorje.

V tej disertaciji predstavljamo model dolgoročnih terminskih pogodb na podlagi spremenljivk, ki vplivajo na ponudbo in povpraševanje po električni energij in na premijo tveganja. Specifični model za dolgoročne terminske cene električne energije iz skandinavske borze Nord Pool pogojuje informacijska množica, ki vsebuje dolgoročne terminske cene surove nafte, premoga, zemeljskega plina, emisijskih dovolilnic, uvožene električne energije ter aluminija. Premijo tveganja modeliramo kot funkcijo časa trajanja do dobave. Vse spremenljivke imajo tedensko resolucijo.

Na podlagi spremenljivk v informacijski množici sestavimo vektorski avtoregresijski model (VAR). Kointegracijska analiza razkrije eno kointegracijsko povezavo med spremenljivkami, v kateri so vse spremenljivke značilne razen cene zemeljskega plina. Zaradi naključnih sil se cene teh dobrin kratkoročno lahko gibljejo neodvisno od drugih cen, vendar med njimi obstaja dolgoročno ravnotežje, ki sili te cene k skupnemu dolgoročnemu ravnovesju. Cena z borze Nord Pool se poveča približno v razmerju 1:1 s ceno surove nafte, malo več kot v razmerju 1:1 s ceno aluminija in malo manj kot v razmerju 1:1 s ceno premoga. Po drugi strani cena z borze Nord Pool pade v razmerju 1:1, ko cena z borze EEX naraste in pade za 0.15% , ko cena emisijskih dovolilnic naraste za 1%. Kointegracija sicer nakazuje neučinkovitost trga dolgoročnih terminskih pogodb električne energije, vendar je ta lahko tudi posledica nihanja premije tveganja. Ugotovimo tudi, da cene surove nafte, premoga, zemeljskega plina in aluminija vplivajo na cene emisijskih dovolilnic in obeh cen električne energije, vendar obratni vpliv ne obstaja.

Strukturna analiza razkrije močno enosmerno Grangerjevo vzročnost vseh spremenljivk na cene emisijskih dovolilnic in na obe ceni električne energije. V sistemu prevladuje dolgoročna dinamika, medtem ko je kratkoročna dinamika precej šibka. Kovariančna matrika preostankov kaže na zelo močno sočasno dinamiko, kar nakazuje da se večina kratkoročnih nihanj dogaja v času krajšem od enega tedna. Strukturni VAR pokaže, da večina dinamike dolgoročnih terminskih cen električne energije izhaja iz šokov v ceni surove nafte, ki je najbolj dominantna spremenljivka v sistemu. Ugotovimo tudi, da je čas trajanja do dobave značilen samo pri ceni emisijskih dovolilnic in ceni električne energije iz borze EEX. To nakazuje, da je v obdobju, ki ga analiziramo, dinamika premije tveganja v cenah iz borze Nord Pool konstantna glede na čas trajanja do dobave.

Ključne besede: cene električne energije, trg električne energije, dolgoročne terminske cene, temeljni modeli, vektorska avtoregresija, kointegracija, strukturni VAR.

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Contents

List of Tables

List of Figures

List of Symbols

Symbol	Description
t	(observation) time
T	delivery, maturity period
T_m	$:=T-t$, time to maturity/delivery
N	sample size, time series length
s	lag length
nd	normal distribution
n	dimension of a stochastic process or time series
r	interest rate
r^p	risk premium - explicit parametrization
$I(\cdot)$	order of integration
t-value	test statistics distributed with Student's t-distribution
LLF	log-likelihood function
$l(\cdot)$	log-likelihood function
β_s	beta of the security
λ	risk premium - implicit parametrization
Δ	differencing operator
E	expectation
var	variance
cov	covariance
MSE	mean square error
RSS	residual sum of squares
LR	likelihood ratio
p	probability
$\chi^2(m)$	χ^2-distribution with m degrees of freedom
$F(m,n)$	F-distribution with m numerator and n denominator degrees of freedom
$t(m)$	Student's t-distribution with m degrees of freedom
k	number of lags in autoregressive process
L	lag operator
VAR(k)	vector autoregressive process of order k
VECM	vector error correction model
$\mathbf{A}'$	transpose of $\mathbf{A}$
$\mathbf{A}_\perp$	orthogonal complement of $\mathbf{A}$
$\otimes$	Kronecker product
vec	column stacking operator
$\mathbf{I}_n$	$(n \times n)$ identity matrix
0	0 or null matrix or vector
diag($\mathbf{A}$)	vector with diagonal entries of matrix $\mathbf{A}$
S_t	price, spot price
F_t	forward price
δ_t	convenience yield

$\mathbf{e}_i$	i-th unit vector
$\mathbf{Y}$	vector of endogenous variables
$\mathbf{Z}$	vector of exogenous variables
$\mathbf{D}$	vector of dummy variables
$\mathbf{A}_0$	VAR matrix of constants
$\mathbf{A}$	VAR coefficient matrix for $\mathbf{Y}$
$\mathbf{\Psi}$	VAR coefficient matrix for $\mathbf{Z}$
$\mathbf{\Theta}$	VAR coefficient matrix for $\mathbf{D}$
$\mathbf{\Phi}$	coefficient matrix of MA representation of VAR
$\mathbf{u}$	VAR residuals
$\boldsymbol{\varepsilon}$	structural shocks
$\boldsymbol{\alpha}$	loading matrix of VECM
$\boldsymbol{\beta}$	cointegration matrix
$\mathbf{\Pi}$	$:= \boldsymbol{\alpha\beta}$
$\mathbf{\Gamma}$	short-run coefficient matrix of VECM
$\mathbf{\Omega}$	residual variance-covariance matrix
$\mathbf{\Sigma}$	variance-covariance matrix of parameter estimates
$\mathbf{P}$	the lower-triangular of Choleski decomposition of $\mathbf{\Omega}$

Introduction

Electric power system has always been regarded as natural monopoly due to the huge and expensive grid system necessary for its operation. Only in the last decade of the 20^{th} century, the electricity market has been partially deregulated. While electricity supply, demand, and trading is deregulated, the transmission and distribution grid operation is still under strong regulation. Electricity has become a traded commodity, but with a few important distinctions, which require electricity to be treated at least slightly different from other commodities. Most electricity markets are still evolving and we have witnessed a large progress in competition since the beginning of the deregulation process. Because of the limitations in electricity production, transport and storage the price of electricity has become one of the most volatile among traded commodities. High uncertainty of electricity prices put a strong incentive on the investors to reduce the price-related risks.

Deregulation brought forth another important aspect of energy markets. In more mature markets, we can observe a significant connection between different prices of energy, which serve either as fuel for generating electricity or as a substitute in electricity use. This strongly indicates that energy markets are integrated both in terms of price and geographical inter-dependence. High energy prices and hence increasing price elasticity have significantly strengthened the integration between different energy markets. Electricity market therefore cannot be treated as individual anymore. Therefore, they require to be modelled as a part of regional or global energy market.

When it comes to decision-making (trading, investment, portfolio and asset management, etc.), each market is dominated by information, which influences individual decisions. The quality of this information largely depends on market transparency, which has, in case of electricity market, improved significantly with the introduction of electricity exchanges. Although some still struggle with low liquidity in the spot market and even low trading frequency in forward market, they do provide at least some information about the competitive market price. Investors in electricity market are trying to reduce the price-related risk with forwards and options. Instead of waiting till the start of delivery period and risk unfavourable price outcomes, the investors are rather willing to enter contracts long before the delivery period if expectations about the price during that delivery period are favourable. A variety of forward and option contracts are traded on the exchange with time-to-delivery spanning from one week to several years, giving investors a variety of choices to hedge against the price-related risk. The volume of forward trading on electricity exchanges is increasing in general and in some markets the volume of forward trading far exceeds the volume of physical trading on the spot market. Still, a high liquidity of the spot market is the first prerequisite for a successful operation of the forward market.

Motivation

The environment that investors face in electricity market is changing rapidly. Since deregulation, the excess capacity in electricity market has gradually been reduced and the investments in new capacity could not follow the growth in consumption. This caused the overall trend of increasing wholesale prices in this period. In recent years, we have also witnessed a shift in electricity production technology - from traditional coal and nuclear to natural gas and renewable sources. The shift is therefore from fuels with reasonably stable fuel prices to fuels with very unstable fuel prices. The market also changed dramatically with the introduction of emission allowances, which have a strong influence on the operation of carbon emitting power plants. Such structural changes in the market increases the long-term uncertainty of electricity prices, hence the risks in portfolio and asset management increase.

We identify four motives to study the dynamics of long-term electricity forward prices:

1. **Long-term positions in electricity market can only be properly hedged by entering long-term electricity forward markets**.

   Commodity forward markets are normally focused on contracts with time-to-maturity up to 1.5 years. Since the correlation between short-term and long-term prices is high in many markets, long-term risks can be hedged with roll-over hedging using short-term and mid-term forwards and futures. Unlike most commodities, electricity cannot be stored to any great extent. In the empirical analysis of forwards from Nord Pool, Koekebakker and Ollmar [1] show that the correlation between short-term and long-term electricity futures is low. They conclude that short-term contracts are not appropriate for hedging long-term exposures in electricity markets, such as long-term procurement costs and production revenues. While far-maturity exposures can normally be hedged with short-term positions, electricity companies can only properly hedge them with long-term trading.

2. **Existing short-term models fail to properly capture the dynamics of long-term electricity forward prices**. Due to different dynamics between short- and long-term electricity forward prices, a model designed to capture the dynamics of electricity spot prices or short-term forward prices cannot be applied in forecasting long-term forward prices. Prices of long-term and short-term forward contracts are driven by different market forces, and therefore a model to capture the long-term forward prices should be designed differently from short-term electricity price models.

3. **Investors could benefit from models that would be able to forecast the expected price changes in long-term electricity forwards**.

   While the efficient market hypothesis states that future changes in asset value cannot be forecasted [2], prices of similar commodities are often found to be cointegrated, which reflects the temporary arbitrage opportunities between the markets [3]. If such arbitrage opportunities exist, the future changes of electricity prices could be forecasted by, for example, looking at the past values of prices of similar commodities. Investors could thus benefit from identifying how the expected changes are influenced by these prices. Hence, they would be able to find a model that converts this information to a forecast of the future change.

4. **Modern asset valuation prefers using forward prices instead of forecasted future spot prices**.

   Investors in electricity market need to have relevant information on electricity prices in future, in order to properly value existing and planned assets. Long-term information about electricity prices is one of the most important variables in these asset valuations. Investors in new production capacity need to estimate the long-term expected spot prices. Alternatively, modern asset pricing suggests using forward prices instead of expected spot prices, since forward prices already incorporate the appropriate level of market price of risk. Unfortunately, the current information on prices in distant future, e.g. 10 years ahead, cannot be observed in the market, since forward contracts with these delivery periods are not traded on the exchange. The term structure of forward contracts typically ends 5 to 6 years ahead. Investors usually follow two basic strategies when estimating the value of the long end of any price term-structure. The first would be to perform technical analysis of past electricity prices movements and use e.g. extrapolation to estimate the value of forward contracts beyond the traded horizon. This strategy is often referred to as technical analysis or chartism [4]. Since the prices of forward contracts beyond the traded horizon may be driven by different or additional market forces than the prices of exchange traded contracts, fundamentalist predict these prices and make trading decisions with the help of underlying fundamental factors, which govern the prices beyond the traded horizon (see, e.g. Povh et al. [5]). In case of electricity, the later strategy usually gives better results [6].

The most accurate information about the value of electricity with a certain delivery period can only be obtained in case of liquid forward market. The absence of accurate information indicates a need for trading horizons on electricity exchanges to be extended to future as far as possible. This would result in better market transparency, which is important not only for the purpose of investment analyses mentioned above, but also for the purpose of global benchmarking. Publicly available long-term price indicators in a particular electricity market would also be useful for investors and other institutions outside the market.

Goals

We define the following major goals of this thesis:

1. The most important goal of this thesis is to identify what drives the dynamics of the long-term forward prices. Most of the previous research in this area is

focused on the short-term forward market, while the long-term forward market is usually ignored, since the reliable long-term data was usually insufficient for serious analysis, till recently. Identification of dynamics is important to understand the price behaviour of long-term forward prices in the past as well as for better forecasting price dynamics in future.

2. In order to capture the dynamics of long-term electricity forward prices we need to set up a theoretical framework of the underlying process in terms of expected electricity supply and demand as well as risk premium. The information set, which the expected price change is conditioned upon should include fundamental variables that influence the expected supply and demand for electricity as well as the risk premium. Since some of these variables may be unobservable, our goal is to find suitable proxies for their values. The goal is not to identify each and every variable that expected price changes are conditioned upon, but to identify only the main drivers and variables that can capture them.

3. Depending on information set we intend to design a model that should be able to capture the underlying dynamics of the long-term electricity forward price formation. The model should capture the short-run dynamics, so that it is able to forecast short-term expected changes, as well as the long-run dynamics in order to forecast forward prices beyond the traded horizon.

4. The obtained model will be useful for identifying the hidden structure of the relationships between the variables in the information set. We intend to identify these relationships and what they can tell us about the future price dynamics.

The lack of reliable market data is the greatest drawback, when focusing on the long-term markets. Information which influences the long-term electricity forward prices is often not reliable and asymmetric or private. We design a specific model for Nordic electricity market, which is one of the oldest electricity markets with high transparency and easily accessible financial and operational data needed for the analysis.

Contributions

In this thesis the results of the above mentioned work are presented and the contributions of our work can be summarised as follows:

1. **Stochastic model for long-term electricity forward price**. Theoretical background on long-term forward price process is presented in terms of expected supply and demand for electricity as well as of the risk premium. For each of these components, we find suitable variables that capture most of the influences as implied by the theory. The model is general enough to be upgraded with additional information.

2. **Estimation procedure for model parameters**. Our model is based on variables that influence each other. To avoid a priori assumptions on causal

structure and exogeneity, we design a vector autoregressive model with 7 endogenous variables, one exogenous variable and 8 intervention dummy variables. We show that this model is suitable for empirical tests and analyses.

3. **Modelling the uncertainty structure for long-term electricity forward prices**. The uncertainty structure is identified with the detailed analysis of the residuals obtained from the vector autoregressive model. We identify the common trends and cycles driving the permanent and transitory part of long-term electricity forward prices. Impulse response analysis, forecast error variance decomposition and identification of structural shocks are used to identify the hidden structural relationships between the variables in the short- in long-run.

4. **Stochastic model for long-term electricity supply, demand and generation fuel prices**. We set up a theoretical framework in which these three components influencing the long-term electricity forward prices are formed. We define the variables which influence these components as implied by the theory. These variables are then divided into three types of information. The first we call *high resolution common-knowledge information*, which we use to set up the long-term electricity forward price model. The second is *low resolution common knowledge information*, which could be used in the model, but would require larger estimation sample. The third we call *asymmetric information*, which involves private information and thus cannot be used for modelling.

These contributions are also appearing in two papers. The first titled *Modelling long-term electricity forward prices* was published in *IEEE Transactions on Power Systems* [7], while the second titled *Modelling the structure of long-term electricity forward prices at Nord Pool* was accepted for publication in *Power System Handbook* [5]. Both papers are attached at the end of this thesis.

Outline

In the next chapter, we briefly present the works related to the modelling of long-term electricity forward prices. We restrict ourselves mainly valuation of assets, commodities and commodity derivatives. At the end of the chapter, we present the basic idea behind this thesis and how it differs from related works. The Chapter 2 provides a basic introduction to electricity markets and asset valuation. We then present theoretical concepts of modelling commodity forward prices and briefly discuss some of the main challenges for the case of electricity forwards.

Chapter 3 provides our definition of long-term electricity forward prices and the data generating process behind them. This is followed by the model setup based on the risk adjustment of the expected spot price. This approach separates the modelling of forward price into modelling the expected spot price and modelling the risk premium. The formation of expected spot price is further elaborated into modelling expected long-term electricity supply and demand and into general framework how to model them. Next, we present the fundamental factors which influence the supply and demand. We also provide an overview of the risk premium in the long-term electricity forwards and review the empirical findings on the risk premium in the electricity market. The chapter ends with the definition of the information set, the variables that are included and their unconditional distribution.

In Chapter 4, we discuss and present the data we choose for variables in the information set. The basic descriptive statistics is included along with a principal component analysis. At the end of this chapter the variables are tested for their order of integration. Chapter 5 centers around the model setup based on the multivariate autoregressive framework. The main focus of this chapter is on the cointegration test followed by the identification of cointegration space and testing the restrictions on cointegration and adjustment dynamics.

In Chapter 6, we perform several structural analyses on the given model. These include Granger causality test, impulse response analysis, variance decomposition and identification of common trends and cycles. The results of these analyses are used to identify the structural representation of the model. Structural VAR framework is then presented and two types of structural identifications are estimated. The first focuses solely on the contemporaneous interdependencies between variables, while the second identification separates the structural shocks into permanent and transitory shocks. Economic interpretation of the resulting structural shocks is discussed and possible implications of this structure are given. At the end of Chapter 6, we focus on the short-run structure of the model and possible applications of the model. Based on the findings in previous chapters, a simplified model is estimated, which captures only the most important and significant short-run and long-run features of the model. In Chapter 7, we summarise the main findings of this thesis and provide possible directions for future work.

Chapter 1

Related Work

Modelling long-term commodity forward prices is not often discussed in academic literature. In most markets, the focus on long-term forward market is not essential since the long-term risks can be hedged with the so-called roll-over hedging. Consequently, the feasibility of hedging long-term forward commitments with short-term futures contracts has received an increased attention in the last decade. Under certain conditions roll-over hedging has several important advantages and this provides some explanation for the lower liquidity and large bid-ask spreads in the available long-term forward contracts. Koekebakker and Ollmar [1], however, show that the correlation between the prices of short-term and long-term electricity forward contracts is very low. This suggests that hedging long-term commitments with short-term forward contracts may prove disastrous in case of the electricity market.

Modelling short dated commodity derivatives received a lot of academic attention in past. However, the literature on energy and energy derivatives is quite recent. Among them, the focus on long-term evolution of oil prices is worth mentioning. Schwartz [8] employs multi-factor models estimated on short-term oil futures and tests the performance of these models on the available long-term oil futures obtained from Enron. Pindyck [9] models long-term evolution of oil, coal and natural gas prices with mean reversion models, using the theory of depletable resources. Schwartz and Smith [10] use long-term forward prices as an equilibrium to which short-term prices revert. They assume that commodity prices are mean reverting in general, but there is also the uncertainty about the equilibrium (long-term) price to which (short-term) prices revert. In case of electricity derivatives, the research literature can be summoned into three groups. One line of research is focused purely on the empirical analysis of observable electricity futures and forward prices. Byström [11] investigates the hedging performance of electricity futures from Nordic electricity market using the minimum variance hedge ratio. Solibakke [12] uses GARCH model specification for prices of one-year-ahead forward contracts from Nord Pool.

The second line of research in electricity prices is the use of multi-factor models. The state variables in these papers are modelled as non-observable, where the first variable is needed to capture randomness in the short-term movement and the second variable to capture the randomness in the long-term price trend (reverting level). Lucia and Schwartz [13] were probably the first to use multi-factor models for electricity spot prices. They extended the model from Schwartz and Smith [10] by incorporating seasonality with deterministic sinusoidal function. Further extension of this model is presented in Villaplana [14] with the inclusion of jump diffusion process with non-constant intensity, while the risk premium is captured with seasonally dependent jump intensity. Bjerksund et al. [15] and Koekebakker and Ollmar [1] were the first to use Heath, Jarrow and Morton [16] framework to model the electricity forward curve. Koekebakker and Ollmar [1] show that unlike the other commodity markets where two or three factors successfully explain over 95% of variations in forward curve, a two factor model can explain only about 75% of variations in electricity forward curve.

The third line of research is based on more fundamental approach that originates in valuation of electricity with production cost models. Unlike the use of non-observable state variables in the works mentioned above, the movements in electricity prices can be explained with several (mostly) observable state variables that influence the supply and demand. These variables have strong theoretical background; their interpretation is much easier, while models based on these variables usually lead to the results that are more consistent with observable market prices. This line of research is pursued by Eydeland and Geman [17], Pirrong and Jermakyan [18], Barlow [19], Eydeland and Wolyniec [6], Bessembinder and Lemmon [20], Longstaff and Wang [21], Skantze and Ilic [22], Johnsen [23] and Skantze et al. [24]. Pirrong and Jermakyan [18] present a supply/demand model with two state variables; fuel price (natural gas) and electricity demand. They introduce the risk premium with the explicit time and load dependent function. They find that the price skewness induced by spikes is the main driver of the risk premium. Right skewness which dominates during high demand induces the long hedging pressure, whereas higher price variance which dominates during low demand induces the short hedging pressure. Similar results were obtained by Bessembinder and Lemmon [20] who presented the risk supply/demand model with fuel price and demand as state variables. One of their main findings is that forward price tends to be higher than expected spot price whenever the expected demand or volatility is high due to the right skewness of the spot price distribution. Eydeland and Wolyniec [6] go a step further and present a model where (beside fuel prices and demand) the outages, weather and emission allowances are also considered as stochastic variables. The purpose of this so-called hybrid model is to incorporate all the basic drivers that influence the electricity production costs, while the transformation from production costs to spot prices or forward prices can be obtained by multiplication and scaling parameters, which are calibrated on real market data. Longstaff and Wang [21] tried to establish the relationship between the risk premium and the difference between available generating capacity and expected demand. Finally, Barlow [19], Skantze and Ilic [22], Johnsen [23] and Skantze et al. [24] use models with similar state variables without any consideration of forward contracts. It should be noted that one possible problem in this class of models is the unavailability of consistent historical data on state variables.

Apart from long-term oil model from Pindyck, or short-term models applied on long-term contracts by Schwartz [8] and Schwartz and Smith [10], there are not any serious publications on modelling long-term forward prices. Niemeyer, however, [25] reports on a forward price and volatility forecasting model that combines the risk adjustment and external long-term electricity price forecasting models. He points out the huge importance of reliable long-term information on electricity forward prices for asset valuation; however, no details about the model structure are presented. This indicates that such models, if successful, are often considered as corporate comparative advantage and therefore often kept secret.

In this thesis, we base our research on fundamental models. We follow a general-to-specific approach (see Hendry and Mizon [26]), where a general theoretical model that drives the long-term electricity forward prices is defined first. We assume that the dynamics in the long-term forward prices can partly be explained with observable fundamental factors influencing long-term supply, demand and risk premium. The other part of the dynamics is treated as completely stochastic, and can be described as independently identically distributed error process. The general model is then designed to match the specifics of the long-term forward prices from Nord Pool. Hence, fundamental variables are chosen to best represent the theoretical influences. While the short-run relationship between fundamental drivers is of the main interest in some of the works outlined above, our contribution is to analyse how the same or similar fundamental drivers interact in the long-run, and whether they can be used to predict a part of the future dynamics of the long-term electricity forward prices.

Chapter 2

Theoretical background

Forward contract is a type of security in a term market, which guaranties the owner a future delivery of the underlying at an agreed price. In Nordic electricity market this does not imply physical delivery of electricity, only the difference between realised spot price during the delivery period and the contract price is exchanged. Unlike other commodities where forwards are specified for a delivery at specific future date, electricity is a flowing commodity and electricity forwards are specified for a delivery period. Since the underlying in these contracts is the electricity spot market price, with hourly resolution, electricity forwards can be viewed as a portfolio of individual forwards with delivery time at a specific hour. Hence, the underlying of electricity forwards is a time average of the electricity spot prices during the delivery period. Futures contract is a more standardised type of forward contract, usually traded on an exchange. While forwards are settled only during delivery period, futures are settled during trading and delivery period through the use of margin accounts.

Since futures with long delivery periods require significant amount of cash in margin accounts, investors seem to prefer forward type of settlement for long-term contracts. To increase the liquidity of these contracts, Nord Pool provides trading with standardised forward contracts with delivery period of a month, a quarter and a year, whereas weekly and daily contracts are traded as futures. Forwards from Nordic electricity market are therefore not typical forward contracts, since the settlement occurs periodically during delivery period, e.g. monthly. These contracts therefore correspond to the definition of swaps in which two counterparties agree on the exchange of periodic payments on a given notional amount of money for a given length of time. These payments for example depend on the difference between a fixed interest rate and a particular market determined floating rate. Since forwards and futures differ only in terms of settlement, they can be treated as the same if deterministic interest rate is assumed (Cox, Ingersoll & Ross [27]).

2.1 Why are forward prices important?

Securities like forwards, futures or swaps are primarily intended to offer investors an opportunity to hedge the uncertain future price of the underlying. In electricity term markets, investors are motivated by hedging uncertain future cash flows or they are speculating on various derivative markets and spot market. Electricity producers are typically facing uncertain future production revenues, while electricity consumers (retailers) are facing uncertain future procurement costs. Producers and consumers are therefore motivated to invest in the forward market in order to decrease the risk associated with the uncertainty of the future spot price. In case forwards are purely financial contracts without obligation of physical delivery, this attracts outside speculators who are willing to bear some of the future spot price risk for a hope of receiving a risk premium.

Since risk management is the driving force of each forward market, risk premium depends on the risk preferences of investors, whereas the liquidity of the forward market depends also on the specifics of the underlying. While commodity forward markets are normally focused on contracts with time-to-maturity up to 1.5 years, the focus horizon of some electricity forward markets is up to 5 years into future and still increasing. Since the correlation between the short-term and long-term prices is high in many markets, long-term risks can be hedged with roll-over hedging using short-term and mid-term forwards and futures. Unlike most commodities, electricity cannot be stored to any great extent. In an empirical analysis of forwards from Nord Pool, Koekebakker and Ollmar [1] show that the correlation between short-term and long-term electricity futures is low and conclude that short-term contracts are not appropriate for hedging long-term exposures in electricity markets, such as long-term procurement costs and production revenues. While far-maturity exposures can normally be hedged with short-term positions, electricity companies can only properly hedge them with long-term trading.

2.1.1 Modern asset pricing

Forward prices can also serve as important information carriers in that they provide valuation signals for strategic decisions like investments and mergers & acquisitions. Modern asset or project evaluation methods like Modern Asset Pricing (MAP) (see, e.g. Salahor [28]) or the theory of real options (Dixit and Pindyck [29]) suggest using forward prices instead of projected future spot prices. Investments are traditionally valued with Discounted Cash-Flows (DCF), in which projected future spot prices are used to estimate the expected future cash-flows of an investment. The main drawback of DCF is that it considers the risk in ad hoc through the choice of a discount rate, while the influence of risk on the value is often assessed with some sensitivity analysis of valuation results. Although some organizations use Capital Asset Pricing Model (CAPM) to determine their corporate discount rate or cost of capital, CAPM is not always successful in measuring the market price of risk. Forward prices, on the other hand, represent the market value of future delivery, and they properly measure the risk related to the uncertainty of the future spot prices. The use of forward prices therefore bypasses the problem of ad hoc selection of appropriate discount rates, allowing the assets to be valued over time with a risk-free interest rate.

2.2 Valuation of commodity forwards

Valuation of commodity derivatives originates from Keynes hypothesis that investors who are motivated to hedge unfavourable future price movements sell futures contracts to speculators [30]. He and Hicks developed a theory of *normal backwardation*, that is, futures prices tend to rise over the life of a futures contract, because hedgers tend to be short in the futures market and since speculators must be motivated to sell futures [31], [32]. Since then, several views on commodity forwards were introduced and developed. Fundamentally they differ only in terms of modelling the expected value of future payoffs, and in terms of adjustment of this expected value for a time value of risk. A classic view on the value of commodity and commodity forwards is given by risk-return finance models. Among them, Capital Asset Pricing Model (CAPM) and its variants are most widely applied. Option pricing theory offers another view on the value of commodity forwards. Cox and Ross [33] made an important contribution with their method of risk neutral valuation. By definition, a risk-neutral probability measure is a measure under which the current price of a security equals the present value of the discounted expected value of its future payoffs given a risk-free interest rate. While the risk neutral dynamics of commodity prices is not directly observable, one can infer some information about this dynamics from the observable prices of commodity forward and spot prices. Risk neutral dynamics is therefore obtained, based on certain assumptions about the evolution of commodity spot prices, while derivative prices are obtained through no-arbitrage principle and convenience yield. Equilibrium models offer alternative solution to model risk neutral dynamics. They involve modelling equilibrium spot prices from the underlying fundamentals of supply and demand. Commodity forward prices can also be valued with the theory of storage. The value of storable commodities is transferred between periods which inherently influence the equilibrium in commodity prices.

2.3 Modelling risk neutral price dynamics

A complex structure of observed commodity prices has lead to different approaches to the problem of identifying the risk neutral probability distribution. These approaches can be broadly classified as parametric and nonparametric. Parametric methods choose a distribution family and then try to identify functional specification and parameters for this distribution that are consistent with the observed prices. Non-parametric techniques do not pre-specify functional relations and offer more flexibility by allowing more general functional forms. Regardless of the approach how to obtain the risk neutral probability distribution, the success of these techniques is always measured against the observed prices of commodities and their properties. Empirical research on commodity prices has documented the following properties (adapted from Seppi [34]):

- mean reversion in prices (Bessembinder et. al [35], Schwartz [8])
- random backwardation (when spot prices are high and revert down to mean) and contango (when spot prices are low and revert up to mean) in futures prices (Litzenberger and Rabinowitz [36])

- less than perfect correlation between short- and long-term forward prices (Clewlow and Strickland, [37])
- higher volatility in short-term forward prices and lower volatility of long-term forward prices (Bessembinder et. al [35])

Electricity prices show similar properties. However, few distinct properties are worth mentioning:

- strong mean reversion with seasonal and weather dependent mean price and volatility (Kaminski [38])
- high (low) volatility when prices are high (low) suggesting heteroscedasticity (Kaminski [38])
- price spikes (jumps followed by strong mean reversion to normal levels) and skewed price distribution (Kaminski [38])
- low correlation between short- and long-term forward prices (Koekebakker and Ollmar [1]).

2.3.1 Single factor models

A classic model for risk neutral dynamics, proposed by Black and Scholes [39] identify the risk-neutral dynamics as a Geometric Brownian motion

$$\frac{dS_t}{S_t} = \mu dt + \sigma dw_t \tag{2.1}$$

where S_t is the spot price, μ is the drift rate of S_t equal to risk-free interest rate, σ the volatility equal to true annualised standard deviation and w_t is the Wiener process. Black and Scholes formula does not involve mean reversion, seasonality, heteroscedasticity, or price spikes and is therefore hardly appropriate to model electricity prices. Generalizations of Black-Scholes formula may include replacing σ with time and price dependent volatility $v(S_t, t)$ and adjusting μ for a convenience yield δ_t

$$\frac{dS_t}{S_t} = (\mu - \delta_t)dt + v(S_t, t)dw_t. \tag{2.2}$$

or mean reversion

$$\frac{dS_t}{S_t} = a(b(t) - S_t)dt + v(S_t, t)dw_t. \tag{2.3}$$

which is known as Ornstein-Uhlenbeck process. In (2.3) $b(t)$ is the time-varying reverting price, while a is the speed of reversion. These generalizations of Black-Scholes formula are still largely incompatible with prices observed in electricity markets. The specifics of electricity prices outlined at the beginning of this section, call for a risk-neutral process that would accommodate and interpret most properties of the observable market and empirical data. An extension of mean reverting model (2.3) is the mean-reverting jump-diffusion model (MRJD)

$$\frac{dS_t}{S_t} = a(b(t) - S_t)dt + v(S_t, t)dw_t + \phi(S_t, t)dJ_t \tag{2.4}$$

where $\phi(S_t, t)$ is the jump magnitude and dJ_t the Poisson process (see, e.g. Clewlow and Strickland [37]). MRJD model therefore explain the stochastic dynamics of S_t with two sources of randomness dw_t and dJ_t. This models is better in replicating the skewed distribution of observable electricity spot prices, however it requires a high speed of mean reversion in order to reduce the spot price following a large positive jump. This has the effect of removing too much variability accompanying the dynamics when electricity spot prices are high.

2.3.2 Multifactor models

The true dynamics of different derivative prices obtained on risk neutral dynamics of single-factor models are highly correlated, since the underlying risk neutral dynamics is driven by only one random process. Schwartz [8] show that commodity derivatives are better explained with multiple sources of randomness, while Koekebakker and Ollmar [1] show that perhaps up to 10 factors would be needed to capture 95% variation in electricity forwards. The idea of multifactor models is that prices, although driven by a variety of information, can always be summarised with a relatively small number of factors.

Gibson and Schwartz [40] present a two-factor model in which the commodity spot price (S_t) is given by a Geometric Brownian motion whose rate of growth is corrected by a stochastic mean-reverting convenience yield (δ_t):

$$\begin{aligned} dS_t &= (r_t - \delta_t)S_t dt + \sigma_s S_t dw_s \\ d\delta_t &= a_\delta(b_\delta - \delta_t)dt + \sigma_\delta S_t dw_\delta \end{aligned} \tag{2.5}$$

where r_t is interest rate, a_δ the mean reversion speed of convenience yield, b_δ the mean reverting convenience yield, σ_s and σ_δ volatility of both factors, while dw_s and dw_δ are correlated increments of Brownian motion processes.

A model from Schwartz and Smith [10] is also based on two factors, the first (X_t) representing the short-run price deviations and the second (ξ_t) representing the long-run equilibrium price level:

$$\begin{aligned} dX_t &= -a_x X_t dt + \sigma_x dw_x \\ d\xi_t &= -\mu_\xi dt + \sigma_\xi dw_\xi \end{aligned} \tag{2.6}$$

where a_x is the speed of mean reversion, μ_ξ the drift in equilibrium price, σ_x and σ_ξ volatility of both factors, while dw_x and dw_ξ are correlated increments of Brownian motion processes. Schwartz [8] extend the Gibson and Schwartz model with a third factor representing a stochastic instantaneous interest rate. He finds that including stochastic interest rates does not explain a lot of additional variation in commodity prices. Examples of multi-factor spot price models are also structural models where demand and supply are modelled separately with stochastic factors. A three-factor electricity price model from [34] models the temperature, fuel costs and capacity as stochastic variables

$$\begin{aligned} dx_t &= \mu_x(x_t, t)dt + v_x(x_t, t)dw_x \\ dy_t &= \mu_y(y_t, t)dt + v_y(y_t, t)dw_y \\ dm_t &= a_m(b_m(t) - m_t)dt + v_m(m_t, t)dJ_t \end{aligned} \tag{2.7}$$

where x_t and y_t are temperature and fuel costs respectively, whereas m_t is capacity modelled as mean reverting process with Poisson jump volatility. Similarly Eydeland and Geman [17], Eydeland and Wolyniec [6] and Pirrong and Jermakyan [18] also use multiple factors which represent different components of electricity supply and demand.

Next step in modelling commodity forward prices is to model the whole forward curve rather than the price of a single contract. A class of multi-factor continuous forward price models such as Heath, Jarrow, and Morton [16] model generally has the following form

$$dF_{t,T} = \mu(F_t, t, T)dt + \sum_j \sigma_j(F_t, t, T)dw_t^j, \qquad 0 \leq t \leq T \tag{2.8}$$

where $\mu(F_t, t, T)$ is the risk neutral drift function and $\sigma_j(F_t, t, T)$ is the j-th factor sensitivity of the forward price to a common set of uncorrelated Wiener processes dw_t^j. For each forward price and each delivery date a separate specification of (2.8) is be made.

Under Efficient Market Hypothesis (EMH) developed by works of Samuelson [41] and Fama [42], efficient markets make forward prices a random walk. EMH implies that market prices fully reflect the information that is available to investors. Expected future changes can therefore only be the result of completely unpredictable events i.e. random walk. EHM implies that $\mu(F_t, t, T) = 0$, which leads to the general Heath, Jarrow and Morton (HJM) model framework

$$dF_{t,T} = \sum_j \sigma_j(F_t, t, T)dw_t^j, \qquad 0 \leq t \leq T \tag{2.9}$$

This representation allows that the empirical changes in forward prices are not perfectly correlated along the forward curve or with the spot prices. The short-term forward prices can thus be more sensitive than long-term forward prices

$$\sigma(F_t, t, T) > \sigma(F_t, t, \tau), \qquad T < \tau. \tag{2.10}$$

The HJM model evolution of forward prices implicitly induces the spot price dynamics, since the spot price is a special case of the forward price when $t = T$. The factors with bigger influence have larger sensitivities which allow the prices to be heteroscedastic. Sensitivities can also change over time, taking into account seasonalities or the current shape of the forward curve F_t itself. They can be calibrated directly from empirical correlations of forward price changes.

2.4 Cost-of-carry

Prices on storable commodities can also be set up based on the assumption that a rational, profit maximizing investor can carry the good as inventory from the current to future periods. Deaton and Laroque [43] show that in the absence of inventory, the spikes observed in commodity prices are solely determined by the production and consumption at that time. Commodity futures prices, on the other hand, are often backwardated indicating that they decline with time-to-maturity. Backwardation implies that immediate ownership of the physical commodity entails some benefit or convenience which a deferred ownership via a long forward position does not. This benefit is expressed as convenience yield and the relationship between the ownership of physical commodity and forward position can be expressed with the standard cost-of-carry model

$$F_{t,T} = S_t e^{(r+c-y)(T-t)} \tag{2.11}$$

where r is the risk-free interest rate, c the storage costs and y the convenience yield all expressed in relative (i.e. percentage) values. The case $r + c - y < 0$ gives rise to backwardation and may be interpreted as a positive net convenience yield, and vice versa. The theory of storage of Kaldor [44] and Working [45], [46] explains convenience yields in terms of a time option, since the holder of a storable commodity can decide when to consume it. If it is optimal to store a commodity for future consumption, then it can be priced as an asset, but if it is optimal to consume it immediately, then it can be priced as consumption good. Thus, a storable commodity's spot price is the maximum of its current consumption value and asset value. In contrast, forward prices of storable commodities represent solely the asset value of the deferred right to consume at the time of delivery. Inventory decisions are important for commodities because by influencing the relative current and future scarcity of the good they link its current consumption and expected future asset values. Since inventory is always physically constrained, this is link imperfect. Deaton and Laroque (1992) model the spikes in commodity prices as arising from stockouts. Thus, stockouts break the link between the current consumption and expected future asset values of a good, which results in backwardation and positive convenience yields. Due to physically constrained inventory, the probability of stockouts tends to increase with time-to-delivery. This implies that the link between the spot and forward commodity price gets weaker with increasing time-to-delivery.

2.5 Risk adjustment

The risk adjustment involves changing the distribution of the underlying price (spot price) S_t in a way that it includes the risk associated with its uncertainty. Assuming the price S_t follows a geometric Brownian motion (2.1), the price of risk, defined as the excess return per unit of volatility, is then

$$\lambda = \frac{\mu - r}{\sigma} \tag{2.12}$$

where r is the risk-free interest rate, while μ and σ are the drift and volatility in (2.1). The value of λ depends solely on the dynamics of S_t and not on the specifics of the derivative. The derivatives with the underlying variable S_t should therefore be valued with following risk neutral process

$$\frac{dS_t^*}{S_t^*} = (\mu - \lambda\sigma)dt + \sigma dw_t. \tag{2.13}$$

This risk neutral process therefore entails implicit specification of risk, since the risk neutral probability measure already incorporates the effect of risk. Eydeland and Wolyniec [6] argue that electricity forward price process should have the explicit parameterization of the risk premia. The default assumption of zero risk-neutral drift function in (2.9) relies on the ability to replicate the payoff of the forward contract with trading on the spot market. This ability also relies on the assumption that the forward price and the spot price converge at expiration. In case of electricity, this is not true in the strict sense, since the average spot price during the delivery period can be very different than the forward price at expiration. In this case, it would be beneficiary to express the risk premium explicitly that is estimating the expected value of the spot price during the delivery period $\mathrm{E}_t[S_T]$ and adjusting this value for risk $r^p_{t,T}$

$$F_{t,T} = \mathrm{E}_t[S_T] + r^p_{t,T}. \tag{2.14}$$

Since electricity cannot be stored, this implies that it cannot be valued as an asset. Therefore the risk neutral dynamics used for asset valuation is different than the one in valuation of options on non-storable commodities. While the risk-neutral expected return in asset valuation equals the risk-free interest rate, the expected risk neutral return in options on non-storable commodities have no connection to the risk-free rate, since these commodities cannot be carried from one time period to another [34]. Hence, instead of cost-of-carry model (2.11), forwards on non-storable commodities are more appropriately valued with

$$F_{t,T} = \mathrm{E}_t[S_T]e^{(r-\lambda)(T-t)} \tag{2.15}$$

where λ is the risk premium (in relative value), also referred to as the *market price of risk*. In (2.15) the risk premium is therefore expressed explicitly indicating that one needs to first obtain the expected future evolution of the spot price and then based on the probability distribution of this expectation and based on the risk preferences of investors estimate the risk premium λ. We outline two approaches to estimate λ. The first is based on explicit parametric modelling, where the risk premium is expressed as a function of different variables, while the parameters are calibrated on real market data. The second is based on Capital Asset Pricing Model (CAPM) and its variants.

2.5.1 Capital Asset Pricing Model

Capital Asset Pricing Model describes the relationship between the expected payoff of a security and its risk. CAPM says that investors in risky term contracts, such as electricity forwards, need to be compensated for the time value of money and risk. The risk premium is defined as the difference between the expected spot price and forward price

$$r^p_{t,T} = F_{t,T} - \mathrm{E}_t[S_T]. \tag{2.16}$$

Black [47] shows that, in CAPM, the risk premium for term contracts is

$$r^p_{t,T} = \beta_s(\mathrm{E}(r_m) - r) \tag{2.17}$$

where $\mathrm{E}(r_m)$ is the expected return in a general capital market, r the risk-free interest rate, while β_s is beta of the security defined as

$$\beta_s = \frac{\mathrm{cov}(\mathrm{E}_t[S_T], r_m)}{\mathrm{var}(r_m)}. \tag{2.18}$$

In case of electricity, beta is therefore the sensitivity of electricity price to a return in the general capital market. For CAPM, forward price is therefore

$$F_{t,T} = \mathrm{E}_t[S_T] + \beta_s(\mathrm{E}(r_m) - r). \tag{2.19}$$

A risk associated with the ownership of an asset is comprised of *systematic* or *undiversifiable* risk and *unique* or *diversifiable* risk (see, e.g. Brealey and Myers [48]). The first cannot be avoided, since it is correlated with the movements in general economic activity and leads to non-zero risk premium. The second is not correlated to the movements in general economic activity, has no influence on diversified investors and leads to zero risk premium. Thus, securities with highly non-diversifiable risk will have beta close to 1, while securities with completely diversifiable risk will have beta around 0.

Although CAPM is able to capture the price of risk, it is not well suited for pricing electricity forwards, since it assumes that (financial) electricity market is also used for diversification of general investors. In financial electricity markets, the participation of investors outside the industry is weak; hence, the dynamics of the risk premium is mainly driven by producers and consumers, who are motivated by hedging production and consumption. Bessembinder and Lemmon [20] show that in the absence of outside speculators, different levels of risk aversion of producers and consumers lead to non-zero risk premium in electricity forwards. Non-zero risk premium attracts participants from outside the industry to include forwards in their portfolios, which would gradually decrease the level of risk premium. Since financial electricity markets are still in the process of removing the barriers to attract more outside participation, one cannot expect that CAPM betas, which are estimated on electricity prices, to properly measure the actual risk premium in electricity forwards.

Chapter 3

Long-term electricity forward price process

Electricity term markets consist of various contracts with different delivery periods and times-to-delivery. In general, the prices of these contracts depend on the variables that influence the expected supply, demand and the risk premium. The fact that electricity term prices behave differently in the short- and the long-run should not be taken as evidence that both are conditioned on different variables. Deaton and Laroque [43], [49] and Chambers and Bailey [50] show that the possibility of storage tend to smooth the impact of supply and demand shocks on the spot price. This smoothing, in turn, has an impact on the unconditional volatility of forward prices. The costs of storage in case of electricity can be considered as extremely high, therefore, one would expect even zero correlation between the short- and the long-term forward prices. Small correlation which can be observed between these prices is the result of the fact, that electricity can be partly stored at least indirectly, by storing fuels or water for which the storage costs are considerably lower. The transmission of supply and demand shocks in the spot price to forward prices therefore depends on the possibilities of storing the supply and demand variables. In case these variables are also traded as long-term forwards, the information on their storage possibilities is already included in their forward prices. However, if there is no trusted long-term information on these variables, the shocks they are causing to spot prices is transferred to forward price only up to the time when their reserves are expected to deplete. Beyond this horizon these variables have no influence on the long-term forward prices.

3.1 Definition

In many commodity term markets, the prices of short-maturity contracts and prices of far-maturity contracts behave similarly. In such case market participants can hedge long-term risks with roll-over hedging using short-maturity contracts. When maturity date closes, they are replaced with the next closest contract. Empirical findings from Nordic electricity markets report that electricity term prices do not behave that way [1]. Although prices always reflect the expected supply and demand at the time of maturity, in case of electricity one cannot store electricity in times of low prices to hedge against the high prices in future. For this reason, the investors facing the long-term electricity price risk can only properly hedge these risks by entering the long-term electricity market.

In Nordic electricity exchange Nord Pool, electricity is traded on spot market and derivatives market. Here, the spot market is referred to day-ahead market where market participants buy or sell contracts for physical delivery of electricity for each delivery hour in the next day. For this reason the day-ahead market is sometimes also referred to as short-term forward market. The Nord Pool forward market consists of forward contracts with time-to-maturity ranging from 1 day to 5 years. Contracts with shorter times-to-maturity and delivery periods are called *futures*, while contracts with longer times-to-maturity and delivery periods are called *forwards*. Beside time-to-maturity and delivery periods, they also differ in terms of settlement, since futures are settled during delivery period as well as trading period, whereas forwards are settled only during delivery period. The relevant benchmark for these contracts is the spot price. Therefore, their cash flow depends on the difference between the realised spot price and the fixed contract price. Since forwards are not settled during trading period and are traded till the beginning of delivery period, they actually correspond to the definition of swaps, although we will continue to denote them forwards.

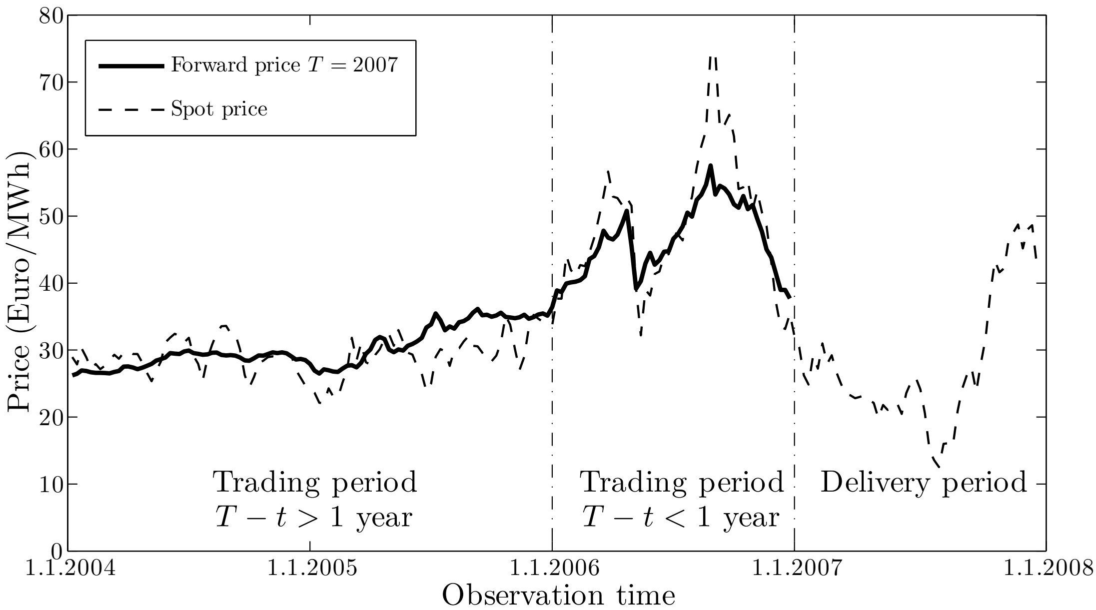

Figure 3.1: Electricity spot price and long-term forward price dynamics from Nord Pool.

Figure 3.1 presents an example of the price dynamics for a forward contract from Nord Pool with the delivery year $T = 2007$. Figure 3.1 demonstrates that the forward-price dynamics is different from the spot-price dynamics when time-to-maturity is high. As the maturity closes, the forward-price dynamics becomes more similar to the spot-price dynamics. Long-term forward prices and short-term forward prices (spot prices), therefore, appear to be governed by different laws, which indicate the need to model them separately. The prices of short-maturity contracts are obviously very volatile due to short-term influence of weather, which influences the production of hydro-power plants and also electricity demand. Their influence is particularly high in markets with large share of hydro-power plants and their possibility to store water for longer periods of time. Such is the Nordic electricity market where hydro-power plants can store water up to one 1 year and more. Far-maturity contracts are not influenced by weather since weather forecasts are not available that far ahead and the influence of present precipitation on water storage is insignificant that far in future. For this reason, we define long-term electricity forward prices as prices of electricity forward contracts with a delivery period of one year and time-to-maturity of more than one year ($T - t \geq 1$ year).

3.2 Data generating process

In this thesis, we consider electricity forward prices as an outcome of the actively traded financial electricity market, where investors are motivated by hedging production and consumption or by speculation. Producers and consumers who face uncertain future cash flows that depend on future evolution of electricity prices are motivated to hedge their positions by trading on financial electricity market. Speculators, on the other hand, include electricity derivatives in their portfolios with the sole expectation of profit. Both types of motivation generate supply and demand for forward contracts. Both are influenced by information that, according to investors, influence either the expected evolution of the future spot prices or the risk premium assessed on the basis of their risk preference over the estimated probability distribution of the future spot prices. These preferences may also depend on the correlation between these expected prices with the general capital market.

We assume the investors operate in a noisy rational expectation economy in which some investors are better informed than others. Investors have rational expectation about the link between the current signals and the current price, as in Grossman and Stiglitz [51] and Admati [52], and the link between the current signals and the next period's price, as in Lucas [53]. When it comes to linking the current signals to next period's price, we distinguish between two types of investors:

- Uninformed investors. They possess only public information, usually referred to the common knowledge of the unconditional distribution of the asset value. We assume all uninformed investors possess common public information set comprising mostly of market data, such as past prices and volumes and publicly available information on fundamental factors.

- Informed investors. They possess private information about the asset's fundamental value and observe its price. Informed investors' information set therefore consists of public information set as well as individual private information.

A state when investors have different information about the current and future value is called *asymmetry of information*. Information asymmetry is not the only source of diversity in the trading process. We also assume that the structure of the data generating process is not perfectly known to all investors; therefore some investors may interpret the information in their information sets differently. While some investors have homogeneous beliefs, others have heterogeneous beliefs and this divergence of beliefs does not vanish in time. In equilibrium, the outputs of the trading process are signals that show diverse private information of investors as well as heterogeneous beliefs.

We assume that any realised price change $\Delta f_{t+\Delta t}$ is the sum of two unobservable components

$$\Delta f_{t+\Delta t} = \Delta a_{t+\Delta t} + \Delta b_{t+\Delta t} \tag{3.1}$$

where $\Delta a_{t+\Delta t}$ is expected component driven by public information set and homogenous beliefs, while $\Delta b_{t+\Delta t}$ is random component driven by private information and heterogeneous beliefs (see Figure 3.2). In efficient market, the expected component $\Delta a_{t+\Delta t}$ evolves as a random walk in which each innovation reflects updates to the public information set. The expected component can therefore be termed as efficient price. Assuming investors are rational in a way that they learn from past experience and use all available information to get the best expectation of the price change, the random component of the price change should be *identically and independently distributed* (*iid*). When the realised price change is known, it becomes a part of new information set available to all investors. Information sets are continuously updated and the forward prices are continuously adapted to new information sets. Market participants are continuously trying to guess the future evolution of the forward price by seeking as much possible information that could influence the future evolution and continuously trying to update their beliefs based on past experience.

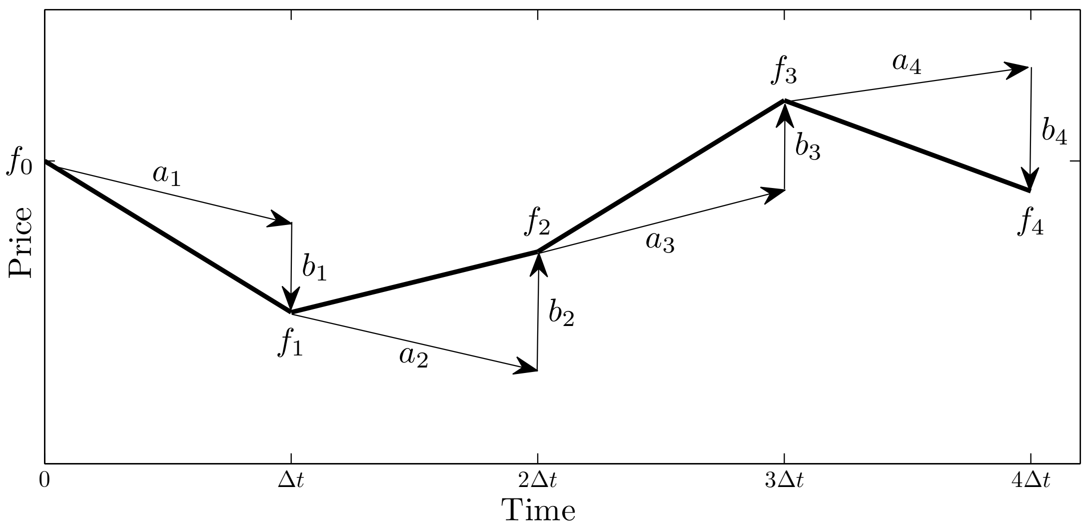

Figure 3.2: Data generating process

3.3 Setup

The forward price data generating process is governed solely by demand and supply for forward contracts. Therefore, an investor trying to predict the future change in the forward price must interpret the information that influences supply and demand. For valuation of forward contracts, we decompose the forward price into to aggregate market expectation about the future evolution of the spot price, which we denote as *expected spot price*, and the aggregate risk preference over the probability distribution of the future spot prices, which we denote as *risk premium*. While the cost-of-carry arbitrage is usually applied in valuation of commodity forwards, it cannot be used in case of electricity forwards, since electricity cannot be bought today at the spot price S_t and stored for subsequent sale at the forward price $F_{t,T}$. As an alternative to the cost-of-carry arbitrage, we use risk-adjusted valuation where one needs to model the expected spot price against which the forward price is settled and adjust it with the risk premium. We set up the model for electricity forward prices with discrete-time version of (2.15) (see, e.g. McDonald [54])

$$F_{t,T} = S_{t,T}(1 + r - \lambda)^{(T-t)} \tag{3.2}$$

where the forward price $F_{t,T}$ is the expected spot price during delivery period $S_{t,T}$ discounted with the risk premium λ and the risk-free interest rate r. The risk-free interest rate is included, if the payment at delivery T is assumed. Transforming (3.2) to logs gives

$$\ln F_{t,T} = \ln S_{t,T} + (T - t)\ln(1 + r - \lambda). \tag{3.3}$$

Assuming the constant risk-free interest rate r and λ and writing time-to-maturity $T - t$ as T_m, (3.3) can be rewritten to

$$\ln F_{t,T} = \ln S_{t,T} + \xi T_m \tag{3.4}$$

where ξ we call a risk premium parameter, while the risk premium is modelled as a function of time-to-maturity ξT_m. In (3.4) the forward price process is therefore the sum of expected spot price and risk premium. The expected spot price is an equilibrium process between the expected supply and demand for electricity during delivery period, whereas the risk premium is an equilibrium process between the supply and demand for bearing risk related to the uncertainty of the expected spot price.

3.4 Expected long-term electricity spot price

We define the long-term electricity forward price as the price of yearly forward contracts with time-to-maturity of more than 1 year. Based on this definition, the correct interpretation of the expected long-term spot price $S_{t,T}$ in (3.4) is the expected price of 1 MW of base-load electricity during the delivery year T. Expected electricity spot price some years ahead is influenced by the expected supply and demand at the delivery period T. The supply and demand, however, are not observable variables and therefore need to be modelled themselves. We will not attempt to model supply and demand directly, but instead we define the information influencing supply and demand and consequently the expected spot price.

3.4.1 Long-term electricity demand

In this thesis, electricity demand is considered as a function of electricity price with other factors influencing demand being constant. The electricity consumption is an equilibrium quantity at a given price and is therefore the realised consumption given the realised price. We define the long-term consumption as the total demand during a delivery period of $T = 1$ year, whereas the long-term electricity demand is a total quantity within T as a function of average electricity price within T. In electricity markets short- and long-term electricity consumption is a well-understood process (see, e.g. Stoll [55]), since one of the key issues in regulated electricity markets was the short- and long-term planning of the power system. The electricity demand, on the other hand, has traditionally been viewed as completely inelastic, thus electricity consumption always equals electricity demand. Long-term consumption is observable variable, since exact data on historical long-term consumption is available. There is, however, no transparent information on the long-term expected consumption, and different market players use different models and forecasts to obtain this information. Long-term electricity consumption is often tackled academic literature (see, e.g. Taylor [56]) and it mostly depends on the following information:

1. Economic activity (gross domestic product, income...)
2. Demographics (population, migration...)
3. Weather (temperature, wind, humidity, luminosity...)
4. Prices of alternative energy sources (oil price, natural gas price...)
5. Consumption of energy intensive industries (aluminium and steel smelters...)

To convert the long-term consumption to long-term demand, one needs to include the long-term price elasticity for electricity and to allow a possibility that electricity export can also be a part of demand. To estimate the expected long-term electricity demand, it is necessary to build a model that includes the above information.

3.4.2 Long-term electricity supply

We assume the long-term electricity supply is influenced by similar information as long-term electricity cost function, which is commonly used in production cost models (Baleriaux, Jamoulle and Guertechin [57]). The long-term cost function is represented by generation units sorted in ascending order according to their variable generation costs. In typical production cost model, cost function consists of average long-term production costs and respective annual production quantity of generating units (see Figure 3.3). Supply function can be obtained with simple multiplication and scaling of the cost function (Eydeland and Wolyniec [6]) and also allowing the possibility that part of the demand can be met with the import. Equivalently to cost function, we define the long-term electricity supply based on the long-term average supply costs and operational constraints.

Long-term average supply costs

Each supplying unit has different supply costs that depend on the type of fuel and fuel costs or import prices. Some production units have very low or no costs of fuel

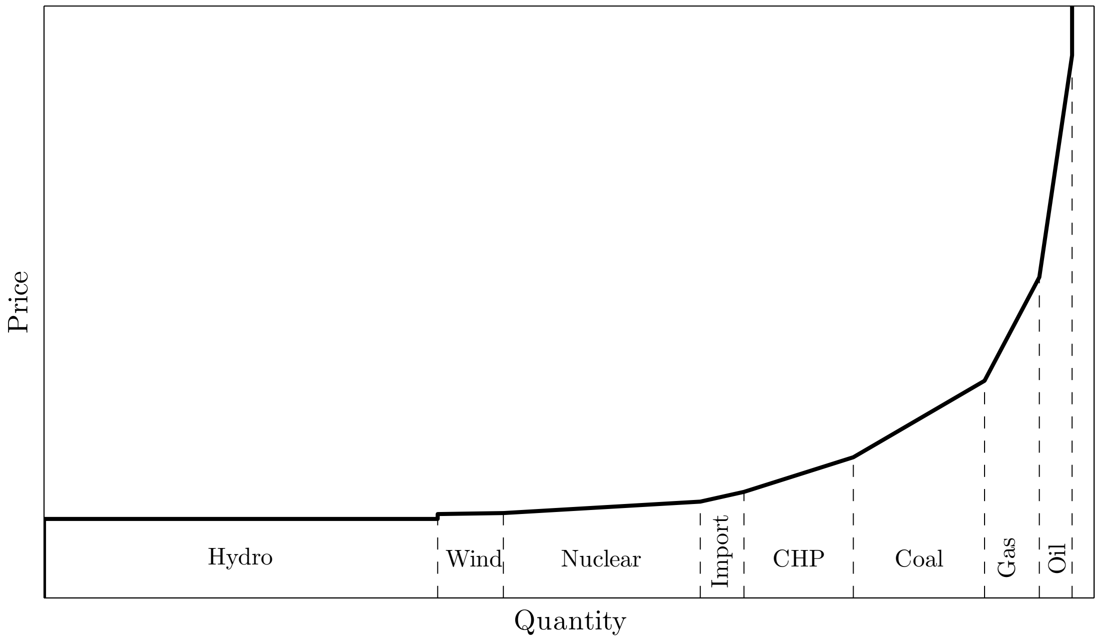

Figure 3.3: Typical long-term cost function of electricity market

at all (e.g. wind, hydro, nuclear), while other units have considerable and uncertain costs of fuel (e.g. coal, natural gas, oil derivatives). Similar to Eydeland and Wolyniec [6], we distinguish between three groups of supply cost variables. In the first group, there are non-tradable fuels such as water, uranium, wind and biomass. Because there is no liquid market and no long-term price information for these fuels, we assume that the costs for these fuels are constant i.e. their long-term production costs are the same as in present. The second group constitutes of tradable fuels, mostly coal, natural gas and oil derivatives. These fuels are traded on several international exchanges, whereas for some of these fuels fairly liquid forward market is also developed. Forwards on these fuels provide transparent information on the expected value of long-term fuel prices. The third group of supply cost variables includes other costs of supply, namely emission allowances and imported electricity. The European Emission Trading Scheme (EU ETS), which started in 2005 introduced restrictions on CO_2 emissions for all industrial emitters including electricity producers. This increased the production costs for fossil-fuel producers and consequently electricity prices. The forward contracts of CO_2 allowances are traded on major European electricity exchanges. Their expected long-term prices are therefore readily available. Imported electricity plays an important part in the supply function, since electricity markets and prices are still modelled regionally due to limited cross-border capacities and different market rules. Neighbouring market can be modelled as a specific type of producer. The price of this producer can be the average price in the market from which the electricity is imported, while the size of such a producer can be defined as the available border capacity between markets.

Operational constraints

Operational constraints reflect the total amount of electricity that can be supplied (generated or imported) to the market during a particular period. For each power plant, existing or planned, the available annual production can be specified and derived from historical utilization data, which depends on water reservoir storage, maintenances, outages, etc. Similarly, the available amount of imported electricity can also be defined, based on forecasted available border capacity between neighbouring markets. To estimate the available production and import, the average reservoir storage, maintenances and outages are considered as well as other operational constraints such as reserves, transmission constraints and import cross-border capacities. The operational constraints therefore include the following information:

1. Installed capacity,
2. Water reservoir resources,
3. Available cross-border capacity between price zones,
4. Maintenance including generation reserve margins,
5. Outages,
6. Operational constraints of generators.

This information includes historic and present data on existing assets as well as forecasted data on planned investments, such as new generator units or new power lines.

3.5 Risk adjustment

In modelling fixed income markets, foreign exchange markets or commodity markets ξ in (3.4) is commonly modelled as constant, implying that the risk premium depends only on time-to-maturity. Assuming the risk premium represents mostly the supply and demand for hedging uncertain future electricity price outcomes and not so much the correlation of electricity prices with the general capital market, a negative ξ would indicate a *backwardation*, i.e. the forward price is below the expected future spot price, while positive ξ would indicate *contango* i.e. the forward price is above the expected future spot price.

Similarly, in case of electricity, a constant risk premium assumption is often applied [61], despite some empirical findings which indicate that the risk premium in short-term electricity forwards might be time-varying. Besides time-to-maturity, the following information regarding the expected spot prices might be influencing:

1. Probability of price spikes - strongly related to load seasonality (Bessembinder and Lemmon [20], Longstaff and Wang [21]) and capacity shortage,
2. The level of expected spot prices (Ollmar [58]),
3. Seasonal observation time i.e. time of the year from January 1st (Ollmar [58]).

Empirical findings from electricity term markets indicate that short-term contracts are mainly used for hedging uncertain future consumption and electricity price spikes. This results in an excess demand for futures contracts and translates into a positive risk premium [58], [59], [60].[1] While some findings also indicate that this is true for long-term contracts [60], other indicate that these are mainly used for hedging long-term production of producers and therefore a negative risk premium is found [58]. Investigations, focused on far-maturity electricity contracts, also indicate that the magnitude and the variability of the risk premium in these forwards are low [58], [62]. In this thesis, we assume that the risk premium in long-term electricity forwards is a function of time-to-maturity only.

3.6 Information sets

In a perfect market, all investors possess perfect information set I_t that includes all variables that influence the forward price in the next period $t+\Delta t$. Such information set would include all the information discussed in Section 3.4. Real financial markets are not perfect and not all investors have access to all information from the perfect information set I_t. For example, some investors may obtain certain information as private or inside information, while others may acquire some information before other investors do. The information set, known to general investors at time t, is therefore incomplete and investors, in such case, are called *incomplete information investors*. An incomplete information set will generally vary from one investor to another, since they result from investors' private information acquisition activity. Since some information that influences the forward price is public, we assume that this information is common knowledge to all investors. Each information set therefore includes a part of information that is symmetric and therefore available in all information sets, while the other part of information is asymmetric and is the result of investors' private information acquisition. There is also a third kind of information that is used only by a limited pool of investors. This information is either accessible only to some investors or alternatively to all investors, however, only some of them believe that this information has an influence on the forward price. We denote this type of information as partly symmetric information.

3.6.1 Symmetry of information

An information to be classified as symmetric or partly symmetric must satisfy two conditions. Firstly, it must be accessible to all investors, implying that such information, when it becomes available, also becomes common knowledge among investors. Secondly, the information must also be reliable. Different information on the same influencing factors is sometimes provided by different institutions. Based on past experience, investors use only information which they believe is the most reliable. We assume information to be asymmetric if it is neither accessible to all investors nor is it reliable. To model the long-term electricity forward prices, we use only

[1] Since some studies use different definitions of risk premium, we adapt these results to match our definition.

symmetric and partly symmetric information from the information sets. We divide all information discussed in previous chapter in three groups with respect to the degree of symmetry.

1. **High-resolution common-knowledge information**. This information is completely public and usually the result of market forces, such as commodity exchange trading, OTC indexes or common knowledge information such as time-to-maturity. We assume this information is completely symmetric and can be used for broad high-resolution statistical analysis.

2. **Low-resolution common-knowledge information**. Changes of rules, decisions, policy interventions, aggregated data and estimated information, such as forecasted electricity consumption and gross domestic product are also public information and can therefore be termed as symmetric or partly symmetric. While this information is commonly used in analyses spanning the period of few decades, the usefulness of this data in high resolution statistical analysis is limited, since such analyses are focused on a period of only a few years, in which this information changes only a few times. With this respect, these changes can often be captured with intervention dummy variables.

3. **Asymmetric information**. This information is either too unreliable to be generally found in significant number of investors' information sets or it is completely private information obtained through private information acquisition activity. Such information cannot be used for broad statistical analysis.

To model the long-term forward prices we therefore use only the high-resolution common-knowledge information. Our information set includes the following information:

1. **Electricity forward prices**
The dynamics of electricity forward price up to time t helps to explain the expected change in $t + \Delta t$. Electricity forward price also serves as a proxy for expected long-term spot price, which has an influence on long-term electricity demand, assuming long-term price elasticity of demand. It can also have an influence on the risk premium as a proxy to the general level of electricity forward prices.

2. **Crude oil prices**
Crude oil price serves as a global indicator for the value of all oil derivatives. Oil derivatives influence the supply costs of electricity, since they are seldom used as marginal sources of electricity production. Alternatively, oil derivatives' prices also serve as a heating source alternative to electricity and therefore they influence the long-term electricity demand.

3. **Coal prices**
Coal is an important source in electricity production and therefore its price has a significant impact on electricity price. Despite the fact that most coal-fired power plants use local coal resources, the value of this coal is measured by benchmarking it with global coal prices usually valid for major nearby ports.

4. **Natural gas prices**
   Similar to oil derivatives, natural gas price also has an important influence since some of electricity is produced from natural gas. A lot of reserve units use natural gas as the generating source. Natural gas can be found as an alternative to electricity in heating and it has the influence on long-term electricity demand.

5. **Emission allowance prices**
   In case emissions produced by generating units are limited, such as CO_2 emissions in Europe, the price for additional emission allowances represents the additional costs when generating electricity from these units.

6. **Prices of electricity in neighbouring markets**
   Part of the electricity supply can also be the electricity imported from neighbouring markets. As a result, the imported electricity increases, which will likely increase local electricity price. Alternatively, part of the demand that is supplied by local generators can also come from electricity export. The choice whether neighbouring electricity price influences electricity demand or supply depends on the connectivity between the two markets.

7. **Aluminium price**
   Although expected electricity consumption, provided as a forecast by some institutions, cannot be used in the information set due to its low resolution, aluminium price is a proxy for a significant part of electricity consumption, particularly when a significant share of electricity is consumed for aluminium smelting. We expect that increased price of aluminium indicates a higher electricity consumption.

8. **Time-to-maturity**
   According to our model, we expect the time-to-maturity to have an influence on the risk premium.

3.6.2 Unconditional distribution of variables

When estimating the expected spot price in the long-term, we seek reliable information on the expected values of the fundamental variables influencing the expected spot price. In the short-term $T \approx t$ these fundamental variables can be predicted with high, though not complete, accuracy. As the time-to-maturity increases $T \gg t$, the variance of these variables increases and their mean values are harder to predict. Still, there is a difference between variables that are considered stationary and integrated variables. With stationary variables, their unconditional distribution is bounded and the unconditional mean is based on the historical average value and the expected growth. Hydro reservoir levels, supply capacity and electricity consumption can be predicted on the basis of historical average value and expected long-term growth. Integrated variables, on the other hand, have no unconditional distribution in the long-run. The shocks in these variables will persist and their unconditional distribution is therefore unbounded. In our case these are fuel prices, prices of electricity in neighbouring markets and possibly emission allowance prices. Fortunately, the market offers securities to hedge their uncertain future evolution. Among these securities, we use long-term forward prices of fuels, emission allowances and electricity in neighbouring markets to explain the dynamics of long-term electricity prices.

Chapter 4

Data analysis

In the previous section, we define a general long-term forward price process conditional on information set that includes the variables that influence either the expected long-term electricity supply and demand or the long-term risk premium. In this chapter, we transform a general model to specific one by finding the exact information set that influences the dynamics of long-term electricity forwards from Nord Pool. Our aim is to find observable variables, which prices of long-term forwards from Nord Pool are conditioned upon. Although the real information set may include variables, which are not observable, some variables can be replaced by proxy variables, for which the correlation between the real variables and these proxies is sufficiently high. Ideally, the information set should contain the variables which are also used by investors (or at least majority of them) when estimating their individual expectations on the future price changes.

4.1 Data

To model the long-term electricity forward price, we use the data valid for the common Nordic electricity market. The market comprises 4 countries; Norway, Sweden, Finland and Denmark, and is one of the oldest electricity markets, with Nord Pool, the oldest electricity exchange. In 2005, most of the electricity in the Nordic electricity market was supplied by hydroelectric plants (54%), with the rest coming from nuclear (22%), renewable (8%), coal (6%), natural gas (5%), imports (3%), oil (1%) and other sources (1%). In 2007, the Nord Pool financial market volume was 1060 TWh, physical volume was 292.2 TWh, whereas the total production in the market was 397.3 TWh, while the consumption was 400.9 TWh. The Nordic electricity market is convenient for modelling purposes due to the following reasons:

- The market is very isolated with less than 10% of electricity imported or exported. This allows a simpler identification of variables that influence electricity prices and their interpretation.
- The transparency of the market is fair.
- High share of hydro production with big storage reservoirs and small concentration on the supply side gives market participants an insignificant power to influence prices individually.

- The liquidity of the spot market and forward market at Nord Pool is fair, giving enough trust in the quoted prices.

The market went through a number of structural changes, the latest being the introduction of European emission trading scheme (ETS) in 2005. Since our information set includes also the emission allowance prices, we analyse only the prices of contracts traded from the start of 2005 to the end of 2007. Our data sample is constructed in a way to include only prices of yearly contracts with time-to-maturity between 1 year and 2 years (as shown in Table 4.1). For observation period 2005, ENOYR07 is used, and this contract is replaced with ENOYR08 with the start of 2006 and with ENOYR09 with the start of 2007. This way, we avoid the price shift when two consecutive contracts are rolled over. Since contracts with delivery period of 2 and 3 years ahead move very similarly, the difference between them is very small. Thus, the price shift at roll-over dates is minimal. For other variables, defined in the following of the thesis, we use forward prices with the same observation time and maturity period as electricity forwards. The analysis of high-resolution financial data

Table 4.1: Sample construction

Contract	Maturity period T	Observation period t
ENOYR-07	2007	2005
ENOYR-07	2008	2006
ENOYR-07	2009	2007

often involves the problem of non-synchronous trading. The prices in our analysis are quoted at different times, and due to the time mismatch the integration between them is not clear. We use a weekly resolution instead of daily resolution since the relative time mismatch is much lower in the case of weekly sampling. Although the weekly sampling tends to smooth out the magnitude of price jumps, the volatility structure should not be significantly different to that when using daily sampling. We use the closing price from each Wednesday as the reference weekly price for all the variables, giving the sample size of $N = 156$. As shown in Figure 4.1, there are no significant shifts at the time of rollover. The sample, however, shows a significant price shock in April 2006 corresponding to observations 67 to 70. Before this shock, CO_2 emission allowance prices were pushing electricity prices up significantly. When the report on actual emissions in EU was published in April 2006, the prices of emission allowances dropped dramatically, which had a significant effect on electricity prices (see Figure 4.1). We will investigate this effect by testing whether this shock can be considered as a structural break in the relationship between the variables.

Crude oil price

For the crude oil price we use the New York Mercantile Exchange (NYMEX) West Texas International (WTI) light sweet crude oil data. Although the Brent crude oil data from the Intercontinental Exchange (ICE) might be a better choice for Nordic countries, availability of long-term oil prices is better at NYMEX. The

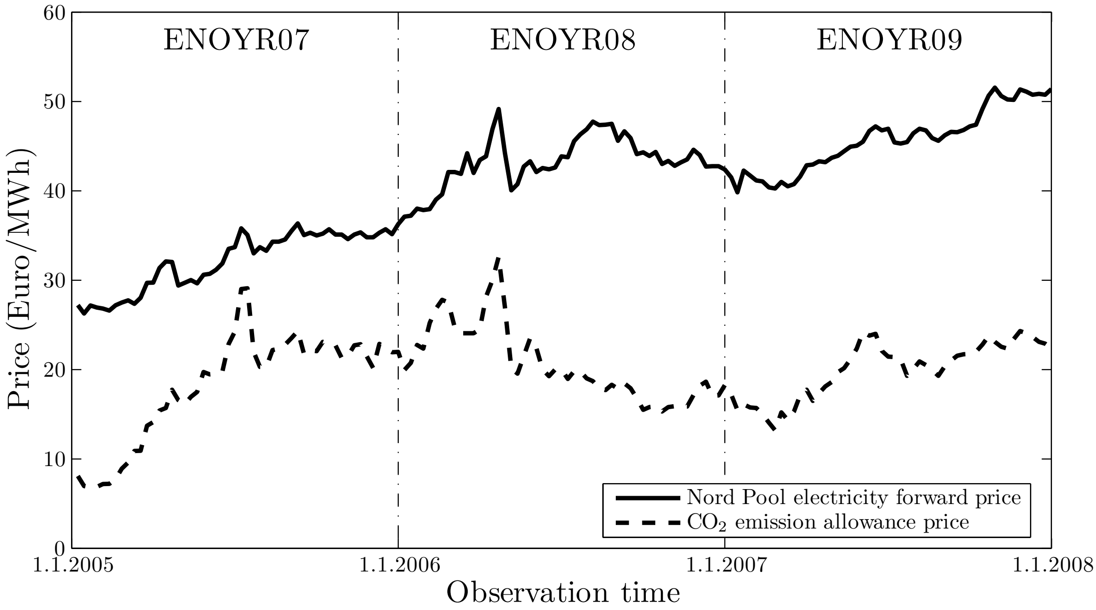

Figure 4.1: Estimation sample for long-term electricity forward price.

long-term crude oil price is influenced by the global long-term supply and demand. The long-term NYMEX WTI price therefore represents a global price indicator of the world oil price.

Coal price

The steam-coal market cannot be characterised as a global market like crude oil. The majority of coal is still traded over the counter, mostly because coal is hard to standardise due to its different energy values. Exchange forward trading with coal is still in its early stages. Instead, we use the TFS API2 index as a reference for coal prices in the Nordic area. TFS API2 is a price index for coal delivered in Amsterdam, Rotterdam and Antwerp harbour and should, therefore, also represent the coal prices in OTC market in the Nordic area.

Natural gas price

The natural gas consumed in the Nordic area comes mainly from the North Sea resources. Natural gas forwards of the North Sea gas is also traded on ICE. We use the ICE quarterly prices of natural gas forwards to construct the yearly forward prices for natural gas.

Emission allowance price

The European emission trading scheme (ETS) was introduced in 2005 for carbon oxide (CO_2) emissions. Electricity producers received a limited amount of free CO_2 allowances, whereas additional allowances can be purchased in the market. Since there are no other major environmental constraint products traded in the market, the price of emission allowances in our model is, therefore, simply the price of the European CO_2 emission allowance. We use the data on EUA prices from Nord Pool, which is available in daily resolution. Since Nord Pool began trading with EUAs in March 2005, we use the Spectron EUA prices, which precede that date. Combining the EUA price from two different exchanges is possible, since CO_2 allowance is a global commodity that can be purchased and used anywhere in Europe. The difference in the EUA prices between Spectron and Nord Pool is negligible.

Imported electricity price

The Nordic electricity market imports electricity from Russia, Germany and Poland. We have no information on import prices from Russia, so we use only the European Energy Exchange (EEX) long-term forward price as a reference price for the electricity imported from Germany and Poland. The neighbouring-market price is, therefore, the EEX long-term electricity forward price, which is available in daily resolution. We expect that the EEX price represents a rich source of information. Firstly, it influences the total market price through import and export and secondly, it could be influenced by similar information that influences the Nord Pool electricity price.

Aluminium price

London Metal Exchange (LME) is world's biggest and most important metal exchange. For aluminium price we use aluminium futures from LME.

Time-to-delivery

Time-to-maturity, also addressed to as time-to-delivery, is the time to date at which the forward contract expires. Since electricity is a commodity with continuous delivery, the term delivery period is used instead delivery time. The expiration date is therefore defined as the average date during the delivery period and time-to-maturity is therefore

$$T_m = \frac{t_s + t_e}{2} - t \tag{4.1}$$

in which t_s is the delivery start date, t_e the delivery end date and t the observation time. T_m is expressed in years.

All data originally quoted in currencies other than EUR are converted to EUR using forward exchange rates.

4.2 Descriptive analysis

In Figure 4.2, we present a time series plot for variables in our information set. We denote Nord Pool electricity forward price as *np*, NYMEX WTI crude oil forward price as *oil*, TFS API2 steam-coal forward price index as *coal*, ICE natural gas forward price as *gas*, EU ETS emission allowance forward price from Nord Pool as

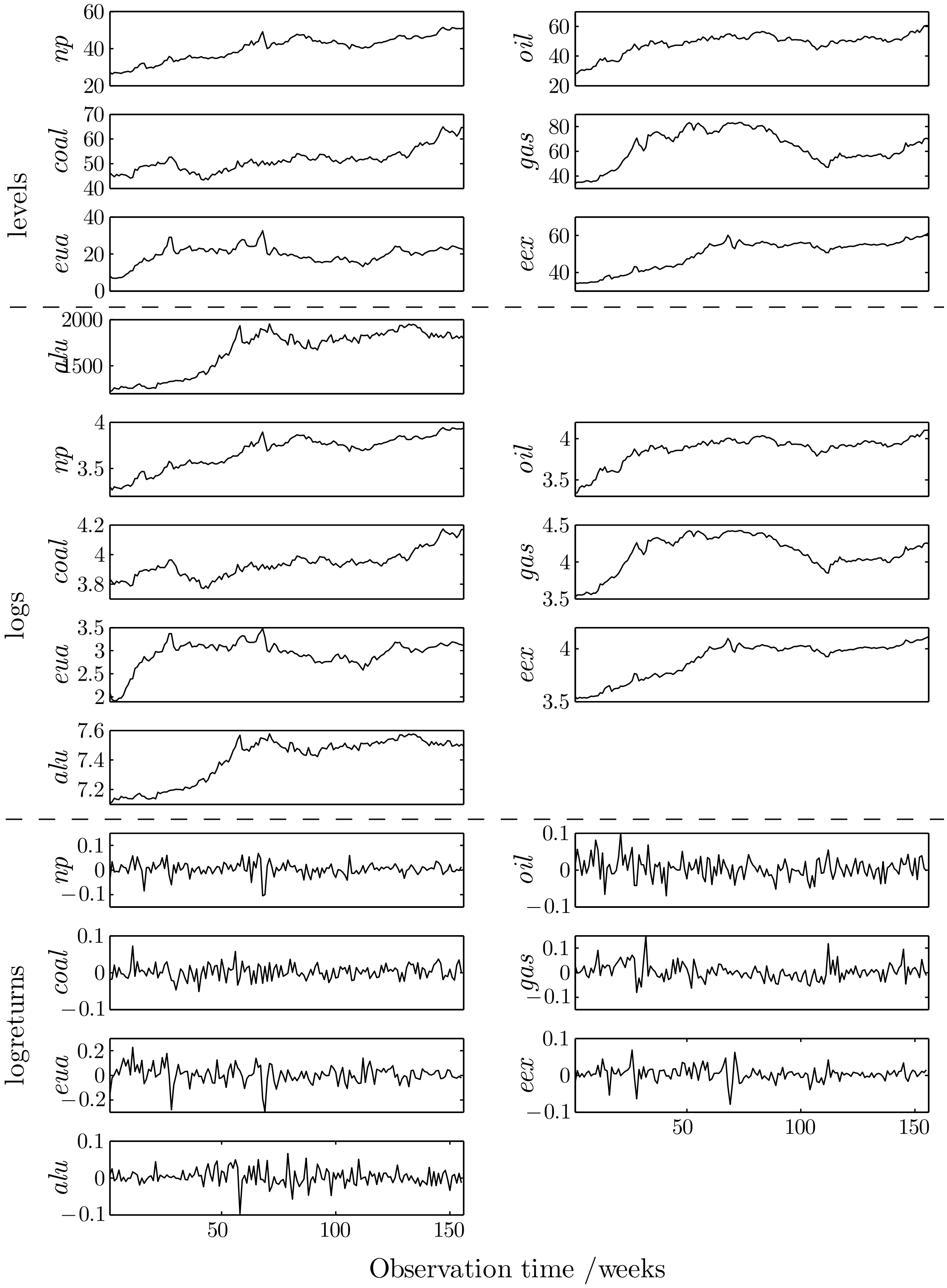

Figure 4.2: Time-series plot for variables in levels, logarithms and logreturns

eua, EEX electricity forward price as *eex* and LME aluminium forward price as *alu*. At certain observation time, all variables have the same delivery period. The three categories in the figure represent the plot of variables in ordinary levels, logarithms (logs) and in logreturns (first differences of logarithms). The variables have a clear upward time trend during the period we analyse. Among other similarities, a similar trend between *np* and *eex* stands out and *eua* price shock in April 2006 is also seen in *eex*.

Tables 4.2 to 4.4 give a descriptive statistics for variables in levels, logs, and logreturns respectively. In all three cases, the normality can be clearly rejected. In case of levels and logs, all variables except *coal* are negatively skewed, whereas kurtosis depends on the type of transformation. In case of logreturns all variables except *oil* and *coal* are strongly leptokurtic, i.e. a strong peak around mean and fat tails. Logreturns of *np*, *eua*, *eex* and *alu* are negatively skewed, while the logreturns of *oil*, *coal* and *gas* are positively skewed.

Table 4.2: Descriptive statistics for data in levels

Statistic	*np*	*oil*	*coal*	*gas*	*eua*	*eex*	*alu*
Mean	40.378	48.641	51.453	63.449	19.634	50.056	1661.09
Standard deviation	6.708	6.764	4.769	13.638	4.694	7.783	235.27
Skewness	-0.439	-1.325	0.920	-0.392	-0.591	-0.781	-0.694
Kurtosis	-0.793	1.431	0.828	-0.700	0.952	-0.830	-1.134

Correlation matrix

Variable	*np*	*oil*	*coal*	*gas*	*eua*	*eex*	*alu*
np	1.000	0.877	0.800	0.434	0.452	0.964	0.885
oil	0.877	1.000	0.590	0.747	0.667	0.879	0.784
coal	0.800	0.590	1.000	0.074	0.287	0.697	0.603
gas	0.434	0.747	0.074	1.000	0.696	0.504	0.388
eua	0.452	0.667	0.287	0.696	1.000	0.440	0.351
eex	0.964	0.879	0.697	0.504	0.440	1.000	0.945
alu	0.885	0.784	0.603	0.388	0.351	0.945	1.000

Correlation matrices show the most interesting properties of the variables in the information set. As expected, the correlation matrix for levels and logs in general indicates a positive correlation between all the variables. Particularly high correlation is found between *np*, *eex* and *alu*, whereas *gas* is significantly correlated only with *oil* and *eua*. The later is probably more of a coincidence rather than showing any theoretical meaning. A more conservative correlation structure is obtained by checking the correlation matrix of logreturns. Here, a strong correlation only between *np*, *eua* and *eex*is found. Other correlations seem less significant in this case. The correlation structure implies a strong interdependence of *np*, *eua* and *eex*, indicating that these variables are likely to influence each other.

Figure 4.3 presents autocorrelation function (ACF) and partial autocorrelation function (PACF) for logreturns. ACF explains how a series is correlated with its own lags, therefore ACF value for lag s represents a correlation between all observations

Table 4.3: Descriptive statistics for data in logs

Statistic	*np*	*oil*	*coal*	*gas*	*eua*	*eex*	*alu*
Mean	3.683	3.873	3.937	4.124	2.941	3.900	7.404
Standard deviation	0.177	0.158	0.090	0.236	0.292	0.169	0.151
Skewness	-0.706	-1.702	0.653	-0.836	-1.703	-0.923	-0.776
Kurtosis	-0.479	2.462	0.411	0.056	3.514	-0.572	-1.044

Correlation matrix

Variable	*np*	*oil*	*coal*	*gas*	*eua*	*eex*	*alu*
np	1.000	0.886	0.781	0.539	0.546	0.972	0.905
oil	0.886	1.000	0.574	0.811	0.762	0.882	0.790
coal	0.781	0.574	1.000	0.151	0.336	0.694	0.620
gas	0.539	0.811	0.151	1.000	0.765	0.584	0.462
eua	0.546	0.762	0.336	0.765	1.000	0.524	0.419
eex	0.972	0.882	0.694	0.584	0.524	1.000	0.955
alu	0.905	0.790	0.620	0.462	0.419	0.955	1.000

Table 4.4: Descriptive statistics for data in logreturns

Statistic	*np*	*oil*	*coal*	*gas*	*eua*	*eex*	*alu*
Mean	0.004	0.005	0.002	0.005	0.007	0.004	0.002
Standard deviation	0.026	0.027	0.019	0.034	0.071	0.019	0.022
Skewness	-0.911	0.192	0.198	0.821	-0.658	-0.410	-0.240
Kurtosis	3.487	0.759	0.516	2.140	3.250	3.809	2.473

Correlation matrix

Variable	*np*	*oil*	*coal*	*gas*	*eua*	*eex*	*alu*
np	1.000	0.275	0.255	0.264	0.611	0.746	0.099
oil	0.275	1.000	0.187	0.300	0.198	0.235	0.288
coal	0.255	0.187	1.000	0.360	0.186	0.213	0.154
gas	0.264	0.300	0.360	1.000	0.404	0.399	0.075
eua	0.611	0.198	0.186	0.404	1.000	0.714	0.094
eex	0.746	0.235	0.213	0.399	0.714	1.000	0.107
alu	0.099	0.288	0.154	0.075	0.094	0.107	1.000

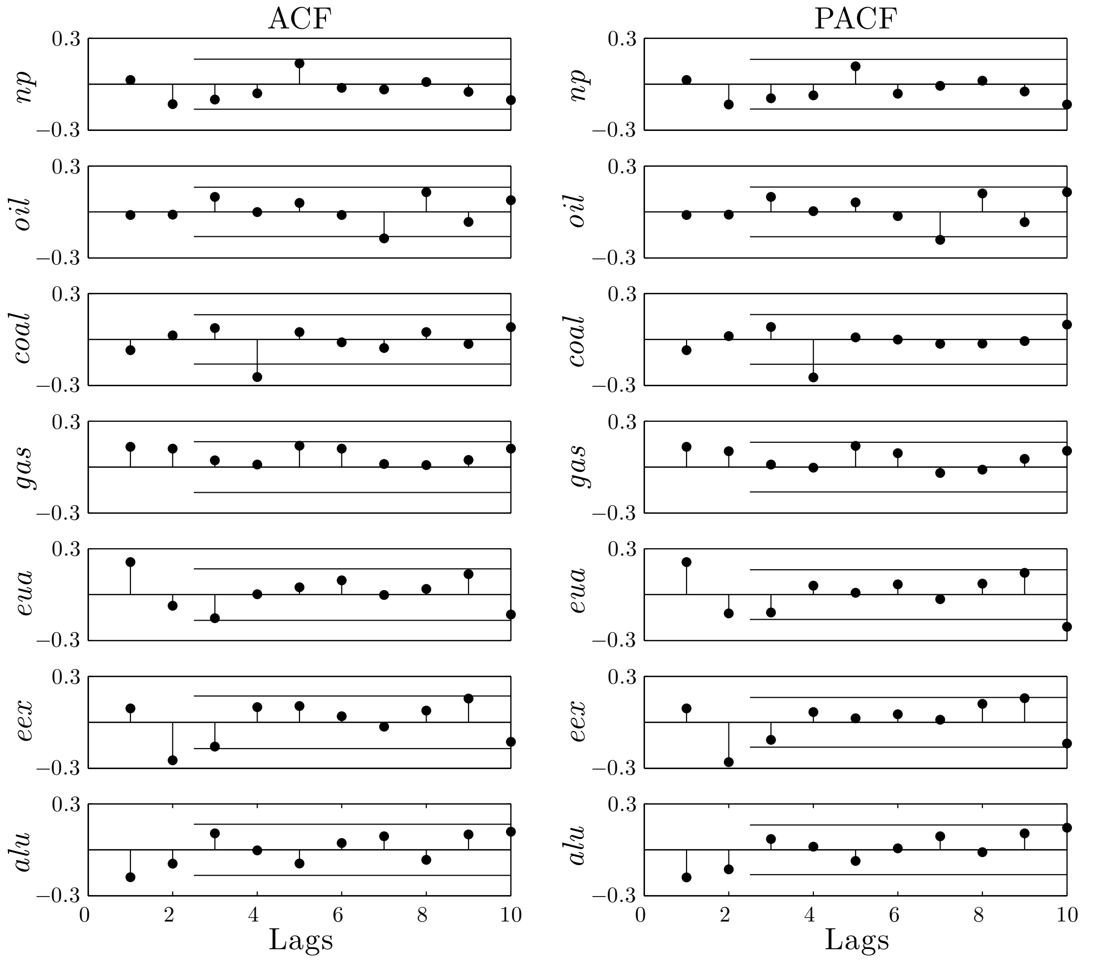

Figure 4.3: Autocorrelation and partial autocorrelation function

in a series with a distance of s. PACF also estimates the same correlation, however, only after removing the correlation at lags 1 to $s-1$. Here, two standard deviations are used as a measure of significance of correlation, which accounts for approximately 95% of confidence interval. The plot reveals a low degree of autocorrelation and partial autocorrelation in all variables. Since we use weekly observations, most of autocorrelation, if present, has probably already died out between two consecutive observations. A barely significant autocorrelation is found at first lag in *eua*, at second lag in *eex* and at fourth lag in *coal*.

Since our interest is in the influence of the variables in the information set on the Nord Pool long-term forward price, we are also interested in the cross-correlation function (CCF). CCF represents how two series are correlated between different lags. CCF value for lag s therefore represents how observations in two series with a distance of s observations are correlated. Cross-correlation at lag 0 is therefore equal to correlation coefficient presented in Table 4.4. Here again, two standard deviations are used to indicate a confidence interval. CCF plot in Figure 4.4 reveals only a first lag cross-correlation in some variables. A model constituting only of logreturns would therefore have low explanatory power, which is also evident from

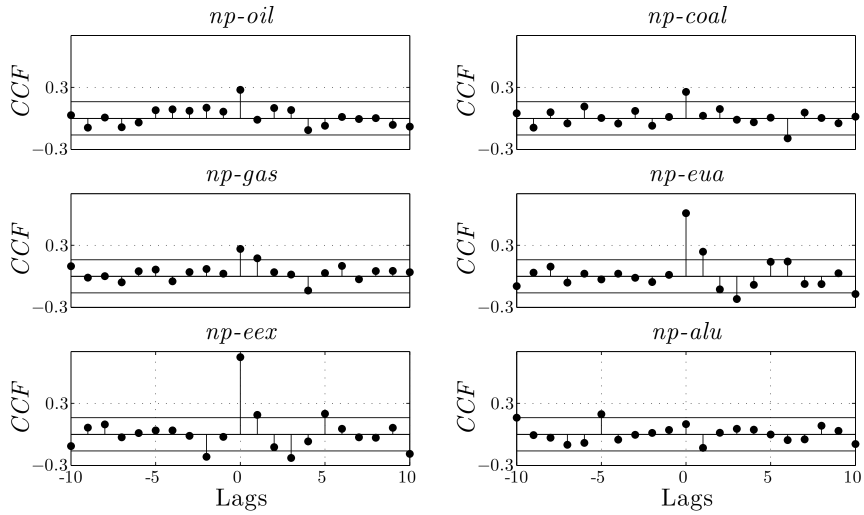

Figure 4.4: Cross-correlation function (CCF) for *np*

the correlation in levels and logs.

4.3 Principal component analysis

The principal component analysis models the variance structure of a set of variables using linear combinations of the variables. These linear combinations, or components, may be used in the subsequent analysis, whereas the combination coefficients may be used in interpreting the components. The principal components of the set of variables are obtained by computing the eigenvalues decomposition of the observed variance matrix. The first principal component is the unit-length linear combination of the original variables with the maximum variance. Subsequent principal components maximise variance among unit-length linear combinations that are orthogonal to the previous components.

In Tables 4.5 and 4.6, the principle component analysis results are presented for variables in logs only. The results show that the first principal component (PC1) accounts for 73% of the total, while the second contributes with 17%, and the third with 7% of the total variance. Only the first two components have eigenvalues higher than 1.0, whereas the first three components together generate 96% of the global variance.

Weights of the first three components show that all variables have a significant impact on the variance structure and none can be excluded at this point. Interestingly, in all three components variations in *coal* have the highest impact on the variance. Since weights for all variables are of quite similar magnitude and the variables represent price from different markets, any economic interpretation of these three components is not possible. The first three principal components are also presented in Figure 4.5. No resemblance to any of the variables plotted in Figure 4.2 is found.

Table 4.5: Principal component analysis for variables in logs

	Eigenvalues	%Variation	%Cumulative
PC1	5.077	72.53	72.53
PC2	1.193	17.04	89.56
PC3	0.463	6.61	96.17
PC4	0.171	2.44	98.62
PC5	0.054	0.77	99.39
PC6	0.034	0.48	99.87
PC7	0.009	0.13	100.00

Table 4.6: Eigenvectors and weights for variables in logs

	Eigenvectors			Weights		
Variable	PC1	PC2	PC3	PC1	PC2	PC3
np	-0.426	-0.214	0.034	-2.418	-1.215	0.191
oil	-0.428	0.169	0.027	-2.717	1.073	0.175
coal	-0.312	-0.485	-0.657	-3.485	-5.420	-7.334
gas	-0.318	0.584	0.177	-1.354	2.486	0.755
eua	-0.318	0.503	-0.518	-1.093	1.730	-1.781
eex	-0.426	-0.176	0.262	-2.534	-1.050	1.559
alu	-0.393	-0.254	0.446	-2.618	-1.692	2.966

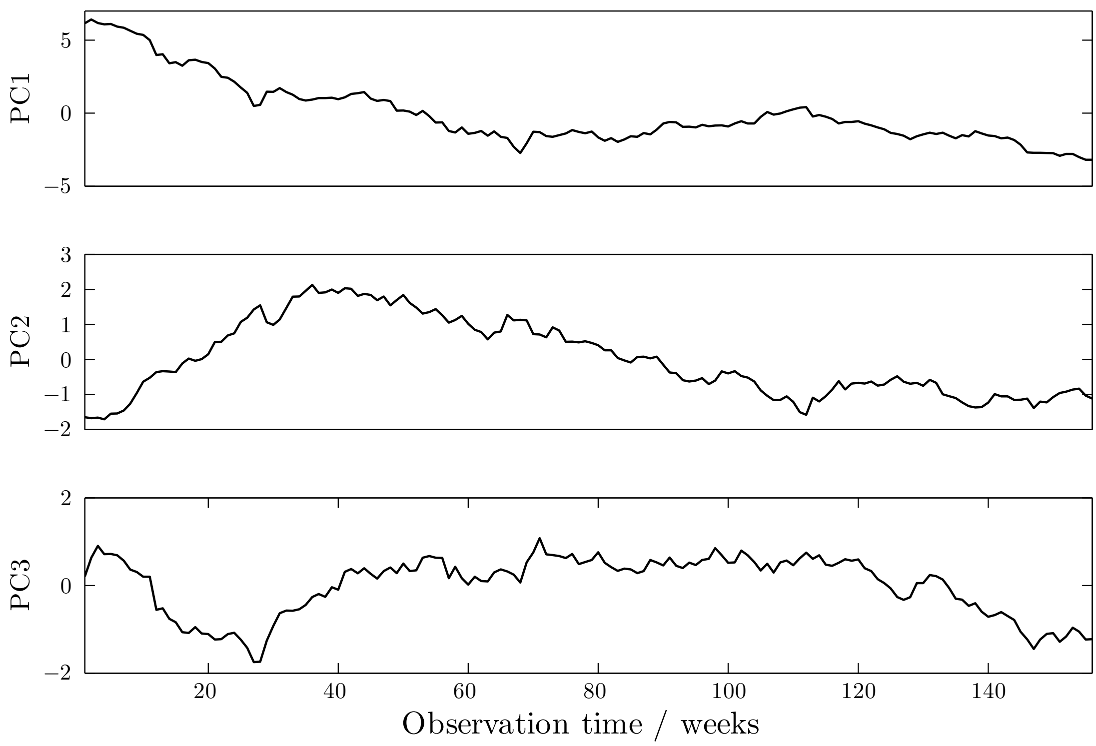

Figure 4.5: Principal components

4.4 Testing for unit roots

Chapter 3 also discusses the difference in unconditional distribution of stationary and integrated variables. A variable is said to be stationary if its mean, variance, and covariance are all invariant with respect to time. If a variable must be differenced d times to become stationary, such variable is said to be integrated of order $I(d)$. In this section, we test each variable in the information set for their order of integration, which equals the number of unit roots. Integration of variables has tremendous impact on the modelling strategy, since any shock that occurs in stationary variable will eventually die out, whereas in integrated variable the shock will persist. Consequently a regression of integrated variables is likely to result in spurious regression implying a false identification of a meaningful economic relationship. Standard regression estimators are valid only for stationary variables; therefore, integrated variables must be differenced as many times as the number of unit roots until they become stationary.

In principle, there are two types of stationarity, the first being stationarity with intercept and the second stationarity with time trend. A stationary series will cross their mean value (intercept stationary) or trend value (trend stationary) frequently, whereas the integrated series experience that only occasionally. A first look at the variables in Figure 4.2 shows that variables in logs appear to be integrated, whereas logreturns clearly look stationary. A more sophisticated method to detect stationarity is the testing for a unit root. Here we employ two most widely used tests for a unit root. The first is the standard Augmented Dickey-Fuller (ADF) test of a unit root (see Dickey and Fuller [63], [64]). This test is based on the t-statistic for $a_2 = 0$ in the regression given by

$$\Delta y_t = a_0 + a_1 t + a_2 y_{t-1} + \sum_{j=1}^{s} c_j \Delta y_{t-j} + u_t. \tag{4.2}$$

Typically, the rejection of the null hypothesis $\mathrm{H}_0 : a_2 = 0$ would be taken as a strong evidence of trend stationarity, whilst the non-rejection would infer that the series is non-stationary. The second test is Phillips-Perron (PP) test for a unit root, which is similar to standard Dickey-Fuller test; however, it involves a non-parametric correction of the t-statistics to account for the autocorrelation in residuals (see Phillips and Perron [65] for details).

Table 4.7 present the results of testing for a unit root for variables in levels, logs and logreturns. Trend stationary test is employed in all cases except for *oil* and *eua* , for which the test with intercept only is applied. Results for levels and logs consistently show that all variables are integrated, except perhaps for *oil* and *eua*, which may be stationary, but close to unit root. The logreturns are strongly stationary in all cases indicating that none of the variables are integrated of order $I(2)$ and all variables except *oil* and *eua* are therefore integrated of order $I(1)$. Since unit root test results may also suffer from the low power of both tests, we will treat all variables as $I(1)$ even though *oil* and *eua* may be stationary and close to unit root. Unit root test for T_m show that T_m is also non-stationary with $ADF = -2.49$ and $PP = -2.54$, while ΔT_m is stationary ($ADF = -12.49^{**}$ and $PP = -12.49^{**}$). This is not in line with the expectations since according to the

Table 4.7: Results of testing for unit root (t-values)

	Levels		Logs		Logreturns	
Variable	ADF	PP	ADF	PP	ADF	PP
np	-2.42	-2.37	-2.33	-2.21	-12.08**	-12.36**
oil	-2.58	-2.60	-3.46*	-3.50**	-12.58**	-12.58**
coal	-1.70	-1.61	-2.03	-2.01	-13.29**	-13.45**
gas	-1.97	-1.97	-2.32	-2.28	-10.79**	-10.87**
eua	-3.08*	-2.68	-3.84**	-2.94*	-8.77**	-9.99**
eex	-1.49	-1.58	-1.54	-1.52	-10.76**	-11.27**
alu	-1.66	-1.45	-1.46	-1.28	-14.77**	-14.80**

*reject the null at 5% significance **reject the null at 1% significance

data sample, the values of T_m is always between 2.5 years and 1.5 years, therefore in theory T_m should be stationary. Obviously, unit-root tests fail to identify stationarity in this case, since the data series crosses its mean value only 3 times, hence using larger sample for T_m would surely result in stationary series. We will continue to treat T_m as stationary.

4.5 Conclusion

In this chapter, we condition the long-term forward prices from Nord Pool on long-term forward prices of crude oil, coal, natural gas, emission allowances, imported electricity, aluminium price and time-to-maturity. Descriptive statistics shows non-normality and positive skewness, whereas correlation structure shows strong correlation between variables in levels and weak correlation between logreturns. This is a combination typically found in integrated and/or cointegrated time series, which will be tested in the next chapter. The autocorrelation and partial autocorrelation function reveal a rather poor autocorrelation structure, which is somewhat sensible since weekly observation time resolution is used and most of the autocorrelation might die out in one week. We also find no significant cross-correlations between variables except at lag 0. Unit root analysis shows that all variables are integrated of order $I(1)$, except *oil* and *eua* which may be near integrated processes. A proper modelling strategy is therefore to use variables in their first differences for subsequent analyses.

Chapter 5

Multivariate model

While our theoretical model focuses on identifying the economic relationships behind the long-term electricity forward price process, the data analysis in the previous chapter reveal that the variables on which these prices are conditioned are likely to be influenced by other variables in the system including the long-term electricity forward prices. This is known as the endogeneity between variables and is often found in co-dependent and substitutable commodity prices. Endogeneity does not allow for a variable of interest to be modelled with univariate regression models in which the dependent (endogenous) variable is modelled as a linear function of present and past values of independent (exogenous) variables and past values of dependent variable. Assuming the "true"economic model underlying behaviour satisfies a first-order linear approximation, endogeneity is typically handled with multivariate regression models. Simultaneous Equation Model (SEM) and Vector Autoregressive Model (VAR) are the two most representative members of this class of regression models.

5.1 Vector autoregressive model

Based on data analysis we employ Vector Autoregressive model framework developed by Sims [66]. Before the wide-spread use of VAR, the relationships between endogenous variables were typically modelled with Simultaneous Equation Models. Major problems with SEM's are the identification restrictions, which require for each equation in SEM at least one exogenous variable. SEM therefore requires the ad-hoc assumptions on exogeneity. Hence, the variables need to be pre-tested for exogeneity using for instance Hausman test or Granger-causality testing framework. Sims [66] raised several objections to the traditional way of identifying macro econometric models:

- Exclusion restrictions were routinely imposed and the decision whether a variable should be regarded as exogenous with respect to the system was made rather arbitrarily.

- Identification was often achieved without solid economic or econometric arguments.

- No economic variable can be deemed as exogenous in a world of rational forward looking investors.

VAR is a powerful tool to model the relationship between variables, since it does not require any ex-ante assumptions on endogeneity or exogeneity of variables. All variables in VAR are treated as endogenous and exogeneity can be tested in ex-post analysis. In the unrestricted general form, the VAR model is essentially only a reformulation of the covariances of the data. VAR also allows a powerful testing for cointegration using Johansen cointegration test, which can result in more than just one cointegrating relationship, usually found in univariate regression models. However, VAR has also some drawbacks and among them interpretability of results and a large number of parameters are most serious. A standard *reduced form n* dimensional k-th order VAR is

$$\mathbf{Y}_t = \mathbf{A}_0 + \mathbf{A}_1\mathbf{Y}_{t-1} + \mathbf{A}_2\mathbf{Y}_{t-2} + \ldots + \mathbf{A}_k\mathbf{Y}_{t-k} + \mathbf{u}_t \tag{5.1}$$

where $\mathbf{Y}_t$ is the vector of endogenous variables (in e.g. levels), $\mathbf{A}_0$ is the $(n \times 1)$ vector of intercept terms allowing the possibility of non-zero mean $\mathrm{E}(\mathbf{Y}_t)$, $\mathbf{A}$ is the $(n \times n)$ coefficient matrix, and $\mathbf{u}_t$ is the $(n \times 1)$ dimension vector of error terms satisfying:

$$\begin{aligned} &\mathrm{E}(\mathbf{u}_t) = 0 \\ &\mathrm{E}(\mathbf{u}_t\mathbf{u}_t') = \mathbf{\Omega} \\ &\mathrm{E}(\mathbf{u}_t\mathbf{u}_{t-k}') = 0 \end{aligned} \tag{5.2}$$

According to Hendry and Richard [67], the conditional mean $\boldsymbol{\mu}_t$ in VAR model is

$$\boldsymbol{\mu}_t = \mathrm{E}_{t-1}(\mathbf{Y}_t|\mathbf{Y}_{t-1},...,\mathbf{Y}_{t-k}) = \mathbf{A}_1\mathbf{Y}_{t-1} + ... + \mathbf{A}_k\mathbf{Y}_{t-k} \tag{5.3}$$

and it represents the investors' expectations at time $t-1$ given the available information at that time $\mathbf{Y}_{t-1}, \ldots, \mathbf{Y}_{t-k}$. Assuming investors are rational in the sense that they use all information when they make plans for time t and do not make systematic forecast errors, the difference between the expected value and realizations should be *normally independently identically distributed errors* (*Niid*), also called a white noise process.

$$\mathbf{Y}_t - \boldsymbol{\mu}_t = \mathbf{u}_t \sim Niid(0, \mathbf{\Omega}) \tag{5.4}$$

A white noise process simply implies that there is no information left in residuals $\mathbf{u}_t$ that could be used for better forecast of the conditional mean. If such information exists, rational investors would use this information for better forecasts and such information would thus diminish in the long-run.

VAR in (5.1) does not include any contemporaneous influences. To include contemporaneous influences suppose a following bivariate VAR(1)

$$\begin{bmatrix} Y_{1,t} \\ Y_{2,t} \end{bmatrix} = \begin{bmatrix} A_{10} \\ A_{20} \end{bmatrix} + \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} Y_{1,t-1} \\ Y_{2,t-1} \end{bmatrix} + \begin{bmatrix} -a_{12} & 0 \\ 0 & -a_{21} \end{bmatrix} \begin{bmatrix} Y_{1,t} \\ Y_{2,t} \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix} \tag{5.5}$$

which has the same form as the simultaneous equation model. This model is not identified since both dependent variables appear on the right hand side (RHS).

To overcome the identification problem a zero restriction on at least one of the coefficients a_{12}, a_{21} must be imposed. Alternatively, one can rewrite (5.5) to

$$\begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix} \begin{bmatrix} Y_{1,t} \\ Y_{2,t} \end{bmatrix} = \begin{bmatrix} A_{10} \\ A_{20} \end{bmatrix} + \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} Y_{1,t-1} \\ Y_{2,t-1} \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix} \tag{5.6}$$

or

$$\mathbf{A}\mathbf{Y}_t \;=\; \mathbf{A}_0 + \mathbf{A}_1\mathbf{Y}_{t-1} + \mathbf{u}_t. \tag{5.7}$$

Multiplying both sides in (5.7) with $\mathbf{A}^{-1}$ gives

$$\mathbf{Y}_t = \mathbf{A}^{-1}\mathbf{A}_0 + \mathbf{A}^{-1}\mathbf{A}_1\mathbf{Y}_{t-1} + \mathbf{A}^{-1}\mathbf{u}_t \tag{5.8}$$

or

$$\mathbf{Y}_t \;=\; \mathbf{A}_0^* + \mathbf{A}_1^*\mathbf{Y}_{t-1} + \boldsymbol{\varepsilon}_t \tag{5.9}$$

which is known as a Structural VAR (SVAR) with residuals $\boldsymbol{\varepsilon}_t = \mathbf{A}^{-1}\mathbf{u}_t$, and $\mathbf{A}_0^* = \mathbf{A}^{-1}\mathbf{A}_0$, $\mathbf{A}_1^* = \mathbf{A}^{-1}\mathbf{A}_1$. Structural VAR residuals $\boldsymbol{\varepsilon}_t$ must satisfy conditions (5.2) as well as orthogonality $\mathrm{E}(\mathbf{u}_{it}\mathbf{u}_{jt}') = 0$ in order for SVAR to be identified. Identification restrictions are imposed on matrix $\mathbf{A}^{-1}$. Structural VAR therefore freely estimates only some contemporaneous relations, while other relations must be restricted in order to achieve identification. The application of Structural VAR will be discussed further in Chapter 6.

5.2 VAR setup

Defining a vector $\mathbf{Y}_t$ of n potentially endogenous variables, a general Gaussian vector autoregressive model takes the following form

$$\mathbf{Y}_t \;=\; \mathbf{A}_0 + \sum_{i=1}^{k} \mathbf{A}_i\mathbf{Y}_{t-i} + \boldsymbol{\Psi}\mathbf{Z}_t + \mathbf{u}_t \tag{5.10}$$

in which $\mathbf{Z}_t$ is a vector of variables that are a priori known to be exogenous, $\mathbf{A}_0$, $\mathbf{A}_i$ and $\boldsymbol{\Psi}$ are parameter matrices and $\mathbf{u}_t$ the matrix of residuals. Each variable in $\mathbf{Y}_t$ is therefore regressed on lagged values of both itself and other endogenous variables as well as exogenous variables.

At this point, we form a seven dimensional VAR ($n = 7$) in which $\mathbf{Y}_t$ includes seven potentially endogenous variables, whereas $\mathbf{Z}_t$ include only time-to-maturity T_m for which we know to be exogenous. All variables are in log-levels, while T_m is in levels. We could also assume that *oil* is exogenous, since one cannot expect that supply and demand on local electricity market can influence global crude oil price. These assumptions are however not necessary and treating all variables as endogenous should have no influence on the final outcome of the model.

Table 5.1 includes the diagnostic test for seven dimensional VAR with lag length $k = 2$. In order to assure that VAR residuals indeed satisfy the conditions in (5.2), we perform several diagnostic tests which involve the following single equation residual tests (adopted from Doornik and Hendry [68]):

- F-test on variable significance (F_{sig}). Under the null hypothesis all coefficients belonging to a variable under test are zero, so these test whether the variable at hand is significant in the system. The test statistic is distributed with $F(n, N-k+1-n)$.

- F-test on residual autocorrelation (F_{ar}), against 4^{th} order autocorrelation. This is Breusch-Godfrey Lagrange-multiplier test for serial correlation, where the null hypothesis is no autocorrelation up to tested order of autocorrelation. The F-statistic is $N\mathrm{R}^2$, where test R^2 is estimated on auxiliary regression of residuals on original variables and its own lags. The test is distributed with $F(s, N-nk-s)$, where s is the number of tested residual lags in the auxiliary regression and k is the lag length.

- χ^2 test normality (χ^2_{nd}). This test is described by Doornik and Hansen [69] and it involves a joint test that transformed skewness and kurtosis are 0 and 3 respectively. The test is approximately distributed with $\chi^2(2)$, since two parameters are jointly tested.

- F-test on heteroscedasticity (F_{het}). This is based on White [70] and involves an auxiliary regression of residuals on a constant, original regressors and all their squares. The null hypothesis is unconditional homoscedasticity against the alternative that residuals are heteroscedastic. The test is distributed with $F(s, N-s-1-k)$, where $s+1$ is the number of regressors in the auxiliary regression.

- F-test on autoregressive conditional heteroscedasticity (F_{arch}), against 4^{th} order. This is a Lagrange-multiplier test with the null hypothesis that the variance is independent of the squared residuals up to the tested order. The test statistic and the corresponding distribution have the same form as F_{ar}. ARCH effects are however not detrimental for the cointegration analysis as shown by Dennis, Hansen, and Rahbek [71].

- Residual skewness. Gonzalo [72] shows that non-normality of residuals is more of a problem if residual skewness diverges from the normality assumption rather than kurtosis. The non-normality problem also tends to decrease with large sample size since t, F and χ^2 statistic will converge to normal distribution asymptotically.

Single equation tests focus on residuals of each single equation as if they are estimated in a single equation. These tests are valid only if other residuals in the system are also without problems. Besides single equation tests, vector tests are also presented. These involve testing the misspecification of the system as a whole, thus all tests are performed on all residuals in the system, giving a more general picture of the system properties. Vector tests involve:

- F-test on vector error autocorrelation (F_{ar}). This is a Lagrange-multiplier test based on Godfrey [73] and uses Rao's F-approximation (see Doornik [74] for details). Under the null hypothesis the residuals of the system are serially correlated.

- χ^2 test for vector normality (χ^2_{nd}). This test involves transforming the residuals from single equations. Then, univariate transformed skewness and kurtosis of residuals are used in vector test statistic, distributed with $\chi^2(2n)$.
- F-test on vector heteroscedasticity (F_{het}). This test is based on Kelejian [75] extension of White's test [70] developed for simultaneous equations framework. It is a Lagrange-multiplier test with Rao's F-approximation.

Table 5.1: VAR(2) diagnostic tests

Variable	$F_{ar}(4,135)$	$\chi^2_{nd}(2)$	Skewness	$F_{het}(28,110)$	$F_{arch}(4,131)$	SE
np	1.756	20.39**	-0.283	3.772**	7.023**	0.0248
oil	1.331	2.09	-0.107	1.010	0.404	0.0264
coal	3.981**	5.56	0.280	0.838	0.585	0.0193
gas	0.905	20.42**	0.722	0.878	5.805**	0.0314
eua	0.550	9.54**	0.159	1.686*	4.247**	0.0620
eex	1.268	26.32**	0.457	1.806*	7.859**	0.0168
alu	1.287	8.92*	-0.042	1.133	2.719*	0.0215
Vector tests: $F_{ar}(196,727) = 1.258$*, $\chi^2_{nd}(12) = 63.93$**, $F_{het}(784,1780) = 1.054$						
Constant: $F_{sig}(7,132) = 5.726$**, T_m: $F_{sig}(7,132) = 1.218$, $LLF = 2615.1$						

*reject the null at 5% significance **reject the null at 1% significance

The results in the Table 5.1 show that all endogenous variables are significant. Significance tests (F_{sig}) show that the constant is strongly significant, while exogenous time-to-maturity T_m is not significant. Misspecification tests reveal several problems in residuals, particularly when vector tests are considered. Significant autocorrelation is present in *coal*, which also has the effect on the vector autocorrelation test. Heteroscedasticity is found in *np*, *eua* and *eex*. The vector test on heteroscedasticity is, however, just below significance. Autoregressive conditional heteroscedasticity (ARCH) test and non-normality are strongly significant in all equations except for *oil* and *coal*. Since VAR specification is more sensitive to skewness than to kurtosis, skewness of residuals is also reported, showing that skewness is not a big problem. This indicates that residuals are more or less normally skewed, thus the kurtosis is the major source of non-normality.

Autocorrelation and heteroscedasticity are of particular concern, since they imply that there is still some information left in residuals which could be used for better forecasting. Autocorrelation can often be removed by increasing the lag length of VAR, however, in our case it cannot, since the second lag is barely significant and the higher lags have no significance at all. On the other hand, autocorrelation and heteroscedasticity in residuals can generally imply two of the following:

1. **Omission of important explanatory variable**. If a part of the process is influenced by some other variables, which are not included in the model, this will reflect itself in autocorrelated and heteroscedastic residuals. In practice,

commodity prices are complex processes influenced by a large number of variables and it is often difficult to find an information set that would include all relevant variables. In case such information set is found, the number of variables might lead to huge number of parameters and problems with the degrees of freedom.

2. **Influence of policy shocks, structural breaks, statistical outliers etc**. These undesired properties are usually found in real application data and some measures exist to overcome them. Particularly in low resolution data samples spanning through the period of several years or decades are often subject to changes in market rules, policy interventions, changes in regulation or structural changes in the influence between variables. Such would be the case if investors find that their perception on how certain variable influence other variables was wrong in the past and they adjust their expectations. This is called a structural break.

We believe that there is still some information that is omitted from our information set that may have a significant impact on the price dynamics of electricity forwards. We also believe that a large part of this information is, however, low resolution information. Although it may have a permanent impact on the variables in the system, we hope their influence is marginal compared to the variables chosen in our information set. At this point we could not find any additional high resolution common knowledge variables that we could potentially include in the model. Another aspect of residual autocorrelation is that some variables may in fact be exogenous. Our system is, of course, not designed to capture the dynamics in, e.g. coal prices, therefore the autocorrelation in coal price could be removed by including some variable that influence only the coal price. Alternatively, adding such variables would not be necessary if coal prices are exogenous with respect to electricity prices, since in this case there would be no need to model the coal price itself.

Besides checking for variable omission, we also scan the data for presence of policy shocks, structural breaks and statistical outliers. First, we introduce a blip dummy to set the residuals from 67 to 70, to zero, which corresponds to *eua* price shock in April 2006. A blip dummy D_{b67} of a type (...0,1,0...) with three lags is used for this purpose. As seen in Figure 4.2 this shock had also a significant impact on the Nord Pool forward price and the EEX forward price. Since shocks are known to induce erratic behaviour and nonlinear dynamics, it is best to remove their influence in linear model applications with dummy variables. A smaller shock also occurred in week 27 in 2005. This is a typical transitory shock when price increases for a period of time and then decrease again to previous levels. To account for this, we add one transitory dummy D_{tr} of a type (...0,1,0,-1,0...) to remove the effect of transitory shock in observations 27 and 29. Additionally, three blip dummies D_{b33}, D_{b57} and D_{b117} are used to remove the largest outliers. Using these additional dummies, we adapt 5.10 to form the following VAR specification

$$\mathbf{Y}_t = \mathbf{A}_0 + \sum_{i=1}^{k} \mathbf{A}_i \mathbf{Y}_{t-i} + \mathbf{\Psi} \mathbf{Z}_t + \mathbf{\Theta} \mathbf{D}_t + \mathbf{u}_t \tag{5.11}$$

in which $\mathbf{D}_t$ are the intervention dummies to render the residuals well-behaved and $\mathbf{\Theta}$ the vector of dummy parameters. The diagnostics of VAR(2) that includes these additional intervention dummies is presented in Table 5.2.

Table 5.2: VAR(2) diagnostic tests

Variable	$F_{ar}(4,126)$	$\chi^2_{nd}(2)$	Skewness	$F_{het}(39,90)$	$F_{arch}(4,122)$	SE
np	1.124	13.61**	-0.195	0.825	0.478	0.0201
oil	2.311	4.41	-0.111	0.768	0.427	0.0256
coal	2.856*	4.15	0.027	0.886	1.350	0.0183
gas	0.272	19.40**	0.126	0.718	0.403	0.0289
eua	0.950	4.12	0.224	0.856	1.443	0.0536
eex	0.578	4.61	0.075	1.144	0.222	0.0136
alu	2.732*	10.13*	0.004	0.676	1.063	0.0210
Vector tests: $F_{ar}(196,665)=1.125$, $\chi^2_{nd}(12)=43.43$**, $F_{het}(784,1780)=0.691$						
Constant: $F_{sig}(7,132)=7.542$**, T_m: $F_{sig}(7,132)=1.857$, $LLF=2731.0$						

*reject the null at 5% significance   **reject the null at 1% significance

Intervention dummies significantly improve the properties of VAR in Table 5.2. Most single equation and vector misspecification tests are improved. There is still a slight autocorrelation present in *coal* and *alu*, but we will not pursue this further, since we expect these two variables are weakly exogenous and they do not have to be modelled themselves. The vector tests, on the other hand, in overall reject the autocorrelation and heteroscedasticity in residuals. While strict normality is still not achieved, single equation and vector tests are improved a little. We also managed to reduce the skewness, which is more critical than the kurtosis. This model specification is now adequate for trusting the statistical inferences on tests we perform in the following of this thesis.

5.2.1 Parameter constancy

We test the specification in Table 5.2 for parameter constancy. In particular, we are interested in the influence of *eua* price shock in April 2006. The shock had a significant effect on the Nord Pool forward price as seen in Figure 4.1 and also on the EEX forward price. To test whether this shock or any other shock experienced during the period in our sample changed the overall structure of the data generating process, we use the Chow test for structural break [76]. A single equation recursive Chow break-point test has the following form of F-test (see Doornik and Hendry [68])

$$\frac{(RSS_N - RSS_{t-1})(t-k-1)}{RSS_{t-1}(N-t+1)} \tag{5.12}$$

where k is the lag length, RSS_N the residual sum of squares estimated on the whole sample N, while RSS_{t-1} is estimated recursively on series of a subsamples $1,\ldots,t,\ t=M,\ldots,N$, where M represents a break point. The test is distributed with $F(N-t+1, t-k-1)$. A system breakpoint F-test uses Rao's F-approximation (see, e.g. Doornik [74]) with R^2 computed as

$$1 - e^{(-2\hat{l}_{t-1}+2\hat{l}_N)}, \quad t = M, \ldots, N. \tag{5.13}$$

where $\hat{l}_t$ is a following log-likelihood function

$$\hat{l}_t = -\frac{1}{2}\log\left|\frac{t}{N}\hat{\boldsymbol{\Omega}}_{\mathbf{t}}\right|. \tag{5.14}$$

Figure 5.1 shows recursive break-point Chow test for each equation in the system and for the system as a whole. The statistics in (5.12) and (5.13) are scaled by 1% critical values from F-distribution as an adjustment for changing degrees of freedom, so that significance values become a constant line throughout the sample. The 1% significance level of the break-point test is never exceeded indicating that parameters of individual equations and of the system as a whole are constant throughout the sample. The *eua* price shock can therefore be considered as a transitory shock, which can be removed with intervention dummies, rather than a structural break.

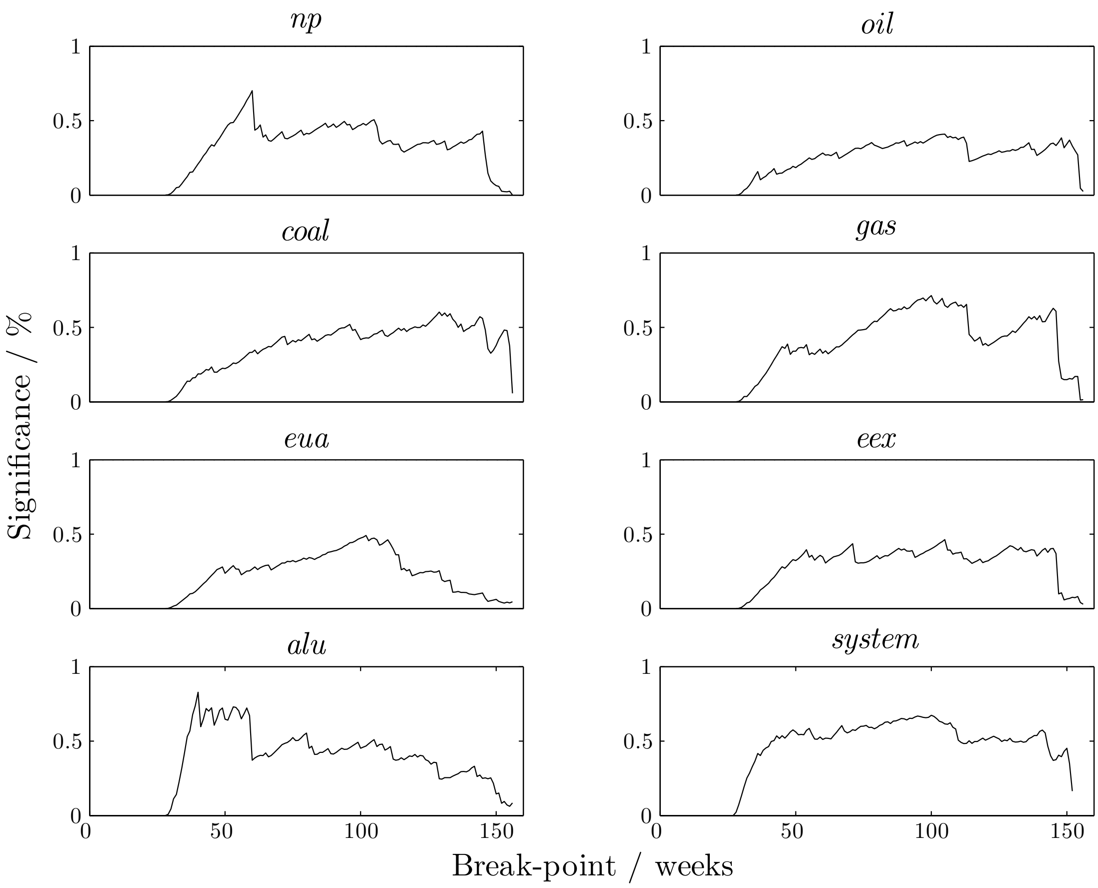

Figure 5.1: Recursive break-point Chow test

5.2.2 VAR stability

A general VAR(k) can always be expressed in the so-called *companion form* where the n-dimensional VAR(k) in (5.1) is written as $(n \times k)$ dimensional VAR(1)

$$\begin{bmatrix} \mathbf{Y}_t \\ \mathbf{Y}_{t-1} \\ \vdots \\ \mathbf{Y}_{t-k+2} \\ \mathbf{Y}_{t-k+1} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 & \dots & \mathbf{A}_{k-1} & \mathbf{A}_k \\ \mathbf{I}_n & 0 & \dots & 0 & 0 \\ 0 & \mathbf{I}_n & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & \mathbf{I}_n & 0 \end{bmatrix} \begin{bmatrix} \mathbf{Y}_{t-1} \\ \mathbf{Y}_{t-2} \\ \vdots \\ \mathbf{Y}_{t-k+1} \\ \mathbf{Y}_{t-k} \end{bmatrix} + \begin{bmatrix} \mathbf{u}_t \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \tag{5.15}$$

or

$$y_t = a + \phi y_{t-1} + u_t \tag{5.16}$$

If $|\phi| < 1$ there exists a covariance stationary process with finite variance. The stability of a VAR can be examined by checking the eigenvalues (or the roots) of the companion matrix ϕ in (5.15), therefore

$$\phi = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 & \dots & \mathbf{A}_{k-1} & \mathbf{A}_k \\ \mathbf{I}_n & 0 & \dots & 0 & 0 \\ 0 & \mathbf{I}_n & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \mathbf{I}_n & 0 \end{bmatrix}. \tag{5.17}$$

The roots of companion matrix give the necessary information about the stability and stationarity of VAR. The necessary and sufficient condition for stationarity is that all the roots of the companion matrix lie inside the unit circle. In case some roots lie inside the unit circle and some roots on the unit circle, then VAR is non-stationary. If any root is outside the unit circle, then VAR is explosive and therefore unstable. There are $k \times n = 14$ roots of the companion matrix and Table 5.3 presents 10 largest roots with their modulus.

Table 5.3: Roots of the companion matrix

Roots	Real	Imaginary	Modulus
1	0.981	-0.025	0.982
2	0.981	0.025	0.982
3	0.948	-0.066	0.950
4	0.948	0.066	0.950
5	0.895	0.000	0.895
6	0.854	0.000	0.854
7	0.361	-0.309	0.476
8	0.361	0.309	0.476
9	-0.294	0.000	0.294
10	-0.052	-0.264	0.269

None of the roots of the companion matrix lie outside the unit circle indicating that this specification of VAR is stable. This is consistent with our expectation that the system is not $I(2)$. At least four roots are close to unity while additional two roots are at the borderline of unity (0.9). This indicates that the system as a whole is $I(1)$ and the variables should therefore be differenced once to get the stationary process $I(0)$. The roots of the companion matrix also suggest that there is between 1 and 3 stationary linear relationships between the variables in the system; however, to define the exact number additional tests are necessary.

5.3 Vector equilibrium correction model

Converting VAR specification (5.11) to first difference model would simply require using logreturns in $\mathbf{Y}_t$ instead of log levels. Such model would sufficiently explain the short-run dynamics of the system, however the long-term relationships between variables, if they exist, would be lost. The long-term relationships between variables are important when variables are cointegrated. The principle of cointegration is that if two variables are cointegrated, they may "wander away" from each other in the short-run due to the impact of random forces, but they will eventually approach each other again due to the effect of long-run equilibrium forces. This means that even if the unconditional distribution of variables is unbounded, the difference between them is bounded. A first difference model that is able to capture the long-run relationship between variables is called *vector equilibrium correction model* (VECM) which is obtained by converting (5.11) to

$$\Delta \mathbf{Y}_t \;=\; \mathbf{A}_0 + \mathbf{\Pi} \mathbf{Y}_{t-1} + \sum_{i=1}^{k-1} \mathbf{\Gamma}_i \Delta \mathbf{Y}_{t-i} + \mathbf{\Psi} \Delta \mathbf{Z}_t + \mathbf{\Theta} \mathbf{D}_t + \mathbf{u}_t. \tag{5.18}$$

VECM in (5.18) has the same innovation process $\mathbf{u}_t$, since no restrictions have been imposed by this transformation. $\mathbf{A}_0$ is unrestricted constant which accounts for a constant in the short-run model (trend in levels) and a constant in cointegration space. In (5.18) the RHS contains information about the short- and the long-run adjustment to changes in $\mathbf{Y}_t$.

VECM formulation has several advantages over VAR. It significantly reduces the problem of multicollinearity, since the first differences are much more orthogonal than the levels. The interpretation of the results is much more intuitive, since parameter estimates can be classified into short-run and long-run effects, which enables to explain what drives the realised changes between two periods. Finally, all the long-run dynamics in VECM is summarised in the matrix $\mathbf{\Pi}$, which is very convenient since the problem of cointegration is then focused only on the properties of $\mathbf{\Pi}$. In (5.18) we also convert T_m to first differences, since the unit root test on this variable show non-stationarity. While a non-stationary exogenous variable could in principle be restricted to cointegration space, T_m is the risk premium parameter and therefore should not be cointegrated with variables in $\mathbf{Y}_t$ since it is in fact stationary. For this reason, T_m is not restricted to cointegration space.

5.4 Cointegration test

If $\mathbf{Y}_t$ in (5.18) contains $I(1)$ variables, then $\Delta\mathbf{Y}_{t-i}$ is $I(0)$, while $\mathbf{\Pi}\mathbf{Y}_{t-1}$ must also be $I(0)$ for $\mathbf{u}_t$ to be a white noise process. Obviously, $\mathbf{\Pi}$ cannot have a full rank $r = n$, since this would imply inconsistency that each stationary variable $\Delta\mathbf{Y}_t$ is a linear combination of non-stationary variables $\mathbf{Y}_{t-1}$ and other components in (5.18), which are all stationary. $\mathbf{\Pi}$ must therefore have a reduced rank $r \leq n-1$. Matrix $\mathbf{\Pi}$ can be decomposed to $\mathbf{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$ where $\boldsymbol{\alpha}$ represents the speed of adjustment to disequilibrium and $\boldsymbol{\alpha}$ is the matrix of long-run coefficients such that $\boldsymbol{\beta}'\mathbf{Y}_{t-1}$ represents up to $n-1$ stationary cointegrating relationships, which ensure that $\mathbf{Y}_t$ converge to their long-run steady state solution. A note, however, is necessary that since $\mathbf{Y}_t$ contains two variables that are possibly $I(0)$ in levels, they may form a cointegrating relation by itself adding to the total number of cointegrating relations. To test for cointegration between variables, we employ Johansen maximum likelihood approach [77] which provides n eigenvalues $\lambda_1 > \lambda_2 > \ldots > \lambda_n$ and their corresponding eigenvectors $\mathbf{V} = (v_1, v_2, \ldots v_n)$ of matrix $\mathbf{\Pi}$. Johansen cointegration test concentrates on testing which eigenvalues of matrix $\mathbf{\Pi}$ in (5.18) are significantly different from 0. These eigenvalues form linear combinations of stationary relationships $\boldsymbol{\beta} = (\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \ldots, \boldsymbol{\beta}_r)$. Cointegration is present if $\mathbf{\Pi}$ has a reduced rank $r \leq (n-1)$, indicating that there are r stationary cointegrating relationships between non-stationary variables in VAR. If $r = n$, this would indicate that all variables are stationary, while $r = 0$ would indicate no stable cointegrating relationships and the VAR with first differences only would be adequate. To determine the rank r, we use the trace test statistics and maximum eigenvalues statistics. Trace test statistics is defined as

$$\lambda_{trace} = -N \sum_{i=r+1}^{n} \log(1-\hat{\lambda}_i), \tag{5.19}$$

whereas maximum eigenvalue test statistic is defined as

$$\lambda_{max} = -N \log(1-\hat{\lambda}_{r+1}), \tag{5.20}$$

where N is the sample size and $\hat{\lambda}_i$ are the eigenvalues of $\mathbf{\Pi}$, estimated using the Maximum Likelihood Estimator (MLE) developed by Johansen [78]. Eigenvalues $1 > \hat{\lambda}_1 > \hat{\lambda}_2 > \ldots > \hat{\lambda}_n, \ \hat{\lambda}_{n+1} = 0$ solve

$$|\lambda\mathbf{S}_{11} - \mathbf{S}_{10}\mathbf{S}_{00}^{-1}\mathbf{S}_{01}| = 0 \tag{5.21}$$

where

$$\mathbf{S}_{ij} = \frac{1}{N}\sum_{t=1}^{N} \mathbf{R}_{it}\mathbf{R}'_{jt}, \quad i,j = 0,1 \tag{5.22}$$

are the product moment matrices of the residuals $\mathbf{R}_{0t}$ and $\mathbf{R}_{1t}$ obtained from regressing $\Delta\mathbf{Y}_t$ and $\mathbf{Y}_{t-1}$ on the lagged differences $\Delta\mathbf{Y}_{t-1}, \ldots, \Delta\mathbf{Y}_{t-k+1}$, exogenous variables and unrestricted deterministic variables (see Johansen [78] for details).

Both tests (5.19) and (5.20) involve the test of null hypothesis that there are r cointegration vectors against the alternative that there are $r+1$ cointegration vectors. The asymptotic critical values for trace test and maximum eigenvalue test are reported in Doornik [79], although these are rather indicative if the sample size is small compared to the number of parameters estimated or if additional unrestricted dummy variables are used in the model. Table 5.4 presents the MLE eigenvalue estimates sorted in descending order and the corresponding log-likelihood functions (LLF) for each rank r. The magnitude of eigenvalues shows that only the first eigenvalue appears to be significant, while other eigenvalues are below the indicative borderline of significance, which is around 0.2.

Table 5.4: Eigenvalues of matrix $\mathbf{\Pi}$

Rank	Eigenvalues	LLF
0		2644.8
1	0.3715	2680.5
2	0.1955	2697.3
3	0.1766	2712.2
4	0.1140	2721.5
5	0.0739	2727.5
6	0.0272	2729.6
7	0.0184	2731.0

The results of the trace test, presented in Table 5.5, show that the first hypothesis $r = 0$ is strongly rejected in both tests. The second hypothesis $r = 1$ is marginally rejected in case of trace test, whereas with maximum eigenvalue test this hypothesis is not rejected. The third hypothesis $r = 2$ is not rejected in neither of them. The trace test therefore suggests two cointegrating vectors, although the second is only marginally significant. Maximum eigenvalue test, on the other hand, suggests only one cointegrating vector.

Table 5.5: Trace and maximum eigenvalue statistic

$H_0 : r \leq$	λ_{trace}	Prob.	λ_{max}	Prob.
0	172.50	0.000**	71.52	0.000**
1	100.98	0.019*	33.50	0.236
2	67.47	0.074	29.92	0.140
3	37.55	0.326	18.63	0.456
4	18.92	0.509	11.82	0.578
5	7.11	0.572	4.25	0.827
6	2.85	0.091	2.85	0.091

*reject the null at 5% significance **reject the null at 1% significance

Apparent contradiction of results in Table 5.5 is not uncommon in cointegration analysis, since the interventions dummies tend to influence the underlying distribution of the test statistic such that the critical values of these tests depend on the number of intervention dummies included in the model. The problem of small sample compared to the number of estimated parameters is also reported by Reimers [80], in which case the critical values of Johansen cointegration test tend to over-reject the null hypothesis when true. Reimer suggests adjusting the degrees of freedom to the number of estimated parameters and thus replacing the sample size N in (5.19) and (5.20) with $N - nk$, where n is the model dimension and k the lag length of VAR. Table 5.6 presents the result of adjusted trace and maximum eigenvalue test. Using Reimer's adjustment of degrees of freedom, both test show that there is only one cointegrating vectors in the system, therefore $r = 1$.

Table 5.6: Adjusted trace and maximum eigenvalue statistic

$H_0 : r \leq$	$\lambda_{trace}(N - nk)$	Prob.	$\lambda_{max}(N - nk)$	Prob.
0	156.82	0.000**	65.02	0.000**
1	91.80	0.089	30.46	0.409
2	61.34	0.197	27.20	0.261
3	34.14	0.499	16.94	0.595
4	17.20	0.634	10.74	0.680
5	6.46	0.646	3.87	0.866
6	2.59	0.107	2.59	0.107

*reject the null at 5% significance **reject the null at 1% significance

As asymptotic distribution critical values suffer from small sample sizes and a number of deterministic variables, a bootstrapping simulation is often used as an alternative method for test statistic of the null hypothesis (see, e.g. Davidson and MacKinnon [81]). In the bootstrap test, one first computes a test statistic $\hat{s}$ obtained from the underlying test and estimates whatever parameters are needed to obtain a data generating process (DGP) that satisfies the null hypothesis. The distribution of the random variable s of which $\hat{s}$ is a realization under this "bootstrap DGP" serves to define the theoretical or ideal bootstrap p-value, $p^*(\hat{s})$, which is just the probability that $s > \hat{s}$ under the bootstrap DGP. Normally, this probability cannot be calculated analytically, and it is thus estimated by simulation, as follows. One draws B bootstrap samples from the bootstrap DGP, each of which is used to compute a bootstrap test statistic s_j^* in exactly the same way as the real sample was used to compute $\hat{s}$. For a one-tailed test with a rejection region in the upper tail, the bootstrap p-value may then be estimated by the proportion of bootstrap samples that yield a statistic greater than $\hat{s}$:

$$\hat{p}^*(\hat{s}) \equiv \frac{1}{B} \sum_{j=1}^{B} \mathcal{I}(s_j^* > \hat{s}), \tag{5.23}$$

where $\mathcal{I}(\cdot)$ is the indicator function. As $B \to \infty$, the estimated bootstrap p-value $\hat{p}^*(\hat{s})$ will approach to the ideal bootstrap p-value $p^*(\hat{s})$. We estimate the bootstrapped critical values on a reduced rank VAR, where $H_0 : \boldsymbol{\beta} = \boldsymbol{\beta}^c$, generating the pseudo data on the basis of the estimated reduced rank parameter estimates and a set of random noises.

Table 5.7 gives the estimated bootstrapped critical values for the trace test statistics with $B = 1000$ simulations for each hypothesis in question. The critical values and the corresponding p-values show that the bootstrapping method gives more strict critical values than asymptotic distribution. The corresponding p-value for the trace statistic in Table 5.7 shows that $H_0 : r < 1$ is not rejected with the bootstrap p-value of 0.074, whereas to the asymptotic p-value in Table 5.5 was 0.019. Similar to Reimer's small sample adjustment, these results again indicate one significant cointegrating relationship, while the second is marginally rejected.

Table 5.7: Bootstrap critical values for trace statistic

$H_0 : r \leq$	1%	5%	10%	p-value
0	147.50	135.99	129.51	0.000
1	115.75	104.65	97.50	0.074
2	82.04	74.04	70.35	0.129
3	60.76	51.05	47.34	0.446
4	37.68	32.33	29.63	0.622
5	20.42	15.61	13.08	0.563
6	10.06	6.30	5.32	0.317

Since the choice of cointegration rank is crucial in modelling cointegrated systems, we look for additional indicators for determining r. Juselius [82] outline three additional indicators which might help to choose the right cointegration rank.

First indication lies in the moduli of the largest roots of the companion matrix, and how they are changing for the hypotheses in question, i.e. $r = 1, 2, \ldots, n-1$. Table 5.8 shows the moduli of the characteristic roots for all hypotheses in question. Only at the first hypothesis the largest moduli is clearly away from the unit circle, whereas moduli of 0.871 or 0.910 could be considered as close to unit root. These results again indicate one strong cointegrating relationship, whereas the second is on the borderline of significance.

Table 5.8: Moduli of the characteristic roots

Rank	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$	$r = 6$
0	1.000	1.000	1.000	1.000	1.000	1.000
1	1.000	1.000	1.000	1.000	1.000	0.967
2	1.000	1.000	1.000	1.000	0.954	0.967
3	1.000	1.000	1.000	0.962	0.954	0.949
4	1.000	1.000	0.910	0.962	0.946	0.822
5	1.000	0.871	0.910	0.786	0.815	0.477
6	0.405	0.431	0.478	0.471	0.466	0.477

The second indication suggested by Juselius are the t-values of α_i coefficients of the $r^{th}+1$ cointegrating vector. If any loading matrix coefficient for $r^{th}+1$ cointegrating vector is significant, then including this vector in the model might help to improve the explanatory power. The critical values for loading matrix coefficients α_i follow Student's t distribution only if the corresponding vector $\beta_i \mathbf{Y}_t$ is stationary,

Table 5.9: t-values of loading matrix coefficients

Variable	$\alpha_{i,1}$	$\alpha_{i,2}$	$\alpha_{i,3}$	$\alpha_{i,4}$	$\alpha_{i,5}$	$\alpha_{i,6}$
np	**-4.946**	-0.231	0.212	-0.273	0.316	1.045
oil	1.180	**-4.199**	0.850	0.082	-0.820	1.108
coal	1.429	0.430	-1.708	-0.811	1.907	1.319
gas	-1.059	-2.284	0.632	**-3.277**	1.165	0.380
eua	**-3.791**	-1.628	**-3.311**	-1.939	-0.512	0.035
eex	**-7.291**	-0.851	-0.527	-1.053	0.106	0.831
alu	1.190	1.306	0.250	-1.854	-2.139	0.980

otherwise Dickey-Fuller critical values are more appropriate. Table 5.9 reports t-values of loading matrix coefficients. The significance of loading matrix coefficients give less intuitive results compared to the findings above. The second cointegrating vector is found significant only in *oil* equation, the third only in *eua* equation and the fourth only in *gas* equation.

Finally, we look at the graph of the first six cointegrating vectors presented in Figure 5.2. The first cointegrating vector looks stationary and the last four are clearly not. The second cointegrating vector looks stationary but may be close to unit root. Figure 5.2 does not provide additional support at the difficult choice between $r = 1$ and $r = 2$. The choice with the cointegration is often difficult when near-integrated variables are in the model. Johansen [78] argues that there is little need to pre-test the variables in the system to establish their order of integration, since the cointegration space is spanned by the number of stationary variables in the model. Many authors, however, show that this is valid only for a pure unit-root assumption, but not in case of near-integrated variables. Based on the unit root test in Table 4.7, we have a reason to believe that at least one of the variables in our system is near-integrated and this has a strong influence on the trace test and the maximum eigenvalue test statistics. Although our sample shows that *oil* or *eua* are near-integrated, forward prices are usually found to be purely integrated processes (see, e.g. Messe and Singleton [83]) so we treat them as such. If *oil* or *eua* would be considered as stationary, the right choice for cointegration rank would be $r = 2$, however, based on the tests reported above, $r = 1$ better reflect the data and the economic theory behind it.

5.5 Restrictions on β and α

5.5.1 Identification of cointegration space

Setting $r = 1$, we estimate the cointegrating vector β and loading vector α. Since no restrictions are imposed on cointegrating vector at this point, β is not identified. To achieve exact identification, β is normalised with respect to *np*. In Table 5.10 we present the values of cointegrating vector β normalised with respect to *np* and coefficients of loading vector α. Together with coefficient values, Student distribution

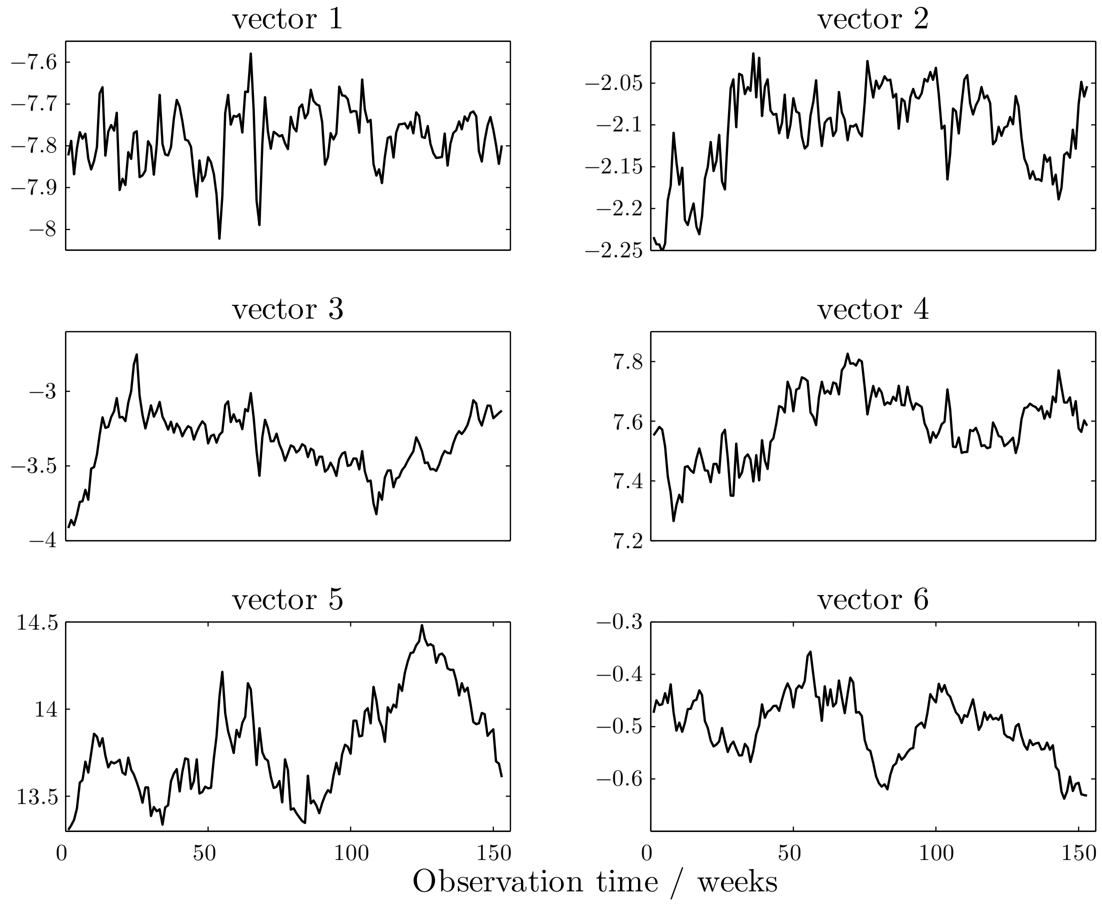

Figure 5.2: Cointegrating vectors

t-values are also reported to get the indication of the significance of parameters.

Based on t-values in Table 5.10 all variables in cointegrating vector appear significant except *gas*. Parameters and corresponding t-values in loading vector $\boldsymbol{\alpha}$ show that, in the short-run model, only *np*, *eua* and *eex* might be important, since weights of the other variables appear statistically insignificant. While asymptotic t-distribution might be a good indication of significance of each parameter in cointegrating vector and loading vector, a proper way to test their significance is by imposing restrictions on certain parameters and performing a standard LR test. Restrictions in cointegration space can be imposed by defining a restricted cointegrating vector $\boldsymbol{\alpha}^c$ as follows

$$\boldsymbol{\beta}^c = (\boldsymbol{\beta}_1^c, \dots, \boldsymbol{\beta}_r^c) = (\mathbf{H}_1\boldsymbol{\varphi}_1, \dots, \mathbf{H}_r\boldsymbol{\varphi}_r) \tag{5.24}$$

in which $\mathbf{H}_i$ are design matrices and $\boldsymbol{\varphi}_i$ are coefficient matrices. Since $r = 1$ we

Table 5.10: Normalised cointegrating vector $\beta_{i,1}$ and loading vector $\alpha_{i,1}$

Variable	$\beta_{i,1}$	t-value	$\alpha_{i,1}$	t-value
np	**1.000**		**-0.147**	**-5.013**
oil	**-1.003**	**-5.236**	0.045	1.119
coal	**-0.920**	**-6.304**	0.039	1.408
gas	-0.106	-1.395	-0.045	-1.012
eua	**0.147**	**3.336**	**-0.301**	**-3.626**
eex	**1.191**	**4.831**	**-0.147**	**-7.350**
alu	**-1.161**	**-5.694**	0.037	1.166

impose the restriction on $\beta_{4,1} = 0$ by setting

$$\boldsymbol{\beta}_1^c = \mathbf{H}_1\varphi_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varphi_{11} \\ \varphi_{12} \\ \varphi_{13} \\ \varphi_{14} \\ \varphi_{15} \\ \varphi_{16} \end{bmatrix} \tag{5.25}$$

This restriction is not rejected with $LR = 1.214$ and p-value based on $\chi^2(1)$-distribution is $p = 0.271$. *Gas* price is therefore indeed insignificant in the long-run model. This reflects the well known fact from energy markets that natural gas price follows the (possibly delayed) information about the crude oil price. Since long-term crude oil price is also included in the model, the additional information on natural gas price does not help to explain the long-run dynamics of the model. Restricting any other parameter in the cointegration space leads to a rejection of LR test. This way the cointegrating vector composing of all variables expect *gas*is exactly identified.

The structure and parameters of cointegrating vector are interesting since this vector represents a linear combination of *np*, *oil*, *coal*, *eua*, *eex* and *alu*. The Nord Pool price increases approximately one to one with oil price, a bit more than one to one with aluminium price and a bit less than one to one with coal price. On the other hand, the Nord Pool price falls a bit more than one to one when EEX price increases and falls by 0.15% if the emission allowance price rises by 1%. The parameters of the cointegrating vector do not imply that there is a negative long-run relationship between *np*, *eua* and *eex*. Since Nord Pool and EEX price are strongly positively correlated, this is rather an indication that if a positive shock occurred in the EEX price in the last period, then a similar positive shock is also likely to have occurred in the Nord Pool price. The cointegrating vector would then pull the Nord Pool price back down in the next period.

We do not test additional restrictions, such as the long-run price homogeneity, since these variables come from different commodity markets and their relationship is established on purely technical basis when investors compare the value of different assets. The price homogeneity would therefore have no theoretical meaning in this case.

5.5.2 Weak exogeneity test

Variables can be classified as weakly exogenous when they have a significant influence on other variables in the long-run, however, they are not influenced by them. Weak exogeneity test is performed on the loading matrix $\boldsymbol{\alpha}$ by testing the restrictions that a particular row in the estimated loading matrix $\boldsymbol{\alpha}$ is insignificantly different from zero. The parameters of $\boldsymbol{\alpha}$ explain how the short-run model is adjusted to the disequilibrium represented by the cointegrating vectors $\boldsymbol{\beta}'\mathbf{Y}_{t-1}$. If the entire row in $\boldsymbol{\alpha}$ is zero this indicates that none of the cointegrating vectors enter the equation associated with this row. The following hypotheses is of interest

$$H_0(r) : \boldsymbol{\alpha} = \mathbf{H}\boldsymbol{\alpha}^c \tag{5.26}$$

in which $\boldsymbol{\alpha}$ is the estimated unconstrained loading matrix, $\mathbf{H}$ is the design matrix and $\boldsymbol{\alpha}^c$ is the constrained loading matrix containing only non-zero coefficients. Based on the results in Table 5.10 we test the hypothesis that *oil*, *coal*, *gas* and *alu* are weakly exogenous in our system. The design matrix therefore includes four restrictions leaving only three free parameters to estimate:

$$\mathbf{H}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}. \tag{5.27}$$

Testing this hypothesis using restricted $\boldsymbol{\beta}^c$ estimated in the previous section shows that the null is not rejected, since $LR = 10.035$ and p-value based on χ^2-distribution is $p = 0.074$. The results of weak exogeneity test are presented in Table 5.11.

Table 5.11: Weak exogeneity test results

Variable	$\beta_{i,1}$	t-value	$\alpha_{i,1}$	t-value
np	**1.000**		**-0.202**	**-6.015**
oil	**-0.958**	**-6.790**	0	
coal	**-0.654**	**-6.740**	0	
gas	0		0	
eua	**0.129**	**3.491**	**-0.365**	**-3.938**
eex	**0.714**	**3.579**	**-0.179**	**-8.248**
alu	**-0.863**	**-5.515**	0	

Variables *oil*, *coal*, *gas* and *alu* are therefore weakly exogenous, while *np*, *eua* and *eex* are endogenous. Fuel prices and aluminium prices therefore have a significant impact on electricity prices and consequently on emission allowance prices, whereas the feedback influence of electricity prices on fuels and aluminium prices is negligible. This is somewhat expected for *oil* and also *gas*, which is highly correlated with *oil*. Obviously global oil and consequently gas prices follow global supply and demand, for which electricity production industry is the marginal consumer of these fuels compared to other consumers. The results, on the other hand, are not so intuitive for coal and aluminium prices. Most of the coal produced today is used for electricity production and aluminium production costs depend mostly on electricity prices. One would therefore expect coal prices and aluminium prices to be tightly connected to electricity prices in a way that electricity prices also influence coal and aluminium prices. Weak exogeneity test results therefore indicate that long-term coal and aluminium prices are governed by other market forces and that the long-term electricity price is not one of them.

The loading parameters in Table 5.11 for *np*, *eex* and *eua* are strongly significant, with *eex* and *np* having a similar speed of adjustment to equilibrium. Around 20% of disequilibrium in *np* and *eex* is corrected in one period, whereas the speed of adjustment for *eua* is significantly higher with about 37% of disequilibrium in cointegrating vector corrected in one period.

Table 5.12 presents the long-run impact matrix $\mathbf{\Pi}$, estimated with constrained α and β, while the t-values of the parameters are given in brackets below. The interpretation of these parameters is rather easy since α and β are both $n \times 1$ vectors, therefore the parameter values are the simple product of the two respective elements in α and β. Endogenous variables adjust negatively to disequilibrium caused by endogenous variables and positively to disequilibrium caused by weakly exogenous variables.

Table 5.12: $\mathbf{\Pi}$ matrix

Variable	*np*	*oil*	*coal*	*gas*	*eua*	*eex*	*alu*
np	−0.202 (−6.015)	0.193 (6.015)	0.132 (6.015)	0 (−)	−0.026 (−6.015)	−0.144 (−6.015)	0.174 (6.015)
oil	0 (−)	0 (−)	0 (−)	0 (−)	0 (−)	0 (−)	0 (−)
coal	0 (−)	0 (−)	0 (−)	0 (−)	0 (−)	0 (−)	0 (−)
gas	0 (−)	0 (−)	0 (−)	0 (−)	0 (−)	0 (−)	0 (−)
eua	−0.365 (−3.938)	0.350 (3.938)	0.239 (3.938)	0 (−)	−0.047 (−3.938)	−0.261 (−3.938)	0.316 (3.938)
eex	−0.179 (−8.248)	0.172 (8.248)	0.117 (8.248)	0 (−)	−0.023 (−8.248)	−0.128 (−8.248)	0.155 (8.248)
alu	0 (−)	0 (−)	0 (−)	0 (−)	0 (−)	0 (−)	0 (−)

5.6 Conclusion

In this chapter, we set up a vector autoregressive model conditional on information set defined in Chapter 4. Due to residual autocorrelation and heteroscedasticity we include additional dummies to make the residuals well behaved. Stability and parameter constancy test reveal that VAR is stable and that parameters are constant throughout the sample. This provides an adequate grounds for testing for cointegration using Johansen cointegration testing framework for $I(1)$ system. Trace and maximum eigenvalue test, together with other indicators, reveal that there is only one significant cointegrating relationship between the variables in the model, while two additional cointegrating vectors are marginally rejected. We estimate the cointegrating vector β and loading matrix α and find that cointegrating vector is not influenced by *gas*, whereas adjustment coefficients for *oil*, *coal*, *gas* and *alu* are insignificant in loading matrix α, implying that these variables are weakly exogenous.

Granger [2] postulates that asset prices, determined in efficient markets, cannot be cointegrated. Fama [84] distinguishes between three forms of market efficiency. Weak-form efficiency implies that future changes in market prices cannot be predicted by analysing only the past values of these prices, therefore technical analysis or chartism cannot not yield any extra returns. Semi-strong efficiency implies that the future prices cannot be predicted with any publicly available information, indicating that neither technical nor fundamental analysis based on public information would yield returns above the risk-free return. Finally, strong-form efficiency implies that all information, public and private, is already included in the price, therefore even the investors with private information cannot predict the future prices. Since we model the long-term forward prices with fundamental variables from other commodity markets, the presence of cointegration could give evidence against semi-strong market efficiency. If prices of two assets are cointegrated, then the price movements of one asset will convey information about the future movements of the other. In this case, it will be profitable to trade across markets of different assets, exploiting movements in the prices of one asset to predict movements in the prices of another asset. Hence, this would imply arbitrage opportunities between markets, known also as cross-sectional market inefficiency. Are therefore electricity and energy markets in general inefficient? Many authors argue that cointegration does not necessarily imply inefficient markets. Copeland [3] shows that cointegration between spot and forward exchange rate is not sufficient condition for market inefficiency, since this might be the result of time varying risk premium. However, under the assumption of risk neutrality, cointegration between different types of asset is in general still considered as enough condition to indicate cross-sectional inefficiency (see, e.g. Dwyer and Wallace [85]). Cointegration found between prices of different sources of energy therefore provides some evidence against the semi-strong form of efficiency. However, it may also imply very volatile risk premium, which we were unable to explain with the variables in the information set.

Chapter 6

Structural analysis

The interpretation of VAR and VECM models is very limited and does not provide all the insight into the dynamics under investigation. Additional concepts and tools were developed to interpret VAR/VECM models more easily. The most important are the causality concepts, impulse response analysis and forecast error variance decomposition. We test one-step-ahead Granger causally, which centers on the identification of how changes in variables cause changes in other variables. While this notion gives short-run causal structure, impulse response analysis and forecast error variance decomposition provide an overall picture of the system dynamics, both in the short- and the long-run. A more advanced structural relationship is given by Beveridge-Nielsen decomposition, which provides the answer on which part of the individual variable is the result of permanent dynamics and which part is only transitory without permanent effect. While permanent components in variables are driven by a few common trends, the transitory components are driven by a few common cycles. We also estimate the Structural VAR, which includes contemporaneous relationships between variables. We identify structural shocks and try to give them a theoretical meaning. At the end of the chapter, we reduce the model size and estimate the short-run structure of the model. Possible applications of the model are also discussed.

6.1 Granger causality

Causality between variables refers to a relationship between two variables, where one variable is a direct consequence of the other variable, therefore one variable can be used to forecast the other. A time series variable x is said to fail to Granger-cause another variable y, if the mean squared error (MSE) of a forecast of y_{t+s} based on information set $I_t = \{x_t, x_{t-1}, \ldots, y_t, y_{t-1}, \ldots\}$ is equal to the MSE of a forecast y_{t+s} based on information set $I_t = \{y_t, y_{t-1}, \ldots\}$, $s > 0$. The conventional Granger-causality test based on a standard VAR-model is defined conditional on the assumption of stationarity. In case of VECM representation, Dolado and Lütkepohl [86] propose a modified Wald statistic which also accounts for rank restrictions. Granger causality can be characterised by specific zero constraints on the VECM coefficients. Thus, if we want to test for Granger-causality in the estimated VECM, we need to test zero constraints for the coefficients that describe the influence of variable Y_i on variable Y_j. Hence, these zero restrictions apply to matrix of short run coefficients $\mathbf{\Gamma}$ as well as to the long-run matrix $\mathbf{\Pi}$ in VECM representation (5.18). In case rank r is reduced, the long run matrix is $\mathbf{\Pi} = \boldsymbol{\alpha\beta}'$. However, when $\boldsymbol{\alpha}$ or $\boldsymbol{\beta}$ are restricted and the corresponding parameters in $\mathbf{\Pi}$ are zero, then the zero restriction only applies to $\mathbf{\Gamma}$. Let $\mathbf{G}$ be a $N \times (nk + d)$ matrix of variables as follows

$$\mathbf{G} = \begin{bmatrix} 1 \\ \mathbf{D}_t \\ \Delta\mathbf{Y}_{t-1} \\ \Delta\mathbf{Z}_t \\ \boldsymbol{\beta}\mathbf{Y}_{t-1} \end{bmatrix} \tag{6.1}$$

where d is the number of deterministic variables plus the number of exogenous variables. Defining $\mathbf{\Phi}$ as

$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{I}_{n(nk+d-r)} & 0 \\ 0 & \boldsymbol{\beta} \otimes \mathbf{I}_n \end{bmatrix} \tag{6.2}$$

and $\mathbf{\Xi}$ is a vector of estimated VECM parameters

$$\mathbf{\Xi} = \mathrm{vec}([\hat{\mathbf{A}}_0 \quad \hat{\mathbf{\Theta}} \quad \hat{\mathbf{\Gamma}} \quad \hat{\mathbf{\Psi}} \quad \hat{\boldsymbol{\alpha}}\hat{\boldsymbol{\beta}}]) \tag{6.3}$$

then the Wald statistic for Granger causality test is

$$\lambda_W = \mathbf{R}'\mathbf{\Xi}'[\mathbf{R\Sigma_\Xi R}']^{-1}\mathbf{R\Xi}, \quad \xrightarrow{d} \quad \chi^2(k) \tag{6.4}$$

where $\mathbf{\Sigma}_\Xi$ is a variance-covariance matrix of VECM parameter estimates

$$\mathbf{\Sigma}_\Xi = \mathbf{\Phi}((\mathbf{ZZ}')^{-1} \otimes \mathbf{\Omega})\mathbf{\Phi}' \tag{6.5}$$

and $\mathbf{\Omega}$ is a variance-covariance matrix of residuals

$$\mathbf{\Omega} = \frac{1}{N}\mathbf{u}_t\mathbf{u}_t'. \tag{6.6}$$

In (6.4) $\mathbf{R}$ is a $k \times (n(n + k + d))$ selection matrix with 1 at the corresponding parameters of $\mathbf{\Xi}$, such that parameters $\mathbf{\Gamma}_{i,j}$, $\mathbf{\Pi}_{i,j}$, $i \neq j$ are selected, and 0 otherwise. The Wald statistic in (6.4) is distributed with χ^2 distribution with k degrees of freedom.

Tables 6.1 to 6.3 give the results for Granger causality tests for three different cases. The first causality test is performed on unrestricted VECM without the rank restriction ($r = 7$). The second test is applied on reduced rank VAR with $r = 1$, whereas the third test applies for reduced rank VAR ($r = 1$) with restrictions on $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ as in Section 5.5. The purpose of three different tests is to identify how rank reduction and constraints on $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ influence causality between variables. The results in Table 6.1 show only few cases of unidirectional causality. These results should, however, be treated only as indicative, since unrestricted VAR still exhibits non-stationarity. Therefore, the conventional Granger-causality cannot be trusted in this case.

Table 6.1: Granger causality test for unrestricted VECM ($r = 7$)

	H$_0$: Variable i does not Granger-cause						
Variable i	*np*	*oil*	*coal*	*gas*	*eua*	*eex*	*alu*
np		1.05	0.62	0.83	5.02	11.32**	2.15
oil	7.83*		0.23	5.97	8.84*	13.86**	1.34
coal	13.23**	2.98		2.37	5.54	20.94**	6.42
gas	0.18	5.97	11.06**		0.83	0.43	4.34
eua	2.53	3.67	2.85	4.62		5.62	0.90
eex	0.93	2.89	0.75	1.14	3.61		2.63
alu	9.30**	0.08	2.57	1.37	8.08*	27.98**	

*reject the null at 5% significance **reject the null at 1% significance

Table 6.2: Granger causality test for unrestricted VECM ($r = 1$)

	H$_0$: Variable i does not Granger-cause						
Variable i	*np*	*oil*	*coal*	*gas*	*eua*	*eex*	*alu*
np		1.74	2.53	1.48	15.62**	61.96**	3.78
oil	28.82**		2.59	8.87*	17.08**	61.30**	2.05
coal	28.47**	3.37		4.59	16.79**	63.28**	2.10
gas	29.17**	7.60*	7.11*		14.90**	61.28**	4.65
eua	31.00**	1.72	2.41	1.25		61.28**	2.20
eex	29.82**	4.78	2.27	1.49	15.02**		1.55
alu	28.49**	1.44	5.52	1.97	14.90**	62.67**	

*reject the null at 5% significance **reject the null at 1% significance

Table 6.3: Granger causality test for restricted VECM ($r = 1$)

	H$_0$: Variable i does not Granger-cause						
Variable i	*np*	*oil*	*coal*	*gas*	*eua*	*eex*	*alu*
np		0.31	0.28	0.31	18.28**	77.81**	2.22
oil	41.34**		0.34	7.65*	19.77**	77.15**	0.51
coal	40.98**	1.93		3.40	19.47**	79.13**	0.55
gas	0.42	6.12*	4.80*		0.00	0.01	3.08
eua	43.52**	0.30	0.17	0.09		77.13**	0.65
eex	42.33**	3.33	0.03	0.32	17.67**		0.01
alu	41.00**	0.02	3.22	0.80	17.55**	78.51**	

*reject the null at 5% significance **reject the null at 1% significance

Restricting $r = 1$ produces very different results of causality test, which are presented in Table 6.2. These results show that all variables Granger-cause *np*, *eua* and *eex*, whereas *oil*, *coal*, *gas* and *alu* are not Granger-caused by any of them, except for some marginal causality between *oil*, *coal* and *gas*. This is consistent with weak exogeneity test results, which show that *oil*, *coal*, *gas* and *alu* influence *np*, *eua* and *eex*, but are not influenced by them. Surprisingly, *gas* also Granger-cause *np*, *eua* and *eex* even though the restrictions on α and β show that *gas* is insignificant. Obvious contradiction of results can be explained by the very nature of both tests. While weak exogeneity test focuses on the long-run causality, the Granger causality test reveals only the short-run causal structure. Luintel and Khan [87] have shown that long-run causality can be examined by testing the cointegrating vectors in the Johansen testing framework for weak exogeneity. Weak exogeneity is sometimes also referred to as long-run Granger non-causality. In the context of cointegrated systems, weak exogeneity is a long-run notion of exogeneity implying that the long-run relations between variables are block triangular. Weak exogeneity means no long-run feedback (insignificance of speed of adjustment coefficients) towards the relevant variable exists and implies a "weak" form of Granger non-causality. Finally, weak exogeneity of a variable in conjunction with absence of Granger-causality in the short-run establishes strong exogeneity for that particular variable (Engle, Hendry and Richard [88]).

Table 6.3 gives the results of Granger-causality test for restricted rank $r = 1$ with additional restrictions on α and β estimated in Section 5.5. Compared to results in Table 6.2, the restrictions on α and β significantly influence the causal structure for *gas*, since the Granger non-causality on other variables is no longer rejected except for the case of *oil* and *coal*. Restrictions imposed on α and β therefore produce a consistent casual structure in the short- and long-run.

6.2 Impulse response analysis

Impulse response analysis is often used to analyse the dynamic interactions between endogenous variables of a VAR/VEC process. In this analysis, exogenous and deterministic variables are treated as fixed and may therefore be dropped from the system. Denoting adjusted endogenous variables as $\mathbf{Y}_t$, estimated VECM parameters as in (5.18) can be converted to VAR(k), while exogenous and deterministic variables can be dropped at this point since these have no influence on impulse responses.

$$\mathbf{Y}_t \;=\; \mathbf{A}_1\mathbf{Y}_{t-1} + \ldots + \mathbf{A}_k\mathbf{Y}_{t-k} + \mathbf{u}_t \tag{6.7}$$

This can be further expressed with the following Wold moving average (MA) representation

$$\mathbf{Y}_t \;=\; \sum_{s=0}^{\infty} \mathbf{\Phi}_s \mathbf{u}_{t-s} \tag{6.8}$$

where $\mathbf{\Phi}_0$ equals the impulse function $\mathbf{I}_K$ and the $\mathbf{\Phi}_s$ can be computed recursively with

$$\mathbf{\Phi}_s = \sum_{m=1}^{s} \mathbf{\Phi}_{s-m}\mathbf{A}_m, \quad s = 1, 2, \ldots \tag{6.9}$$

in which $\mathbf{A}_m = 0$ for $m > k$. The elements of $\mathbf{\Phi}_s$ represent the impulse responses of the components of $\mathbf{Y}_t$ with respect to the innovations $\mathbf{u}_t$. More specifically, parameter $\mathbf{\Phi}_{ij,s}$ represents the expected response of variable i to an unit impulse in variable j occurring s-th period ago. These impulse responses are sometimes called *forecast error impulse responses* because the $\mathbf{u}_t$ are the 1-step ahead forecast errors (see Lütkepohl [89]).

Figure 6.1 presents impulse response analysis in which the response in endogenous variables is measured as a function of unit impulse in endogenous and weakly exogenous variables in our model. A response of a variable to its own impulse always equals one at the first step and then gradually decreases. Since all variables except *gas* are present in cointegrating vector, the signs of parameters in cointegrating vector influence the signs of the response to an impulse. The impulse in weakly exogenous variables *oil*, *coal* and *alu* induces a positive impulse in endogenous variables. The response to the impulse in *np*, *eua* and *eex* is in all cases negative, since their cointegrating vector parameters have all positive signs. The magnitude of long-term response in endogenous variables is directly proportional to the parameters in cointegrating vector multiplied with loading matrix parameters. The short-run responses are also influenced by matrix of short run parameters $\mathbf{\Gamma}$, hence the first step response presents the combination of the long-run response governed by $\mathbf{\Pi}$ and short-run response governed by $\mathbf{\Gamma}$, whereas a unit impulse in *gas* is governed only by the respective short-run parameters in $\mathbf{\Gamma}$.

A major drawback of ordinary impulse response function is the assumption that the underlying shocks are to occur in isolation and therefore the covariance matrix $\mathbf{\Omega}$ is assumed to be diagonal. In most real world cases, the covariance matrix is not diagonal and the shocks to the system are not likely to occur in isolation. Table 6.4 presents the covariance matrix of residuals taken from the restricted VAR. The table shows that $\mathbf{\Omega}$ cannot be considered diagonal in our case, since particularly high covariance can be found between *np* and *eua* and between *eua* and *eex* residuals.

Table 6.4: Covariance matrix $\mathbf{\Omega}$ ($\times 10^{-6}$)

Variable	*np*	*oil*	*coal*	*gas*	*eua*	*eex*	*alu*
np	350	141	97	142	392	149	25
oil	141	648	56	233	184	78	133
coal	97	56	309	195	113	55	29
gas	142	233	195	813	618	169	63
eua	392	184	113	618	2729	348	128
eex	149	78	55	169	348	162	45
alu	25	133	29	63	128	45	408

To take into account the interdependence of shocks, a real covariance matrix must be considered in impulse response function. A popular way how to achieve this is to use Cholesky decomposition of the covariance matrix $\mathbf{\Omega}$. Denoting $\mathbf{P}$ a lower triangular matrix such that $\mathbf{\Omega} = \mathbf{PP}'$, the orthogonalised shocks are given by $\boldsymbol{\varepsilon}_t = \mathbf{P}^{-1}\mathbf{u}_t$, for which the covariance matrix is diagonal. Hence, in the stationary

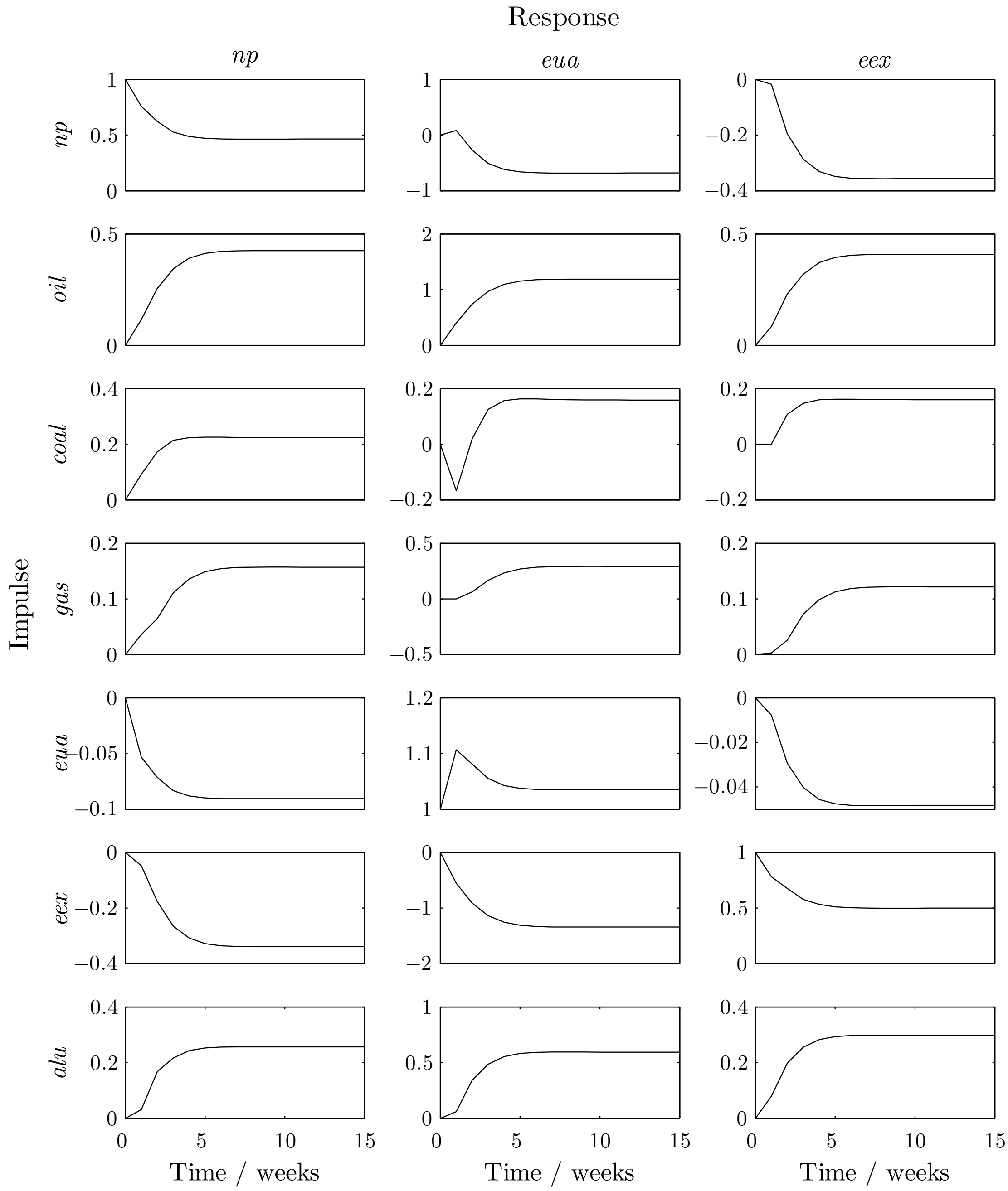

Figure 6.1: Impulse responses. In the upper left corner a unit impulse in *np* results in decreasing long-run response of *np* to approximately 0.5, while no short-run response is visible. In the middle figure of the upper row a unit impulse to *np* results in a positive short-run response in *eua*, while the negative long-run response prevails from the second week forward.

case we get

$$\mathbf{Y}_t = \sum_{s=0}^{\infty} \mathbf{\Psi}_s^o \boldsymbol{\varepsilon}_{t-s}. \tag{6.10}$$

Here the response function for an impulse in variable j is given with

$$\mathbf{\Psi}_{j,s}^o = \mathbf{\Phi}_s \mathbf{P} \mathbf{e}_j \tag{6.11}$$

where $\mathbf{e}_j$ is an $m \times 1$ selection vector with unity at j-th element and zeros elsewhere. Since $\mathbf{P}$ is a lower triangular, a shock $\boldsymbol{\varepsilon}_t$ in the first variable may have an instantaneous effect on all variables, whereas a shock in the second variable cannot have an instantaneous impact on the first variable, but only on the other variables and so on. Cholesky decomposition therefore imposes a recursive causal structure from the top variables to the bottom variables, but not the other way around. Hence, the effects of a shock may depend on the way the variables are arranged and the results may be very different if non-diagonal values of covariance matrix are high.

In the view of this non-uniqueness of the impulse responses, Pesaran and Shin [90] proposed the generalised impulse response function (GIRF), for which the (scaled) response to an impulse in variable j is given with

$$\mathbf{\Psi}_{j,s}^g = \frac{\mathbf{\Phi}_s \mathbf{\Omega} \mathbf{e}_j}{\sqrt{\sigma_{jj}}} \tag{6.12}$$

where σ_{jj} is j-th diagonal element of covariance matrix $\mathbf{\Omega}$. Instead of controlling the impact of correlation among residuals, GIRF follows the idea of nonlinear impulse response function and computes the mean impulse response function. When $\mathbf{\Omega}$ is diagonal, GIRF equals the impulse response function, whereas when compared to the orthogonalised impulse response function, GIRF does not depend on variable ordering. Figure 6.2 presents the generalised impulse responses together with their confidence intervals for the three endogenous variables. In general, the results show similar impulse responses as in Figure 6.1. However, their interdependence is changed. In contrast to orthogonalised impulses or unit impulses, generalised impulses are the corresponding columns of the residual covariance matrix scaled by the standard deviation of the of impulse variable residuals. Generalised responses can therefore be directly compared in relative terms. The generalised impulse responses also provide us with measure how quickly the long-run relations converge to their steady state values. Figure 6.2 shows that in all cases, responses to generalised impulses are the highest in *eua* and lowest in *eex*. Steady state or the long-run generalised impulses are achieved after approximately 5 weeks. Their values together with t-values are also presented in Table 6.5.

6.3 Forecast error variance decomposition

The dynamic interactions between the variables in the system can also be summarised with the Forecast Error Variance Decomposition (see, e.g. Hamilton [91], Franses [92], Chapter 9, and Lütkepohl [93]). While the impulse response function records the effects of a shock in one variable on other variables in VAR, the Forecast Error Variance Decomposition (FEVD) separates variation in an endogenous

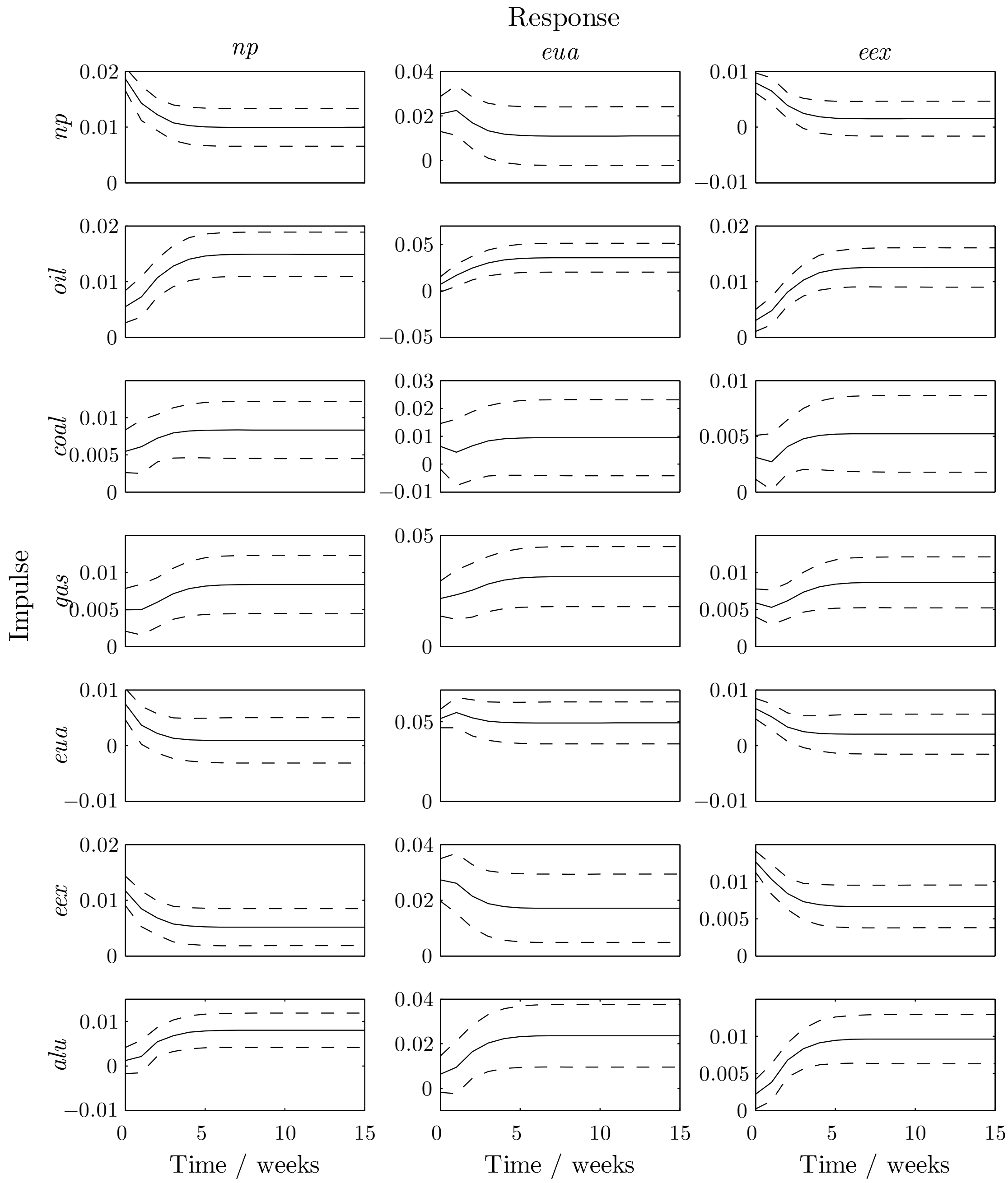

Figure 6.2: Generalised impulse response with confidence intervals. In contrast to Figure 6.1 these responses do not start with 1 or 0, since the system is hit simultaneously by a generalised impulse in all variables, which is proportional to residual covariances.

Table 6.5: Long-run generalised impulse responses ($\times 10^{-4}$), t-values in brackets

Response	Impulse *np*	*oil*	*coal*	*gas*	*eua*	*eex*	*alu*
np	100 (5.76)	149 (7.33)	83 (4.26)	84 (4.18)	10 (0.46)	52 (3.05)	80 (4.07)
oil	83 (3.58)	238 (9.52)	9 (0.33)	101 (3.75)	56 (2.05)	60 (2.70)	44 (1.69)
coal	55 (3.47)	30 (1.56)	166 (10.37)	87 (4.77)	23 (1.22)	49 (3.25)	1 (0.06)
gas	98 (3.11)	178 (4.76)	88 (2.43)	320 (9.79)	129 (3.51)	152 (5.24)	45 (1.26)
eua	110 (1.64)	358 (4.50)	95 (1.36)	314 (4.56)	493 (7.30)	171 (2.74)	236 (3.29)
eex	15 (0.95)	126 (6.97)	52 (2.97)	87 (4.94)	21 (1.13)	67 (4.55)	96 (5.68)
alu	11 (0.69)	43 (2.36)	19 (1.06)	38 (2.12)	22 (1.24)	37 (2.60)	158 (10.71)

variable into component shocks to the system. Using the elements $\mathbf{\Phi}_s$ of the impulse response function in (6.9) and denoting the ij-th element of the impulse response coefficient matrix as $\mathbf{\Phi}_{ij,s}$, the relative contribution of variable j to the s-step forecast error variance of variable i is

$$\theta^o_{ij,s} = \frac{\sum_{m=0}^{s-1}(\mathbf{e}_i'\mathbf{\Phi}_m\mathbf{P}\mathbf{e}_j)^2}{\sum_{m=0}^{s-1}(\mathbf{e}_i'\mathbf{\Phi}_m\mathbf{\Omega}\mathbf{\Phi}_m'\mathbf{e}_i)} \tag{6.13}$$

where $\mathbf{P}$ is the lower triangular of Cholesky decomposition of the covariance matrix $\mathbf{\Omega}$, whereas $\mathbf{e}_i$ and $\mathbf{e}_j$ are two $m\times 1$ selection vectors with unity at i-th an j-th element respectively and zeros elsewhere. The numerator in (6.13) can be interpreted as the contribution of variable j to the s-step forecast error variance of variable i, whereas the denominator is the sum of contributions from all variables to the forecast error variance of variable i.

Similar as orthogonalised impulse response function, FEVD also uses Cholesky decomposition of the covariance matrix and the resulting variance decomposition very much depends on ordering of variables. Following the work on GIRF, Koop et al. [94] and Pesaran and Shin [90] developed Generalised Forecast Error Variance Decomposition (GFEVD), which is invariant to variable ordering. In GFEVD, the relative contribution of variable j to the s-step forecast error variance of variable i is

$$\theta^g_{ij,s} = \frac{\sum_{m=0}^{s-1}(\mathbf{e}_i'\mathbf{\Phi}_m\mathbf{\Omega}\mathbf{e}_j)^2}{\sigma_{jj}\sum_{m=0}^{s-1}(\mathbf{e}_i'\mathbf{\Phi}_m\mathbf{\Omega}\mathbf{\Phi}_m'\mathbf{e}_i)}. \tag{6.14}$$

Important difference between FEVD and GFEVD is that, in case of orthogonalised variance decomposition, the sum of relative contributions to s-step forecast error variance of variable i always equals one, whereas in GEFVD it does not.

$$\sum_{j=0}^{n}\theta^o_{ij,s} = 1, \qquad \sum_{j=0}^{n}\theta^g_{ij,s} \neq 1. \tag{6.15}$$

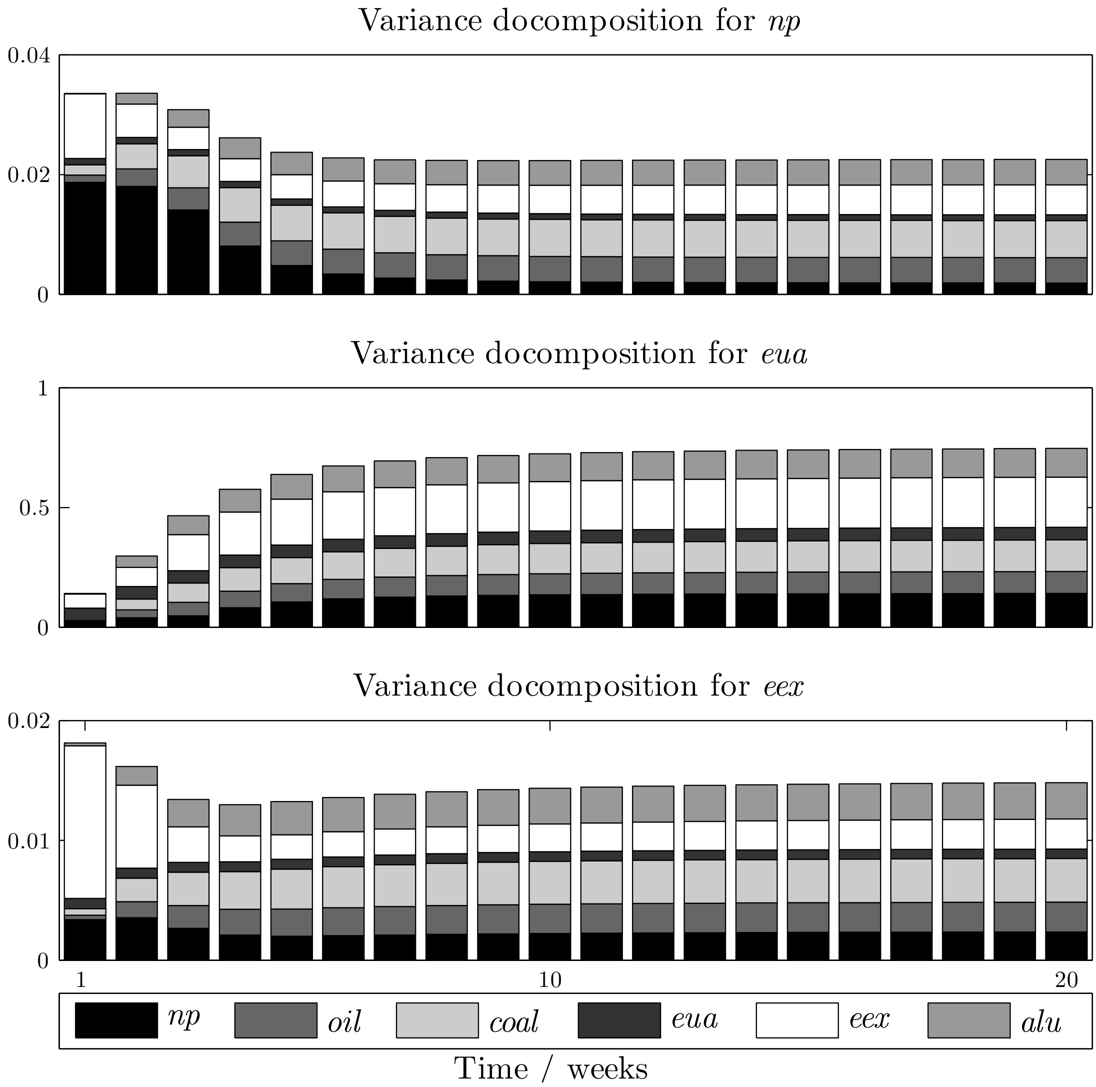

Figure 6.3: Generalised forecast error variance decomposition

Figure 6.3 presents the generalised forecast error variance decomposition for *np*, *eua* and *eex*. In short-run, the variance of variables is mostly influenced by its own shocks, particularly in case of *np* and *eex*, in which more than 50% of the variance is caused by its own shocks. In *eua*, the short-run forecast error variance is smaller than the long-run forecast error variance. In the long-run, the variance structure is more equally distributed among all the variables. While the variation in *eua* has the smallest impact on variation of variables, the impacts of other variables are quite significant. *Oil*, *coal* and *alu* have similar if not higher influence on the variance structure, compared to the influence of *np*, *eua* and *eex*. This indicates that there is no dominating variable in any of the endogenous variable forecast error variances. Among weakly exogenous variables, the influence of *coal* is consistently higher than the influence of *oil* and *alu*. A small difference can also be observed when compared to the influence between *np* and *eex*. The influence of *eex* on *np* forecast error variance is higher than the influence of *np* on *eex* forecast error variance. Also the forecast error variance of *eua* is influenced more by *eex* than it is by *np*. Thus, in terms of forecast error variance, *eex* is more important than *np*.

6.4 Permanent and transitory decomposition

6.4.1 Permanent and transitory components

Any non-stationary series can be decomposed into permanent (or trend) and transitory (or cycle) components. Several permanent-transitory decompositions have been developed and used in empirical and theoretical analyses. These include multivariate extension of Beveridge-Nelson decomposition proposed by Stock and Watson [95], the observable permanent-transitory decomposition of Gonzalo and Granger [96] where the components are identified as being the combinations of observable prices, while Engle and Kozicki [97] introduced the concepts of serial correlation common features in cointegrated VAR model. Here, we employ a permanent-transitory decomposition proposed by Gonzalo and Granger [96] that focuses on identifying $n-r$ $I(1)$ common stochastic trends.

Consider an n-dimensional stochastic process $\mathbf{Y}_t$ of integrated variables and its error correction (VECM) representation

$$\mathbf{\Gamma}(L)\Delta\mathbf{Y}_t \;=\; \boldsymbol{\delta}\mathbf{\Lambda} + \mathbf{\Pi}\mathbf{Y}_{t-1} + \mathbf{u}_t \tag{6.16}$$

where $\mathbf{\Gamma}(L) = \mathbf{I}_n - \sum_{i=1}^{k-1}\mathbf{\Gamma}_i L^i$, $\Delta = 1 - L$ is the difference operator, and L the lag operator, $\mathbf{\Lambda}$ is a matrix of deterministic variables $\mathbf{\Lambda} = [1 \quad \mathbf{D}_t]$ and $\boldsymbol{\delta}$ is the matrix of their corresponding parameters, while $\mathbf{u}_t$ is a vector of random disturbances from individual equations with variance-covariance matrix $\mathbf{\Omega}$. Since the elements of $\mathbf{Y}_t$ are $I(1)$, the Wold theorem assures that its first differences have an infinite vector moving average representation, showing the way disturbances of previous periods affect the current value of variables:

$$\Delta\mathbf{Y}_t = \mathbf{C}(L)(\boldsymbol{\delta}\mathbf{\Lambda} + \mathbf{u}_t) \tag{6.17}$$

where $\mathbf{C}(L) = \mathbf{I}_n + \sum_{i=1}^{\infty}\mathbf{C}_i L^i$. Defining

$$\mathbf{C}^*(L) = (\mathbf{C}(L) - \mathbf{C})(1 - L)^{-1} \tag{6.18}$$

(6.17) can be expressed as:

$$\Delta\mathbf{Y}_t = \mathbf{C}\boldsymbol{\delta}\mathbf{\Lambda} + \mathbf{C}\mathbf{u}_t + \mathbf{C}^*(L)\Delta\mathbf{u}_t. \tag{6.19}$$

Johansen [78] has demonstrated that $\mathbf{C}$ is the long-run impact matrix defined as

$$\mathbf{C} = \boldsymbol{\beta}_\perp(\boldsymbol{\alpha}'_\perp(\mathbf{I} - \sum_{i=1}^{k-1}\mathbf{\Gamma}_i)\boldsymbol{\beta}_\perp)^{-1}\boldsymbol{\alpha}'_\perp, \tag{6.20}$$

where $\boldsymbol{\alpha}_\perp$ and $\boldsymbol{\beta}_\perp$ are orthogonal complements to $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, such that $\boldsymbol{\alpha}'\boldsymbol{\alpha}_\perp = 0$ and $\boldsymbol{\beta}'\boldsymbol{\beta}_\perp = 0$. Integrating (6.19) gives a multivariate version of the Beveridge-Nelson decomposition of $\mathbf{Y}_t$

$$\mathbf{Y}_t = \mathbf{Y}_0 + \mathbf{C}\boldsymbol{\delta}\sum_{i=1}^{t}\mathbf{\Lambda}_t + \mathbf{C}\sum_{i=1}^{t}\mathbf{u}_i + \mathbf{C}^*(L)\mathbf{u}_t. \tag{6.21}$$

Here, $\alpha_{\perp}$ gives the vectors defining the space of the common stochastic trends, and therefore should be informative about the key driving variable(s) in the system. The $\beta_{\perp}$ vector gives the loadings associated with $\alpha_{\perp}$ i.e. the series which are driven by the common trends. Thus, the $\mathbf{C}$ matrix measures the combined effects of these two orthogonal complements.

Equation (6.21) shows the decomposition of matrix polynomial $\mathbf{C}(L)$ into a permanent part $\mathbf{C}$ and a transitory lag distribution $\mathbf{C}^*(L)\Delta\varepsilon_t$. The second term on the right-hand side of (6.21) consists of n random walks $\sum_{i=1}^{t}\mathbf{u}_i$ which are multiplied by a matrix $\mathbf{C}$ of rank $n-r$. Thus, there are actually $n-r$ stochastic trends driving the system. One may call $\mathbf{Y}_t$ an $I(1)$ process if there are actually $I(1)$ trends (random walks) in the representation (6.21).

Following (6.21), the permanent component of $\mathbf{Y}_t$, adjusted for exogenous variables is

$$\mathbf{Y}_t^P = \mathbf{Y}_0 + \mathbf{C}\delta\sum_{i=1}^{t}\mathbf{\Lambda}_t + \mathbf{C}\sum_{i=1}^{t}\mathbf{u}_t + \mathbf{F}\mathbf{\Psi}\sum_{i=1}^{t}\Delta\mathbf{Z}_t \tag{6.22}$$

where $\mathbf{F}$ is

$$\mathbf{F} = \mathbf{J}\left[\mathbf{I}_{n+r} - \mathbf{A}\right]^{-1}\begin{bmatrix}\mathbf{\Psi}\\ \beta\mathbf{\Psi}\end{bmatrix} \tag{6.23}$$

$\mathbf{J}$ is a selection matrix $\mathbf{J} = [\mathbf{I}_n \quad 0]$ and $\mathbf{A}$ is $(n+r)\times(n+r)$ matrix

$$\mathbf{A} = \begin{bmatrix}\mathbf{\Gamma} & \alpha\\ \beta'\mathbf{\Gamma} & \beta'\alpha\end{bmatrix}. \tag{6.24}$$

The transitory component can thus be estimated with

$$\mathbf{Y}_t^T = \mathbf{Y}_t - \mathbf{Y}_t^P. \tag{6.25}$$

The $\mathbf{C}$ matrix contains useful information on the overall effects of the stochastic driving forces in the system. The columns show how the cumulated residuals from each VAR equation load into the variables individually. Hence, a column of insignificant coefficients means that the corresponding variable has only exhibited transitory effects on the variables in the system and significant coefficients in a column mean that the variable in question has affected some variables of the system permanently. The rows of the $\mathbf{C}$ matrix show the weights with which each of the variables in the system have been affected by cumulated empirical shocks.

Table 6.6 presents the parameters of the $\mathbf{C}$ matrix with their Students's t-values below in brackets. The variable *np* is permanently influenced by all shocks and among them the permanent influence of shocks in u_{np}, u_{oil} and u_{eex} appears to be the highest and most significant. Similarly, we can find a significant permanent influence of shocks in endogenous variables on all endogenous variables, except for the case of shocks in u_{eua}, which does not seem to have a permanent impact on *eex*. Shocks in weakly exogenous variables also permanently influence endogenous variables, with the exception of u_{coal} with insignificant permanent influence on *eua*. Shocks in u_{gas} also seem to have significant permanent influence on endogenous variables, although the parameter values are small and t-values are slightly above significance.

Table 6.6: **C** matrix (with t-values below)

Variable	u_{np}	u_{oil}	u_{coal}	u_{gas}	u_{eua}	u_{eex}	u_{alu}
np	0.464 (5.27)	0.425 (6.55)	0.224 (2.62)	0.157 (2.91)	−0.091 (−2.35)	−0.338 (−2.78)	0.257 (3.90)
oil	0.145 (1.29)	0.904 (11.43)	−0.218 (−1.94)	0.150 (2.09)	0.034 (0.67)	−0.232 (−1.48)	−0.075 (−0.88)
$coal$	0.025 (0.32)	0.021 (0.38)	0.880 (11.36)	0.104 (2.09)	−0.018 (−0.50)	0.008 (0.07)	−0.076 (−1.29)
gas	0.042 (0.28)	0.328 (3.06)	−0.256 (−1.67)	1.103 (11.32)	−0.016 (−0.23)	−0.016 (−0.07)	−0.030 (−0.26)
eua	−0.682 (−2.03)	1.187 (4.46)	0.159 (0.52)	0.291 (1.54)	1.035 (7.83)	−1.341 (−2.95)	0.594 (2.45)
eex	−0.356 (−4.61)	0.408 (7.20)	0.160 (2.12)	0.122 (2.55)	−0.048 (−1.42)	0.499 (4.66)	0.298 (5.15)
alu	−0.040 (−0.54)	−0.012 (−0.24)	−0.010 (−0.13)	0.082 (1.75)	−0.014 (−0.43)	0.074 (0.72)	0.773 (13.93)

Figures 6.4, 6.5 and 6.6 present the actual, permanent and transitory component for three endogenous variables *np*, *eua* and *eex*. The permanent and actual components are closely related in all three cases. The permanent components therefore show a common long-run (permanent) impact due to the shocks in all variables.

A closer look at permanent components reveals that permanent components are somehow preceding the movements of the actual component. In 2005 and in most of 2006, when energy prices were increasing, the actual components were lagging behind the permanent components and the transitory components were negative on average. At the end of 2006 and in beginning of 2007, a period of decreasing prices was again preceded by the permanent components and the transitory components were positive on average during this period. Only in the second half of 2007, the actual and the permanent components were closely related. The transitory component in *eua* is approximately 5 times higher than the transitory components in *np* and *eex*. A similarity can be observed in the movements of all three transitory components. This implies that they are governed by few common cycles.

6.4.2 Common trends and cycles

The decomposition of the **C** matrix is similar to that of the **Π** matrix and establishes the dual property between the **C** matrix and **Π** matrix. If we set

$$\boldsymbol{\beta}_\perp^* = \boldsymbol{\beta}_\perp(\boldsymbol{\alpha}_\perp'(\mathbf{I} - \sum_{i=1}^{k-1} \boldsymbol{\Gamma}_i)\boldsymbol{\beta}_\perp)^{-1} \tag{6.26}$$

then we can rewrite

$$\mathbf{C} = \boldsymbol{\beta}_\perp^* \boldsymbol{\alpha}_\perp' \tag{6.27}$$

which is similar to decomposition of **Π**. In the **Π** matrix form, $\boldsymbol{\beta}$ determines the common long-run relationships and $\boldsymbol{\alpha}$ the loadings, whereas in the moving average representation $\boldsymbol{\alpha}_\perp'$ determines the common stochastic trends driving the long-run relation out of equilibrium and $\boldsymbol{\beta}_\perp^*$ defines the loadings (Juselius [82]). The non-stationarity

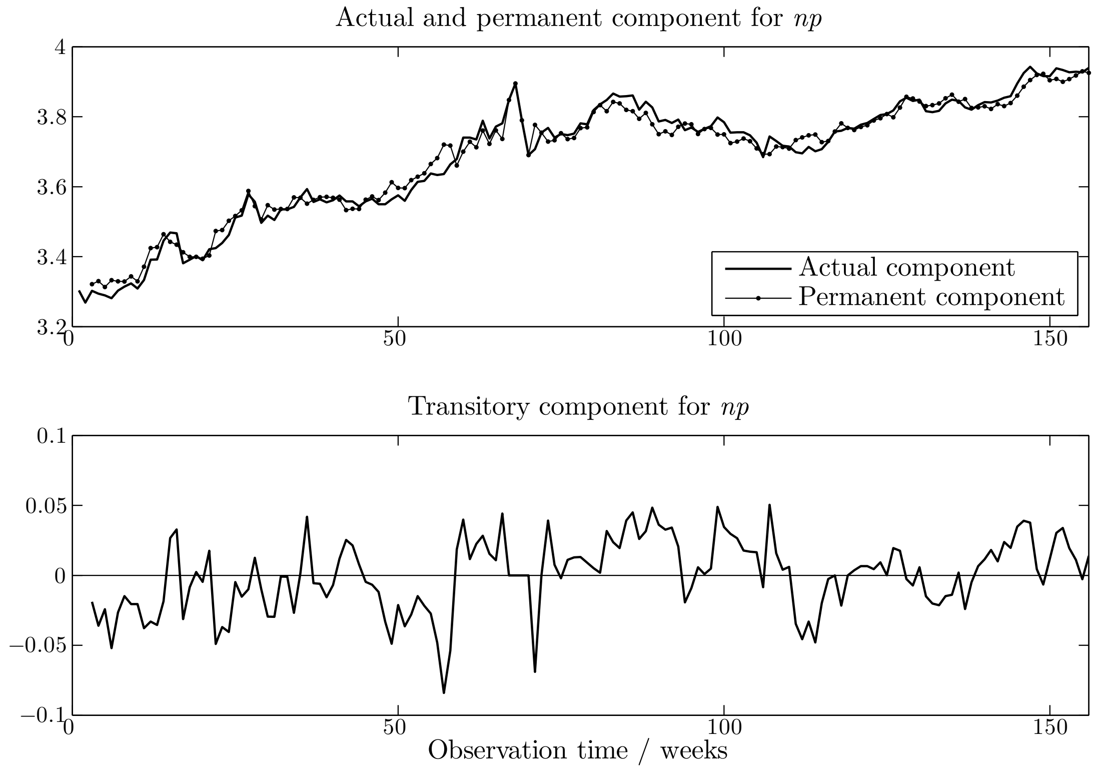

Figure 6.4: Permanent and transitory component for *np*

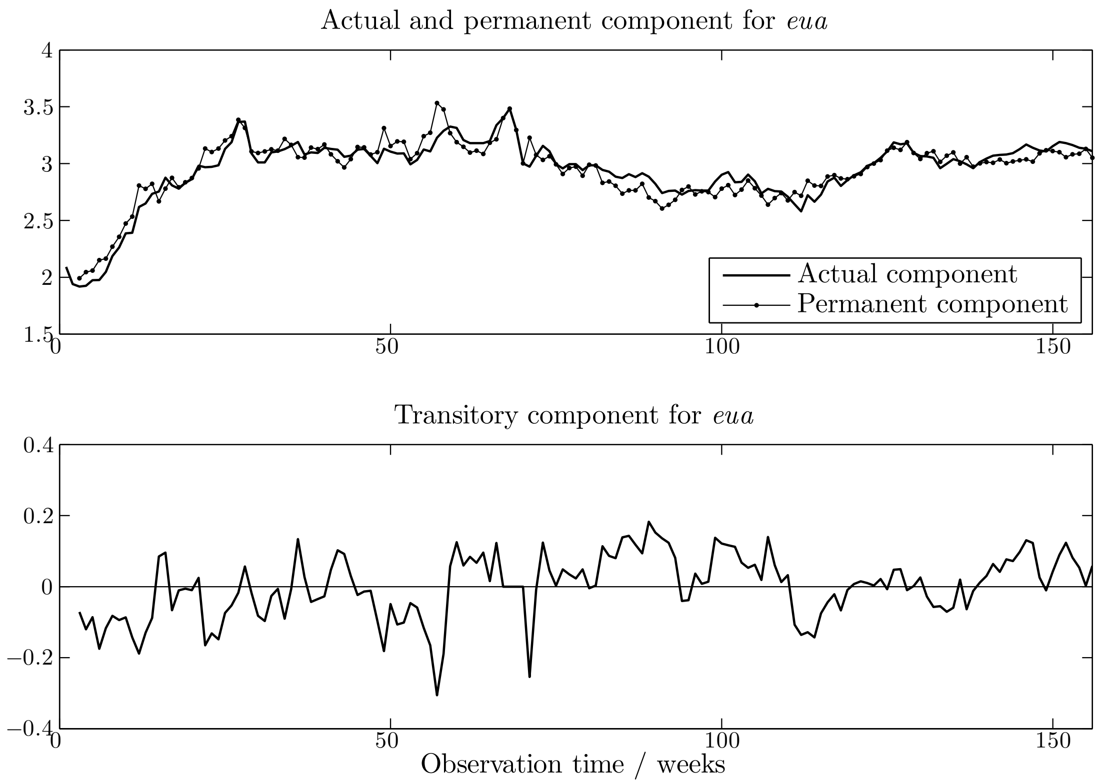

Figure 6.5: Permanent and transitory component for *eua*

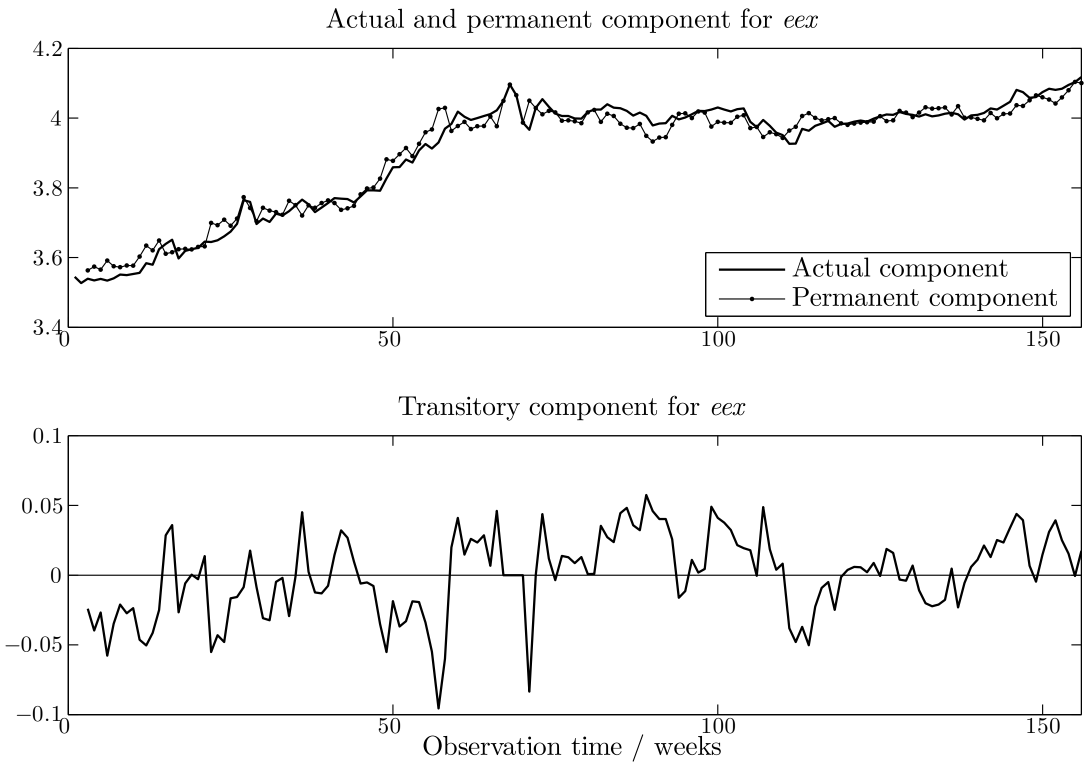

Figure 6.6: Permanent and transitory component for *eex*

of the $\mathbf{Y}_t$ therefore originates from the cumulative sum of $n - r$ combinations of $\alpha'_\perp \sum_{i=1}^{t} \mathbf{u}_t$. These are the common driving trends of the variables in $\mathbf{Y}_t$. A note however is necessary, that these trends are just one of many possible representations of non-stationarity, since the linear combinations of these common stochastic trends also form a common stochastic trend. In (6.28) and (6.29), we present $n-r$ matrices $\alpha'_\perp$ and $\beta^*_\perp$. A zero row in α corresponds to a unit vector in $\alpha'_\perp$.

$$\alpha'_\perp = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -0.804 & 0 & 0 & 0 & 0.552 & -0.220 & 0 \\ -0.395 & 0 & 0 & 0 & -0.220 & 0.892 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6.28}$$

$$\beta^*_\perp = \begin{bmatrix} 0.425 & 0.223 & 0.157 & -0.349 & -0.465 & 0.257 \\ 0.904 & -0.218 & 0.149 & -0.046 & -0.272 & -0.075 \\ 0.020 & 0.880 & 0.103 & -0.031 & 0.001 & -0.076 \\ 0.328 & -0.256 & 1.103 & -0.039 & -0.027 & -0.030 \\ 1.187 & 0.159 & 0.291 & 1.415 & -1.154 & 0.594 \\ 0.408 & 0.160 & 0.122 & 0.150 & 0.597 & 0.298 \\ -0.012 & -0.010 & 0.082 & 0.008 & 0.085 & 0.773 \end{bmatrix} \tag{6.29}$$

Among the $n-r$ stochastic trends, two represent a linear combination of cumulated shocks u_{np}, u_{eua} and u_{eex}. The remaining stochastic trends represent the cumulated shocks in weakly exogenous variables u_{oil}, u_{coal}, u_{gas} and u_{alu}. $\beta_{\perp}^{*}$ represents the loading weights how each of the $n-r$ stochastic trends enter each equation in (6.22). Results in (6.29) show that stochastic trends in weakly exogenous variables are mostly influenced by its own cumulated shocks. Endogenous variables, however, show a strong influence of the two combined stochastic trends and to a less degree the influence of cumulated shocks in weakly exogenous variables. Among them, the shocks in u_{alu} have a particularly significant influence on common trends in endogenous variables.

In 6.30 and Table 6.7 we present a restricted $\beta_{\perp}^{*}$ and the corresponding $\mathbf{C}$ matrix. The estimates are obtained by restricting the least significant coefficients in $\beta_{\perp}^{*}$, until the LR test on restricted log-likelihood function is rejected. Since these restrictions apply only to $\beta_{\perp}^{*}$, $\alpha_{\perp}$ is unchanged. The results in equation 6.30 and Table 6.7 are obtained with 23 over identifying restrictions on $\beta_{\perp}^{*}$ which gives 27 zero parameters in the $\mathbf{C}$ matrix. The LR-test statistic is $LR = 27.651$ and the corresponding p-value is $p = 0.275$. The results of the restricted $\mathbf{C}$ show interesting features of the common stochastic trends in the system. Cumulated shocks u_{gas} have significant influence only on *gas*. The cumulated shocks in three weakly exogenous variables influence all endogenous variables, except for the insignificant influence of u_{coal} on *eua*. Beside the influence on endogenous variables and its own, the cumulated shocks u_{oil} also influence *gas*, but not *coal*. Weakly exogenous variables *oil*, *coal* and *alu* are therefore influenced only by own cumulated shocks. The influence of cumulated shocks u_{oil} and u_{alu} on *eex* and *np* show strikingly identical impact to both variables. Cumulated shocks in u_{coal} however have a bit higher impact on *np* compared to the impact on *eex*.

The cumulated shocks in the three endogenous variables influence all three endogenous variables and none of the weakly exogenous variables. Due to restrictions on α this is rather expected. While the impact of own cumulated shocks on endogenous variables is positive, the impact of the other two endogenous variables is negative. This corresponds to the strong contemporaneous correlation between these three variables and implies that these three variables strongly cointegrate. A similar influence is found in cointegrating vector β, which was in our case normalised with respect to *np*. However, normalizing β with respect to *eua* or *eex* would yield negative parameters for the remaining two variables. The parameters in Table 6.7 also show that *eua* is more prone to shocks in other variables since the parameters in *eua* row are at least two times higher than the parameters in *np* and *eex*. Variable *eex* also has higher a influence of cumulated shocks compared to *np*, indicating that, among the three endogenous variables, *eua* is the most sensitive to shocks in the system, while the sensitivity of *np* is the lowest. A relative interdependence of *np* and *eex* also provides an interesting result. The impact of u_{eex} on *eex* is about 39% higher than the impact of u_{np} on *eex*, whereas the impact of u_{np} on *np* is about 70% higher compared to the impact of u_{eex} on *np*. This implies that, in relative terms, the common trend in *eex* is more prone to the shocks in *np* than common trends in *np* to the shocks in *eex*. A part of this can be explained by the fact that *eex* price is constantly higher than *np* due to tighter supply in continental Europe. This indicates that the elasticity of supply might be lower in continental Europe.

Table 6.7: **C** matrix

Variable	u_{np}	u_{oil}	u_{coal}	u_{gas}	u_{eua}	u_{eex}	u_{alu}
np	0.385	0.407	0.410	0	−0.102	−0.226	0.332
	(10.30)	(9.05)	(10.41)	(−)	(−7.66)	(−4.74)	(10.22)
oil	0	0.891	0	0	0	0	0
	(−)	(13.50)	(−)	(−)	(−)	(−)	(−)
coal	0	0	0.934	0	0	0	0
	(−)	(−)	(14.83)	(−)	(−)	(−)	(−)
gas	0	0.302	0	0.882	0	0	0
	(−)	(3.71)	(−)	(17.20)	(−)	(−)	(−)
eua	−0.791	1.239	0	0	1.059	−1.266	0.665
	(−3.54)	(5.92)	(−)	(−)	(11.67)	(−4.11)	(3.70)
eex	−0.394	0.401	0.282	0	−0.049	0.544	0.336
	(−10.75)	(10.64)	(7.71)	(−)	(−3.62)	(11.80)	(9.46)
alu	0	0	0	0	0	0	0.762
	(−)	(−)	(−)	(−)	(−)	(−)	(14.87)

$$\boldsymbol{\beta}_{\perp}^{*} = \begin{bmatrix} 0.407 & 0.410 & 0 & -0.316 & -0.331 & 0.332 \\ 0.891 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.934 & 0 & 0 & 0 & 0 \\ 0.302 & 0 & 0.882 & 0 & 0 & 0 \\ 1.239 & 0 & 0 & 1.500 & -1.049 & 0.665 \\ 0.401 & 0.282 & 0 & 0.170 & 0.652 & 0.336 \\ 0 & 0 & 0 & 0 & 0 & 0.762 \end{bmatrix} \tag{6.30}$$

Figure 6.7 presents a graph of the six common stochastic trends as defined by $\boldsymbol{\alpha}_{\perp}$. A visual inspection shows that the first three and the last common trend have very similar dynamics to *oil*, *coal*, *gas* and *alu* respectively, which are presented in Figure 4.2. These common trends are therefore mainly influenced by own cumulated errors. Weak exogeneity restrictions on $\boldsymbol{\alpha}$ imply that the dynamics in these variables is explained only by the short-run matrix $\boldsymbol{\Gamma}$ and deterministic variables which are all $I(0)$. The source of integration in these variables therefore cannot be explained by any other variable but its own past values, adjusted for the I(0) components. A small exception is *gas*, which is also partly influenced by residuals in *oil*. Contrary to this, the three endogenous variables show the influence of all common stochastic trends except the third (*gas*) and the second (*coal*) in case of *eua*.

While permanent components in variables can be reduced down to few common stochastic trends, the transitory components can also be reduced to few common stochastic cycles. Vahid and Engle [98] gave conditions under which the transitory component of the multiple time series can be represented by few common stochastic cycles. Warne [99] shows that testing for s common cycles is equivalent to testing for a rank of matrix $\boldsymbol{\Theta} = [\boldsymbol{\Gamma}_1, \ldots, \boldsymbol{\Gamma}_{k-1}, \boldsymbol{\alpha}]$. Assuming that $\mathbf{Y}_t$ is subject to s common cycles, it follows that $\boldsymbol{\Theta}$ has less than $n(n(k-1)+r)$ unique parameters. To test for the null hypothesis that $\boldsymbol{\Theta}$ has rank s against the alternative that it has rank $s+g$, restrictions have to be imposed on $\boldsymbol{\Gamma}_i$ and $\boldsymbol{\alpha}$. Warne [99] shows that the number of restrictions is $q = g(nk + r - 2s - g)$ for $1 \leq s \leq n - s$ and that $q > 0$ only when $k \geq 2$. Using the two stage algorithm proposed by Warne [99], a standard LR testing framework can be used to test the hypotheses. The test for the null hypothesis that

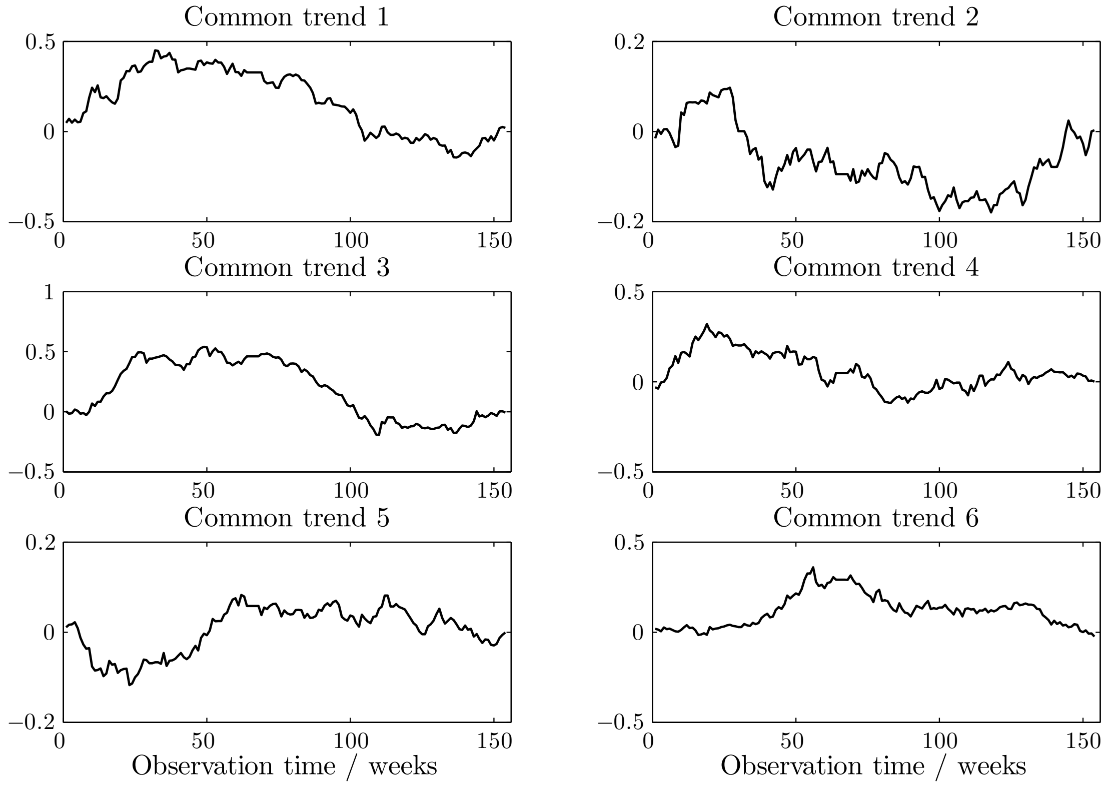

Figure 6.7: Common trends

there are at most s common cycles in the system against the alternative that there are more than s common cycles is presented in Table 6.8. The results show that our system has at most 3 common cycles, although the hypothesis $s \leq 3$ is marginally accepted. This indicated that transitory components in Figures 6.4, 6.5 and 6.6 are composed of three components (cycles). The identification of these common cycles is discussed in works of Vahid and Engle [98], Hecq et al. [100], [101], Kugler and Neusser [102], Vahid and Issler [103] and Paruolo [104]. We leave this for future work.

Table 6.8: Test for common cycles

$s \leq$	LR	df	p-value
1	97.931	48	0.000
2	61.136	35	0.004
3	34.938	24	0.069
4	18.653	15	0.230
5	6.015	8	0.646
6	3.156	3	0.368

6.5 Structural VAR

The impulse response analysis uncovered important relations between the variables in our VECM model, while on the other hand, there are some obstacles in their interpretation. In particular, impulse responses are generally not unique and it is often not clear which set of impulse responses actually reflects the ongoings in the system. Vector autoregressive models are actually reduced form models and structural restrictions are required to identify the relevant innovations and impulse responses. In this chapter, we introduce contemporaneous influences in our VECM specification and impose some identifying restrictions on these relations in order to identify the structural shocks driving the system. While these restrictions can only be set arbitrarily, they must at least follow some economic theory and expectations behind the model.

6.5.1 Setup

A conventional approach to finding a model with instantaneously uncorrelated residuals is to model the instantaneous relations between the observable variables directly. In Section 5.1, we have shown that contemporaneous influences can be modelled directly with a following Structural VAR (SVAR) model:

$$\mathbf{Y}_t \;=\; \mathbf{A}_0 + \sum_{i=1}^{k} \mathbf{A}_i \mathbf{Y}_{t-i} + \mathbf{\Psi} \mathbf{Z}_t + \mathbf{A}^{-1} \mathbf{u}_t \tag{6.31}$$

where $\mathbf{A}$ is a matrix of contemporaneous influences with ones on diagonal (diag$\mathbf{A} = 1$). The $\mathbf{A}$ matrix converts the VAR residuals $\mathbf{u}_t$ to SVAR residuals $\boldsymbol{\varepsilon}_t$ with:

$$\boldsymbol{\varepsilon}_t = \mathbf{A}^{-1} \mathbf{u}_t \tag{6.32}$$

where VAR residuals can be correlated with the residual covariance matrix $\mathbf{\Omega}_u$ i.e. $\mathbf{u}_t \sim (\mathbf{0}, \mathbf{\Omega}_u)$, whereas the SVAR residuals are assumed to be uncorrelated, standardised normal variables with zero mean $\boldsymbol{\varepsilon}_t \sim (\mathbf{0}, \mathbf{A}\mathbf{\Omega}_u\mathbf{A}')$. Thus, for a proper choice of $\mathbf{A}$, $\boldsymbol{\varepsilon}_t$ will have a diagonal covariance matrix. From the relation

$$\mathbf{\Omega}_\varepsilon = \mathbf{A}\mathbf{\Omega}_u\mathbf{A}' \tag{6.33}$$

and the requirement that $\mathbf{\Omega}_\varepsilon$ is a diagonal matrix, we need to impose exactly $n(n-1)/2$ restrictions on $\mathbf{A}$. Since the diagonal elements of $\mathbf{A}$ are assumed to be normalised to 1, additional restrictions can be imposed on the upper off-diagonal elements of $\mathbf{A}$. If they are restricted to 0, matrix $\mathbf{A}$ has the following form

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ a_{21} & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & 1 \end{bmatrix}. \tag{6.34}$$

This indicates that variable $Y_{1,t}$ can have contemporaneous impact on all other variables, variable $Y_{2,t}$ can have contemporaneous impact on all other variables, except $Y_{1,t}$ and so on. If (6.31) is transformed to MA representation such as (6.7), the resulting impulse responses are qualitatively the same as the orthogonalised impulse responses based on the Cholesky decomposition of (6.11) which were considered in Section 6.2. The only difference is that, for the latter case, the impulses have unit variances which is not the case for the presently considered impulses.

Alternative formulation of Structural VAR model is represented by matrix $\mathbf{B}$ which is obtained by relaxing the definition of $\mathbf{A}$ so that the diagonal elements of $\mathbf{A}$ are no longer assumed to be normalised to have unitary coefficients

$$\mathbf{Y}_t = \mathbf{A}_0 + \sum_{i=1}^{k} \mathbf{A}_i \mathbf{Y}_{t-i} + \mathbf{\Psi} \mathbf{Z}_t + \mathbf{B} \boldsymbol{\varepsilon}_t \tag{6.35}$$

The $\mathbf{B}$ matrix converts the VAR residuals $\mathbf{u}_t$ to SVAR residuals $\boldsymbol{\varepsilon}_t$, usually called *structural shocks*, with:

$$\boldsymbol{\varepsilon}_t = \mathbf{B}^{-1} \mathbf{u}_t \tag{6.36}$$

where $\mathbf{\Omega}_u = \mathbf{B} \mathbf{\Omega}_\varepsilon \mathbf{B}'$. Normalizing the variances of the structural shocks to one i.e. assuming $\boldsymbol{\varepsilon}_t \sim (\mathbf{0}, \mathbf{I}_n)$ gives

$$\mathbf{\Omega}_u = \mathbf{B}\mathbf{B}'. \tag{6.37}$$

Similar to matrix $\mathbf{A}$, we need to impose $n(n-1)/2$ restrictions on $\mathbf{B}$ in order for SVAR in (6.35) to be identified. Again, choosing $\mathbf{B}$ to be lower-triangular, for example, provides sufficiently many restrictions. Such identifying restrictions are, however, different to Cholesky decomposition used in orthogonal impulse response function. The recursive structure of $\mathbf{B}$ is chosen only if it has some theoretical justification, so that $\boldsymbol{\varepsilon}_t$ can be regarded as orthogonal structural shocks, which can be labelled with some real or theoretical variables. This specification also allows zero restrictions in $\mathbf{B}$, other than the one needed for exact identification, leading to over-identified Structural VAR. Using the VECM MA representation we can rewrite (6.21) to:

$$\mathbf{Y}_t = \mathbf{Y}_0 + \mathbf{C}\boldsymbol{\delta} \sum_{i=1}^{t} \mathbf{\Lambda}_t + \mathbf{C}\mathbf{B} \sum_{i=1}^{t} \boldsymbol{\varepsilon}_i + \mathbf{C}^*(L)\mathbf{B}\boldsymbol{\varepsilon}_t. \tag{6.38}$$

Since the term $\mathbf{C}\mathbf{B}\sum_{i=1}^{t} \boldsymbol{\varepsilon}_i$ measures the permanent common trends in $\mathbf{Y}_t$, then $\overline{\mathbf{C}} = \mathbf{C}\mathbf{B}$ is a total impact matrix, which measures the long-run effects of the structural shocks. Imposing the restrictions directly on matrix $\mathbf{B}$ is in fact not necessary for identifying the structural innovations and impulse responses. Another type of restrictions is followed by Blanchard and Quah [105]. They considered the accumulated effects of structural shocks to the system by focusing on the total impact matrix $\overline{\mathbf{C}}$. They assumed that some shocks do not have any total long-run effects, which corresponds to zero restrictions in matrix $\overline{\mathbf{C}}$. If restrictions are placed solely on $\mathbf{B}$ or $\overline{\mathbf{C}}$, in both cases exactly $n(n-1)/2$ restrictions are needed for exact identification. Combining the restrictions on $\mathbf{B}$ and $\overline{\mathbf{C}}$ is also possible. If, for example, m structural shocks have no long-run effect on p variables, this corresponds to mp zero restrictions on $\overline{\mathbf{C}}$, while the remaining $n(n-1)/2 - mp$ restrictions must be imposed on $\mathbf{B}$. Matrix $\mathbf{B}$ and $\overline{\mathbf{C}}$ are estimated with the Maximum Likelihood estimation algorithm proposed by Amisano and Giannini [106].

6.5.2 Contemporaneous restrictions

In the following, we try to identify the structural shocks and their meaning by placing different restrictions on **B** and $\overline{\mathbf{C}}$, which are subject to theoretical expectations. In first case, we assume only contemporaneous restrictions i.e. all structural shocks are assumed to have long-run impact. We use standard recursive restrictions on **B** by assuming some recursive causal structure between variables. Here, we use the findings from Granger causality test, impulse response analysis, variance decomposition and covariance matrix. Granger causality test gives a clear signal that endogenous variables *np*, *eua* and *eex* are Granger caused by all other variables, therefore these variables should come last in the causal structure. Among the weakly exogenous variables, bidirectional causality is found between *oil* and *gas*. To set their causal order, we use a standard assumption that *oil* influences *gas* and not vice versa. Since *gas* may also Granger cause *coal* we place *oil* first, *gas* and *coal* third, whereas *alu* is not Granger caused by any variable in the system, therefore it can be placed fourth.

Determining the causal structure among endogenous variables is less straightforward. We place *eua* fifth, since we assume that, theoretically, *eua* should influence both electricity prices more than vice versa. The same causal ordering was derived and assumed by Fezzi [107]. The last choice between *eex* and *np* is quite arbitrarily since we have found no strong indicators of unidirectional causality between them. For this reason, we estimate two different **B** matrices, using the same causal structure from *oil* to *eua*, and with two different ordering between *np* and *eex*.

The results of the contemporaneous restrictions on **B**, using the causal structure described above are presented in Table 6.9. The table presents parameters of **B** with their t-values below in brackets. The first structural shock is a direct replication of *oil* reduced form residuals. This shock can be termed as *oil shock*. The second shock is a linear combination of *oil* and *gas* reduced form residuals and both parameters are significant. It represents the residual information in *gas* residuals which is not already present in *oil* residuals. A similar structure can be found in the third and the fourth structural shock. They appear to be a linear combination of corresponding reduced form residuals and *oil* or *gas* residuals. Since these variables are weakly exogenous, their reduced form residuals consists mostly of their past shocks. Therefore, the first four structural shocks represent mainly the shocks in *oil*, *gas*, *coal* and *alu* respectively. However, the second, the third, and the fourth structural shock are slightly corrected for the instantaneous influence of *oil* or *gas*. Without any loss of generality, these three structural shocks can be labelled as *residual gas, coal* and *alu* shocks respectively.

A similar, though more complex, picture can be observed in the last three structural shocks. They are, again, dominated by the respective reduced form residuals, however, adjusted for the structural shocks above the causal chain. Therefore, we cannot label them in any other way than *residual eua, eex* and *alu* shocks. Another way of looking at this structure is to explain how much of the variation in reduced form residuals is the result of new information in own variable and how much of variation is simply the result of variation in other variables above the causal chain structure. We can observe that *alu* shocks have no significant contemporaneous influence on any other variable, while *coal* shocks influence only *np*. *Oil* and *gas* shocks have contemporaneous influence on all variables except for *coal* and *alu*, respectively.

We can also observe that the amount of variation in *np* and *eex* reduced form residuals due to own shocks is relatively small, compared to other reduced form residuals. This is partly explained by the chosen causal structure; however, any other causal structure would have less theoretical meaning.

Table 6.9: **B** matrix ($\times 10^{-3}$)

Variable	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7
u_{oil}	**25.4** **(17.55)**	0 (−)	0 (−)	0 (−)	0 (−)	0 (−)	0 (−)
u_{gas}	**9.1** **(4.10)**	**26.9** **(17.55)**	0 (−)	0 (−)	0 (−)	0 (−)	0 (−)
u_{coal}	2.2 (1.57)	**6.5** **(4.77)**	**16.2** **(17.55)**	0 (−)	0 (−)	0 (−)	0 (−)
u_{alu}	**5.2** **(3.25)**	0.6 (0.36)	0.8 (0.52)	**19.4** **(17.55)**	0 (−)	0 (−)	0 (−)
u_{eua}	**7.2** **(1.73)**	**20.4** **(5.10)**	−2.2 (−0.57)	4.1 (1.08)	**47.2** **(17.55)**	0 (−)	0 (−)
u_{eex}	**3.0** **(3.02)**	**5.2** **(5.48)**	0.9 (0.98)	1.3 (1.44)	**4.6** **(5.36)**	**10.1** **(17.55)**	0 (−)
u_{np}	**5.5** **(3.75)**	**3.4** **(2.37)**	**3.9** **(2.77)**	−0.4 (−0.32)	**6.2** **(4.65)**	**8.2** **(6.89)**	**13.6** **(17.55)**

In Figure 6.8 the graph of the cumulated structural shocks, estimated as $\sum_{i=1}^{t} \boldsymbol{\varepsilon}_i$, is presented. It represents two cases of different causal ordering between *np* and *eex*, where the first corresponds to the matrix **B** in Table 6.9. The first four structural shocks, therefore, represent mostly the cumulation of *oil*, *gas*, *coal* and *alu* reduced form residuals, multiplied by the respective parameters in $\mathbf{B}^{-1}$. The figure shows that different causal structure does not influence the first five structural shocks. The last two shocks, however, have switched places and the difference between them is rather small. This indicates that the causal ordering between *eex* and *np* has little influence on structural shocks. This indicates that a direct contemporaneous influence between these two variables is significant, however, because these two variables have very similar dynamics, the causal chain between them does not significantly change the results. The long-run impact matrix $\overline{\mathbf{C}}$ with t-values, which corresponds to **B** matrix in Table 6.9 is presented in Table 6.10. The parameters show that weakly exogenous variables are permanently influenced mostly by the corresponding residual structural shocks, except for *gas*, where *oil* shocks are also quite significant. The situation in case of endogenous variables is quite the opposite. Variable *eua* is permanently influenced by residual *oil* and *eua* by all residual shocks except residual *coal* shocks. Variables *eex* and *np* are dominated by *oil* shocks, whereas the influence of other residual shocks is less significant or not significant at all. Interestingly, while residual *eex* shocks have significant contemporaneous influence on *np* residuals, it has no long-run impact on *np*.

In Figure 6.9 we present a graph of the cumulated shock components for *np*, estimated as $\overline{\mathbf{C}} \sum_{i=1}^{t} \boldsymbol{\varepsilon}_i$. These show how estimated structural shocks influence *np*. As shown in Table 6.10, all parameters in row 7 are significant, except for residual *eex* shock parameter. Consequently, the 6$^{\text{th}}$ component is very small compared to other components. Changing the causal structure between *eex* and *np* has little

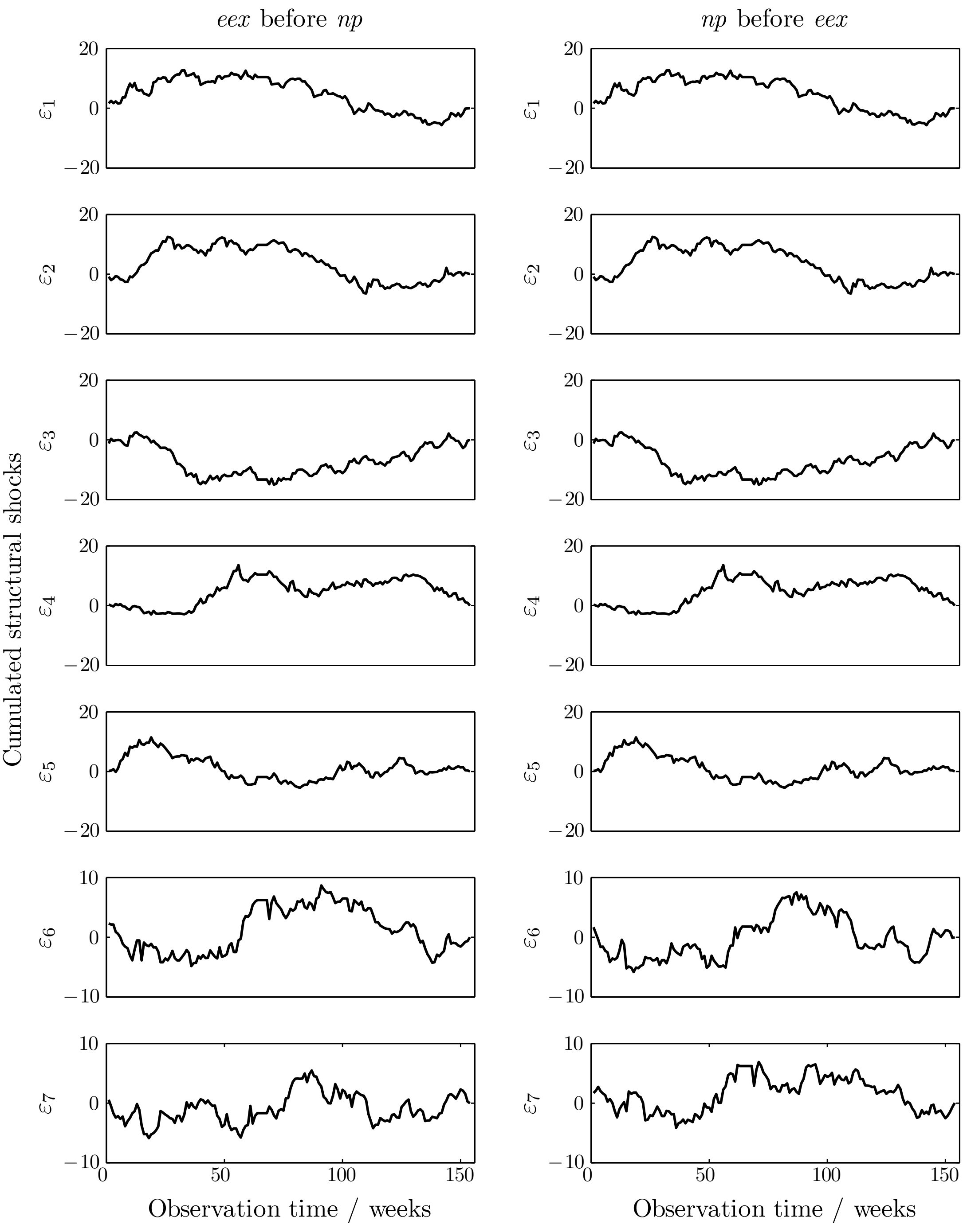

Figure 6.8: Structural shocks

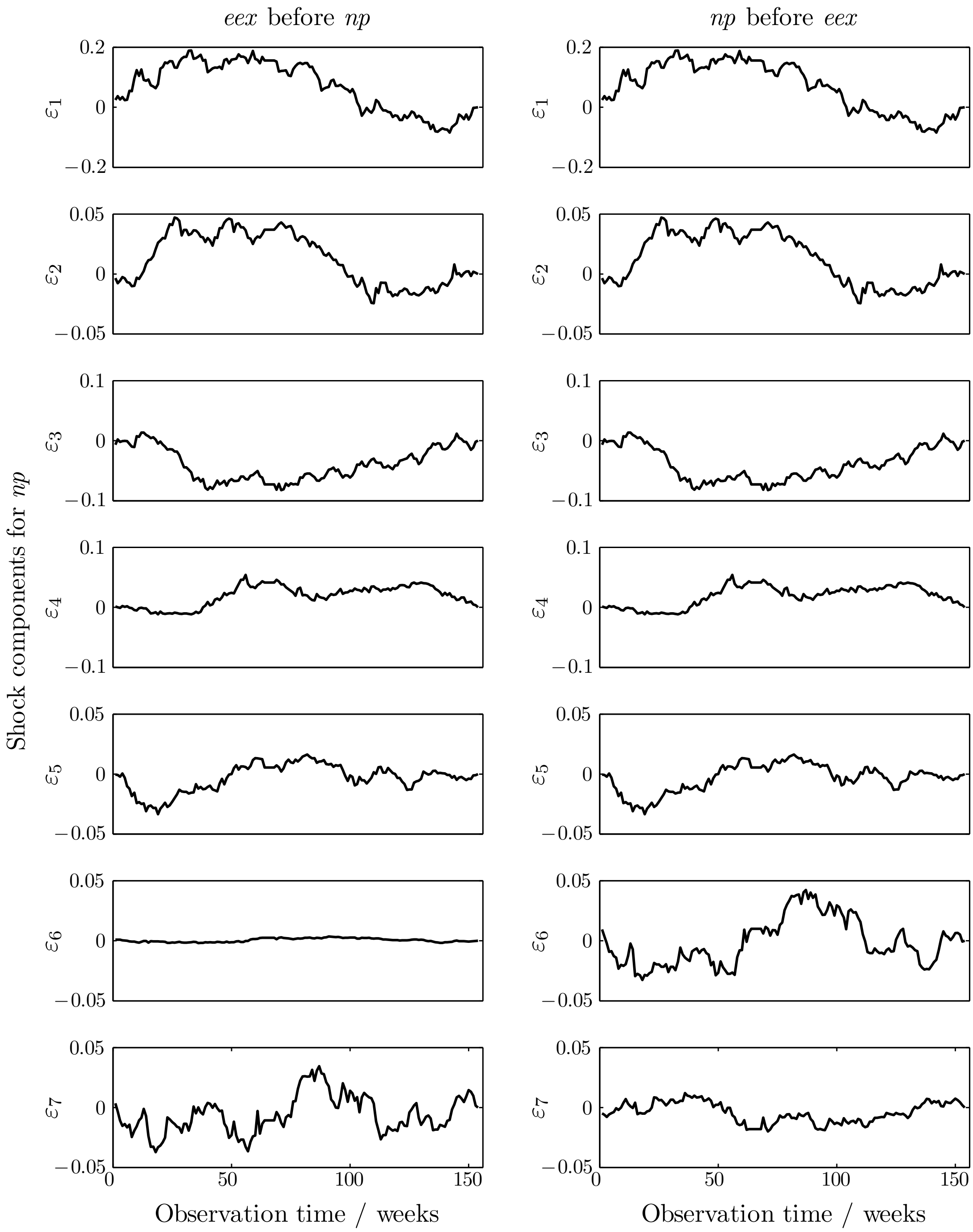

Figure 6.9: Shock components for *np*

Table 6.10: $\overline{\mathbf{C}}$ matrix ($\times 10^{-3}$)

Variable	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7
oil	**23.8**	2.6	**−3.3**	−1.7	1.4	−1.2	2.0
	(9.52)	(1.46)	**(−1.77)**	(−0.98)	(0.74)	(−1.03)	(1.29)
gas	**17.8**	**27.8**	−4.0	−0.7	−0.6	0.2	0.6
	(4.76)	**(9.80)**	(−1.58)	(−0.30)	(−0.21)	(0.13)	(0.28)
coal	3.0	**8.2**	**14.3**	−1.5	−0.6	0.3	0.3
	(1.56)	**(4.76)**	**(9.43)**	(−1.32)	(−0.48)	(0.38)	(0.32)
alu	**4.3**	2.5	0.4	**15.1**	−0.6	0.4	−0.5
	(2.36)	(1.52)	(0.25)	**(10.76)**	(−0.46)	(0.59)	(−0.54)
eua	**35.8**	**21.0**	−3.0	**14.4**	**38.5**	**−19.1**	**−9.3**
	(4.50)	**(3.59)**	(−0.50)	**(2.42)**	**(6.04)**	**(−4.00)**	**(−2.01)**
eex	**12.6**	**4.9**	2.0	**6.4**	−2.2	**2.1**	**−4.9**
	(6.97)	**(3.59)**	(1.42)	**(4.97)**	(−1.56)	**(2.21)**	**(−4.46)**
np	**14.9**	**3.8**	**5.5**	**4.0**	**−2.9**	0.4	**6.3**
	(7.33)	**(2.50)**	**(3.53)**	**(2.78)**	**(−1.83)**	(0.37)	**(5.04)**

effect on the impact of residual *np* shocks. However, the impact sign of the residual *eex* shock is negative (-2.9×10^{-3}), while the parameter t-value is more significant ($t = 2.74$) in case the *np* is ordered before *eex*. This indicates that, in terms of contemporaneous influences, these two variables are equal. However, when the long-run impact is considered, residual *eex* shocks have no long-run impact on *np*, whereas the residual *np* shocks have significant long-run impact on *eex*.

6.5.3 Permanent and transitory shocks

In previous section, we have shown that $\mathbf{C}$ has rank $n-r$ corresponding to $n-r$ common trends in the system having a permanent impact on $\mathbf{Y}_t$. Since $\boldsymbol{\varepsilon}_t$ represents a regular random vector with non-singular covariance matrix, the matrix $\mathbf{B}$ has to be non-singular. Hence, from $\overline{\mathbf{C}} = \mathbf{CB}$ it follows that $\overline{\mathbf{C}}$ has also rank $n-r$ and there can only be r zero columns in this matrix. If there are r transitory shocks, we can restrict r columns of $\overline{\mathbf{C}}$ to zero. Since $\overline{\mathbf{C}}$ has reduced rank $n-r$, each column of zeros stands for $n-r$ independent restrictions only. The r transitory shocks therefore represent only $r(n-r)$ independent restrictions on $\overline{\mathbf{C}}$, while the remaining $(n(n-1)/2 - r(n-r))$ must be imposed on contemporaneous matrix $\mathbf{B}$. Based on the results of previous section, we set the restrictions on $\overline{\mathbf{C}}$ such that the first 6 structural shocks may have a permanent effect on all variables, while the last shock is assumed to be transitory i.e. without any permanent impact. This corresponds to the following 6 restrictions $\overline{C}_{i,7} = 0, i = 1...6$. The remaining restrictions are imposed on $\mathbf{B}$ using the same causal structure as in previous section, however, leaving parameters in the last column free. Tables 6.11 and 6.12 present the resulting $\mathbf{B}$ and $\overline{\mathbf{C}}$. Matrix $\mathbf{B}$ is unchanged from *oil* to *alu*, while significant changes can be observed in the parameters and their significance in the last three rows. It is important to note that different restrictions on $\mathbf{B}$ and $\overline{\mathbf{C}}$ do not change structural shocks as long as these restrictions provide exact identification (i.e. exactly $n(n-1)/2$ linearly independent restrictions). The interpretation of these shocks may however be changed, since in this case they represent a different combination of reduced form residuals. For this reason, giving structural shocks some theoretical meaning is in most cases arbitrary.

Structural shocks represent some hidden dynamics, which cannot be observed on the market. Therefore, in best case, they represent some uncorrelated unobservable variables that jointly influence the variables in the system. The interpretation of the first four structural shocks in Table 6.11 remains unchanged, while the interpretation of the last three structural shocks is even more difficult than in the previous section. When $\mathbf{B}^{-1}$ is considered, these represent a linear combination of all reduced form residuals, although the last three are statistically most significant. Since reduced form residuals are correlated, the same structural shocks can be obtained through the different linear combinations of reduced form residuals. In general, the interpretation of all structural shocks should remain the same, regardless of the restrictions and corresponding linear combinations of reduced form residuals. Among the last three structural shocks, one therefore represents the residual *eua* shock, while the last two represent residual long-term information on electricity supply and demand, which is not included in previous structural shocks. The first is less significant and has a permanent impact on *eex* and *np*, while the second, which is more significant, has only transitory impact.

Table 6.11: $\mathbf{B}$ matrix for permanent and transitory shocks ($\times 10^{-3}$)

Variable	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7
u_{oil}	**25.4** **(17.55)**	0 (–)	0 (–)	0 (–)	0 (–)	0 (–)	0 (–)
u_{gas}	**9.1** **(4.10)**	**26.9** **(17.55)**	0 (–)	0 (–)	0 (–)	0 (–)	0 (–)
u_{coal}	2.2 (1.57)	**6.5** **(4.77)**	**16.2** **(17.55)**	0 (–)	0 (–)	0 (–)	0 (–)
u_{alu}	**5.2** **(3.25)**	0.6 (0.36)	0.8 (0.52)	**19.4** **(17.55)**	0 (–)	0 (–)	0 (–)
u_{eua}	**7.2** **(1.73)**	**20.4** **(5.10)**	−2.2 (−0.57)	4.1 (1.08)	**41.6** **(10.76)**	0 (–)	**22.1** **(4.57)**
u_{eex}	**3.0** **(3.02)**	**5.2** **(5.48)**	0.9 (0.98)	1.3 (1.44)	−0.6 (−0.40)	1.9 (1.20)	**10.9** **(16.55)**
u_{np}	**5.5** **(3.75)**	**3.4** **(2.37)**	**3.9** **(2.77)**	−0.4 (−0.32)	0.5 (0.23)	**−11.9** **(−6.63)**	**12.2** **(8.15)**

Table 6.12 presents the long-run impact matrix $\overline{\mathbf{C}}$ subject to restrictions on the last structural shock which corresponds to zero last column. Again the impact of first four structural shocks remains unchanged. The fifth column is changed only a little, hence the shock components in Figure 6.10 are almost the same as in Figure 6.9. The sixth column now includes significant long-term impacts for both *eex* and *np*, while in case of contemporaneous restrictions only, the sixth shock had no long-run impact on *np*. An interesting case is presented in the value of the long-run impact parameters for *eex* and *np* for sixth structural shock which, according to Table 6.11, represents only the residual information in *np*. This shock has long-run impact on both variables and changing the variable ordering between them does not change these findings. This indicates that part of the contemporaneous variation in *np* helps explaining the long-run variation in *eex* and not vice-versa i.e. the contemporaneous variation in *eex* has only transitory impact on *np*. Interestingly, the parameters

of both long-run impacts for sixth structural shock have different sign, which is a response to the cointegrating relationship. We have shown in the impulse response analysis that a positive shock in *np* has a negative long-run response in *eex* and *np*.

Table 6.12: $\overline{\mathbf{C}}$ matrix for permanent and transitory shocks ($\times 10^{-3}$)

Variable	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7
oil	**23.8**	2.6	**−3.3**	−1.7	1.6	−2.1	0
	(9.52)	(1.46)	**(−1.77)**	(−0.98)	(0.74)	(−1.38)	(−)
gas	**17.8**	**27.8**	−4.0	−0.7	−0.6	−0.5	0
	(4.76)	**(9.80)**	(−1.58)	(−0.30)	(−0.21)	(−0.25)	(−)
coal	3.0	**8.2**	**14.3**	−1.5	−0.7	−0.3	0
	(1.56)	**(4.76)**	**(9.43)**	(−1.32)	(−0.48)	(−0.26)	(−)
alu	**4.3**	2.5	0.4	**15.1**	−0.7	0.6	0
	(2.36)	(1.52)	(0.25)	**(10.76)**	(−0.46)	(0.59)	(−)
eua	**35.8**	**21.0**	−3.0	**14.4**	**43.6**	5.6	0
	(4.50)	**(3.59)**	(−0.50)	**(2.42)**	**(7.03)**	(1.02)	(−)
eex	**12.6**	**4.9**	2.0	**6.4**	−2.5	**5.2**	0
	(6.97)	**(3.59)**	(1.42)	**(4.97)**	(−1.58)	**(4.79)**	(−)
np	**14.9**	**3.8**	**5.5**	**4.0**	**−3.3**	**−6.1**	0
	(7.33)	**(2.50)**	**(3.53)**	**(2.78)**	**(−1.85)**	**(−4.70)**	(−)

6.6 Short-run structure

The focus of analyses in previous sections was mostly on the long-run relationships between the variables in the model. We also analysed few short-run indicators, namely Granger causality, impulse response analysis, forecast error variance decomposition and structural shocks. These indicators explain how the variables in the system respond to instantaneous changes in the variables or to changes in variables a few time periods before. Our VAR model consists of $k = 2$ lags, whereas in VECM representation only $k-1$ lags of variables in first differences are sufficient to capture the short-run dynamics. Since the observation time resolution is one week, $k = 2$ indicates that the short-run dynamics consists of two weeks. The above mentioned analyses show a rather poor short-run structure and a very strong long-run dynamics governed by one cointegrating relationships. In this section, we focus only on the short-run structure and treat cointegrating relationship as predetermined variable.

6.6.1 Identification and estimation

In Chapter 5, we estimate $r = 1$ cointegration relations between $n = 7$ variables. While the cointegration relations explain how the variables interact in the long-run, the short-run structure consists of n equations between n variables $\Delta\mathbf{Y}_t$ which are explained by $n(k-1)$ lagged (predetermined) variables $\Delta\mathbf{Y}_{t-i}$, r lagged equilibrium errors $\boldsymbol{\beta}'\mathbf{Y}_t$ plus the exogenous variables and deterministic terms. In the identification of the long-run structure, there are no predetermined components, whereas in case of the short-run structure $\Delta\mathbf{Y}_{t-i}$ and $\boldsymbol{\beta}'\mathbf{Y}_t$ are predetermined, as well as

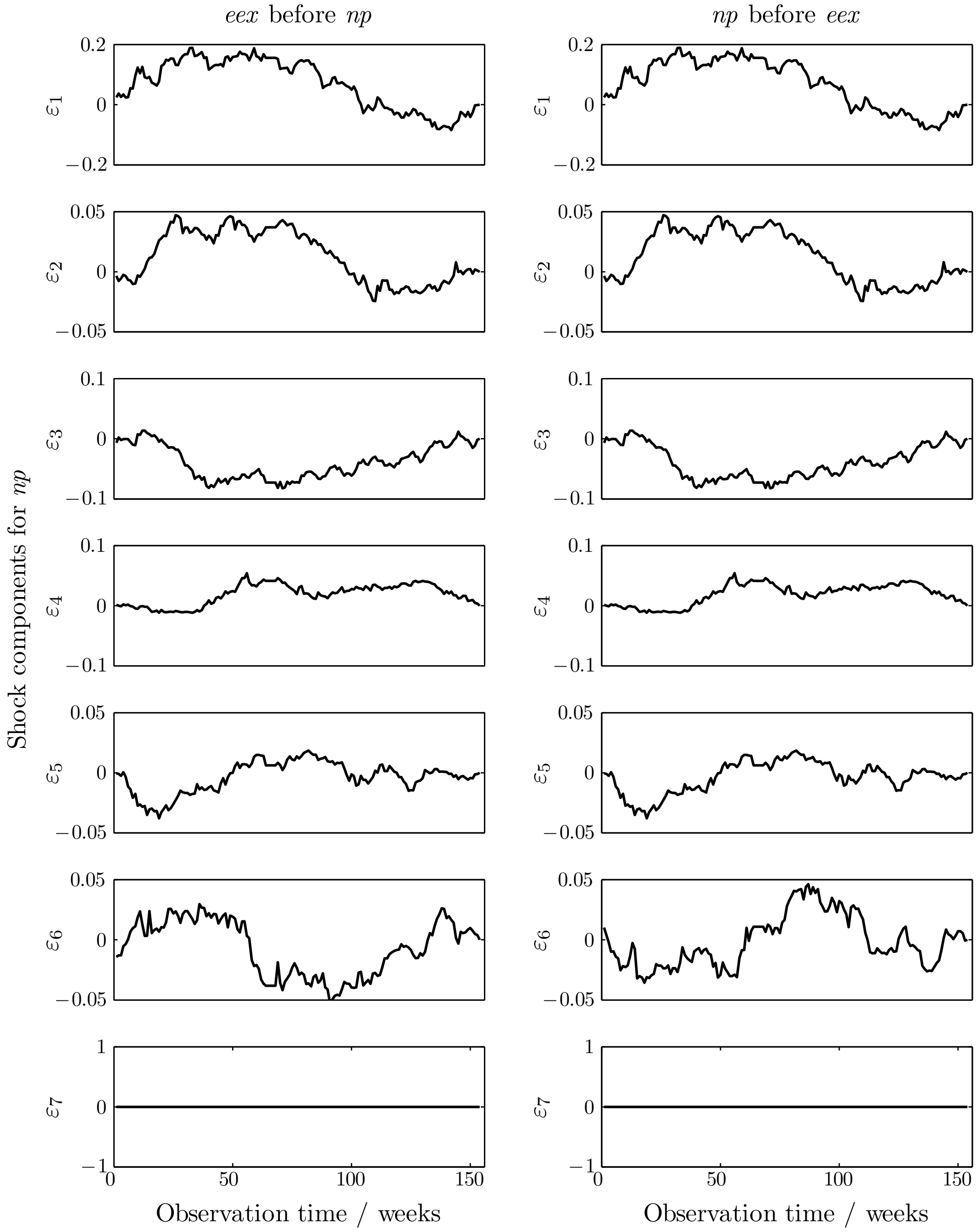

Figure 6.10: Shock components for *np* with permanent and transitory shocks

exogenous and deterministic variables. VECM parameters are estimated with two stage estimation, where in the first stage $\widehat{\beta}$ is estimated, while in the second stage the following VAR is estimated

$$\Delta \mathbf{Y}_t = \mathbf{A}_0 + \alpha\widehat{\beta}'\mathbf{Y}_{t-1} + \sum_{i=1}^{k-1} \mathbf{\Gamma}_i \Delta \mathbf{Y}_{t-i} + \mathbf{\Psi}\Delta \mathbf{Z}_t + \mathbf{\Theta}\mathbf{D}_t + \mathbf{u}_t. \quad (6.39)$$

Estimated $\widehat{\beta}$ is called superconsistent estimator of β, since the speed of convergence to the true value β is proportional to N as $N \to \infty$, while the convergence speed of the estimators of parameters in (6.39) is proportional to $\sqrt{N}$ as $N \to \infty$. If $\widehat{\beta}$ is superconsistent estimator, it can be substituted for the true β and all the other parameters may be estimated in the second stage. Second stage estimation can also be used in case the individual restrictions are put on α, $\mathbf{\Gamma}_i$, $\mathbf{\Psi}$ and $\mathbf{\Theta}$.

Based on cointegration test and weak exogeneity test, we form a VAR as in (6.39). Since *oil*, *coal*, *gas* and *alu* were found to be weakly exogenous, there is no necessity for these variables to be modelled themselves. Hence, the vector of endogenous variables $\mathbf{Y}_t$ now includes only *np*, *eua* and *eex*, while $\mathbf{Z}_t$ includes the first lag of weakly exogenous variables *oil*, *coal*, *gas* and *alu* in first differences, and time-to-maturity in first differences ΔT_m. Estimation of (5.18) therefore includes one lag of three endogenous variables in first differences, the first lag of cointegrating vector, the first lag of differences in three exogenous variables, ΔT_m, eight dummy variables and a constant, giving 9, 3, 12, 3, 24 and 3 parameters respectively, a total of 54 parameters to estimate. Table 6.13 reports the block significance F-tests for individual variables. These tests reveal that among the lags of endogenous and exogenous variables only the first lag of Δnp is close to being significant. Furthermore, cointegrating vector, constant and time-to-maturity in first differences are all strongly significant. Among the dummy variables, only the shock dummies are significant, whereas outlier dummies are not.

Table 6.13: VECM significance test

Variable	$F_{sig}(3,134)$	p-value	Variable	$F_{sig}(3,134)$	p-value
Δnp_{t-1}	2.452	0.066	D_{b67}	**4.689**	**0.004****
Δoil_{t-1}	1.677	0.175	D_{b68}	**5.954**	**0.001****
$\Delta coal_{t-1}$	1.224	0.304	D_{b69}	**6.750**	**0.000****
Δgas_{t-1}	0.403	0.751	D_{b70}	**9.017**	**0.000****
Δeua_{t-1}	1.930	0.128	D_{tr}	**16.608**	**0.000****
Δeex_{t-1}	0.394	0.757	D_{b33}	0.493	0.688
Δalu_{t-1}	0.939	0.423	D_{b57}	1.471	0.225
$Const.$	**16.911**	**0.000****	D_{b117}	1.242	0.297
$\widehat{\beta}'\mathbf{Y}_{t-1}$	**17.044**	**0.000****	ΔT_m	**5.902**	**0.001****
$LLF = 1149.2$	$R^2(LR) = 0.738$	$N = 154$	$m = 54$		

*reject the null at 5% significance **reject the null at 1% significance

The results in Table 6.13 indicate that among the lags of endogenous and weakly exogenous variables only the first lag on *np* in first differences is close to statistical significance in equations for *np*, *eua* and *eex*. We find that *gas* is also insignificant in the short-run structure; therefore, it can be completely removed from the system. This implies that the long-term electricity forwards respond only to prices of crude oil, whereas a gas price is already included within crude oil prices. Also, the gas price used in our model is from the U.K. market, which might not be entirely correct future price indicator for the Nordic market after all. Another reason for the gas price insignificance may be the relative immaturity of the long-term gas market, which might not give enough trust to investors' expectations. Predetermined cointegration vector is strongly significant, with similar value of F-test as constant. Time-to-maturity ΔT_m is also strongly significant in the system. The three outlier dummies D_{b33}, D_{b57} and D_{b117} are insignificant, which indicates that these were necessary to remove the outliers in weakly exogenous variables. *Eua* shock dummies D_{b67}, D_{b68}, D_{b69} and D_{b70} as well as transitory dummy D_{tr} are all strongly significant, since they were included to remove the shocks in two electricity price variables and *eua*.

6.6.2 Reduction

Based on the results in Table 6.13, we reduce the model size with the standard F-test, by excluding the insignificant variables from the system. The reduction is performed sequentially by removing the variable with the lowest block significance in each step until the likelihood ratio test on reduction is rejected. The final model specification is presented in 6.14. The 3-dimensional model now includes only 27 parameters and the value of F-test on reduction is $F(27, 391) = 1.077$ with p-value $p = 0.364$. The reduction test is therefore not rejected, whereas further reductions are rejected at 5% significance level. The reduced model includes the first lag of Δnp, cointegrating vector $\widehat{\boldsymbol{\beta}}'\mathbf{Y}_{t-1}$, a constant, ΔT_m and five dummies only.

Table 6.14: VECM parameters and their significance

Equation	*np*		*eua*		*eex*	
Variable	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
$Const.$	**-0.899**	**0.000****	**-2.137**	**0.000****	**-0.780**	**0.000****
Δnp_{t-1}	-0.059	0.381	**0.474**	**0.011***	0.062	0.181
$\widehat{\boldsymbol{\beta}}'\mathbf{Y}_{t-1}$	**-0.155**	**0.000****	**-0.367**	**0.000****	**-0.134**	**0.000****
ΔT_m	-0.007	0.633	-0.100	0.011*	**-0.031**	**0.001****
D_{b67}	**0.077**	**0.000****	0.085	0.124	**0.035**	**0.012***
D_{b68}	**0.062**	**0.003****	0.080	0.155	**0.052**	**0.000****
D_{b69}	**-0.080**	**0.000****	**-0.155**	**0.007****	-0.014	0.329
D_{b70}	**-0.104**	**0.000****	**-0.235**	**0.000****	**-0.070**	**0.000****
D_{tr}	**0.057**	**0.000****	**0.213**	**0.000****	**0.061**	**0.000****

*reject the null at 5% significance **reject the null at 1% significance

Table 6.14 gives the parameter values and their corresponding p-values for individual equations in VECM. These results show important properties of system dynamics. Constant and cointegrating vector are strongly significant in all three equations. The influence of Δnp_{t-1} is significant only in *eua* equation, indicating that changes in *np* price cause changes in *eua* price in the next period. Other variables therefore significantly influence the endogenous variables only through cointegrating relationship. Most of the dynamic structure is, therefore, either instantaneous or long-run. The first is defined with strong correlation between the variables in the system, while the second is described with the cointegrating relationship and the respective weights for each equation. Due to the reduction of the short-run structure and different estimation technique, the values of the adjustment parameters for the cointegrating vector have changed significantly for *np* and *eex*, compared to the values in Table 5.11. On a contrary, the adjustment parameter for *eua* remained approximately the same. Parameter for ΔT_m is significant only in Δeua and Δeex equation. Assuming that time-to-maturity T_m directly represents the influence of the risk premium, this indicates that the influence of the risk premium can be found only in *eex* and *eua*, while the significance is strongly rejected in *np* ($p = 0.633$). The remaining five dummy variables are all statistically significant, although not in each equation.

6.6.3 Diagnostics

We perform diagnostic test for reduced VECM model in Table 6.14 using the same tools as in Section 5.2. The diagnostic tests presented in Table 6.15 show that after reduction from 54 to 27 parameters, the main properties of residuals remained unchanged with standard errors very close to values in Table 5.2.

Table 6.15: Reduced VECM diagnostic and significance test

Variable	$F_{ar}(4,141)$	$\chi^2_{nd}(2)$	Skewness	$F_{het}(12,132)$	$F_{arch}(4,137)$	SE
Δnp	1.067	18.726**	0.146	0.890	1.042	0.0197
Δeua	1.615	4.145	0.402	0.355	5.122**	0.0541
Δeex	1.461	8.219*	0.141	2.016*	1.151	0.0134
Vector tests: $F_{ar}(36,387) = 1.351$, $\chi^2_{nd}(6) = 23.42^{**}$, $F_{het}(72,696) = 1.097$						
Δnp_{t-1} :	$F_{sig}(4,141) = 4.119^{**}$		$Constant$:		$F_{sig}(7,132) = 15.30^{**}$	
$\widehat{\beta}'\mathbf{Y}_{t-1}$	$F_{sig}(7,132) = 15.46^{**}$		ΔT_m :		$F_{sig}(4,141) = 5.346^{**}$	
$LLF = 1133.1$ $R^2(LR) = 0.677$ $N = 154$ $m = 27$						

*reject the null at 5% significance **reject the null at 1% significance

Since *oil*, *coal*, *alu* and *gas* equations are removed from the system, the autocorrelation in the *coal* and *alu* is no longer a problem. Vector autocorrelation shows a slightly higher autocorrelation, yet still insignificant. Small heteroscedasticity is found in *eex* residuals, which might be due to the reduction and removal of three outlier dummies. Since vector heteroscedasticity is still below significance, we do not consider this as a problem. Normality tests show little improvement and this only due to removal of some variables from the system. Skewness in *eua* residuals increased a little. Vector normality test is decreased notably; however, it is still

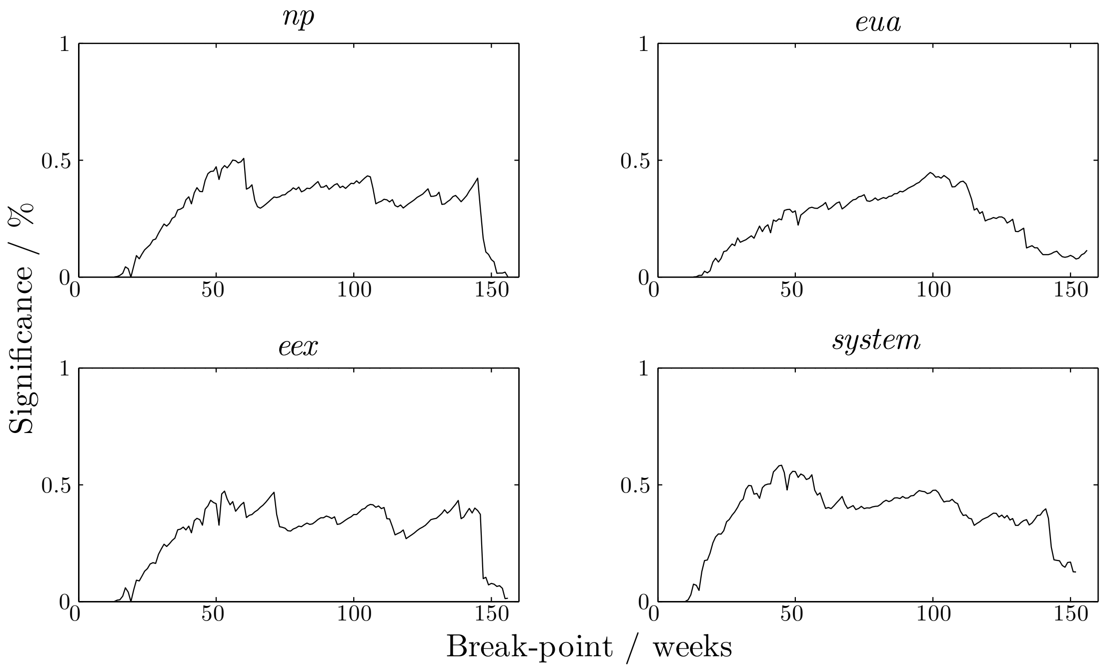

Figure 6.11: Recursive break-point Chow test for reduced VECM

rejected. Contrary to VAR(2) specification in Table 5.2, significant autoregressive conditional heteroscedasticity is found in *eua* residuals. Since *eua* residuals also have a high standard error this implies that with the variables in the system we were unable to explain a large part of *eua* dynamics. Trading with EUA CO_2 emission allowances began only in 2005, implying that the market was more or less immature during this period and proper pricing mechanisms were probably not established for quite some time. The *eua* price shock in April 2006 is also an indication that *eua* market was lacking proper information and knowledge on the part of investors that could establish efficient pricing of EUA CO_2 emission allowances.

To test for the parameters constancy we again perform recursive breakpoint Chow test. Figure 6.11 presents for each equation the results for recursive F-test scaled by 1% critical values from F-distribution as an adjustment for changing degrees of freedom, so that significance values become a constant line throughout the sample. The figure shows that parameters in each individual equation and in the system as a whole are constant regardless of the position of the break-point.

6.7 Applications

The estimated model with long-run structure estimated in Chapter 5 and short run-structure estimated in previous section is mainly designed to capture the dynamics of long-term electricity forwards from Nord Pool. The model could also be applied to model and forecast the other two endogenous variables. It should be noted, however, that the variables in the information set were chosen to match the data valid for Nordic electricity market; therefore it might be the case that different variables would be more appropriate when focusing on EEX electricity market or EU CO_2

emission allowance market. Additional variables might also be beneficial, although we show that the residuals of these two equations satisfy white noise conditions. Regardless of the possibility that all endogenous variables could be modelled and forecasted with this model, in this section we outline only two possible applications of this model with respect to the long-term electricity forwards from Nord Pool.

6.7.1 One-step-ahead forecast

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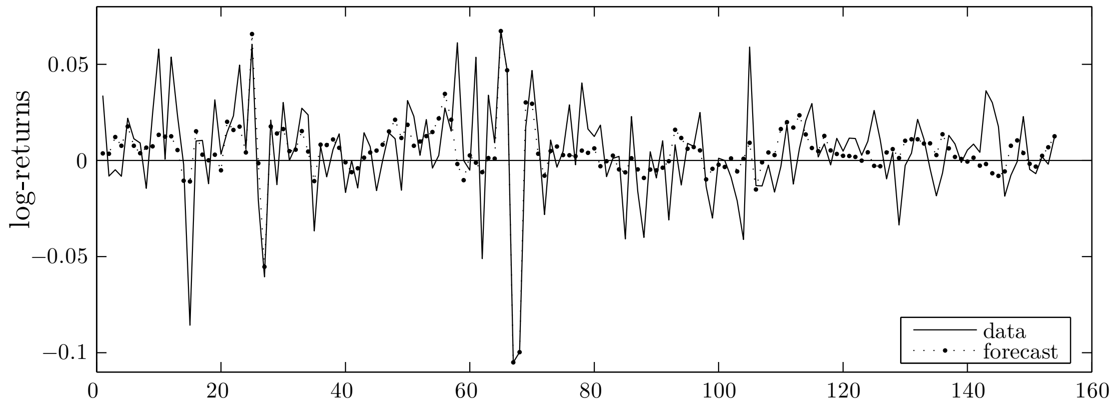
Cointegration between the variables in the system implies that variables do not fluctuate freely without regard to the other variables, but are instead bound together by a long-run equilibrium. If due to random forces some variable deviates from the equilibrium defined by the cointegrating vector β, the adjusting vector α has a tendency to decrease the disequilibrium until the equilibrium state is achieved again. This can have significant implications on the short-term strategy when buying or selling electricity forwards on the exchange. The model could thus be able to estimate the expected value for the next period i.e. the next Wednesday's closing price in our case, based on the information from the current week. We show that this expected value is composed of the long-run equilibrium adjusted with the loading vector α as well as the short-run structure. Besides the expected value, the information on the long-run equilibrium and the disequilibrium existing in current week prices is also very valuable.

The sign of disequilibrium shows the expected sign of the price change in the next week. The realised price might, however, be of different sign, due to random forces or the short-run structure. In Figure 6.12, we present the actual and forecasted value for *np* in log-returns and log-levels. The figure shows that a large part of the realised one-step-ahead change in *np* cannot be forecasted. However, the model is able to forecast a small part of the realised change. The target of the model is not to correct the while disequilibrium in one week, since we show that only about 20% of the disequilibrium in *np* is adjusted in each week. Since the sign of the expected change is also important, we compare the sign of the realised and forecasted one-step-ahead price change. Out of $N - k = 154$ observations the model produces the correct sign of the change in 102 cases. This result must, however, be considered as an in-sample test result, since the short-run parameters and cointegrating vector are estimated on the whole sample.

A true picture of the forecasting performance can only be given by the out-of-sample test. We also perform an out-sample test which is presented in Figure 6.13. While the most accurate picture would be obtained if at each one-step-ahead forecast the model parameters would be updated to match the latest data, we estimate the model parameters only on the in-sample consisting of data from 2005 and 2006, and test the performance of the model on the out-sample consisting of data from 2007. We find that out of 52 out-sample observations the model gives correct sign of the price changes in 32 cases. Figure 6.13, however, shows that the forecasting performance is significantly worse on the out-sample. Based on price dynamics in 2006 and 2007 the model gives slightly higher forecasted prices through most of the 2007. Nonetheless this is only one part of the conclusion. We can also observe that



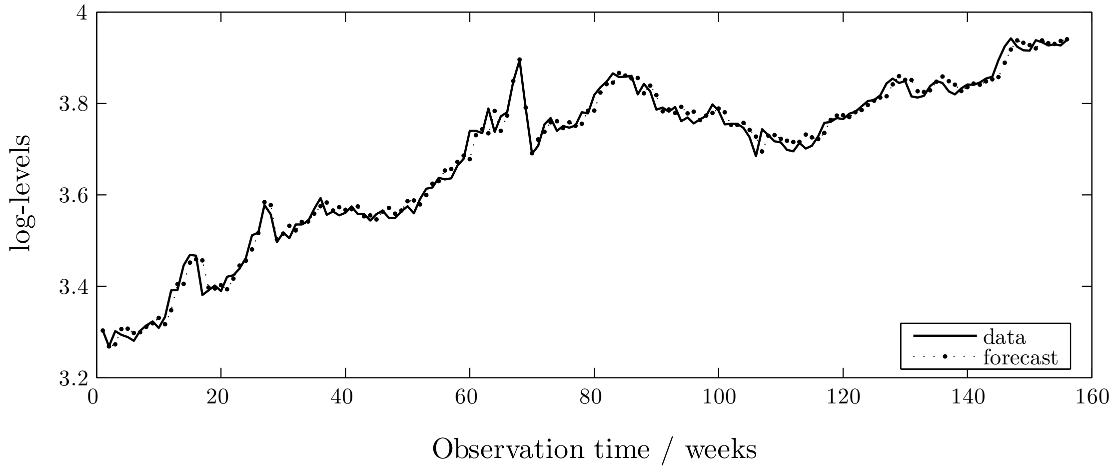


Figure 6.12: In-sample one-step-ahead forecast for *np*. In the upper picture the actual data and one-step-ahead forecast is presented. Out of 154 observations, the sign of actual and forecasted value are the same in 102 cases.

at the end of the year the model is able to return to the realised price level. Even though the cointegrating relationship was estimated one year ago (end of 2006), the model does not make a systematic error in the long-run.

6.7.2 Multi-step-ahead forecast

The estimated model can also be used for multistep-ahead forecast. These forecasts are used to estimate the expected price far in the future, e.g. 5 years ahead. Since the model is in the first differences, price changes are estimated recursively, i.e. the n-step-ahead forecast result is used for forecasting the price change at $n + 1$.

In Figure 6.14, we present an example for multi-step ahead forecast with $n = 260$ weeks, which equals approximately to 5 years. Here we make no assumptions on the dynamics of the weakly exogenous variables, i.e. their values are the same as at the end of our in-sample. In this case, the slope of the forecast is therefore dependent only on the constant and time-to-maturity parameter. Although we show that time-

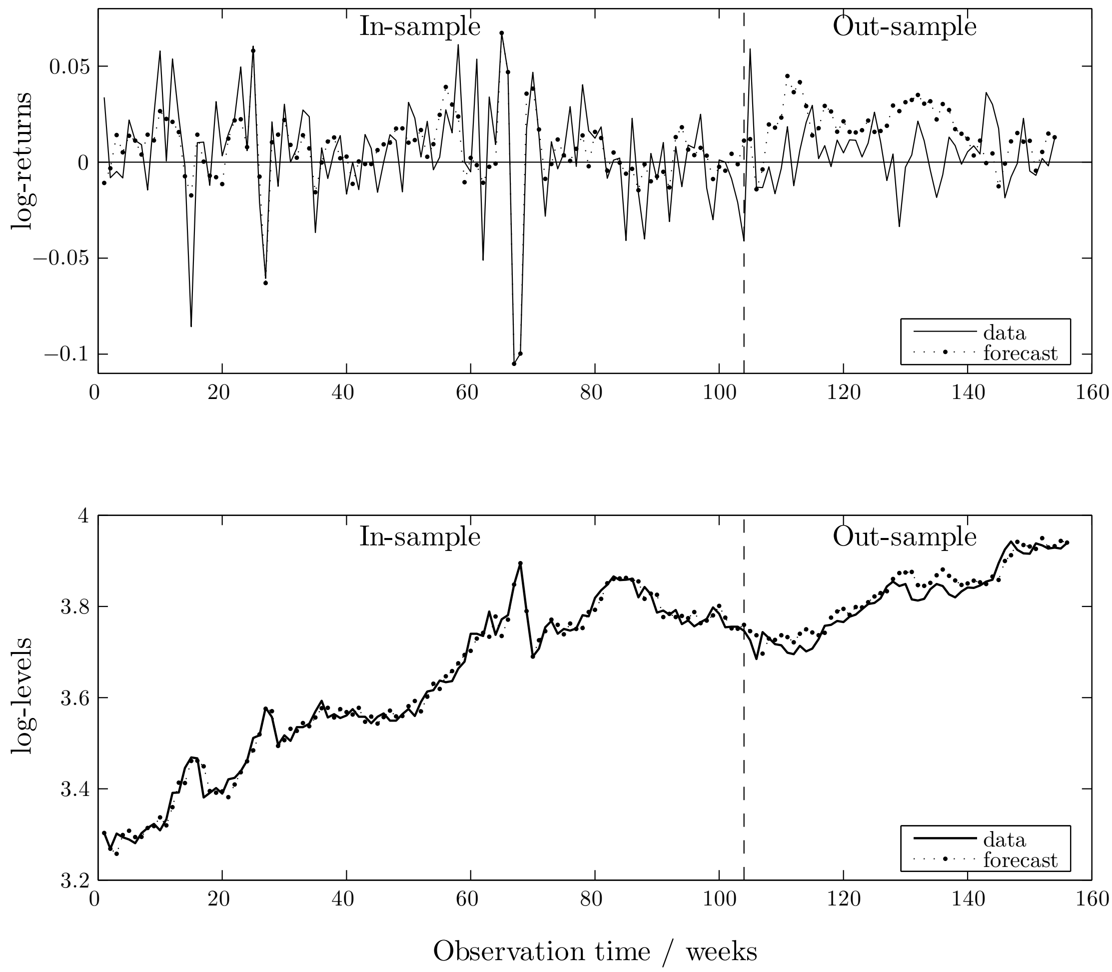


Figure 6.13: Out-sample one-step-ahead forecast for *np*. In the upper picture the actual data and one-step-ahead forecast is presented. The model is estimated on the in-sample consisting for data from 2005 and 2006 and tested on the out-sample consisting on data from 2007. Out of 52 out-sample observations, the sign of actual and forecasted value are the same in 32 cases. In the lower picture the same values are presented in log-levels.

to-maturity is insignificant in *np* equation it is still included in the model with parameter value of -0.007, which influences the discrete change in the forecast at the beginning of each year. Alternatively, we could also take a look at the existing forward curve of weakly exogenous variables and include this data in the forecast. The resulting forecast could be compared to the existing electricity forward price curve observed on e.g. Nord Pool. One could also include the information on the existing forward curves for endogenous variables, in which case the model would be useful to forecast the forward curve beyond the traded horizon. This forecasting method would be the alternative to a simple regression model presented in [5].

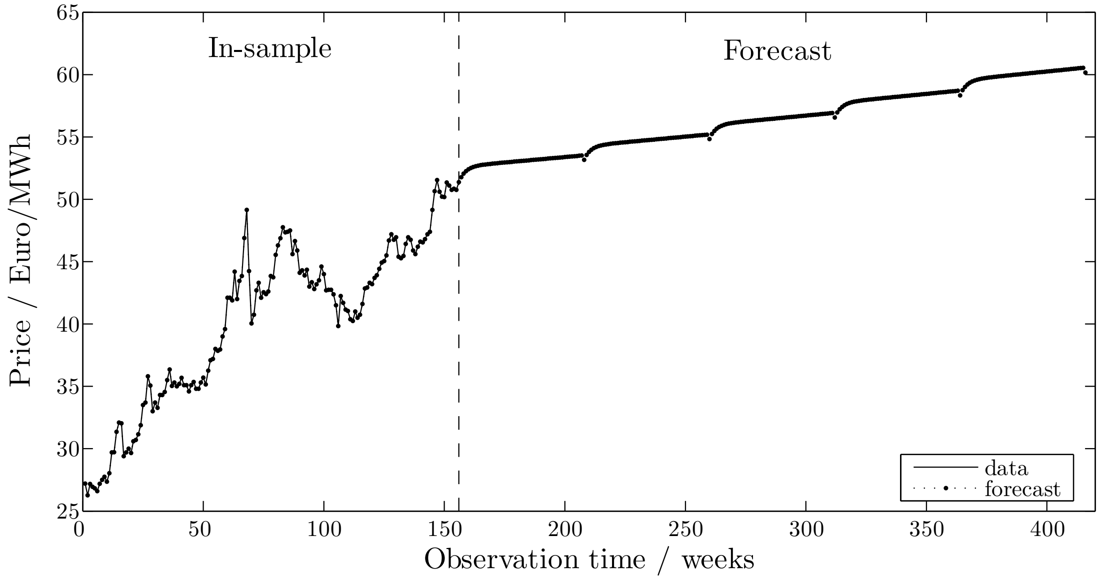


Figure 6.14: Multi-step-ahead forecast for *np*. The forecast assumes no price changes in weakly exogenous variables, therefore the slope of the forecast is determined only by the constant, while the shifts at the beginning of each year are caused by a small time-to-maturity parameter.

6.8 Conclusion

In this chapter we investigate the dynamics driving the VECM system estimated in Chapter 5. First, we perform Wald tests for Granger causality between individual pairs of variables in the system. Granger causality concept, which measures the short-run causal structure, show that endogenous variables are Granger caused by all variables, however, when VECM based on restricted α and β is tested, causality from *gas* is rejected. We also find some causality influences between exogenous variables. Next, impulse response analysis is performed, based on single unit impulse responses and generalised impulse responses. Impulse response analysis shows that the system is mainly governed by long-run impact matrix $\mathbf{\Pi}$, while short-run coefficient matrix $\mathbf{\Gamma}$ significantly dominates the system only in the first week, after which the long-run impact matrix prevails. After an impulse, a steady state of the system is reached in approximately 5 weeks. A similar situation is found with generalised forecast error variance decomposition. The short-run variance is influenced mostly by the past variance in own variable, whereas in the long-run, all the variables have similar influence on the forecast error variance and there is no dominating variable in this respect.

Next we focus on identification of permanent and transitory variable components and system cycles. We identify permanent and transitory components for each variable using multivariate Beveridge-Nielsen decomposition. These show that endogenous variables are mostly governed by permanent components. The identification of common trends shows that four common trends represent cumulated shocks in weakly exogenous variables residuals, whereas two trends represent a linear combination of cumulated shocks in the three endogenous variables residuals. We also find that the former load into weakly exogenous variables, whereas all of them, except cumulated shocks in *gas* , load into endogenous variables. A test for the common cycles shows that transitory variable components compose of at most three common cycles.

A classic reduced form VAR analysed in previous chapters was upgraded in this chapter to Structural VAR representation. The sole purpose of SVAR is to identify the uncorrelated structural shocks, which fundamentally influence the variation in variables in contrast to reduced form residuals which are assumed to be only a linear combination of structural shocks. To identify these structural shocks, we adopt two kinds of causal structure, the first being the contemporaneous causal structure only, while the second identification is obtained by partial restrictions on long-run structural shock impact matrix. Since the last structural shock should have no long-run impact, we restrict the last column of the long-run impact matrix to zero. Both identification structures yield the same structural shocks, since these are independent on the type of identification. The shock components give a slightly different structure for the two types of identification. The results show that the structural shocks cannot be given a specific theoretical meaning. Our interpretation is that they represent the residual or additional information present in each variable that is not already included in the variables above the causal chain structure. We find that these results are relatively robust to different causal structure, since the first four variables are weakly exogenous, whereas the last three structural shocks, representing additional information in *np*, *eua* and *eex* have a minor influence on the variation of these variables. We also find that residual information in *np* has a long-run impact on *np* and *eex*, while residual information in *eex* has only transitory impact on both.

At the end, we identify the short-run structure of our VECM model. The block significance test shows that this structure is very poor when it comes to the influence of past realised changes of exogenous and endogenous variables on future changes of endogenous variables. We reduce the model from 18 parameters in each equation down to 9 parameters. Only the first lag of *np* in first differences is significant in explaining the future changes in *eua*. This indicates that the data generating process might actually be governed by the dynamics shorter than one week since we have shown this by examining the contemporaneous relations between the variables. While choosing a finer observation time resolution might result in richer short-run structure, it would also bring up a number of other difficulties associated with high-resolution data in inefficient markets. The resulting model might have to include a lot more variables to get rid of the serial correlation and heteroscedasticity. Since our focus is more on the long-run structure, we do not attempt to decrease the observation time resolution. The long-run structure captured in cointegrating vector is very significant in these three variables. We also show that time-to-maturity is significant in *eex* and *eua* equation, but not in *np* equation. This implies that during the period we analyse, the risk premium in *np* can be considered as a constant with respect to time-to-maturity.

Chapter 7

Conclusions and future work

In this thesis we model the dynamics of long-term electricity forward prices. In contrast to other commodity markets, the term structure of electricity prices displays no connection between spot- or short-term forward prices and long-term forward prices. Although they are governed by the same laws of supply and demand, the obvious difference is caused by the impossibility of storage, which implies that the supply and demand shocks in the spot market cannot be transferred to the forward market. Instead, the long-term electricity forward prices with certain delivery period are conditioned on the expected values of variables influencing supply, demand or risk premium, with the same delivery period. Since the expected value and unconditional distribution of these variables may be different in the short- and the long term, the resulting expectations about short and long-term electricity forward prices can differ a lot.

We define a general and specific model based on the information set, which includes the past values of variables influencing electricity supply (prices of crude oil, steam coal, natural gas, CO_2 emission allowances and imported electricity), demand (aluminium prices) and past values of long-term prices from Nord Pool themselves. Risk premium is modelled as a function of time-to-maturity only. We use vector autoregressive modelling framework to study the dynamics and interactions between these variables.

We find the following evidences:

1. Cointegration test reveals one significant cointegrating relationship between all variables except *gas*. This implies that investors rather use *oil* as a proxy for the future value of natural gas. However, the share of natural gas in electricity production is also very low in Nordic electricity market. The presence of cointegration gives some evidence against the semi-strong form efficiency of long-term electricity forward market, although, the apparent temporary arbitrage opportunities that seem to exist between different commodity prices could also be the result of very volatile risk premium, in which case the cointegration is not sufficient condition to imply that this market is inefficient. Weak exogeneity test reveals that *np*, *eua* and *eex* are endogenous, whereas *oil*, *coal*, *gas* and *alu* are weakly exogenous. These variables therefore influence *np*, *eua* and *eex*, but are not influenced by them.

2. Granger causality tests show that, in the short-run, endogenous variables are

Granger caused by all other variables, while marginally significant causality between exogenous variables is found only between *oil* and *gas*. Impulse response analysis reveals that the system is dominated by long-run equilibrium forces, while short-run forces dominate the system only in the first step. Steady state of the system after a shock is reached after approximately 5 weeks. Forecast error variance decomposition shows that short-run variance is influenced mostly by the past variance in own variable, whereas in the long-run, all the variables have similar influence on the forecast error variance and there is no dominating variable in this respect. Multivariate Beveridge-Nielsen decomposition shows that endogenous variables are mostly governed by permanent components. Six common trends are identified; four represent cumulated shocks in weakly exogenous variables' residuals, whereas two represent a linear combination of cumulated shocks in three endogenous variables' residuals. We also find that the former load only into weakly exogenous variables, whereas all of them except cumulated shocks in *gas* load into endogenous variables. The test for common cycles shows that transitory variable components compose of at most three common cycles.

3. We estimate Structural VAR and identify uncorrelated structural shocks, which fundamentally influence the variation in variables, whereas reduced form residuals are assumed to be only a linear combination of structural shocks. We adopt two kinds of causal structures, the first being the contemporaneous causal structure only, while the second identification is obtained by combining the contemporaneous structure and long-run impact structure. While both identification structures yield the same structural shocks, the shocks' components for each variable give a slightly different structure for both types of identification. The results show that the structural shocks cannot be given a specific theoretical meaning. Our interpretation is that they represent the residual or additional information present in each variable that is not already included in the variables above the causal chain structure. We find that these results are relatively robust to different causal structures, since the first four variables are weakly exogenous, whereas the last three structural shocks, representing additional information in *np*, *eua* and *eex* , have a minor influence on the variation of these variables. We also find that residual information in *np* has a long-run impact on *np* and *eex*, while residual information in *eex* has only transitory impact on both.

4. In contrast to long-run structure, the short-run structure of the system is very poor. Among the variables only the first lag of *np* in first differences is significant in explaining the future changes in *eua*. This indicates that the data generating process might actually be governed by dynamics shorter than one week, since we show that contemporaneous relations between the variables are very strong. We also show that time-to-maturity is significant in *eex* and *eua* equation, but not in *np* equation. This implies that the risk premium in *np* can be considered as constant with respect to time-to-maturity.

Future work

Our model is specifically designed to capture the dynamics of long-term electricity forward prices from Nord Pool. An interesting task for the future would be to design the information set and the respective model to match other electricity markets. Perhaps even in the case of EEX market, the information set might include slightly different or even additional variables. An interesting case would be to analyse how using different proxy variables influences the results and modelling accuracy. A major future challenge would also be the incorporation of low-resolution common-knowledge information as discussed in section 3.6. This would at least require converting this information to higher resolution. However, for the proper inference on results the sample size would need to increase too.

Alternatively to modelling weekly dynamics, we expect the focus on daily dynamics would yield richer short-run structure and perhaps a more accurate model for day-ahead forecasting purposes. Such model could be based on daily resolution data or perhaps even intra-day resolution data if available. As already pointed out, higher resolution sampling brings up a number of difficulties with the data construction. Since such model would typically involve a lot of autoregressive terms, the structural interpretation might be even more difficult.

The forecasting properties of the resulting model were only briefly presented in this thesis. Forecasting is particularly interesting for day-to-day trading as well as extending the existing term structure beyond the traded horizons on the exchange. As we outlined in the introduction, term-structure for electricity prices 10 years ahead or even more would be beneficiary for modern asset pricing methods, in which forward prices instead of forecasted future spot prices play a central role. Although we present a simple forecast example in which we assume the future prices of weakly exogenous variables are constant, we could also use existing forward curves on exogenous and endogenous variables to produce a long-term forecast of electricity forward prices.

More analytical work could also be done in risk premium dynamics. In this thesis we assume simple time-to-maturity dependence, however, at least in the short-term forward prices the risk premium dynamics is much richer, and many authors find additional driving forces. The result of this search might not produce any additional long-term risk premium drivers, since as we show that reduced form residuals do not display significant signs of autocorrelation or heteroscedasticity. Nevertheless additional tests could be performed in order to identify whether a cointegration between the commodity prices in our information set implies arbitrage opportunities or is this only the result of time-varying risk premium.

Although VAR modelling framework is most widely used in multivariate systems, there are a number of promising alternatives to analyse the dynamics and relationships between the variables in our information set. Extensions of classic VAR are Bayesian Vector Autoregression (BVAR), Time-varying Vector Autoregressive model (TVAR), Vector Autoregressive Moving Average models (VARMA) or Markov-switching Autoregressive Model (MVAR). More sophisticated techniques are State-Space models and Kalman Filters which can be used to fit unobserved component time series models, Dynamic Factor Models (DFM), Factor Augmented

VAR (FAVAR) and Dynamic Stochastic General Equilibrium models (DSGE). These may give different additional information about the system dynamics, since some of them are not bound by stationarity, normality or heteroscedasticity.

Chapter 8

Razširjen povzetek

8.1 Uvod

Lastništvo premoženja je neločljivo povezano z njegovim vrednotenjem. Vrednost premoženja je v splošnem odvisna od pričakovanih finančnih tokov v prihodnosti, ki jih to premoženje ustvarja. Finančni tokovi povezani z lastništvom premoženja so skoraj vedno negotovi, saj nanje vpliva vrsta zunanjih dejavnikov na katere lastnik nima vpliva. Ker imajo tveganja povezana s temi negotovostmi za lastnike (fizične ali korporacije) lahko zelo hude posledice, na trgu obstaja povpraševanje po zavarovanju teh tveganj. To povpraševanje lahko zadostijo investitorji z nasprotnim tveganjem ali finančne institucije, ki so takšna tveganja pripravljeni sprejemati v zameno za neko dodatno premijo. Ponudba in povpraševanje po zavarovanju tovrstnih tveganj imenujemo terminski trg, udeležence tega trga pa v tej disertaciji imenujemo investitorji.

Investitorji na terminskem trgu dobrin se vseskozi ukvarjajo s vprašanjem kakšna je današnja cena določene dobrine z dobavo v prihodnosti? Ta je neločljivo povezana z današnjo ceno (*v nad. promptna cena*) in stroški skladiščenja. Deaton and Laroque [43], [49] pokažeta, da v kolikor so stroški skladiščenja dobrine majhni se nihanja promptne cene, ki jih povzročijo šoki v trenutni ponudbi in povpraševanju, zelo močno prenašajo na terminske cene, zato je v teh primerih gibanje terminskih cen za nekaj let v prihodnosti zelo podobno gibanju promptne cene. Investitorji to zakonitost izkoriščajo tako da izpostavljenost dolgoročnim tveganjem cene dobrin zavarujejo s kratkoročnimi izvedenimi finančnimi instrumenti, čemur pravimo strategija obnovitve (*ang. roll-over*).

Električne energije žal ni mogoče ekonomsko učinkovito shranjevati, saj so stroški skladiščenja izredno visoki. Posledica tega je, da se šoki v trenutni ponudbi in povpraševanju ne morejo prenašati na terminski trg. Koekebakker in Ollmar [1] pokažeta, da je korelacija med kratkoročnimi in dolgoročnimi cenami električne energije majhna, kar pomeni da je lahko strategija zavarovanja dolgoročnih tveganj s kratkoročnimi terminskimi pogodbami ve primeru električne energije zelo tvegana. Kljub visokim stroškom direktnega shranjevanja, je del električne energije možno dokaj poceni shraniti posredno in sicer preko shranjevanja vode ali goriv za proizvodnjo električne energije. To pomeni da se šoki na promptnem trgu električne energije vseeno delno prenašajo na terminski trg, zato korelacija med temi cenami obstaja, vendar je ta precej manjša kot pri terminskih trgih dobrin, ki jih lahko skladiščimo.

V tej disertaciji raziskujemo dinamične zakonitosti oblikovanja dolgoročnih terminskih cen električne energije. Ker je dinamika teh cen drugačna od kratkoročnih terminskih cen, predstavljamo splošen model za dolgoročne terminske cene električne energije. Ta je v nadaljevanju nadgrajen v specifičen model za modeliranje dolgoročnih terminskih cen električne energije iz skandinavske borze Nord Pool.

8.2 Dolgoročne terminske cene električne energije

Eksplicitna definicija dolgoročnih terminskih cen ne obstaja, zato v tej disertaciji definiramo dolgoročne terminske cene električne energije kot cene terminskih pogodb električne energije, ki imajo čas trajanja do dobave $T - t$ večji od enega leta. Ta definicija je odvisna od specifičnih razmer posameznega trga, predvsem pa od tega kakšne so zmožnosti trga pri posrednem shranjevanju električne energije, kot je shranjevanje vode ali goriv. Definicija dolgoročnosti je v tem primeru torej odvisna od funkcije stroškov shranjevanja v odvisnosti od obdobja hranjenja. Dlje časa, ko je možno poceni shranjevati vodo ali goriva, dlje v prihodnost se šoki na promptnem trgu prenašajo na terminski trg. Definicijo dolgoročnih terminskih cen zato pogojujemo na čas trajanja do dobave $T - t$, ko se šoki na promptnem trgu že iznihajo, kar pomeni da gre za takšen čas trajanja do dobave, pri katerem se shranjevanje današnjih zalog vode ali goriv več ne izplača.

Terminske cene električne energije lahko pravilno vrednotimo le tako, da izračunamo pričakovano ceno električne energije za čas dobave in to pričakovanje prilagodimo za premijo tveganja. Odnos med njima podaja enačba (3.2). Na ta način proces oblikovanja terminske cene ločimo na dva dela. V prvem delu se pričakovana promptna cena $S_{t,T}$ oblikuje na podlagi ravnotežja med pričakovano ponudbo in povpraševanjem po električni energiji v obdobju dobave. V drugem delu na podlagi negotovosti pričakovane promptne cene in na podlagi odnosa investitorjev do te negotovosti definiramo ponudbo in povpraševanje po zavarovanju tveganj povezanih s temi negotovosti, kar v ravnotežju privede do ravnotežne premije tveganja, ki jo imenujemo tudi tržna cena tveganja.

8.2.1 Model

Za modeliranje pričakovane dolgoročne cene električne energije sledimo konceptu temeljnih modelov. Dolgoročno ponudbo in povpraševanje modeliramo s temeljnimi faktorji, ki vplivajo na ponudbo in povpraševanje. Na tej podlagi sestavimo model pričakovane promptne cene, ki ga nato nadgradimo s premijo tveganja, da dobimo model dolgoročnih terminskih cen električne energije. Ker ponudba in povpraševanje nista opazni spremenljivki, ju zato ni mogoče modelirati eksplicitno. Namesto tega sestavimo ravnotežni model dolgoročnih terminskih cen električne energije na podlagi temeljnih faktorjev, ki vplivajo na ponudbo, povpraševanje in premijo tveganja. Takšna strategija modeliranja omogoča iskanje neposrednih povezav med opaznimi spremenljivkami. S tem se izognemo modeliranju neopaznih spremenljivk, ki so v samem bistvu zgolj teoretične kategorije, ki sicer pomagajo razložiti ozadje določenega procesa, a v realnem svetu pogosto nimajo posebnega pomena.

Pričakovano dolgoročno povpraševanje po električni energiji definiramo kot pričakovano dolgoročno porabo električne energije, ki jo prilagodimo z dolgoročno cenovno elastičnostjo porabe. Proces dolgoročne porabe električne energije je precej dobro poznan iz časov reguliranega trga. Nanj vplivajo, ekonomska aktivnost (bruto domači proizvod, prihodki), demografija (prebivalstvo, migracije), vreme (temperatura, veter, vlažnost, osvetljenost), cene alternativnih virov energije (cene nafte, zemeljskega plina) in poraba energetsko intenzivne industrije (tovarne aluminija, železarne).

Pričakovano dolgoročno ponudbo modeliramo z dvema globalnima spremenljivkama in sicer s pričakovanimi dolgoročnimi stroški dobave in z obratovalnimi omejitvami. Podobno kot Eydeland in Wolyniec [6], pričakovane dolgoročne stroške dobave razdelimo v tri skupine. V prvi skupini so netrgovana goriva, ki predstavljajo goriva pri proizvodnji električne energije, ki se ne trgujejo na odprtem trgu, zato o njihovi vrednost ne obstaja transparenten in tržno določen cenovni signal. V to kategorijo spadajo veter, voda, obnovljivi viri energije ipd. V drugi skupini so trgovana goriva, ki predstavljajo goriva pri proizvodnji električne energije, ki se v različnih oblikah trgujejo na svetovnih trgih. Mednje spadajo surova nafta in naftni derivati, zemeljski plin, premog in uran. V tretji skupini so drugi stroški dobave, med katere uvrščamo stroške uvoza električne energije in dodatne stroške proizvodnje, kot so cene emisijskih dovolilnic, koncesij ipd.

Obratovalne omejitve vplivajo na pričakovano skupno količino električne energije, ki jo je možno v določenem obdobju dobaviti na trg. Nanjo vpliva instalirana moč, velikost in stanje bazenov hidroelektrarn, razpoložljive čezmejne prenosne zmogljivosti, načrtovani in naključni izpadi, zahtevane rezerve elektroenergetskega sistema, ter tehnične omejitve generatorjev. Te informacije vključujejo zgodovinske in sedanje podatke o elementih elektroenergetskega sistema (elektrarne, elementi omrežja) in podatke o pričakovanih parametrih novih elementov elektroenergetskega sistema.

Na podlagi dolgoročne ponudbe in povpraševanja lahko v ravnotežju izračunamo dolgoročno ceno električne energije. Za modeliranje dolgoročne terminske cene električne energije uporabimo (3.2), ki ob logaritmiranju postane (3.4). To pomeni, da k pričakovani dolgoročni ceni električne energije prištejemo premijo tveganja, ki je v tem primeru funkcija časa trajanja do dobave $T - t$, ki ga v tej disertaciji krajše označujemo s T_m. Empirične raziskave o premiji tveganja na trgu električne energije ugotavljajo različne vplivne dejavnike na premijo tveganja. Poleg časa trajanja do dobave so različni avtorji kot vplivne dejavnike identificirali še verjetnost nastopa cenovnih konic [20], [21] ter splošni nivo cen električne energije in čas opazovanja [58]. Nekateri avtorji, ki v svojih raziskavah delno analizirajo tudi dolgoročne terminske cene, ugotavljajo, da je pri teh cenah dinamika premije tveganja relativno majhna [58], [62].

8.2.2 Informacijska množica

Dolgoročna terminska cena električne energije $F_{t,T}$ je pogojena s teoretično informacijsko množico I_{t-1}. Ta vsebuje vse informacije, ki vplivajo na $F_{t,T}$, kar v splošnem vključuje vse spremenljivke navedene zgoraj. Teoretična informacijska množica je dejansko presek vseh individualnih informacijskih množic, ki jih investitorji uporabljajo pri oceni $F_{t,T}$. Te se lahko med investitorji precej razlikujejo, saj vsebujejo informacije, ki so splošno znane, kot tudi zasebne ali notranje informacije, ki se posledica zasebnih aktivnosti pridobivanja informacij. Neenakost informacijskih množic imenujemo nesimetrija informacij. V tej disertaciji gibanje $F_{t,T}$ pogojujemo z informacijsko množico, ki vsebuje samo simetrične informacije z visoko resolucijo. Izmed zgoraj naštetih vplivnih faktorjev smo v našo informacijsko množico vključili naslednje informacije:

- Terminska cena električne energije z borze Nord Pool, ki jo označujemo z *np*. Bodoče gibanje terminske cene električne energije je pogojeno tudi z gibanjem te cene v preteklosti.

- Terminska cena surove nafte WTI z borze NYMEX, ki jo označujemo z *oil*. Ta predstavlja nadomestek za cene naftnih derivatov, ki se uporabljajo pri proizvodnji električne energije.

- Cenovni indeks premoga TFS API2, ki ga označujemo kot *coal*. Ta predstavlja pričakovano ceno premoga za dobavo v nizozemskih pristaniščih, kar predstavlja dovolj dober približek za skandinavski trg.

- Terminska cena zemeljskega plina z borze ICE, ki jo označujemo z *gas*. Ta predstavlja pričakovano ceno zemeljskega plina iz območja Severnega morja.

- Terminska cena emisijskih dovolilnic CO_2 z borze Nord Pool, ki jo označujemo z *eua*. Ta predstavlja pričakovano ceno emisijskih dovolilnic CO_2 za celotno Evropo.

- Terminska cena električne energije z borze EEX, ki jo označujemo z *eex*. Ta predstavlja terminsko ceno električne energije z dobavo v Nemčiji in je dober približek za vrednost izvožene in uvožene električne energije iz Srednje Evrope v Skandinavijo.

- Terminska cena aluminija z borze LME, ki jo označujemo z *alu*. Precejšen del električne energije v Skandinaviji se porabi za proizvodnjo aluminija, zato cena aluminija predstavlja precejšen del informacije o pričakovani porabi električne energije v prihodnosti.

- Čas trajanja do dobave, ki ga označujemo s T_m. Ta predstavlja vpliv premije tveganja, kot funkcije T_m.

Pri modeliranju uporabljamo tedensko resolucijo spremenljivk. Ker se te spremenljivke trgujejo ob različnih časih, je pri uporabi dnevne ali manjše resolucije potrebno upoštevati točno časovno zaporedje informacij, kar privede do problemov z interpretacijo medsebojnih vplivov. V tej disertaciji zato uporabljamo zaključne vrednosti spremenljivk za vsako sredo.

8.3 Multivariantni model

8.3.1 Analiza podatkov

Opisna analiza spremenljivk pokaže, da imajo vse spremenljivke v povprečju pozitiven časovni trend in nenormalno porazdelitev. S korelacijsko analizo zaznamo visoko korelacijo med spremenljivkami pri absolutnih in logaritemskih absolutnih vrednostih, medtem ko pri prvih diferencah logaritmov najdemo visoko korelacijo le med *np*, *eua* in *eex* (Tabele 4.2, 4.3 in 4.4). Avtokorelacijska (ACF) in križno korelacijska (CCF) analiza na Slikah 4.3 in 4.4) zaznata neizrazito avtokorelacijo in križno korelacijo pri vseh spremenljivkah. Z analizo glavnih komponent, ki je podana v Tabelah 4.5 in 4.6 ugotovimo, da prve tri glavne komponente pojasnjujejo okoli 95% vseh nihanj, medtem ko imata samo prvi dve komponenti lastni vrednosti večji od 1. Uteži spremenljivk, ki pojasnjujejo kako te vplivajo na glavne komponente, kažejo relativno podobne vplive na prve tri komponente, zato te glavne komponente ni mogoče označiti z imenom kakšne opazne spremenljivke.

Stacionarnost spremenljivk testiramo z razširjenim Dickey-Fullerjevim [63], [64] (4.2) in Phillips-Perronovim testom [65]. Rezultati obeh testov v Tabeli 4.7 pokažejo, da so vse spremenljivke integrirane in torej vsebujejo 1 enotski koren (integrirane reda $I(1)$). Testa stacionarnosti za *oil* in *eua* pokažeta, da sta ti dve spremenljivki blizu meje stacionarnosti.

8.3.2 Model vektorske avtoregresije

Na podlagi analize podatkov, posebej korelacijske analize, je spremenljivke potrebno obravnavati kot endogene. To pomeni, da te vplivajo druga na drugo, zato ene spremenljivke ni možno eksplicitno izraziti kot funkcijo preostalih. Spremenljivke je zato potrebno analizirati v sistemu, ki zajema tudi medsebojne in povratne vplive. V tej disertaciji uporabimo model vektorske avtoregresije (VAR), pri katerem vsako spremenljivko zapišemo kot funkcijo preteklih lastnih vrednosti in preteklih vrednosti drugih spremenljivk (5.2). Ker so spremenljivke nestacionarne, jih v obliki absolutnih vrednosti ni mogoče uporabiti za linearno regresijo, ker bi ta privedla do fenomena neprave regresije. V tem primeru je potrebno spremenljivke odvajati (uporabimo prve diference), in sicer tolikokrat, kolikor je enotskih korenov v spremenljivkah. Pri linearni regresiji diferenc izgubimo informacijo o dolgoročni povezanosti oz. kointegraciji med spremenljivkami. Kointegracija je lastnost sistema dveh ali več spremenljivk, ki so integrirane reda $I(r)$, vendar med njima obstaja vsaj ena linearna kombinacija, ki je integrirana reda $I(r-1)$. Kointegracijo najlažje testiramo na VAR modelu z absolutnimi vrednostmi spremenljivk z Johansenovim testom kointegracije [78]. Če ta pokaže da med spremenljivkami ni kointegracije, nadaljujemo z VAR modelom, ki vsebuje samo diference, v kolikor pa kointegracija obstaja pa VAR v (5.2) pretvorimo v vektorski model popravljanja napak (VECM).

Najprej definiramo model VAR(k) (5.2), kjer je $k = 2$ število avtoregresijskih členov, $\mathbf{Y}_t$ sestavlja sedem endogenih spremenljivk *np*, *oil*, *coal*, *gas*, *eua*, *eex* in *alu*, medtem ko $\mathbf{Z}_t$ vsebuje čas trajanja do dobave T_m, za katerega vemo da je eksogena spremenljivka. Analiza VAR modela je ključno odvisna od tega ali imajo preostanki regresije lastnosti belega šuma, t.j. da so neodvisno, enakomerno in normalno porazdeljeni. Diagnostika VAR(2) v Tabeli 5.1 kaže, da ima takšna specifikacija modela nekaj težav z avtokorelacijo (F_{ar}), normalno porazdelitvijo (χ^2_{nd}), heteroskedastičnostjo (F_{het}) ter avtoregresijsko pogojno heteroskedastičnostjo (F_{arch}). Model zato nadgradimo z 8 umetnimi spremenljivkami in dobimo (5.11). Prve štiri odpravljajo nelinearnost *eua* šoka v aprilu 2006 (glej Sliko 4.1), peta umetna spremenljivka odpravlja prehodni šok v točkah 27 in 29, tri umetne spremenljivke pa odpravljajo tri največje regresijske osamelce. Diagnostika tako popravljenega modela v Tabeli 5.2 kaže precej boljšo sliko, saj testi preostankov precej bolje nakazujejo lastnosti belega šuma. Nenormalnost porazdelitve (predvsem zaradi sploščenosti) in majhna avtokorelacija v enačbah *coal* in *alu* nimata pomembnega vpliva na Johansenov test kointegracije.

Stabilnost VAR modela preverimo z lastnimi vrednostmi pridružene matrike (5.17). VAR je stabilen, če vse lastne vrednosti ležijo znotraj enotskega kroga. V kolikor nekatere lastne vrednosti ležijo na enotskem krogu je VAR nestacionaren, v kolikor pa nekatere lastne vrednosti ležijo izven enotskega kroga, potem je VAR eksploziven in torej nestabilen. Lastne vrednosti pridružene matrike v Tabeli 5.3 kažejo, da šest največjih lastnih vrednosti leži blizu enotskega kroga, ostale pa znotraj enotskega kroga, kar pomeni da je VAR stabilen a nestacionaren. Konstantnost parametrov preverimo z rekurzivnim Chow-ovim testom preloma. Ta temelji na rekurzivnem izračunu parametrov modela in kako se ti spreminjajo v odvisnosti od velikosti izbranega vzorca. Slika 5.1 prikazuje da 1% verjetnost testa nikoli ni presežena, torej so parametri posameznih enačb in celotnega sistema konstantni skozi celoten vzorec.

8.3.3 Kointegracija

VAR model (5.11) pretvorimo v VECM (5.18), kjer matrika $\mathbf{\Pi} = \boldsymbol{\alpha\beta}'$ zajema dolgoročne povezave med spremenljivkami, $\boldsymbol{\beta}$ je matrika koeficientov kointegracijskih vektorjev tako da $\boldsymbol{\beta}'\mathbf{Y}_{t-1}$ predstavlja do $n-1$ linearnih kombinacij $\mathbf{Y}_t$ reda $I(0)$, $\boldsymbol{\alpha}$ pa predstavlja naložno matriko, ki pove kako kointegracijski vektorji popravljajo odklon dolgoročnega ravnovesja v vsaki VAR enačbi.

Johansenov test kointegracije temelji na statističnem izračunu ranga matrike $\mathbf{\Pi}$. Rang matrike določimo na podlagi cenilke največjega verjetja (*ang. Maximum Likelihood Estimator - MLE*) za hipotezo, da je največ r lastnih vrednosti matrike $\mathbf{\Pi}$ statistično različnih od 0, proti alternativni hipotezi, da je teh največ $r+1$. Lastne vrednosti matrike $\mathbf{\Pi}$ izračunamo z enačbama (5.21) in (5.22), na podlagi katerih lahko izračunamo test sledi (*ang. trace test*) (5.19) in test maksimalne lastne vrednosti (*ang. maximum eigenvalue test*) (5.20). Rezultati obeh testov so prikazani v Tabeli 5.5. Test sledi kaže, da med spremenljivkami obstajate dve stacionarni kointegracijski povezavi, medtem ko test maksimalne lastne vrednosti kaže da obstaja samo ena. Zaradi dodatnih umetnih in eksogenih spremenljivk kritične asimptotične vrednosti obeh testov niso povsem natančne, zato opravimo še dva dodatna testa. Prvi je Reimerjeva prilagoditev za izgubo prostostnih stopenj [80], pri katerem v enačbah (5.19) in (5.20) zamenjamo T s $T-nk$, kjer je n dimenzija VAR modela in k število avtoregresijskih členov. Rezultati te prilagoditve prikazani v Tabeli 5.6 pri obeh testih kažejo na eno samo kointegracijsko povezavo. Za test sledi izračunamo tudi kritične vrednosti z metodo samovzorčenja (*ang. bootstrap*) [81]. Na podlagi tako določenih kritičnih vrednosti hipoteza $r \leq 1$ ni zavrnjena z verjetnostjo $p = 0.074$.

Juselius [82] za pomoč pri odločitvi glede kointegracijskega ranga predlaga dodatne indikatorje. Prvi je velikost lastnih vrednosti pridružene matrike (5.17) in kako se te spreminjajo ob različnih hipotezah $r = 1, 2, \ldots, n-1$. Rezultati tega indikatorja v Tabeli 5.8 ne dajejo jasnega signala, saj ni mogoče natančno vedeti ali modul lastne vrednosti 0.871 statistično leži znotraj enotskega kroga ali na enotskem krogu. Drugi indikator so statistične t vrednosti koeficientov matrike $\boldsymbol{\alpha}$, ki so prikazane v Tabeli 5.9. Te kažejo da je v prvih štirih stolpcih vsaj en koeficient statistično različen od 0. Tretji indikator leži v samem grafu kointegracijskih vektorjev, ki so prikazani na Sliki 5.2. Iz slike je razvidno, da samo prvi vektor izgleda stacionaren (velikokrat prečka svojo srednjo vrednost), medtem ko je drugi nekje na meji stacionarnosti. Čeprav ti indikatorji ne pomagajo pri odločitvi, verjamemo da hipoteza $r = 1$ najbolje odraža skupno ugotovitev vseh testov.

Na podlagi določenega ranga matrike $\boldsymbol{\Pi}$ lahko izračunamo vrednosti vektorjev $\boldsymbol{\alpha}$ in $\boldsymbol{\beta}$ in postavimo omejitve za koeficiente njihovih vrednosti. V prvem koraku najprej normaliziramo vektor $\boldsymbol{\beta}$, tako da ima parameter *np* vrednost 1 in omejimo parameter *gas* na 0. Ta omejitev, ki jo testiramo s standardnim testom razmerja obetov (*ang. Likelihood Ratio - LR*), ni zavrnjena, medtem ko nadaljnje omejitve parametrov katerekoli druge spremenljivke so zavrnjene. V drugem koraku, skupaj z omejitvijo *gas* parametra omejimo še parametre *oil*, *coal*, *gas* in *alu* v $\boldsymbol{\alpha}$ na vrednost 0. Tudi vse te omejitve skupaj niso zavrnjene, pri čemer je vrednosti razmerja obetov $LR = 10.035$ in verjetnost $p = 0.074$. Spremenljivke *oil*, *coal*, *gas* in *alu* so torej *šibko eksogene*, kar pomeni da v dolgoročnem smislu endogene spremenljivke nanje ne vplivajo, zato jih ni potrebno eksplicitno modelirati v VAR modelu.

Vrednosti omejenih $\boldsymbol{\alpha}$ in $\boldsymbol{\beta}$ so prikazani v Tabeli 5.11, medtem ko Tabela 5.12 prikazuje vrednosti parametrov $\boldsymbol{\Pi}$, skupaj s statističnimi t-vrednostmi spodaj v oklepajih. Vrednosti parametrov $\boldsymbol{\beta}$ so posebej zanimivi. V dolgoročnem smislu *np* narašča približno v razmerju 1:1, nekoliko močneje z *alu* in nekoliko šibkeje s *coal*. Po drugi strani *np* z naraščanjem *eex* pada v nekoliko večjem razmerju kot 1:1 in ob porastu *eua* za 1% pa pade približno za 0.15%. Ker so *np*, *eua* in *eex* močno korelirane, to pomeni, da je ob pozitivnem šoku *eex* po vsej verjetnosti v istem trenutku narasla tudi *np*, zato v tem primeru kointegracijski vektor vleče *np* nazaj k dolgoročnemu ravnovesju.

8.4 Strukturna analiza

8.4.1 Dinamična analiza

Namen strukturne analize je ugotoviti dolgoročno in kratkoročno strukturo in dinamiko sistema. Grangerjev test vzročnosti odkriva kratkoročne vzročno-posledične povezave med spremenljivkami. V primeru VAR ali VECM modela, ta temelji na Waldovi statistiki za ničelno hipotezo, da so parametri matrik $\boldsymbol{\Gamma}$ in $\boldsymbol{\Pi}$, ki podajajo medsebojne vplive med spremenljivkami, enaki 0 (glej enačbe (6.1) do (6.6)). Tabeli 6.2 in 6.3 podajate rezultate Grangerjevega testa vzročnosti za primer omejenega VECM ($r = 1$) brez in z omejitvami na $\boldsymbol{\alpha}$ in $\boldsymbol{\beta}$. Rezultati kažejo da spremembe šibko eksogenih spremenljivk povzročajo spremembe endogenih spremenljivk, tudi v primeru *gas*. Obratne vzročnost nismo našli. Najdemo tudi manjšo dvosmerno vzročnost med *oil* in *gas* ter enosmerno vzročnost od *coal* proti *gas*. V primeru omejenih α in β, *gas* ne povzroča sprememb v nobeni izmed spremenljivk. Ti rezultati dajejo nekoliko drugačno sliko, kot test šibke eksogenosti, saj Grangerjev test vzročnosti zaznava kratkoročno vzročnost, medtem ko test šibke eksogenosti zaznava dolgoročno vzročnost.

Medsebojne povezave spremenljivk v sistemu lahko odkrijemo tudi z impulzno odzivno analizo. Ta podaja odziv vseh spremenljivk na enotski impulz v vseh spremenljivkah, ki ga izračunamo s pretvorbo VAR/VECM modela v obliko drsečega povprečja. Slabost te analize je, da predpostavlja izolirane impulze v spremenljivkah, čeprav kovariančna matrika ostankov v Tabeli 6.4 kaže, da so impulzi oz. šoki med seboj korelirani in torej ni verjetno, da bi se šok v sistemu pojavil izolirano. To problem rešujeta ortogonalna impulzno odzivna funkcija (6.10), (6.11) in posplošena impulzno odzivna funkcija (6.12). Ker prva zelo zavisi od vrstnega reda spremenljivk, na Sliki 6.2 prikazujemo samo posplošeno impulzno odzivno analizo. Odzivi endogenih spremenljivk na impulze vseh spremenljivk kažejo, da je *eua* najbolj občutljiva na impulze spremenljivk, medtem ko sta *eex* in *eua* približno enako občutljiva. Največje odzive povzročajo impulzi v lastni spremenljivki in v *oil*, kar kaže na ta da *oil* zelo močno vpliva na dinamiko sistema. Stabilno stanje sistema je doseženo približno v 5 tednih po impulzu.

Razčlenitev variance je še eno pomembno orodje za analizo dinamike sistemov. Temelji na izračunu relativnega prispevka neke spremenljivke v varianci napake napovedi neke druge spremenljivke. Podobno kot pri impulzno odzivni analizi, tudi tu ločimo med ortogonalno razčlenitvijo variance (6.13) in posplošeno razčlenitvijo variance (6.14). Iz Slike 6.3, ki prikazuje posplošeno razčlenitev variance za tri endogene spremenljivke, je razvidno, da imajo vse spremenljivke pomemben delež pri varianci napake napovedi, le *eua* povzroča nekoliko manjšo varianco napake napovedi.

8.4.2 Skupni trendi in cikli

Nestacionarne časovne serije je v splošnem možno razčleniti na trajne komponente (trende) in prehodne komponente (cikle). Ena od možnih razčlenitev je trajno-prehodna razčlenitev tipa Gonzalo in Granger [96], ki identificira $n - r$, $I(1)$ skupnih stohastičnih trendov, kombinacija katerih predstavlja trajno komponento vsake spremenljivke. Metoda temelji na ortogonalnih vrednostih $\alpha_\perp$ and $\beta_\perp$, ki tvorita matriko dolgoročnih vplivov $\mathbf{C}$ (6.20). $\mathbf{C}$ je možno zapisati tudi kot (6.27), tako da $\alpha'_\perp \sum_{i=1}^{t} \mathbf{u}_t$ predstavlja $(n - r)$ skupnih trendov, kjer so $\mathbf{u}_t$ preostanki sistema. $\beta^*_\perp$, ki jo izračunamo z enačbo (6.26), predstavlja utežno matriko, ki določa trajne komponente kot linearne kombinacije skupnih trendov.

Slike 6.4, 6.5 in 6.6 prikazujejo dejansko, trajno in prehodno komponento za *np*, *eua* in *eex*. Iz slik je razvidno, da trajna komponenta v povprečju nekoliko prehiteva dejansko komponento. Opazimo, da je v času naraščanja cen prehodna komponenta v povprečju negativna, v času padanja cen pa v povprečju nekoliko pozitivna. To nakazuje, da dejanska cena potrebuje nekaj časa da ujame trajno komponento, ki predstavlja stabilno stanje. Oblika prehodnih komponent je pri vseh treh cenah zelo podobna, kar nakazuje da so te komponente linearna kombinacija nekaj skupnih ciklov.

Identifikacija skupnih trendov je podana z matriko $\boldsymbol{\alpha}_\perp$ v (6.28). Vidimo da štirje skupni trendi predstavljajo kumulativne preostanke v enačbah štirih šibko eksogenih spremenljivk, ostala dva skupna trenda pa predstavljata linearno kombinacijo kumulativnih preostankov v enačbah treh endogenih spremenljivk. Obliko teh skupnih trendov prikazuje Slika 6.7, njihov vpliv na vsako spremenljivko podaja $\boldsymbol{\beta}^*_\perp$ v (6.29), njun produkt pa predstavlja matriko $\mathbf{C}$, ki je podana v Tabeli 6.6. Če v matriki $\mathbf{C}$ omejimo parametre z najnižjimi t-vrednostmi na 0, dobimo omejeno matriko $\mathbf{C}$ in pripadajočo omejeno $\boldsymbol{\beta}^*_\perp$, medtem ko je $\boldsymbol{\alpha}_\perp$ indiferentna na te omejitve. Četrti in peti skupni trend vplivata samo na endogene spremenljivke. Preostali skupni trendi, ki predstavljajo zgolj kumulativne preostanke v *oil*, *coal*, *gas* in *alu*, vplivajo večinoma na lastno spremenljivko in endogene spremenljivke, le pri prvem trendu opazimo tudi vpliv na *gas*, medtem ko drugi trend nima vpliva na *eua*. To pomeni da *oil* poleg endogenih spremenljivk vpliva tudi na *gas*, medtem ko *coal* nima vpliva na *eua*. Podobno kot so trajne komponente sestavljene iz skupnih trendov, so tudi prehodne komponente sestavljene iz skupnih ciklov. Test za število skupnih ciklov pokaže, da so prehodne komponente sestavljene iz največ 3 skupnih ciklov. Identifikacijo teh ciklov prepuščamo bodočim raziskavam.

VAR v (5.11) predstavlja skrčeno obliko (*ang. reduced form*) bolj splošnega strukturnega vektorskega avtoregresijskega modela (SVAR). Ta poleg avtoregresijskih členov na desni strani vsebuje še člene, ki predstavljajo sočasne vplive. SVAR je mogoče izraziti z matriko $\mathbf{A}$ (6.31) ali z matriko $\mathbf{B}$ (6.35). Matrika $\mathbf{B}$ pretvori korelirane preostanke skrčene oblike $\mathbf{u}_t$ v ortogonalne strukturne preostanke oz. strukturne šoke (6.36). Ker matrika $\mathbf{B}$ podaja tudi sočasne povezave med endogenimi spremenljivkami, sistem (6.35) ni identificiran, saj $\mathbf{B}$ vnaša n^2 dodatnih parametrov in samo $n(n+1)/2$ dodatnih enačb, kar pomeni da moramo v matriki $\mathbf{B}$ postaviti najmanj $n(n-1)/2$ dodatnih omejitev, ki omogočajo natančno identifikacijo (glej Lütkepohl [89]). Medtem ko omejitve matrike $\mathbf{B}$, predstavljajo kratkoročne vzročne povezave, ki smo jih predstavili v Grangerjevi vzročni analizi, pa je omejitve mogoče postaviti tudi na matriko dolgoročnih vplivov $\overline{\mathbf{C}} = \mathbf{CB}$, če kateri od strukturnih šokov nima dolgoročnega vpliva na katero od spremenljivk.

V prvem koraku postavimo omejitve samo na matriko $\mathbf{B}$, in sicer tako da predpostavljamo vzročno strukturo, ki najbolje odraža rezultate dinamične analize, to je naslednjo smerno vzročnost: *oil*, *gas*, *coal*, *alu*, *eua*, *eex* in *np*. Ta vzročnost pomeni, da *oil* lahko sočasno vpliva na vse spremenljivke, *gas* na vse razen na *oil* in tako dalje. Ob tako razvrščenih spremenljivkah omejimo zgornjo izvendiagonalno trikotno matriko $\mathbf{B}$ na 0, kar predstavlja $n(n-1)/2$ omejitev. Tako omejena matrika $\mathbf{B}$ in pripadajoča matrika $\overline{\mathbf{C}}$ sta predstavljeni v Tabelah 6.9 in 6.10, Slika 6.8 pa prikazuje strukturne šoke $\boldsymbol{\varepsilon}_t = \mathbf{B}^{-1}\mathbf{u}_t$. Prvi štirje strukturni šoki predstavljajo preostalo informacijo spremenljivk, ki ni vsebovana v spremenljivkah nad vzročno strukturo. To pomeni da drugi strukturni šok predstavlja preostale šoke v *gas*, ki niso že vsebovani v *oil*. Interpretacija zadnjih treh šokov je bolj težavna. Tudi v tem primeru gre za preostalo informacijo spremenljivk glede na vzročno strukturo, vendar je njihova oblika nekoliko bolj občutljiva na vzročno strukturo. Kljub temu je velikost komponent teh šokov majhna za *np* (Slika 6.9), kar pomeni, da imajo zadnji trije strukturni šoki majhen vpliv na gibanje *np*. Če zamenjamo vzročno strukturo *eex* in *np*, ugotovimo da *eex* nima dolgoročnega vpliva na *np*, medtem ko dolgoročni vpliv v obratni smeri obstaja.

Na podlagi lastnosti kointegriranih sistemov, vemo da ima samo $n - r$ strukturnih šokov trajni vpliv na spremenljivke medtem ko ima r strukturnih šokov zgolj prehodni vpliv (glej Juselius [82]). Na podlagi te zakonitosti omejimo zadnji stolpec matrike $\overline{\mathbf{C}}$ na 0, preostale omejitve pa postavimo na matriko $\mathbf{B}$. Rezultati takšnih omejitev so podani v Tabelah 6.11 in 6.12. Oblika strukturnih šokov ostane nespremenjena, le nekoliko se spremeni struktura zadnjih treh šokov. Tudi v tem primeru šoki v *np* trajno vplivajo na *eex*, obratni vpliv pa je zgolj prehoden. Čeprav je poimenovanje strukturnih šokov skoraj v vseh primerih zelo poljubno, lahko z zagotovostjo rečemo da prvi štirje šoki predstavljajo preostalo informacijo spremenljivk, ker smo podobne medsebojne vplive našli tudi z dinamično analizo. Interpretacija zadnjih treh šokov je precej bolj komplicirana, zato jo puščamo odprto za prihodnje raziskave.

8.4.3 Kratkoročna struktura

Kratkoročna struktura se sestoji iz treh enačb endogenih spremenljivk, ki so funkcija treh avtoregresijskih členov $\Delta\mathbf{Y}_{t-1}$, enega člena avtoregresijskih ravnovesnih napak $\boldsymbol{\beta}'\mathbf{Y}_t$, štirih členov šibko eksogenih spremenljivk, enega člena eksogene spremenljivke, konstante, ter osmih umetnih spremenljivk. Takšen model ima tako 18 parametrov v vsaki enačbi, kar skupno znaša 54 parametrov. Kratkoročno strukturo VECM modela izračunamo z dvo-stopenjskim izračunom, pri katerem v prvi stopnji izračunamo cenilko $\widehat{\boldsymbol{\beta}}$, v drugi stopnji pa izračunamo (6.39). Rezultati testa značilnosti spremenljivk (F_{sig}) v Tabeli 6.13 kažejo, da so statistično značilne samo konstanta, kointegracijski vektor, ΔT_m in 5 umetnih spremenljivk, medtem ko je avtoregresijski člen *np* na robu značilnosti.

Model skrčimo s standardnim testom razmerja obetov, in sicer tako da sekvenčno izločamo člene z najmanj značilnim parametrom, dokler test razmerja obetov ni zavrnjen. Na ta način iz modela (6.39) izločimo 9 spremenljivk, statistična značilnost zgoraj naštetih spremenljivk pa je tako potrjena. Model ima na koncu samo 27 parametrov, vrednost test razmerja obetov pri krčenju pa je $F(27, 391) = 1.077$ in p-vrednost $p = 0.364$. Vrednosti parametrov v Tabeli 6.14 kažejo, da je *gas* povsem nepomembna spremenljivka v sistemu, medtem ko imajo *oil*, *coal*, *eua*, *eex* in *alu* samo dolgoročni vpliv na endogene spremenljivke. Parameter ΔT_m je značilen samo v enačbi za *eua* in *eex*, kar pomeni da je premija tveganja v teh dveh cenah odvisna od T_m, medtem ko je v *np* konstantna glede na čas trajanja do dobave. Diagnostika skrčenega modela v Tabeli 6.15, kaže da so glavne lastnosti belega šuma preostankov ostale nespremenjene, medtem ko so se vektorski testi nekoliko izboljšali, saj sistem ne vsebuje več preostankov šibko eksogenih spremenljivk. Rekurzivni Chow-ov test preloma, prav tako kaže, da parametri modela ostajajo konstantni skozi ves vzorec.

8.5 Zaključek

Model dolgoročnih cen električne energije smo ga zgradili na podlagi sedmih spremenljivk, ki vplivajo na pričakovano dolgoročno ceno električne energije in časom trajanja do dobave, ki podaja vpliv premije tveganja. Vse spremenljivke so integrirane, med njimi pa najdemo eno kointegracijsko povezavo, v kateri je cena zemeljskega plina statistično neznačilna. Izmed sedmih spremenljivk so štiri šibko eksogene, preostale tri pa endogene. Z vzročno analizo ugotovimo da v kratkoročnem smislu cena surove nafte vpliva tudi na ceno zemeljskega plina, medtem ko cena premoga ne vpliva na ceno CO_2 emisijskih dovolilnic. Strukturni VAR model razkrije, da na cel sistem dominantno vplivajo šoki v ceni surove nafte, šoki v cenah premoga, aluminija, CO_2 emisijskih dovolilnic in električne energije iz borze Nord Pool pa so nekoliko manj izraziti a imajo kljub temi trajen vpliv. Šoki v dolgoročnih terminskih cenah električne energije iz borze EEX nimajo trajnega vpliva na nobeno izmed spremenljivk. Kointegracija med spremenljivkami lahko pomeni, da je del gibanja dolgoročnih terminskih cen električne energije mogoče predvideti, kar nakazuje da bi lahko bili trgi dolgoročnih terminskih pogodb električne energije neučinkoviti, čeprav je prisotnost kointegracije lahko tudi posledica variabilne premije tveganja. Čeprav bi model, ki ga predstavljamo v disertaciji, zgrajen za modeliranje cen električne energije na drugih trgih lahko vseboval drugačne spremenljivke, je ta dovolj splošen, da je mogoče te ugotovitve posplošiti tudi za druge trge električne energije.

8.6 Izvirni prispevki disertacije

Izvirne prispevke disertacije lahko strnemo v naslednje točke:

- stohastični model dolgoročnih terminskih cen električne energije,
- razvoj postopka za kalibracijo modela dolgoročnih terminskih cen električne energije,
- modeliranje strukture negotovosti dolgoročnih terminskih cen električne energije,
- stohastični model dolgoročne ponudbe, dolgoročne porabe, cene goriv in premije tveganja.

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Izjava

Izjavljam, da sem disertacijo napisal sam in da sem avtor rezultatov, ki so v disertaciji predstavljeni kot izvirni znanstveni prispevek disertacije.

Martin Povh

Paper 1: Modelling long-term electricity forward prices

Modeling Long-Term Electricity Forward Prices

Martin Povh and Stein-Erik Fleten

***Abstract*—In contrast to forwards and futures on storable commodities, prices of long-term electricity forwards exhibit a dynamics different to that of short-term and midterm prices. We model long-term electricity forward prices through demand and supply of electricity, adjusted with a risk premium. Long-term prices of electricity, oil, coal, natural gas, emission allowance, imported electricity, and aluminum are modeled with a vector autoregressive model (VAR). For estimation, we use weekly prices of far-maturity forwards relevant for the Nordic electricity market. Although electricity prices experienced a few substantial shocks during the period we analyzed, there is no evidence of a structural break. Cointegration analysis reveals two stationary long-run relationships between all variables except the gas price, indicating that these variables move together over time. We find some influence of the risk premium, however not on the long-term electricity forwards at Nord Pool.**

***Index Terms*—Cointegration, electricity prices, long-term forward prices, vector autoregressive (VAR) modeling.**

I. Introduction

COMMODITY forward markets are normally focused on contracts with time to maturity up to 1.5 years. Since the correlation between the short-term and long-term prices is high in many markets, long-term risks can be hedged with roll-over hedging using short-term and midterm forwards and futures. Unlike most commodities, electricity cannot be stored to any great extent. In an empirical analysis of forwards from Nord Pool, Koekebakker and Ollmar [1] show that the correlation between short-term and long-term electricity futures is low and conclude that short-term contracts are not appropriate for hedging long-term exposures in electricity markets such as long-term procurement costs and production revenues. While far-maturity exposures can normally be hedged with short-term positions, electricity companies can only properly hedge them with long-term trading. Although the liquidity of long-term electricity forwards is still often low, their maturities lie up to six years in the future.

Long-term electricity forward prices also serve as important information carriers, since they provide valuation signals for strategic decisions like investments, mergers and acquisitions, and financing of new long-term generation assets. In recent years, these decisions are also influenced by the environmental pressure on the technology shift from traditional coal and nuclear to natural gas and renewable sources. Investment and disinvestment decisions are triggered by changes in the relative economics of technologies, driven by changes in underlying commodity prices. The real options theory comprehensively described in [2] is appropriate for analyzing such decisions. The real option theory suggests using forward prices instead of projected future spot prices. The use of forward prices bypasses the problem of risk adjustment of the discount interest rates, allowing the assets to be valued over time with a risk neutral pricing.

Since forward contracts are not traded far enough to be used in real asset valuation, the forward prices beyond the traded horizon need to be forecasted. Extrapolation of quoted forward prices might not give the best estimate, since it ignores the available information about the long-term supply and demand. Forward contracts towards the end of the term structure are often illiquid, reducing the trust in extrapolation. More sophisticated models that focus on modeling long-term supply and demand and risk adjustment are therefore necessary to produce a better estimate of forward prices beyond the term structure. Since such models involve the understanding of what influence the prices of traded far-maturity forwards, they might also prove useful in speculative trading.

Long-term electricity prices are traditionally modeled with long-term production-cost models [3], [4]. In a restructured market, however, electricity prices do not necessarily equal production costs. Different extensions of production cost models were proposed to better reflect the real prices observed in the deregulated market. A hybrid approach, in which bottom-up models based on production cost variables are calibrated on market data, has gained increasing attention in recent years [5], [6]. Nonetheless, the literature on long-term electricity forwards is still scant, due to the lack of trusted long-term market data. Long-term forward prices are more often modeled as an extension of short-term forward-price modeling. Schwartz [7] uses models estimated on short-term oil futures and tests their performance on the available long-term oil futures. The correlation between long-term electricity forward prices and short- or midterm electricity forward prices, as shown in Koekebakker and Ollmar [1], is, however, low in many markets. This indicates that short-term models are unable to explain the dynamics of long-term electricity forward prices. An example of long-term electricity forward price modeling is provided in [8], which reports on a forward-price and volatility-forecasting model that combines risk adjustment and external long-term forecasting models.

In this paper, we focus on modeling the dynamic structure of long-term electricity forwards. To model these prices, we try to identify the long-term information that influences the expected long-term electricity supply, demand, and risk premium. We analyze the weekly prices of Nord Pool's long-term electricity forwards and how these are influenced by long-term forward prices of fuels, emission allowances, and imported elec-

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M. Povh is with the Faculty of Electrical Engineering, University of Ljubljana, Ljubljana SI-1000, Slovenia (e-mail: martin.povh@fe.uni-lj.si).

S.-E. Fleten is with the Norwegian University of Science and Technology Department of Industrial Economics and Technology Management, Trondheim NO-7491, Norway (e-mail: stein-erik.fleten@iot.ntnu.no).

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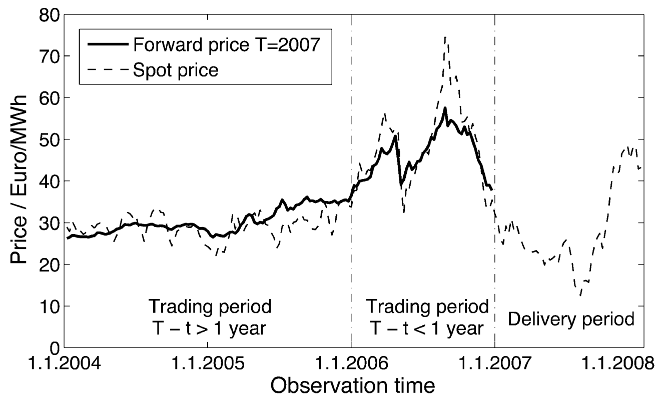

Fig. 1. Electricity spot price and long-term forward price dynamics from Nord Pool.

tricity. Due to possible endogeneity, we use vector autoregressive model, which do not require any ad-hoc assumptions on exogeneity.

This paper is organized as follows. Section II identifies the long-term electricity forward price process. The use of the data and univariate model representation is presented in Section III. Section IV starts with descriptive analysis of variables and multivariate representation. This is followed by cointegration analysis and estimation of vector error correction model. Section V draws the conclusions.

II. Long-Term Forward Price Formation

We define long-term electricity forward prices as prices of electricity forwards with a delivery period of one year and a time to maturity of more than one year ($T-t \geq 1$ year). Fig. 1 presents an example of the price dynamics for a forward contract from the Nordic electricity exchange Nord Pool with delivery in 2007. These contracts, though named forwards, correspond to the definition of swaps, having a stream of cash flows that depends on the difference between the realized spot price and the fixed contract price. We will continue to denote them forwards. Fig. 1 demonstrates that the forward-price dynamics is different from the spot-price dynamics when $T \gg t$. As the delivery period closes ($T \approx t$), the forward-price dynamics becomes more similar to the spot-price dynamics. Long-term forward prices and short-term forward prices (spot prices) are, therefore, governed by somewhat different laws, which indicate the need to model them separately.

A. Setup

The nonstorability of electricity has important implications on electricity trading and the valuation of forward contracts. While the cost-of-carry arbitrage is usually applied in valuation of commodity forwards, it cannot be used in case of electricity forwards, since electricity cannot be bought today at the spot price S_t and stored for subsequent sale at the forward price $F_{t,T}$. As an alternative to the cost-of-carry arbitrage, one can use an equilibrium approach [9]

$$F_{t,T} = S_{t,T}(1+r-\lambda)^{(T-t)} \tag{1}$$

where the forward price $F_{t,T}$ is the (rational) expectation about the spot price at delivery time, $E_t[S_T]$ (or simply $S_{t,T}$) discounted with the risk-free interest rate r and the risk premium λ. Due to the uncertainty of the expected spot price, $S_{t,T}$, market participants require a compensation for bearing the spot-price risk, i.e., they determine their own risk premium. When individual risk preferences are matched (e.g., on the exchange), the aggregated risk premium is obtained; this is also referred to as the market price of risk. In (1), the forward price formation is therefore an equilibrium process. If one is able to obtain an unbiased estimate of the expected spot price $S_{t,T}$, the supply and demand for bearing the spot price risk determines the risk premium λ.

Transforming (1) to logs gives

$$\ln F_{t,T} = \ln S_{t,T} + (T-t)\ln(1+r-\lambda). \tag{2}$$

Assuming constant risk-free interest rate r and risk premium λ and writing time to maturity $(T-t)$ as T_m gives

$$\ln F_{t,T} = \ln S_{t,T} + RT_m \tag{3}$$

where R is the risk premium parameter defined as $\ln(1+r-\lambda)$. In (3), the risk premium therefore depends only on time to maturity, which is a very common assumption in modeling fixed income markets, foreign exchange markets, or commodity markets. In case of electricity, an assumption that risk premium depends only on time to maturity is also often applied [10], despite some empirical findings, which indicate that the risk premium in short-term electricity forwards might be influenced by the probability of price spikes (load seasonality) and the level of prices [11], [12]. However, some investigations, which also extend to far-maturity contracts indicate that the magnitude and the variability of the risk premium in far-maturity electricity forwards is low [13], [14].

In (3), the forward prices are therefore mainly driven by the expected spot prices, subject to information sets available to market participants. We assume that information sets in our case include past information about $F_{t,T}$ as well as the information that influence the expected spot price $S_{t,T}$, i.e., variables that influence the expected supply and demand. We assume all participants (i.e., producers, buyers, and traders) have the same information set.

B. Modeling the Long-Term Expected Spot Price

We define the long-term forwards as the contracts with delivery period of one year, having a payoff that depends on the realized spot price over the delivery year. The long-term expected spot price $S_{t,T}$ therefore represents the expected price of 1 MW of annual base-load electricity. Expected electricity spot prices a few years into the future are influenced by the expected supply and demand at the delivery time T. The supply and demand are however not observable variables. Instead we can use fundamental variables that influence the supply and demand to estimate their influence on the expected spot price. Electricity demand can sufficiently be explained with weather, economic activity, and demography, whereas the variables influencing the supply can be grouped into five groups:

1) fuel prices (coal, natural gas, oil);
2) water-reservoir level in hydro-rich systems;

3) emission allowance prices (CO_2);
4) supply capacity (market structure, available capacity);
5) electricity prices in neighboring markets (in the case that a significant share of electricity is imported or exported).

When estimating the expected spot price in the long-term, we seek reliable information about the expected values of the fundamental variables mentioned above. In the short term ($T \approx t$), the fundamental variables can be predicted with high, though not complete, accuracy. As the time to maturity increases, the variance of these variables increases and their mean values are harder to predict. Still, there is a difference between variables that are considered stationary and integrated variables. With stationary variables, the unconditional variance is bounded and the unconditional mean is based on historical average and expected growth. Such are the hydro reservoir levels, supply capacity, and electricity demand, which can be predicted based on historical average value and expected long-term growth. Integrated variables on the other hand have no unconditional distribution since the shocks in these variables will persist and their unconditional variance is therefore unbounded. In our case, these are fuel prices, emission allowance prices, and prices of electricity in neighboring markets. Fortunately the market offers securities to hedge their uncertain future evolution. Among these securities, we use long-term forward prices of fuels, emission allowances, and electricity in neighboring markets to explain the dynamics of long-term electricity price.

Information on the forward prices of fuels, emission allowances, and imported electricity changes on a daily basis, since these forwards are usually traded each working day. Information on the long-term expected demand and the expected supply capacity changes only when new information on the underlying factors (GDP, construction, and retirement plans) becomes available; this information changes less frequently (e.g., monthly, quarterly, or yearly). The problem with this data is also that it is not as reliable as market-based information. Unless it is published as a part of exchange information, market participants need to estimate the expected demand and supply capacity themselves. The expected spot price is, therefore, influenced by high-resolution market-based information (forward prices of fuels, emission allowances, and neighboring-market electricity) and by low-resolution estimated information (expected demand and supply capacity).

Due to the different resolutions of both types of information, an estimation of the influence of these variables on electricity forward prices is challenging. In this paper, we use only high-resolution market-based information, whereas low-resolution estimated information is the additional source of uncertainty and it influences the variance structure of expected long-term electricity spot prices. Our model for the expected long-term spot price of electricity is therefore

$$\ln S_{t,T} = \sum_i \alpha_i \ln F^{fuel}_{t,T,i} + \sum_j \beta_j \ln F^{ea}_{t,T,j} + \sum_k \gamma_k \ln F^{nm}_{t,T,k} \tag{4}$$

where $S_{t,T}$ is the expected electricity spot price, $F^{fuel}_{t,T,i}$ is the forward price of fuel i, $F^{ea}_{t,T,j}$ is the forward price of the emission allowance j, and $F^{nm}_{t,T,k}$ the forward price of electricity in a neighboring market k.

TABLE I
SAMPLE CONSTRUCTION

CONTRACT	MATURITY PERIOD T	OBSERVATION PERIOD t
ENOYR07	2007	2005
ENOYR08	2008	2006
ENOYR09	2009	2007

III. DATA AND DESCRIPTIVE ANALYSIS

We test the proposed model on the long-term electricity forwards from the Nordic electricity exchange Nord Pool. Nord Pool is one of the oldest electricity exchanges, covering the area of four Nordic countries: Norway, Sweden, Finland, and Denmark. In 2005, most of the electricity in the Nordic electricity market was supplied by hydroelectric plants (54%), with the rest coming from nuclear (22%), renewable (8%), coal (6%), natural gas (5%), imports (3%), oil (1%), and other sources (1%). In 2005, the Nord Pool financial market volume was 786 TWh, physical volume was 176 TWh, whereas the total production in the market was 404 TWh. The market went through a number of structural changes, the latest being the introduction of the European emission trading scheme (ETS) in 2005. Since this changed the overall price formation, we choose to analyze only the prices from the start of 2005 to the end of 2007. Our data sample is constructed in a way to include only prices of yearly contracts with time to maturity between one year and two years as shown in Table I. For observation period 2005, ENOYR07 is used, and this contract is replaced with ENOYR08 with the start of 2006 and with ENOYR09 with the start of 2007. This way, we avoid the price shift when two consecutive contracts are rolled over. Since contracts with delivery period two and three years ahead move very similarly, the difference between them is very small. For other variables, defined in the following of the paper, we use forward prices with the same observation and maturity period as electricity forwards.

The analysis of high-resolution financial data often involves the problem of nonsynchronous trading. The prices in our analysis are quoted at different times, and due to the time mismatch, the integration between them is not clear. We use weekly resolution instead of daily resolution, since the relative time mismatch is much lower in the case of weekly sampling. Although the weekly sampling tends to smooth out the magnitude of price jumps, the volatility structure should not be significantly different to that when using the daily sampling. We use the closing price from each Wednesday as the reference weekly price for all the variables, giving the sample size of $\mathrm{N} = 156$.

As shown in Fig. 2, there are no significant shifts at the time of rollover. The sample, however, shows a significant price shock in April 2006 corresponding to observations 67 to 70. Before this shock, CO_2 emission allowance prices were pushing electricity prices up significantly; however, when the report on actual emissions in EU was published in April 2006, the prices of emission allowances dropped dramatically, which had a significant effect on electricity prices. We will investigate this effect by testing whether this shock can be considered as a structural break in the relationship between the variables.

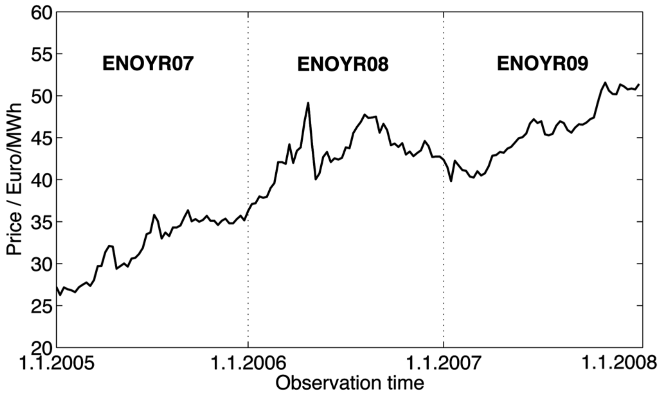

Fig. 2. Estimation sample for long-term electricity forward price.

A. Fuel Prices

Fuel prices can be divided into two groups. In the first group are the fuels that are not traded on an exchange, and so no transparent information about their prices exists. These fuels are uranium, biomass, water, wind, solar, and other renewable sources. Prices of these fuels are uncertain and they influence the variance structure of electricity prices. In the second group are the fuels that are traded on an exchange, and at least some information about their long-term prices is available. These are oil derivatives, natural gas, and coal. Although their use in electricity production in the Nordic market is small, they are often the marginal source of production and can have a significant influence on electricity prices. We model the fuel price $F_{t,T,i}^{fuel}$ with the forward prices for coal $F_{t,T}^{coal}$, natural gas $F_{t,T}^{gas}$, and crude oil $F_{t,T}^{oil}$, which also represents the price of all oil derivatives:

$$\sum_i \alpha_i \ln F_{t,T,i}^{fuel} = \alpha_0 + \alpha_1 \ln F_{t,T}^{oil} + \alpha_2 \ln F_{t,T}^{coal} + \alpha_3 \ln F_{t,T}^{gas}. \tag{5}$$

For the crude-oil price, we use the NYMEX WTI light sweet crude oil data. Although the Brent crude oil data from the Intercontinental Exchange (ICE) might be a better choice for Nordic countries, the availability of long-term oil prices is much better at NYMEX. The long-term NYMEX WTI price represents a global price indicator of the world oil price in the long-term.

The steam-coal market cannot be characterized as a global market like crude oil. The majority of coal is still traded over the counter, mostly because coal is hard to standardize, due to its different energy values. Exchange forward trading with coal is still in its early stages. Instead, we use the TFS API2 index as a reference for coal prices in the Nordic area. TFS API2 is a price index for coal delivered in Amsterdam, Rotterdam, and Antwerp harbor and should, therefore, also represent the coal prices in the Nordic area.

The natural gas consumed in the Nordic area comes mainly from North Sea resources. Natural gas forwards of North Sea gas is also traded on ICE. We use the ICE quarterly prices of natural gas forwards to construct the yearly forward prices for natural gas.

B. Emission Allowance Prices

The price of emission allowances in our model include only the price of the European CO_2 emission allowance (EUA) which were introduced by ETS in 2005 for carbon oxide (CO_2) emissions. The second part of (4) is therefore

$$\sum_j \beta_j \ln F_{t,T,j}^{ea} = \beta_1 \ln F_{t,T}^{eua} \tag{6}$$

where $F_{t,T}^{eua}$ is the forward price of the EUA. We use the data on EUA prices from Nord Pool. Since Nord Pool began trading with EUAs in March 2005, we use the Spectron EUA prices which precede that date. The difference between different EUA prices is negligible, since CO_2 allowance can be purchased and used anywhere in Europe.

C. Neighboring-Market Price

The Nordic electricity market imports electricity from Russia, Germany, and Poland. We have no information on import prices from Russia, so we use only the European Energy Exchange (EEX) long-term forward price as a reference price for the electricity imported from Germany and Poland. The neighboring-market price is, therefore, the EEX long-term electricity forward price:

$$\sum_k \gamma_k \ln F_{t,T,k}^{nm} = \gamma_1 \ln F_{t,T}^{eex}. \tag{7}$$

Combining (3)–(7) gives the following regression model describing the long-term electricity forward prices from Nord Pool:

$$\ln F_{t,T}^{np} = \alpha_0 + \alpha_1 \ln F_{t,T}^{oil} + \alpha_2 \ln F_{t,T}^{coal} + \alpha_3 \ln F_{t,T}^{gas} + \beta_1 \ln F_{t,T}^{eua} + \gamma_1 \ln F_{t,T}^{eex} + RT_m + u_{t,T}. \tag{8}$$

In (8), we include an error term $u_{t,T}$, which represents the uncertainty in the expected spot price and the uncertainty in explanatory variables. We assume the error term $u_{t,T}$ follows a normal distribution.

IV. Multivariate Model

Model (8) defines the univariate relationship between a dependent variable on the left and explanatory variables on the right-hand side. The drawback of such a representation is that explanatory variables are assumed to be exogenous, which is an assumption that should be tested rather than assumed *a priori*. Another drawback of the representation in (8) is that it fails to validly estimate all the long-term relationships between the variables, particularly when variables are nonstationary and cointegration between variables is present. In our model, we cannot assume exogeneity or stationarity, since the prices of interdependent commodities are often cointegrated and nonstationary. A model that overcomes the deficiencies of single equation models is a vector autoregressive model (VAR). A VAR model assumes that all variables are endogenous; hence, all variables are modeled as a function of own past values and past values of other endogenous variables. We define a general Gaussian vector autoregressive model

$$\mathbf{Y}_t = \mathbf{A}_0 + \sum_{i=1}^{k} \mathbf{A}_i \mathbf{Y}_{t-i} + \sum_{j=1}^{m} \Psi_j \mathbf{Z}_{t-j} + \sum_{h=1}^{n} \Theta_h \mathbf{w}_{h,t} + \mathbf{u}_t \tag{9}$$

TABLE II
DESCRIPTIVE ANALYSIS OF VARIABLES

VARIABLE	*np*	*oil*	*coal*	*gas*	*eua*	*eex*
MEAN	3.68	3.87	3.94	4.12	2.94	3.90
STD. DEV.	0.18	0.16	0.09	0.24	0.29	0.17
SKEWNESS	-0.71	-1.70	0.65	-0.84	-1.70	-0.92
EXC. KUR.	-0.48	2.46	0.41	-0.06	3.51	-0.57
ADF TEST	-2.33	-3.46*	-2.03	-2.68	-3.84**	-1.54
PP TEST	-2.21	-3.50**	-2.01	-2.56	-2.93*	-1.51

* rejects the null at 5% significance. ** rejects the null at 1% significance.

TABLE III
VAR(2) DIAGNOSTIC TESTS

TEST	$F_{ar}(4,136)$	$\chi^2_{nd}(2)$	SKEW.	$F_{het}(26,113)$	$F_{arch}(4,132)$	*SE*
np	1.11	19.7**	-0.33	4.56**	6.81**	0.0252
oil	1.35	2.15	-0.14	1.08	0.40	0.0262
coal	3.57**	6.26*	0.26	0.90	0.55	0.0193
gas	0.82	21.3**	0.73	0.99	5.75**	0.0313
eua	0.98	11.4**	0.11	1.66*	3.79**	0.0635
eex	1.84	29.6**	0.36	2.06**	8.33**	0.0179
CONST.: $F_{sig}(6,135) = 2.72$*, T_m: $F_{sig}(6,129) = 0.60$, LLF=2207.1						
VECTOR: $F_{ar}(144,656)$=1.37**, $\chi^2_{nd}(12)$=59.0**, $F_{het}(546,1610)$=1.15*						

* rejects the null at 5% significance. ** rejects the null at 1% significance.

where $\mathbf{Y}_t$ is a vector of endogenous variables, $\mathbf{Z}_t$ vector of exogenous variables, and $\mathbf{w}_{h,t}$ intervention dummies to render the residuals $\mathbf{u_t}$ well-behaved. We use the logs of $F^{np}_{t,T}$, $F^{oil}_{t,T}$, $F^{coal}_{t,T}$, $F^{gas}_{t,T}$, $F^{eua}_{t,T}$, and $F^{eex}_{t,T}$ as endogenous variables, and we denote them as np, oil, $coal$, gas, eua, and eex, respectively. Time to maturity T_m is considered exogenous.

A. Sample Analysis

Table II gives descriptive analysis of variables in (9). The variables are not normally distributed; particularly the skewness and excess kurtosis oil and eua are very high. Nonstationarity cannot be rejected in all cases except for oil and eua. Both stationarity tests strongly reject the nonstationarity in first differences (not presented here). Variables np, $coal$, gas, and eex are therefore integrated of order $I(1)$, while oil and eua may be of order $I(0)$, although the results are not strongly significant.

B. VAR Setup

Based on stationarity test, we will assume that none of the variables is $I(2)$ and the system is therefore adequately modeled as $I(1)$.

The model (9) with lag length set to $k = 2$ is estimated, and results together with diagnostics are presented in Table III. No intervention dummies $\mathbf{w}_{h,t}$ are included at this point. All endogenous variables in VAR are significant. A significance test (F_{sig}) on deterministic components show that the constant is marginally significant, while time to maturity T_m is not significant. The diagnostic tests involve F-tests that there is no residual autocorrelation (F_{ar}, against 4th-order autoregression), that residuals are normally distributed (χ^2_{nd}), that there is no heteroscedasticity (F_{het}), and that there is no autoregressive conditional heteroscedasticity (F_{arch}, against 4th order). Misspecification tests reveal significant problems with all of these tests, particularly when vector tests are considered. Since VAR estimates are more sensitive to skewness than kurtosis, residuals skewness is also reported.

TABLE IV
VAR(2) DIAGNOSTICS TESTS

TEST	$F_{ar}(4,126)$	$\chi^2_{nd}(2)$	SKEW.	$F_{het}(28,101)$	$F_{arch}(4,120)$	*SE*
np	1.12	13.6**	-0.20	1.02	0.48	0.0201
oil	2.31	4.41	-0.11	1.04	0.43	0.0256
coal	2.86*	4.15	0.03	1.13	1.35	0.0183
gas	0.27	19.4**	0.13	0.99	0.40	0.0289
eua	0.95	4.11	0.22	1.09	1.44	0.0536
eex	0.58	4.61	0.08	1.15	0.22	0.0136
alu	2.73*	10.1*	0.00	0.74	1.06	0.0210
CONST.: $F_{sig}(7,124) = 7.54$**, T_m: $F_{sig}(7,124) = 1.86$, LLF=2731.0						
VECTOR: $F_{ar}(196,665)$=1.13, $\chi^2_{nd}(14) = 43.4$**, $F_{het}(784,1601)$=0.86						

* rejects the null at 5% significance. ** rejects the null at 1% significance.

To overcome the undesired properties of residuals in Table III, we first focus on the structural specification of the model. Increasing the lag length k does not help to remove residual autocorrelation. Since residual autocorrelation also suggests an omission of important variables that influence the dynamic structure of our model, we analyze the price movement during this period and search for additional variables that might also be included in the model. Among much nonquantifiable information, we find that aluminum prices also affected the prices of electricity in Scandinavia and Europe during this period. Aluminum prices rose significantly during this period, and this triggered some decisions to postpone the decommissioning of some aluminum smelters, which could sell aluminum under increased long-term aluminum prices, with long-term electricity forwards as hedging instruments. The long-term aluminum prices therefore reflect changes in part of future electricity consumption and influence the demand for long-term electricity forwards. Descriptive analysis for aluminum forward price (alu) from London Metal Exchange also show a non-normal distribution, while the values of ADF and Phillips-Peron test are -1.46 and -1.28 $(\mathrm{cv}_{1\%} = -4.02)$ indicating integration of order $I(1)$.

We also introduce a few dummies to the system to account for the shocks, which are known to induce erratic behavior and nonlinear dynamics. First we introduce a blip dummy to set the residuals from 67 to 70, to zero, which correspond to eua price shock in April 2006. A blip dummy D_{b67} of a type $(\ldots 0, 1, 0 \ldots)$ with three lags is used for this purpose. Next we add one transitory dummy D_{tr} of a type $(\ldots 0, 1, 0, -1, 0 \ldots)$ to remove the effect of transitory shock in observations 27 and 29. Additionally three blip dummies D_{b33}, D_{b57}, and D_{b117} are used to remove the largest outliers. The diagnostics of VAR that include these changes is presented in Table IV.

The results show that aluminum price and dummies help to improve the properties of VAR. Most single equation and vector misspecification tests are improved. There is a still slight autocorrelation present in $coal$ and alu, but we will not pursue this further, since we expect these two variables are weakly exogenous and they do not have to be modeled themselves. The

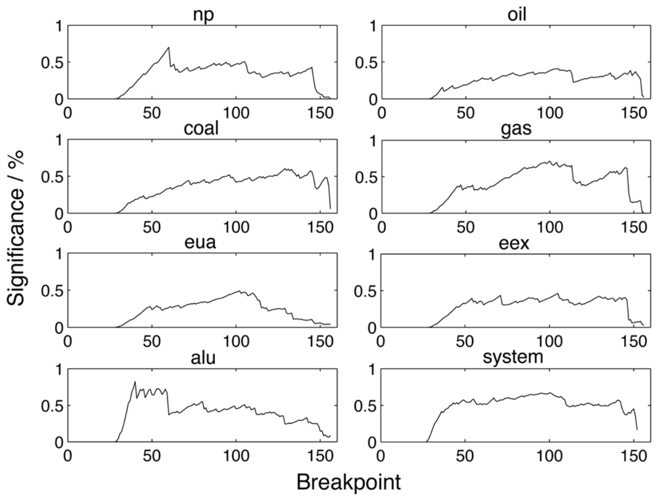

Fig. 3. Recursive break-point Chow test.

vector tests on the other hand reject the autocorrelation and heteroscedasticity in residuals. While strict normality is still not achieved, we managed to reduce the skewness, which is more critical than kurtosis.

We test this specification for parameter constancy. In particular, we are interested in the influence of eua price shock in April 2006. The shock had a significant effect on the Nord Pool forward price as seen in Fig. 2 and also on the EEX forward price. To test whether this shock, or any other shock during this period, changed the overall structure of the data generating process, we use the Chow test for structural break [15]. Fig. 3 shows recursive break-point Chow test for each equation in the system and for the system as a whole. The 1% significance level of the break-point test is never exceeded indicating that parameters of individual equations and the system as a whole are constant throughout the sample. The eua price shock can therefore be considered as a transitory shock, which can be removed with intervention dummies, rather than a structural break.

VAR in Table IV is also tested for stability by checking the roots of the companion matrix. All the roots lie inside the unit circle with the moduli of the three largest roots being 0.981, 0.981, and 0.948, respectively indicating that this representation of VAR is stable.

C. Cointegration Analysis

Since unit root testing indicates that first differences are $I(0)$, we convert the model (9) to first difference model. This model explains only the short-run dynamics of the system, while the long-run relationship between variables, which is important if variables are cointegrated, is lost. Cointegration between nonstationary variables can be captured with *vector equilibrium error correction model (VECM)*:

$$\Delta \mathbf{Y}_t = \mathbf{A}_0 + \Pi \mathbf{Y}_{t-1} + \sum_{i=1}^{k-1} \mathbf{\Gamma}_i \Delta \mathbf{Y}_{t-i} + \sum_{j=1}^{m} \Psi_j \Delta \mathbf{Z}_{t-j} + \sum_{h=1}^{n} \Theta_h \mathbf{w}_{h,t} + \mathbf{u}_t \quad (10)$$

which has the same innovation process $\mathbf{u}_t$, since no restrictions have been imposed by transformation from (9) to (10). In (10), the right-hand side contains information about the short- and the long-run adjustment to changes in $\mathbf{Y}_t$. If $\mathbf{Y}_t$ contains $I(1)$ variables, then $\Delta \mathbf{Y}_{t-i}$ is $I(0)$, while $\mathbf{\Pi}\mathrm{Y}_{t-1}$ must also be $I(0)$ for $\mathbf{u}_t$ to be a white noise process. Matrix $\mathbf{\Pi}$ can be decomposed to $\mathbf{\Pi} = \boldsymbol{\alpha\beta}'$, where $\boldsymbol{\alpha}$ represent the speed of adjustment to disequilibrium and $\boldsymbol{\beta}'$ is the matrix of long-run coefficients such that $\boldsymbol{\beta}'\mathbf{Y}_{t-1}$ represents up to $n-1$ stationary cointegrating relationships, which ensure that $\mathbf{Y}_t$ converge to their long-run steady-state solution. A note, however, is necessary that since $\mathbf{Y}_t$ contains two variables that are possibly $I(0)$ in levels, they form a cointegrating relation by itself adding to the total number of cointegrating relations. A_0 is unrestricted constant which accounts for a constant in the short-run model (trend in levels) and a constant in cointegration space.

To test for cointegration between variables, we employ Johansen testing procedure [16] which concentrates on testing whether the eigenvalues λ_i of the matrix $\mathbf{\Pi}$ in (10) are significantly different from 0. We test whether $\mathbf{\Pi}$ has a reduced rank $r \leq (n-1)$, indicating that there are r stationary cointegrating relationships between nonstationary variables in VAR. If $r = n$, this would indicate that all variables are stationary, while $r = 0$ would indicate no stable cointegrating relationships and the VAR with first differences only would be adequate. To determine the rank r, we use the trace test statistics

$$\lambda_{\text{trace}} = -T \sum_{i=r+1}^{n} \log(1 - \hat{\lambda}_i) \quad (11)$$

where T is the sample size and $\hat{\lambda}_i$ are the estimated eigenvalues of $\mathbf{\Pi}$. The results of the cointegration rank test, presented in Table V, show that λ_1 is strongly significant while λ_2 and λ_3 are on the borderline of significance with p values 0.019 and 0.074, respectively. It is hard to know exactly if they form a stationary cointegrating vector or not. Since the choice of cointegration rank is crucial in modeling cointegrated systems, we look for additional indicators for determining r, as specified in [17]. First we look at the moduli of the largest characteristic roots of the model, and see how they are changing for the hypotheses in question, i.e., $r = 1$, 2, and 3. Table V again show indecisive results for $r = 2$ and $r = 3$. It is hard to know exactly whether a moduli of 0.910 represent a unit root or not. Next we look at the significance of parameters of loading matrix $\boldsymbol{\alpha}$. The t-value of $\alpha_{2,2}$ is -4.20 and for $\alpha_{5,3}$ is -3.28 indicating that the second vector adds additional explanatory power to the

TABLE V
COINTEGRATION RANK TEST AND CHARACTERISTIC ROOTS

H_0: *rank*≤	λ_i	λ_{trace}	*prob*	$r=1$	$r=2$	$r=3$
0	0.372	172.5**	0.000	1.000	1.000	1.000
1	0.196	101.0*	0.019	1.000	1.000	1.000
2	0.177	67.47	0.074	1.000	1.000	1.000
3	0.114	37.55	0.326	1.000	1.000	1.000
4	0.074	18.92	0.509	1.000	1.000	0.910
5	0.027	7.11	0.572	1.000	0.871	0.910
6	0.018	2.85	0.091	0.405	0.431	0.458

* rejects the null at 5% significance. ** rejects the null at 1% significance.

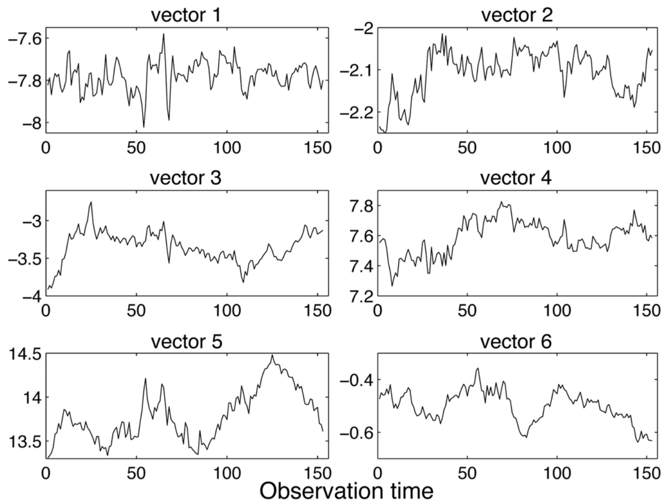

Fig. 4. Cointegrating vectors.

oil equation and third cointegrating vector to the eua equation. Finally we look at the graph of the first six cointegrating vectors presented in Fig. 4. The first two cointegrating vectors look stationary and the last three are clearly not. For the third cointegrating vector, it is hard to decide, so we test the third vector with ADF test and Phillips-Perron test. Both of them reject stationarity with probability of 0.115 and 0.153, respectively. Based on these findings, we choose cointegration rank $r = 2$. Although the third vector might also help to explain the long-term relationship in eua equation, it is not stationary.

To identify the two cointegration vectors, we test the restrictions on estimated $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$. We first rotate the cointegration space by normalizing $\boldsymbol{\beta}$ with respect to np and oil. This way, beta is exactly identified and the significance of each variable in cointegration space can be tested by putting additional restrictions on parameters in $\boldsymbol{\beta}$. These tests based on standard LR test show that in the second vector, only the oil parameter is significant, indicating that the second vector is exactly oil. This is consistent with the unit root test results showing oil being stationary in levels. Gas price is insignificant in both cointegrating vectors and therefore have no long-run explanatory power. Testing all restrictions gives the following representation of $\boldsymbol{\beta}$, with standard errors below.

$$\hat{\boldsymbol{\beta}} = \begin{array}{c} \begin{array}{ccccccc} np & oil & coal & gas & eua & eex & alu \end{array} \\ \begin{bmatrix} \underset{(-)}{1} & \underset{(-)}{0} & \underset{(0.10)}{-0.72} & \underset{(-)}{0} & \underset{(0.03)}{0.13} & \underset{(0.17)}{0.93} & \underset{(0.16)}{-0.96} \\ \underset{(-)}{0} & \underset{(-)}{1} & \underset{(-)}{0} & \underset{(-)}{0} & \underset{(-)}{0} & \underset{(-)}{0} & \underset{(-)}{0} \end{bmatrix} \end{array}.$$

The first cointegrating vector is interesting since it represents a linear combination of np, $coal$, eua, eex, and alu. The Nord Pool price therefore increases approximately one to one with the aluminum price and less than one to one with the coal price. On the other hand, the Nord Pool price falls almost one to one with EEX price increase and falls by 0.13% if the emission allowance price rises by 1%. Since Nord Pool and EEX price are strongly positively correlated, this means that if a positive shock occurred in the EEX price in the last period, then similar positive shock is also likely to have occurred in the Nord Pool price. The cointegrating vector would then pull the Nord Pool price back down in the next period.

TABLE VI
DIAGNOSTICS TEST OF VECM

TEST	$F_{ar}(4,140)$	$\chi^2_{nd}(2)$	SKEW.	$F_{het}(14,129)$	$F_{arch}(4,136)$	SE
Δnp	1.02	22.3**	0.13	0.87	0.93	0.0198
Δoil	0.45	5.97	-0.11	0.86	0.79	0.0258
Δeua	0.85	1.02	0.21	0.82	2.42	0.0527
Δeex	1.56	9.46**	0.13	1.37	0.95	0.0134

Δnp_{-1}: $F_{sig}(4,141) = 3.15$* $\quad$ ΔT_m: $F_{sig}(4,141) = 4.26$**,
$\hat{\boldsymbol{\beta}}_1'\mathbf{Y}_{t-1}$: $F_{sig}(4,141) = 13.7$** $\quad$ $\hat{\boldsymbol{\beta}}_2'\mathbf{Y}_{t-1}$: $F_{sig}(4,141) = 13.6$**
CONST.: $F_{sig}(4,141) = 13.7$** $\quad$ $LLF = 1496.5$

VECTOR: $F_{ar}(64,491)=1.09$; $\chi^2_{nd}(8)=36.3$; $F_{het}(140,1001)=1.08$

* rejects the null at 5% significance. ** rejects the null at 1% significance.

In the second step, we test the restrictions on loading matrix $\boldsymbol{\alpha}$, which is also known as the test for weak exogeneity. The test involves testing the restrictions that particular row in the estimated loading matrix $\boldsymbol{\alpha}$ is insignificantly different from zero. The parameters of $\boldsymbol{\alpha}$ explain how the short-run model is adjusted to the disequilibrium represented by cointegrating vectors $\boldsymbol{\beta}'\mathbf{Y}_{t-1}$. If the entire row in $\boldsymbol{\alpha}$ is zero, this indicates that none of the cointegrating vectors enters the equation associated with this row. Testing these restrictions additionally to restrictions on $\boldsymbol{\beta}$ shows that $coal$ and alu are weakly exogenous in our model, while other variables should be treated as endogenous. The values of $\boldsymbol{\alpha}$ coefficients, reported below, show that oil and gas price have very low and also insignificant speed of adjustment to both cointegrating vectors, even though the weak exogeneity test for these two variables was rejected at 5% significance. The parameters for Nord Pool, EEX, and EUA price are strongly significant. EUA price has the highest speed of adjustment with about 39% of disequilibrium in the first vector corrected in one period:

$$\hat{\boldsymbol{\alpha}} = \begin{array}{c} \begin{array}{ccccccc} np & oil & coal & gas & eua & eex & alu \end{array} \\ \begin{bmatrix} \underset{(0.03)}{-0.18} & \underset{(0.04)}{0.04} & \underset{(-)}{0} & \underset{(0.04)}{-0.09} & \underset{(0.09)}{-0.39} & \underset{(0.02)}{-0.18} & \underset{(-)}{0} \\ \underset{(0.04)}{0.19} & \underset{(0.05)}{-0.10} & \underset{(-)}{0} & \underset{(0.05)}{0.05} & \underset{(0.10)}{0.32} & \underset{(0.02)}{0.18} & \underset{(-)}{0} \end{bmatrix} \end{array}.$$

Based on cointegration test and weak exogeneity test, we form a VECM as in (10). $\mathbf{Y}_t$ now includes endogenous variables np, oil, gas, eua, and eex, while $\mathbf{Z}_t$ includes two weakly exogenous variables $coal$ and alu and time to maturity T_m. Estimation of (10) includes one lag of first differences of endogenous variables, the first lag of two cointegrating vectors, the first lag of weakly exogenous variables, eight dummy variables, and a constant, giving 25, 10, 15, 40, and 5 parameters, respectively, a total of 95 parameters to estimate. We reduce the model size with the standard F-test. The reduced model shows that gas is also insignificant in the short-run model so we completely remove gas from the system. The 4-D model now includes 40 parameters and the value of F-test on reduction is $\mathrm{F}(30,526) = 1.06$. The reduced model includes the first lag of Δnp, two cointegrating vectors, a constant, ΔT_m, and five dummies only. The diagnostic tests presented in Table VI show that the main properties of residuals remained unchanged with standard errors very close to values in Table IV. Since $coal$, alu, and gas equation are removed from the system, vector autocorrelation and heteroscedasticity tests are improved. Normality test, however, shows no improvement. Both cointegrating vectors are highly significant, confirming that the choice of cointegration rank is correct. ΔT_m is significant only in Δeua and Δeex equation, indicating that the system shows some influence of the risk premium; however, the significance is strongly rejected in Δnp $(p = 0.70)$, which is in our interest.

V. Conclusion

We have analyzed the long-term electricity forward prices using a vector autoregressive model. The model is specified based on variables that influence the expected long-term electricity supply, demand, and risk premium. We use the long-term forward prices of oil, coal, natural gas, emission allowances, imported electricity, and aluminum prices to model the dynamic properties of long-term electricity forward prices. The risk premium is modeled as a function of time to maturity.

The model is estimated on weekly data from 2005 to 2007 using variables relevant for the Nordic electricity market. We specify a 7-D VAR with two lags and few intervention dummies. The influence of the emission allowance price shock in April 2006 is analyzed with a Chow breakpoint test, which showed no breaks in constant or trend. The variables in the model are all integrated of order $I(1)$, except oil and emission allowance price, which are $I(0)$ but close to unit root. The system is tested for cointegration using the Johansen cointegration test. The test, together with other indicators, indicates two stationary cointegrating vectors, the first being a linear combination of nonstationary variables. The Nord Pool price increases if aluminum price and coal prices increase in the previous week and decreases if the emission allowance price and the EEX price increase in the previous week. The second cointegrating vector is exactly the oil price, which is stationary in our case. Gas price is found insignificant in both the short-and the long-run model. The model shows some influence of the risk premium; however, its influence on electricity forward prices from Nord Pool is not confirmed. This indicates that the risk premium dynamics in the long-term electricity forwards from Nord Pool is rather low and that the risk premium could be considered as constant. While these results hold for the Nordic electricity market, other markets may have a different maturity level and their price dynamics may respond to other variables. Nevertheless, the general approach could be used for analyzing other electricity markets.

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Martin Povh is pursuing the Ph.D. degree at the Faculty of Electrical Engineering, University of Ljubljana, Ljubljana, Slovenia.

He is a Research Assistant at the University of Ljubljana. His research is focused on modeling electricity markets, electricity prices and their uncertainty, investment analyses under uncertainty, and electricity market operation.

Stein-Erik Fleten received the M.Sc. and Dr.Ing. degrees from the Norwegian University of Science and Technology (NTNU), Trondheim, Norway, in 1995 and 2000, respectively.

He is a Professor in the Department of Industrial Economics and Technology Management at NTNU. His research interests include real options valuation, financial engineering in energy commodity markets, and stochastic programming. Applications typically involve restructured electricity markets.

Paper 2: Modelling the structure of long-term electricity forward prices at Nord Pool

Modelling the structure of long-term electricity forward prices at Nord Pool

Martin Povh, Robert Golob, Stein-Erik Fleten

Abstract This chapter models long-term electricity forward prices with variables that influence the price of electricity. Long-term modelling requires consideration of expected changes in the demand and supply structure. The model combines high-resolution information on fuel costs from financial markets and low-resolution information on the demand/supply structure of the electricity market. We model the latter using consumption and supply capacity, and the former with forward prices of fuels, emission allowances and imported electricity. The model is estimated using data from the Nordic electricity market and global long-term forward prices of energy. Owing to a lack of data on consumption and supply capacity, the estimated results only provide the broad influence of these variables on forward prices. Though extrapolation of the prices observed in Nord Pool may suffer from the influence of short-term variables, such as precipitation and temperature, the model yields robust forecasts of the prices of contracts that are not exchange traded.

1 Introduction

Since the beginning of the deregulation of electricity markets, a significant connection between electricity prices and different energy prices has prevailed. This is because the various energy sources serve either as a fuel for generating elec-

Martin Povh
Faculty of Electrical Engineering, University of Ljubljana, Tržaška 25, SI-1000 Ljubljana, Slovenia, e-mail: martin.povh@fe.uni-lj.si

Robert Golob
Faculty of Electrical Engineering, University of Ljubljana, Tržaška 25, SI-1000 Ljubljana, Slovenia, e-mail: robert.golob@fe.uni-lj.si

Stein-Erik Fleten
Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology, Trondheim NO-7491, Norway, e-mail: stein-erik-fleten@iot.ntnu.no

tricity or as a substitute for electricity use. Transparency in electricity markets has also improved significantly with the introduction of electricity exchanges. Although electricity markets struggle with low liquidity, they do provide information about the competitive market price, often used as an indicator in over-the-counter (OTC) trading. Moreover, because the price of electricity is very volatile, both in the short and the long term, participants in electricity markets attempt to reduce price-related risk with forward trading.

A variety of forward contracts are traded on exchanges with times-to-delivery ranging from 1 week to a few years, and these give market participants a variety of choices for hedging against price risk. In liquid forward markets, forward trading also conveys the present information on the expected price in the future adjusted for the market price of risk. This information is especially important in the long term, not only for hedging purposes but also for different strategic purposes, including investment analysis, asset management, strategic decisions on mergers and acquisitions, state energy policies, etc. Excess capacity has gradually decreased since deregulation and investments in new capacity has not followed the growth in consumption. Apart from the increasing prices of other energy commodities, this has also had an important impact on the overall trend in increasing wholesale prices during this period.

In recent years, we have also witnessed a shift in electricity production technology from traditional coal and nuclear to natural gas and renewable sources. This shift in production requires investors to have relevant information about the electricity market in the future in order to support investment decisions today. Long-term information about electricity prices is one of the most important variables in these analyses. Investors in new production capacity need to estimate long-term expected spot prices. Alternatively, real option theory suggests the use of forward prices instead of expected spot prices, as forward prices already incorporate the appropriate market price of risk. Unfortunately, we do not have current information about prices in the distant future, e.g., 10 years ahead, because forward contracts with these delivery periods are not traded on the exchange. Typically, the term structure of forward contracts ends 5 to 6 years ahead.[1] This, however, does not mean that there is no forward trading beyond this horizon. For example, an OTC forward market with 10-year forward contracts existed for a few years in the Nordic electricity market. This provides evidence that investors seek long-term forward contracts to hedge long-term price related risks properly.

A simple way to estimate the value of the longer end of the term structure of electricity prices would be to extrapolate prices observable at electricity exchanges. Investors often use different rules of thumb to estimate the value of forward contracts beyond the traded horizon. These rules can be based on different historic data, intuition and experience. However, market forces other than the prices of observable exchange traded contracts may drive the prices of forward contracts beyond the traded horizon. This leads to potential errors in these estimates. Finally, the most accurate information about the value of electricity with a certain delivery period can

[1] A forward price term structure is a set of prices of exchange traded forward contracts for various times-to-maturity.

only be provided by a liquid forward market. The absence of accurate information indicates the need to extend the trading horizon on electricity exchanges into the future as far as possible. This would result in better market transparency, which is important not only for the investment purpose mentioned earlier but also for global benchmarking. Publicly available long-term price indicators in a particular electricity market would also be useful for investors and other institutions outside the market itself.

Long-term modelling of commodity forward prices is relatively new, as the availability of the long-term forward data remains low. Schwartz [1] employs models based on short-term oil futures and tests their performance on available long-term oil futures. Pindyck [2] models the long-term evolution of oil, coal and natural gas prices with mean reversion models using the theory of depletable resources. Schwartz and Smith [3] use long-term commodity prices as an equilibrium to which short-term prices revert. Unfortunately, there is relatively little work on the modelling of long-term electricity forward prices.

However, an increasing number of studies focus on the short-term electricity forward market. For example, Byström [4], Lucia and Schwartz [5] and Solibakke [6] focus on the valuation of short-term forward contracts and their hedging performance. Johnsen [7] presents a short-term supply/demand model that we could potentially use in long-term modelling. Bessembinder and Lemon [8] present a model in which the electricity forward price is the equilibrium of the supply and demand for forward contracts. Because electricity prices are influenced by the impossibility of storage and high uncertainty in underlying factors, some researchers suggest a combined approach to modelling electricity price. For instance, Eydeland and Wolyniec [9], and Pirrong and Jermakyan [10], combine the properties of financial models common in stock valuation with the properties of fundamental models where supply and demand are modelled using fundamental factors [11]. Here we follow a similar idea to model the prices of long-term electricity forwards with a regression approach. Our model depends on the assumption that the expected value of a commodity with a certain delivery period depends on expected supply and demand, as well as the supply and demand for risk hedging associated with uncertainty relating to its expected value. Modelling long-term supply and demand requires different fundamental factors from the short term, while structural changes in the market also require consideration. We identify which of these parameters influence the supply and demand for electricity in the long term and then seek long-term information that we can use to model these variables. We estimate the model parameters using the market data available for the Nordic electricity market.

The chapter is organized as follows. Section 2 identifies the long-term electricity forward price process with respect to supply, demand and the risk premium. We identify three groups of variables and present a simple model for each. At the end of this section, we present the model for long-term forward prices. Section 3 discusses the data and parameter estimation process. In Section 4, we provide the estimation results and some indicators of model performance. Section 5 contains some conclusions and recommendations.

2 Long-term forward price process

Modelling forward prices involves two time dimensions. The first is the observation time t, reflecting the time at which the price of a particular forward contract is observed or settled. The second is the delivery period T, which represent a period of time in which the forward contract matures or is delivered. The forward price is therefore denoted as $F_{t,T}$. We can forecast unobserved forward prices using either of these dimensions. The first strategy involves forecasting the future evolution of the price of an observed contract and using the n-step ahead forecast to obtain the value of a contract at time $t+n$. This model would typically involve autoregressive components of dependent and explanatory variables. The expected value of the forecast for the long-term forward price $E[F_{t,T}]$ would therefore be the average forecasted value during the delivery period T. This strategy is less convenient for a long-term forecast as the forecasting horizon could be very large when compared with the estimation sample.

An alternative strategy for forecasting the price of an unobserved forward contract would be forecasting with respect to the delivery period T. This strategy is based on the assumption that the relationships between the dependent and explanatory variables is dimension invariant; i.e., these relationships do not change with the observation time t or delivery period T. Under this assumption, the relationships estimated on observed contracts with delivery period T can be used to forecast the price of a contract with delivery period $T+N$. This strategy also has some advantages when it comes to modelling long-term prices. First, the forecasting horizon is very small if the delivery period T is long enough; e.g., 1 year. Second, we can use the available long-term information on the explanatory variables directly in the forecast as we assume the same relationship for the observed contracts. Third, we can design the underlying model to match the specifics of long-term prices; for example, constructing a specific model for a specific type of contract.

2.1 Definition

We define long-term electricity forward prices as the prices of electricity forward contracts with a delivery period of 1 year and a time-to-delivery of more than 1 year ($T-t \geq 1$ year). Fig. 1 depicts the prices of the yearly Nord Pool contracts ENOYR1, ENOYR2 and ENOYR3 with respective times-to-delivery of 1, 2 and 3 years. Fig.1 demonstrates that the forward price dynamics depend very much on the time-to-delivery $T-t$. Short-term information, such as the level of water reservoirs, strongly influences the price of ENOYR1. Namely, a large proportion of hydropower plants in the Nordic market have the possibility of storing water up to 1 year or longer. There is also some degree of short-term information in ENOYR2 and ENOYR3, however, we assume that the prices of these contracts are mainly driven by the long-term information. Somewhat different laws therefore govern long-term and short-term forward (or spot) prices, and this suggests the need for different

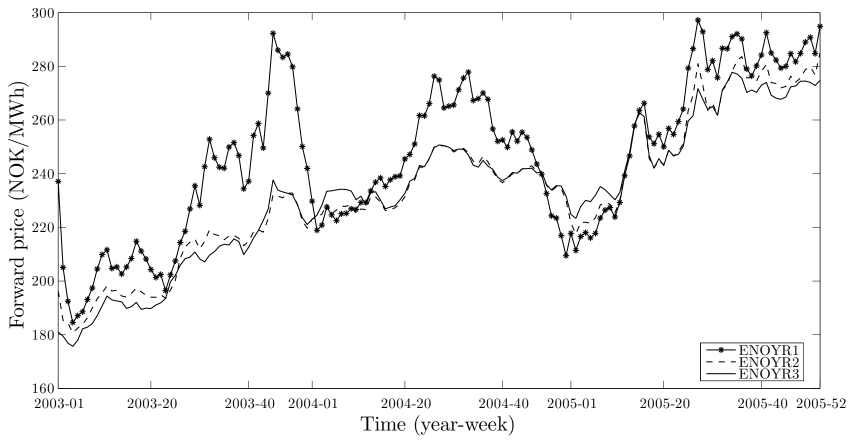

Fig. 1 Average weekly price of yearly forward contracts at Nord Pool

modelling strategies. Using this definition, long-term forward prices in the Nordic electricity market are ENOYR2 and ENOYR3, whereas ENOYR1 is the mid-term and eventually short-term contract as time-to-delivery approaches zero.

2.2 Valuation of forwards

Electricity is a very peculiar commodity because there is no economically efficient means of storage. This suggests that in order to estimate the relation between the forward price and the spot price of electricity, we cannot use standard *cost-of-carry* arbitrage:

$$F_{t,T} = (P_t + C_s)(1 + r - y)^{(T-t)} \tag{1}$$

where the forward price $F_{t,T}$ is the sum of the spot price P_t and storage costs C_s discounted by the difference between the risk-free interest rate r and the convenience yield y. Putting counterparty risk aside, the arbitrage principle implies that there should be no difference between buying the forward contract and buying the commodity in the spot market, storing it and using it during the delivery period. Using the arbitrage principle to value electricity forward contracts is not advisable, because electricity cannot be stored today and consumed later in the future. One possible alternative for valuing electricity forward contracts is a risk-adjusted expected spot price:

$$F_{t,T} = E_t[P_T](1 + r - \lambda)^{(T-t)} \tag{2}$$

where the forward price $F_{t,T}$ is the current expectation about the spot price during the delivery period $E_t[P_T]$ (or simply $P_{t,T}$) discounted by the difference between the risk-free interest rate r and the risk premium λ.

2.3 Risk adjustment

Assuming all investors hold the same expectation about the spot price, this approach (2) basically seeks the equilibrium between supply and demand for reducing risk by using an appropriate risk premium. Another interpretation of risk adjustment is the separation of the forward price data-generating process in two subprocesses. The underlying process of the expected supply and demand for electricity during the delivery period generates the expected spot price during the delivery period. Using this expectation, we obtain the probability distribution of the expected spot price, and this generates the risk premium based on the risk preferences of investors over this probability distribution. Transforming (2) to logs gives:

$$\ln F_{t,T} = \ln P_{t,T} + \ln(1 + r - \lambda)(T - t). \tag{3}$$

While risk-free interest rates can be easily determined by looking at, for example, the prices of government bonds, the assessment of λ, or the market price of risk, has always been a challenging task. A somewhat naive way to determine λ is to use the Capital Asset Pricing Model (CAPM), in which the risk premium in the electricity market depends on the risk premium in the overall capital market and the correlation between movements in the electricity market and movements in the overall capital market. Although CAPM is able to capture the price of risk, it is not well suited for pricing electricity derivatives because it assumes that (financial) electricity market is also used for diversification of general investors. In financial electricity markets, the participation of investors outside the industry is weak; hence, the dynamics of the risk premium is mainly driven by producers and consumers, who are motivated by hedging production and consumption. These investors are generally not diversified in the general capital market, and due to the ownership structure dominated by governmental influence, it is natural to consider them risk-averse. For instance, Bessembinder and Lemmon [8] show that in the absence of outside speculators, different levels of risk aversion by producers and consumers lead to non-zero risk premiums in electricity forwards. These non-zero risk premiums may attract participants from outside the industry to include forwards in their portfolios, and this can gradually decrease the level of risk premium.

To estimate the risk premium, we use past empirical findings about the risk premium in the electricity market. First, Bessembinder and Lemmon [8] and Longstaff and Wang [12] argue that the risk premium in short-maturity electricity forwards is influenced by the probability of price spikes (load seasonality) and the level of prices. A few studies attempting to estimate the risk premium on far-maturity contracts have found that it is much lower than in the short-maturity contracts [13, 15]. [13] also finds a connection between the risk premium and seasonal observation time in short- and far-maturity contracts from Nord Pool. This implies that there is a difference in supply and demand for long-term forwards at different times of the year. One of the reasons could be the yearly liquidity cycle, as the liquidity of yearly contracts from Nord Pool is typically lower at the beginning of the year and increases towards the end of the year when most of the yearly delivery contracts are settled. While this significantly influences the price of the first yearly contract, it also has some influence on the prices of all subsequent yearly contracts. Based on these findings, we use the following model for risk adjustment:

$$\ln F_{t,T} = \ln P_{t,T} + \kappa_0 r_{t,T} + \kappa_1 + \kappa_2 \tau_t \tag{4}$$

where the risk premium in far-maturity contracts is a constant κ_1 plus a seasonal time-dependent term $\kappa_2 \tau_t$ in which τ_t is a seasonal time, defined as the time from January 1 in the observation year. Although the forward price in (4) is the price of a pure financial contract where there is no cash flow between the buyer and the seller at settlement time, there is a strong rationale for the prices of contracts beyond the traded horizon requiring at least some cash flow or financial guarantee at the time of the settlement. Therefore, long-term interest rates will have an influence on the value of the forward contract. We also believe that interest rates influence forward prices through their influence on the fundamental variables discussed in the following section. For this reason, the interest rate is included not only as a discounting variable but also as an explanatory variable for the expected spot price. The risk adjustment in (4) is no longer a function of time-to-delivery $T-t$. Because of the Samuelson effect [14], the volatility of futures and forwards increases as time-to-delivery approaches. Hence, we assume that the volatility of far-maturity contracts does not vary significantly with time-to-delivery. The risk premium in far-maturity contracts is therefore a constant with respect to time-to-delivery.

2.4 Modelling the long-term expected spot price

The expected spot price during a particular period is a simple average of the expected spot prices during the delivery period. To model the long-term expected spot price, we model supply and demand using fundamental factors, whereas we obtain the price in equilibrium by matching supply and demand.

2.4.1 Long-term demand

Electricity demand is a process driven by short-term fundamental drivers, such as the daily and weekly cycle, temperature, the price elasticity of electricity and its substitutes, daylight hours, etc. Economic drivers (gross domestic product, household consumption expenditure) and demographic drivers (population, migration) influence electricity demand in the long term, typically causing demand to grow over time. Unfortunately, long-term information on short-term drivers does not exist. However, historical averages may be a reasonable estimate of their long-term expected value. This implies that these drivers do not influence the expected value of long-term demand, as their expected value is constant. However, we need to model the uncertainty in these variables as a short-term non-persistent error. Contrary to the situation for the short-term drivers, we can predict the long-term drivers to some extent, or alternatively, one can use existing forecasts produced by different institutions. We model long-term electricity demand as long-term electricity consumption adjusted by the price elasticity:

$$\ln q_t^d = \ln c_t + \eta \ln P_t \tag{5}$$

where q_t^d is the quantity of demand, P_t is the spot price, η is the demand elasticity, and c_t is electricity consumption. Long-term consumption c_t is a well-understood process, driven by the long-term variables described earlier. However, because we quite often adequately model expected consumption growth in developed countries as a constant, we employ a simple linear model:

$$\ln c_t = \alpha_0 + \alpha_1 t \tag{6}$$

and the long-term demand function is therefore:

$$\ln q_t^d = \alpha_0 + \alpha_1 t + \eta \ln P_t + u_t^d. \tag{7}$$

In (7) a stochastic error term u_t^d is included to account for short-term non-persistent errors such as temperature, daylight hours, wind, etc.

2.4.2 Long-term supply

Electricity supply is more price elastic than electricity demand, though the underlying factors influencing supply have greater uncertainty. While many variables that influence long-term demand cannot be included in the model because we do not have reliable long-term information, this is not the case for long-term supply. Some important influencing variables are also traded commodities with transparent long-term price information. Nevertheless, long-term information about some variables that also play an important role in electricity supply does not exist. We can divide the variables that influence the shape of the supply function into two groups. The first group includes structural variables that influence the structure of the market. We capture these variables with supply capacity. The second group of supply variables consists of supply cost variables; these cause changes in electricity supply costs.

We model long-term electricity supply as a function of supply capacity, supply costs and price elasticity:

$$\ln q_t^s = a_1 \ln C_t^s + a_2 \ln g_t + a_3 \ln P_t \tag{8}$$

where P_t and q_t^s are the spot price and the supply quantity respectively, g_t is the supply capacity and C_t^s are supply costs.

The supply function derives from the different units that supply the market. Each unit has limited installed capacity and therefore a limited amount of electricity it can produce in 1 year. Electricity producers also face other restrictions, including technical and environmental constraints and the weather. The level of precipitation or the available water for hydro production is a variable that has a significant impact on the supply capacity in some markets. However, we do not have any long-term information about this. Our expectation on hydro production for a particular year in the future instead derives from the historical average. We define supply capacity as the sum of total installed capacity in the market including net transfer capacity. Given that we cannot treat wind production the same as other types of capacity, we adjust the wind capacity with the average utilization factor of 0.25. Supply from neighbouring markets is a specific producer that participates in the market with a net transfer capacity.

Given the current market situation, it is reasonable to assume that until the start of the delivery period T a few years ahead, some new production units will enter the market and some old production units will be decommissioned. In theory, the expected change in supply capacity should depend on the spread between electricity prices and supply costs; however, in reality, new investments are to a large degree driven by political decisions, such as environmental pressures or the change in the level of reserve capacity associated with deregulation. We do not claim that the spread between electricity prices and supply costs provides no incentive for new investments. However, we believe that it is still not a prevailing factor influencing new investments. For this reason, we model the supply capacity g_t during time t with a simple linear model:

$$\ln g_t = \gamma_0 + \gamma_1 t + u_t^g \tag{9}$$

where the error term u_t^g represent the uncertainty in the supply capacity. Alternatively, one can use long-term information about construction plans, which is often available, although in most cases, it is still a rough expectation.

On the supply side, each unit has different production costs, which depend on the type of fuel, fuel costs and efficiency. While some units have very low or no fuel costs (wind, hydro, nuclear), other units have considerable and uncertain costs of fuel (coal, natural gas, oil). Similar to Eydeland and Wolyniec [9], we divide the supply cost variables into three groups. The first group consists of non-tradable fuels such as water, uranium, wind and biomass. As there is no liquid market or long-term price information for these fuels, we assume that their expected costs do not change with t or T. The second group consists of tradable fuels, mostly coal, natural gas and oil derivatives. Long-term information about the prices of these fuels is available in the form of futures and forwards, sometimes traded on the exchange up to 6 years into the future. The third group of supply cost variables includes other costs of supply such as emission allowance prices and the prices of electricity imports. We assume that the introduction of the European Union Emission Trading Scheme (EU ETS) in 2005 increased production costs for fossil-fuel producers, thereby increasing electricity prices. Emission allowance prices should therefore have a significant influence on electricity prices. Imported electricity is also part of the supply function, and we model this as electricity from a specific type of producer. The price for this producer is the average price in the market from where the electricity is imported.

Supply costs C_t^s can then be expressed as a linear combination of the relevant supply cost variables:

$$\ln C_t^s = b_0 + b_1 \ln P_t^{oil} + b_2 \ln P_t^{coal} + b_3 \ln P_t^{ea} + b_4 \ln P_t^{import} \tag{10}$$

where P_t^{oil} and P_t^{coal} are the prices of crude oil and coal, P_t^{ea} is the emission allowance price and P_t^{import} is the price of electricity in the market from which the electricity is imported. In (10) we do not include the price of natural gas as long-term information about the price of natural gas remains scarce and because the price of natural gas is highly correlated with the price of crude oil. We assume that the price of crude oil is a good proxy for the prices of all oil derivatives and natural gas.

Combining (8) and (10), we obtain the following linear supply model:

$$\begin{aligned} \ln P_t = \beta_0 + \beta_1 \ln g_t + \beta_2 \ln q_t^s + \beta_3 \ln P_t^{oil} \\ + \beta_4 \ln P_t^{coal} + \beta_5 \ln P_t^{ea} + \beta_6 \ln P_t^{import} + u_t^s. \end{aligned} \tag{11}$$

To account for all uncertainties in the supply function, such as precipitation and wind, we also include a stochastic error u_t^s.

2.5 Long-term forward price model

The average electricity spot price during time t is obtained by matching the supply and demand function. In equilibrium, the demand quantity q_t^d equals the supply quantity q_t^s, giving:

$$\begin{aligned} \ln P_t = \beta_0 + \beta_1 \ln g_t + \beta_2 \ln q_t^d + \beta_3 \ln P_t^{oil} \\ + \beta_4 \ln P_t^{coal} + \beta_5 \ln P_t^{ea} + \beta_6 \ln P_t^{import} + u_t^s. \end{aligned} \tag{12}$$

Equation (12) is a model for expected spot price for any time t. By replacing spot price variables in (12) with forward-looking variables, we can express the expected spot price during delivery period T a few years ahead as follows:

$$\begin{aligned} \ln P_{t,T} = \beta_0 + \beta_1 \ln g_{t,T} + \beta_2 \ln q_{t,T}^d + \beta_3 \ln F_{t,T}^{oil} \\ + \beta_4 \ln F_{t,T}^{coal} + \beta_5 \ln F_{t,T}^{ea} + \beta_6 \ln F_{t,T}^{import} + u_{t,T}^s. \end{aligned} \tag{13}$$

In (13), $F_{t,T}^{oil}$, $F_{t,T}^{coal}$ and $F_{t,T}^{ea}$ are the forward prices of oil, coal and emission allowances respectively, whereas $F_{t,T}^{import}$ is the forward price of electricity in the market from where the electricity is imported.

The final step in modelling long-term forward prices is the risk adjustment of the expected spot price $P_{t,T}$, which can is obtained with (4). Combining (4), (7), (9) and (13) gives:

$$\ln F_{t,T} = \delta_0 + \delta_1(\gamma_0 + \gamma_1 T) + \delta_2(\alpha_0 + \alpha_1 T) + \delta_3 \ln F_{t,T}^{oil}$$
$$+\delta_4 \ln F_{t,T}^{coal} + \delta_5 \ln F_{t,T}^{ea} + \delta_6 \ln F_{t,T}^{import} + \delta_7 r_{t,T} + \delta_8 \tau_t + u_{t,T} \quad (14)$$

where $\delta_{1,\dots,6}$ equals $\beta_{1,\dots,6}$ divided by $1-\beta_2\alpha_2$, whereas $\delta_{7,8}$ equals $\kappa_{0,2}$ divided by $1-\beta_2\alpha_2$. The error term $u_{t,T}$ is a linear combination of $u_{t,T}^d$, $u_{t,T}^s$ and $u_{t,T}^g$. Expected supply capacity and expected consumption in (14) are modelled with two linear submodels; however, the model structure allows the use of any other information about expected consumption and supply capacity, and hence information that is more accurate can be used if available.

3 Model estimation

The model (14) combines two important properties of long-term forward prices: changes in the structure of the market and changes in supply costs. We estimate the model parameters using data from the common Nordic electricity market, comprising Norway, Sweden, Finland and Denmark. The Nordic electricity market is one of the world's oldest electricity markets, with a high level of competition and with Nord Pool as the longest established power exchange. Nord Pool has high liquidity in the spot market and fair liquidity in the forward market. Because the Nordic electricity market is large (close to 400 TWh in yearly consumption) and the level of concentration on the production side is small, we assume that the market is close to efficient. Similar to most electricity markets, the Nordic electricity market still struggles with the lack of long-term information necessary to estimate the parameters of our model.

3.1 Data

All of the data discussed in this section represent the average information for delivery period T. Most data are available at daily resolution, except for the data on structural variables that are only available on a monthly, quarterly or yearly basis. As we wish to minimize the influence of short-term variations in price due to different short-term factors and, at the same time, produce an adequate data sample to obtain significant results, we use a weekly resolution ($t = 1$ week).

3.1.1 Electricity

Yearly electricity forward contracts at Nord Pool trade up to 5 years ahead, while contracts beyond this horizon trade on the OTC market. Because the liquidity and trading frequency of yearly forwards decreases significantly with time-to-delivery, we only use the prices of the 2-year ahead (ENOYR2) and 3-year ahead (ENOYR3) forward contracts to estimate the model parameters. To estimate the structural parameters, we use monthly average electricity spot prices from Nord Pool.

3.1.2 Electricity consumption and supply capacity

The model for the long-term electricity forward price in (14) also draws on the expected long-term electricity consumption and supply capacity. Their influence, however, is difficult to quantify as we have 3 years of high-resolution electricity forward data and several years of low-resolution consumption and supply capacity data. Instead, we assume that the expected consumption and supply capacity influence the expected spot price in the same way that actual consumption and supply capacity influence the spot price in the past. To estimate the structural parameters, we use historical data on yearly electricity consumption and supply capacity for all four countries. To produce a larger sample, we use monthly frequencies for the yearly data, where the individual observation is the sum or average of the previous 12 months. For supply capacity where only yearly data are available, we use linear interpolation to obtain monthly observations. These data are from Nordel (Organisation for Nordic Transmission System Operators). We modify the supply capacity data to represent the total average supply capacity in the market. We adjust the influence of wind capacity with an average utilization factor of 0.25 and include the net import capacity (available from Nordel). To estimate the demand variables in (7), we use annual data on temperature-adjusted consumption from Nordel. Temperature-adjusted consumption is consumption adjusted to normal temperature conditions using a heating-degree-day index [16].

3.1.3 Oil prices

Most crude oil and oil derivatives are traded on two major international exchanges, London's Intercontinental Exchange (ICE), and the New York Mercantile Exchange (NYMEX). Because there is a small difference in the quoted price of crude oil in both exchanges, we choose NYMEX crude oil data. We also use NYMEX monthly averages of the crude oil spot prices for estimation of the structural parameters.

3.1.4 Coal prices

The international coal market is based on coal prices offered at major shipping ports and freight prices. As a result, there are different prices for different delivery ports across Europe. Few institutions provide OTC coal prices based on reported deals from traders. We use McCloskey's North West European (NWE) Steam Coal Marker Price. This is a respected coal price marker and is particularly valid for the UK market. Given that long-term forward prices for coal are not available, we assume that the NWE coal marker price also provides adequate information about future coal prices.

3.1.5 Emission allowance prices

The EU ETS was implemented in 2005. Because Nord Pool began trading with CO_2 allowances in the beginning of 2005, we assume that the expected CO_2 allowance was constant and equalled approx. 8.7 EUR/ton, which is the price observed during the first days of trading. We base this assumption on the idea that the trading in the market usually starts at a price that is the average of different expectations of investors. We combine European Carbon Index data from the European Energy Exchange (EEX) and the Nord Pool data on European Emission Allowances (EUA) forward contracts. EEX started publishing the European Carbon Index in November 2004, and Nord Pool started trading EUA in March 2005. Given that emission allowances can be purchased and used anywhere in Europe, the differences in prices between the different markets are small.

3.1.6 Imported electricity prices

The Nordic electricity market has become a net importer of electricity in recent years. More than half of imported electricity comes from Russia and the remainder is imported from Germany and Poland. As there is no electricity price signal in Russia, we use only prices from EEX, which is a dominant price indicator for Germany and more or less all of Central Europe. We use data on electricity spot prices and yearly futures (traded up to 6 years ahead) in the estimation.

3.1.7 Interest rates

Interest rates are needed not only as explanatory variables but also for estimating the forward exchange rates for converting US dollars and Euros to Norwegian kroner. For estimation of forward exchange rates, we use interest rate parity. Long-term government bond yields from the Bank of Norway provide the risk-free interest rate for estimation of the forward exchange rate. For estimation of the forward exchange rate for US dollars, we use Treasury bill long-term interest rates from the US Department of Treasury, and for conversion from the Euro, we use Eurozone government benchmark yields from Reuters.

3.2 Data analysis

The data described in the previous section vary in terms of observation time and delivery period resolution. For long-term supply capacity, we use yearly data from 1990 to 2005, whereas for long-term consumption, we use yearly data from 1996 to 2005: this is because temperature-adjusted consumption is only available after 1996. For estimation of the structural parameters δ_1 and δ_2, we use monthly data from January 2001 to December 2005. Weekly parameters are estimated on weekly data from Week 1 in 2003 to Week 52 in 2005. Table 1 presents all of the data used for the model estimation with their observation time, sample size and resolution.

Table 1 Data observation time and sample size

Variable	observation time	Sample size	resolution
$\ln F_{t,T}^{np}$	Week 1, 2003 – Week 52, 2005	314	Weekly
$\ln F_{t,T}^{oil}$	Week 1, 2003 – Week 52, 2005	314	Weekly
$\ln F_{t,T}^{coal}$	Week 1, 2003 – Week 52, 2005	314	Weekly
$\ln F_{t,T}^{eua}$	Week 1, 2003 – Week 52, 2005	314	Weekly
$\ln F_{t,T}^{eex}$	Week 1, 2003 – Week 52, 2005	314	Weekly
$r_{t,T}$	Week 1, 2003 – Week 52, 2005	314	Weekly
τ_t	Week 1, 2003 – Week 52, 2005	314	Weekly
$\ln g_t$	Jan. 2001 – Dec. 2005	60	Monthly
$\ln c_t$	Jan. 2001 – Dec. 2005	60	Monthly
$\ln w_t$	Jan. 2001 – Dec. 2005	60	Monthly
$\ln P_t^{np}$	Jan. 2001 – Dec. 2005	60	Monthly
$\ln P_t^{oil}$	Jan. 2001 – Dec. 2005	60	Monthly
$\ln P_t^{coal}$	Jan. 2001 – Dec. 2005	60	Monthly
$\ln P_t^{eua}$	Jan. 2001 – Dec. 2005	60	Monthly
$\ln P_t^{eex}$	Jan. 2001 – Dec. 2005	60	Monthly
$\ln g_t^y$	1990 – 2005	16	Yearly
$\ln c_t^{ta,y}$	1996 – 2005	10	Yearly
$\ln P_t^{np,y}$	1996 – 2005	10	Yearly

3.2.1 Testing for stationarity

A regression between non-stationary time series may result in a spurious regression, and we cannot rely upon the regression parameters and their confidence intervals. We assume that the structural data, such as the annual consumption and supply capacity, are stationary. Although the small data samples may not reveal stationarity, in theory, these variables have more or less constant growth, and shocks in these variables will eventually die out with respect to long-term growth. Weekly variables, on the other hand, include the forward prices of energy commodities, which in theory are not stationary, so we perform a unit root test only on weekly variables. A standard way to test for stationarity is to run the Augmented Dickey–Fuller (ADF) test;

however, as the weekly data series comprises two contracts, there is a significant shift in the mean where the contracts are rolled over. Here we employ an ADF test that allows for a known structural break [17] at observation $T_b = 157$:

$$\Delta y_t = \alpha_0 + \alpha_1 DU_t + d(DT_b)_t + \beta t + \rho y_{t-1} + \sum_{j=1}^{p} c_j \Delta y_{t-j} + e_t \tag{15}$$

where y_t is the series under test, DU_t is a shift dummy variable $DU_t = 1$ if $t > T_b$, and 0 otherwise and DT_b is an impulse dummy $DT_b = 1$ for $T = T_b + 1$ and 0 otherwise. The optimal number of lags p in (15) minimizes Akaike's information criterion. Table 2 gives the results of the Perron-type ADF test for stationarity with intercept ($\beta = 0$) and trend stationarity. The critical value at the 5% level of significance for the constant ($\beta = 0$) is -2.871, and -3.424 for the constant plus trend. The results show that $F_{t,T}^{np}$, $F_{t,T}^{oil}$, $F_{t,T}^{coal}$, $F_{t,T}^{eua}$ and $F_{t,T}^{eex}$ are probably non-stationary, whereas $r_{t,T}$ and τ_t could be considered as stationary with intercept.

Table 2 Unit root test results

Variable	t-value (constant)	t-value (constant + trend)
$\ln F_{t,T}^{np}$	-1.463 **	-3.103 **
$\ln F_{t,T}^{oil}$	0.729 **	-2.594 **
$\ln F_{t,T}^{coal}$	-2.666 **	-0.575 **
$\ln F_{t,T}^{eua}$	0.183 **	-1.750 **
$\ln F_{t,T}^{eex}$	-2.923 *	-3.211 **
$r_{t,T}$	-4.275	-1.922 **
τ_t	-3.502	-3.720 *

* reject the null hypothesis at the 5% level of significance
**reject the null hypothesis at the 1% level of significance

When forecasting the future evolution of a time series with respect to the observation time t, a regression between non-stationary data would require the use of first differences, providing that the series contain only one unit root $I(1)$ and after testing for cointegration when the series are driven by a common source of non-stationarity. Because stationarity is measured with respect to observation time, we believe that when it comes to forecasting with respect to delivery period T, non-stationarity is not an issue. Here relationships between contemporaneous values of variables with the same delivery period T are estimated, which are then used to forecast the dependent variable $\mathrm{E}[F_{t,T+N}]$. As these relationships are assumed to be independent of t or T, non-stationary data will not produce inconsistent forecasts. One may, for example, randomly rearrange the data sample with respect to t, which would significantly change the results of the stationarity test as well as $\mathrm{E}[F_{t+n,T}]$. However, the coefficient estimates, their confidence intervals, and $\mathrm{E}[F_{t,T+N}]$ are unchanged. For this reason, we continue to employ a regression in levels, even in the presence of non-stationary data.

3.3 Parameter estimation

As shown in Table 1, the estimation data are available at three different resolutions. For this reason, we estimate the model parameters sequentially. In the first step, regressions (7) and (11) are estimated to obtain the expected demand parameters α_0, α_1 and α_2 and the expected supply capacity parameters γ_0 and γ_1. The expected demand and supply capacity parameters are estimated using yearly data. In the second step, the structural parameters δ_1 and δ_2 are estimated. These describe how the expected spot prices change because of changes in the expected supply capacity and expected consumption. We estimate the structural parameters with monthly data. In the third step, the weekly parameters δ_0, δ_3, δ_4, δ_5, δ_6, δ_7 and δ_8 are estimated using weekly data.

In the first regression, we estimate the demand parameters α_0, α_1 and α_2 using (7). Since temperature is short-term non-persistent error, we use data on temperature-adjusted consumption to exclude variations in consumption due to temperature. This way, the demand model (7) yields the demand that would occur under expected normal temperature conditions. For the electricity price P_t^{np}, we use yearly averages of the Nord Pool spot price. The demand parameters in Table 3 show growth in electricity demand equivalent to approx. 1.3% growth in annual consumption. A negative and significant long-term demand elasticity indicates that an important proportion of electricity consumers in the Nordic electricity market adjust their consumption according to the electricity spot price.

Table 3 Demand parameters

Variable	Parameter value	Standard error	p-value
α_0	6.079	0.054	0.000
α_1	0.013	0.001	0.000
α_2	-0.052	0.012	0.003
$N = 10$, $R^2(\text{adj.}) = 0.895$, $SE = 0.012$			
reg. eq. : $\ln c_t^{ta,y} = \alpha_0 + \alpha_1 t + \eta \ln P_t^{np,y} + u_t^d$			

In the second step, we estimate the supply capacity parameters γ_0 and γ_1 in (11). As shown in Fig. 2, we observe constant growth in supply capacity from the Nordic electricity market from 1990 to 2005 except in 1999 when Sweden and Denmark decommissioned some thermal and nuclear power plants. To take this shift into account, we add a shift dummy DU_{1999}^{cap} with 1 at $t \geq 1999$ and 0 otherwise. The supply capacity parameters in Table 4 show that the logged supply capacity annual growth is approximately 1.2% per year.

In the third step, we estimate δ_1 and δ_2 with monthly parameters. The estimation of these parameters is the most difficult part of our model estimation, as we only possess annual data on supply capacity that we can only consistently use after 2000 with the formation of the final geographical scope of the Nordic market. We

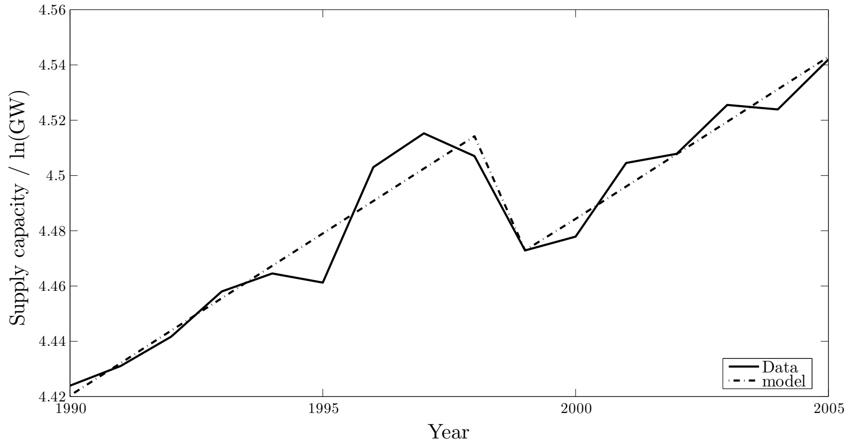

Fig. 2 Supply capacity in Nordic electricity market

Table 4 Supply capacity parameters

Variable	Parameter value	Standard error	p-value
γ_0	4.415	0.005	0.000
γ_1	0.012	0.001	0.000
γ_2	-0.053	0.008	0.000
$N = 16$, $R^2(\text{adj.}) = 0.946$, $SE = 0.008$			
reg. eq. : $\ln g_t^y = \gamma_0 + \gamma_1 t + \gamma_2 DU_{1999}^{cap} + u_t^g$			

assume that expected supply capacity $g_{t,T}$ and expected consumption $c_{t,T}$ influence the expected spot price $P_{t,T}$ in the same way as actual supply capacity g_t and actual consumption c_t influence the average spot price P_t in the past. In this manner, we can use historical average spot prices to estimate δ_1 and δ_2. First we estimate rough approximations of parameters β_0, β_3, β_4, β_5, β_6 in (12). These are estimated with an auxiliary regression (14) where $\delta_1 = \delta_2 = 0$ is assumed, hence $\beta_1 = \beta_2 = 0$ and $\delta_{0,3..6} = \beta_{0,3...6}$. The estimated approximations $\hat{\beta}_0$, $\hat{\beta}_3$, $\hat{\beta}_4$, $\hat{\beta}_5$, $\hat{\beta}_6$ are then used in the following regression:

$$\ln P_t^{np*} = \beta_1 \ln g_t + \beta_2 \ln q_t + \psi \ln w_t \tag{16}$$

where $\ln P_t^{np*}$ is the electricity spot price adjusted for the influence of the spot prices of oil, coal, emission allowances, and imports:

$$\ln P_t^{np*} = \ln P_t^{np} - \hat{\beta}_0 - \hat{\beta}_3 \ln P_t^{oil} - \hat{\beta}_4 \ln P_t^{coal} - \hat{\beta}_5 \ln P_t^{ea} - \hat{\beta}_6 \ln P_t^{eex} \tag{17}$$

and the two structural parameters are estimated with:

$$\delta_1 = \frac{\hat{\beta}_1}{(1-\hat{\beta}_2\hat{\alpha}_2)}, \quad \delta_2 = \frac{\hat{\beta}_2}{(1-\hat{\beta}_2\hat{\alpha}_2)}. \tag{18}$$

In (16), the spot price is also adjusted for the influence of water reservoir levels w_t. In the Nordic electricity market, the levels of hydro power plant reservoirs heavily influence electricity spot prices. We use Nordel data on the average monthly reservoir potential in GWh.

Table 5 Structural parameters

Variable	Parameter value	Standard error	p-value
β_1	-6.992	2.080	0.001
β_2	6.487	1.600	0.000
ψ	-1.680	0.146	0.000
$N = 60,\ \ R^2(\text{adj.}) = 0.689,\ \ SE = 0.152$			
reg. eq. : $\ln P_t^{np*} = \beta_1 \ln g_t + \beta_2 \ln q_t + \psi \ln w_t$			

Although two regressions are needed to estimate the structural parameters, only the results of regression (16) are presented in Table 5. In addition, although both structural parameters appear significant, they are approximations. Namely, we may question the supply capacity parameter, as it was estimated using few data. Nevertheless, the parameters have their expected sign and therefore can at least illustrate their influence. This is because expected forward prices decrease when expected supply capacity increases, and increase when expected consumption increases. The corresponding values of δ_1 and δ_2 when applying (18) are -5.3970 and 4.9554, respectively.

In the last step of the parameter estimation, we run regression (14) in which $F_{t,T}^{np}$ is adjusted for the influence of expected supply capacity and expected consumption:

$$F_{t,T}^{np*} = F_{t,T}^{np} - \hat{\delta}_1(\hat{\gamma}_0 + \hat{\gamma}_1 T) - \hat{\delta}_2(\hat{\alpha}_0 + \hat{\alpha}_1 T). \tag{19}$$

Table 6 provides the estimates of the weekly parameters with their corresponding standard errors and p-values. All parameters have their expected sign as all positively correlate with the electricity forward price. A negative constant parameter compensates for the high values of the structural parameters. The estimated parameters are all significant, with none of the p-values above the 1% level. The parameters in Table 6 also indicate that the EEX price is the most significant parameter. This is somewhat unexpected as only a small share of electricity in the Nordic electricity market is from Central Europe. This implies that the EEX price may serve as a sort of marginal producer in the Nordic electricity market. The EEX price may also serve as one of the benchmarks for investors in the Nordic electricity market. It is also reasonable to assume that other information that we cannot quantify similarly influences the EEX and Nord Pool prices. Specific information, including

Table 6 Weekly parameters

Variable	Parameter value	Standard error	p-value
δ_0	-4.543	0.117	0.000
δ_3	0.088	0.012	0.000
δ_4	0.131	0.011	0.000
δ_5	0.063	0.009	0.000
δ_6	0.343	0.024	0.000
δ_7	0.015	0.004	0.000
δ_8	0.046	0.005	0.000
$N = 314,\quad R^2(\text{adj.}) = 0.962,\quad SE = 0.022$			
reg. eq. : $\ln F_{t,T}^{np*} = \delta_0 + \delta_3 \ln F_{t,T}^{oil} + \delta_4 \ln F_{t,T}^{coal} + \delta_5 \ln F_{t,T}^{ea} + \delta_6 \ln F_{t,T}^{eex} + \delta_7 r_{t,T} + \delta_8 \tau_t + u_{t,T}$			

construction plans for new generation and interconnection capacity or other political decisions that will influence prices in the future, is likely to influence prices in both markets. Because the same or similar variables as we use in the model may influence the price of EEX, there is the indication of a possible problem with multicollinearity. We use Variance Inflation Factors (VIF) to detect multicollinearity in the explanatory variables in question. As a rule of thumb, if any VIF exceeds 10, the corresponding variable is said to be highly collinear [18], in which case, it is inadvisable to regress one on another.

Table 7 Variance Inflation Factor

Variable	VIF value
$\ln F_{t,T}^{oil}$	8.1
$\ln F_{t,T}^{coal}$	5.1
$\ln F_{t,T}^{eua}$	7.5
$\ln F_{t,T}^{eex}$	9.2
$r_{t,T}$	6.0
τ_t	1.2

Although none of the VIF values in Table 7 exceeds the critical value, the model could still suffer from potential multicollinearity. A detailed analysis of the regression results, however, indicates no significant sign of multicollinearity. According to Brooks (2003), typical problems with multicollinearity are as follows.

1. The regression is very sensitive to small changes in the specification, so that dropping or adding one variable in the regression leads to a large change in the level or significance of the other parameters.
2. The estimated parameters have high standard errors and wide confidence intervals.

Detailed analysis of the model specification and standard errors above shows no such problems. Finally, if the model parameters have appropriate sign and significance, and the selected variables have good theoretical background, Brooks suggests that the presence of multicollinearity can be ignored. Although $F_{t,T}^{eex}$ indicates some degree of collinearity with $F_{t,T}^{np}$, we assume it also includes other valuable information that otherwise cannot be modelled, so we choose not to remove it from the regression.

3.4 Model testing

The main obstacle to long-term modelling and forecasting is insufficient market data to estimate the parameters and test model performance. We use the out-of-sample test to test the performance of the model on the available long-term contracts. We use an out-of-sample test to see how the model behaves using data that are not included in the estimation of the parameters. However, choosing the 2005 data for the out-of-sample tests would not provide a true picture of the model because the CO_2 allowance price is assumed to be constant during 2003 and most of 2004, and changes only in 2005. In this case, the model would not produce the true influence of CO_2 allowance prices that were a particularly important driver of the price increase in 2005. Given that the model does not include any autoregressive terms and is not intended to forecast prices with respect to t, it is in principle possible to choose any in-sample or out-of-sample period. Here we estimate the model parameters with in-sample data from 2004 to 2005 and test the model on the out-of-sample data from 2003. Only ENOYR3 is used to present the results of this test. As shown in Fig. 3, the model produces similar levels of prices for out-of-sample data. The standard error of the estimate on the in-sample data is 0.0219, and the standard error of the estimate on the out-of-sample data is 0.0258.

The problem of testing the performance of the model when predicting prices 4 to 10 years ahead offers no adequate solution, as there are no prices for us to benchmark the forecast. To estimate this set of prices, we use the variables with the same delivery period as the price in question. This means that for estimation of the ENOYR5 contract, we use the forward prices of oil, imports, emission allowances and the EEX price with delivery 5 years ahead. Interest rates are available up to 10 years ahead, and the prices of oil, emission allowances and imports are available up to 6 years ahead, whereas the coal price index we use in estimation is only relevant for spot prices. To estimate prices up to 10 years ahead, we forecast the missing variables needed for estimation. Here, for variables for which we have the information available up to 6 years ahead, we extrapolate the difference between the last two prices for the remainder of the forward curve. For the coal price, we assume zero growth in the term structure. Fig. 3.4 provides the forward prices for ENOYR3, ENOYR5 and ENOYR10 and the price for a 10-year contract (ENO10), derived as

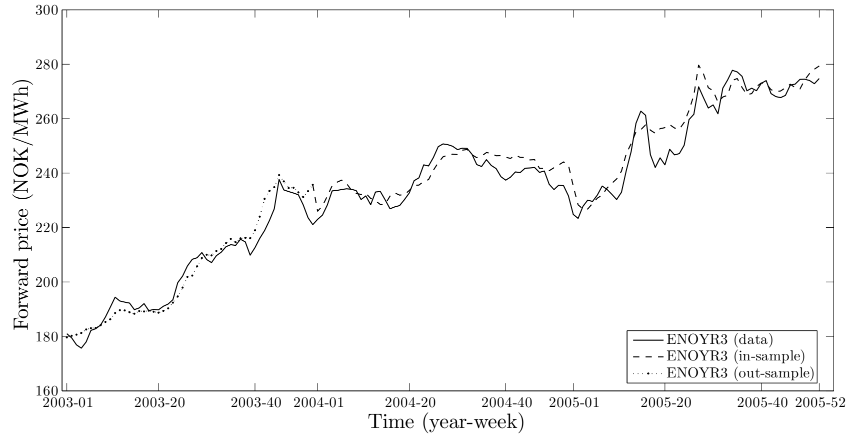

Fig. 3 Out-of sample test on ENOYR3 data

the interest-rate-weighted average price of all yearly contracts. [2] Here we assume that settlement occurs only once a year.

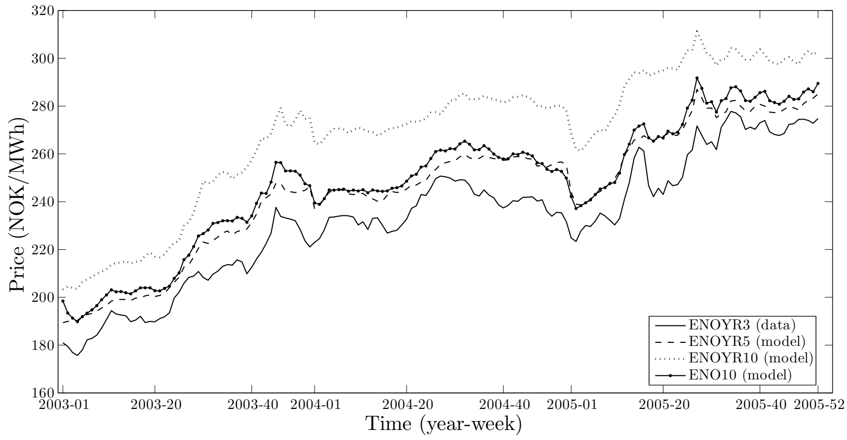

Fig. 4 The prices of ENOYR3, ENOYR5, ENOYR10 and ENO10

[2] If r_{10} is a 10-year interest rate and k is the time to the first settlement, the value of ENO10 with the start of delivery in January next year is given by:

$$\mathrm{ENO10} = \frac{\mathrm{ENOYR1}\frac{1}{(1+r_{10})^{k}} + \mathrm{ENOYR2}\frac{1}{(1+r_{10})^{1+k}} + \ldots + \mathrm{ENOYR10}\frac{1}{(1+r_{10})^{9+k}}}{\frac{1}{(1+r_{10})^{k}} + \frac{1}{(1+r_{10})^{1+k}} + \ldots + \frac{1}{(1+r_{10})^{9+k}}}. \tag{20}$$

While the price of ENO10 suggests reasonable dynamics compared with the ENOYR3 contract, the price level of the ENO10 is also important. ENO10 contracts are only traded OTC, so there are no available data on their prices. We only obtained OTC data on ENO10 prices for the period from Week 48, 2000 to Week 33, 2002, but could not use this in the regression as data on the other variables are missing during this period. Even though these prices are based on known OTC deals and rough estimation, analysis shows that the ENO10 price has dynamics similar to the ENOYR2 and ENOYR3 prices, and is on average 6.9% higher than the ENOYR3 price during that period, whereas our forecast of the ENO10 price is on average 6.6% higher than the ENOYR3 price. While there is no strong reason to believe that the ENO10 price in 2005 should behave similarly to the ENO10 price in 2000–02, it is the only available indicator on how the model behaves when predicting the ENO10 year price. The slopes of the term structures of the explanatory variables influence the differences between the prices 4–10 years ahead. Among these, the slope of expected supply capacity and consumption are particularly important. The expected growth in supply capacity and consumption has not changed significantly since 2002, and therefore the difference between ENOYR3 and ENO10 should not have changed significantly since then. Fig. 5 presents the estimated electricity for-

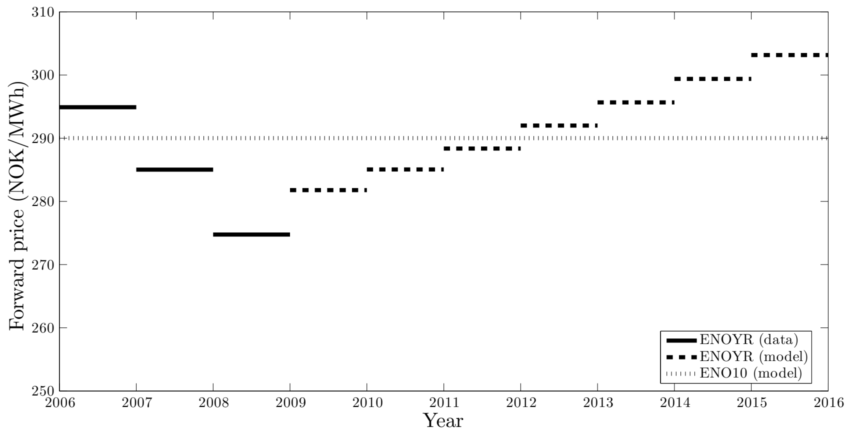

Fig. 5 Estimated term structure for week 52, 2005

ward price term structure for Week 52, 2005. Here the ENOYR1 and ENOYR2 prices are higher than the ENOYR3 price, indicating the higher prices in the spot and short-term forward market typical of low hydrological balances or temperatures. Extrapolation of the first three yearly prices up to 10 years ahead would in this case produce a term structure with a negative slope. As the slope of the far end of the term structure should not depend on short-term variations in weather, we should therefore extrapolate prices with care. Our model, although based on rough estimates of the structural parameters, produces a positive slope for the remainder of the term structure. Here the influence of the structural parameters plays an important role, as these parameters are independent of any short-term influences.

4 Conclusions

We present a regression model for long-term electricity forward prices. We intend the model for forecasting the prices of long-term forward contracts that we cannot observe on the exchange. Long-term forward prices mostly depend on variables that influence the supply costs of electricity. However, as the time horizon in long-term modelling is very long, we also consider expected changes in the structure of the supply and demand for electricity. We present ideas on how to combine high-resolution data on fuel prices from financial markets and low-resolution data on the expected structure of supply and demand, including expected electricity consumption and supply capacity. Combining both types of information yields a relationship for the expected long-term electricity spot price, which when adjusted for a risk premium provides the model for the long-term electricity forward price. Combining data with different observation time resolution and different delivery periods requires a work-around in parameter estimation. We estimate two submodels for expected supply capacity and consumption, whereas their influence on the expected spot price, which is one of the critical parts of the model, is estimated only roughly based on the historical influence of actual supply capacity and actual demand on actual spot price. The changes in supply costs are modelled with the crude oil price, coal prices, the emission allowance price and the price of imported electricity. Although stationarity tests indicate that most variables are likely non-stationary, we argue that this is not relevant in a model used to forecast prices with respect to the delivery period and not with respect to the observation time. We also detect the presence of multicollinearity between variables. However, we observe none of the problems typically associated with multicollinearity.

The estimated model provides the rough influence of expected supply capacity, expected consumption and long-term supply cost variables on long-term electricity forward prices in the Nord Pool. The performance of the model is tested with out-of-sample data on ENOYR3 contracts from Nord Pool, and the results indicate stable parameter estimates. To test the performance of the model on contracts beyond 3 years is not possible as these contracts only trade OTC, and therefore their prices are not publicly available. Although the models suffer from a lack of data on structural variables, particularly expected supply capacity and expected consumption, the forecasts of the prices beyond the traded horizon provide robust and expected results independent of short-term variations (such as precipitation or temperature), unlike the simple extrapolation of current prices. The model also offers many possibilities for improvement, both in the choice of variables and the data underlying them, as well as for parameter estimation. We hope that the availability of these data in terms of accuracy, resolution and longer horizons will improve in the future.

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