THEORETICAL LIDAR POINT DENSITY FOR TOPOGRAPHIC MAPPING IN THE LARGEST SCALES

TEORETIČNA GOSTOTA LIDARSKIH TOČK ZA TOPOGRAFSKO KARTIRANJE V NAJVEČJIH MERILIH

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ABSTRACT

When ordering LiDAR data, LiDAR point density per surface unit is important information with decisive influence on the price of the LiDAR survey. The paper first deals with the theoretical calculation of the minimum LiDAR point density, necessary for the acquisition of topographic data of the largest scales. For this purpose the sampling theorem is used. However, since topographic objects (roads, water bodies, etc.) and phenomena represented on topographic maps and in topographic bases are in many cases located under vegetation, also the rate of laser beam penetration through vegetation for the area where the topographic data are to be gathered has to be known. In a research on a test case conducted in the area of the town Nova Gorica we calculated the rate of laser beam penetration for four different vegetation types: scarce Mediterranean vegetation, thick thermophilic deciduous forest, mixed vegetation (meadows, orchards and forest) and built-up area. By connecting the theoretic minimum LiDAR point density with the rate of penetration, we defined the minimum LiDAR point density for the needs of data acquisition on topographic maps of the largest scales or in topographic bases of comparable detail (from 1 : 1000 to 1 : 10,000).

KEY WORDS

airborne laser scanning, LiDAR, topography, LiDAR point density, sampling theorem

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IZVLEČEK

Gostota lidarskih točk na površinsko enoto je pomemben podatek pri naročanju lidarskih podatkov, ki odločilno vpliva tudi na ceno lidarskega snemanja. V članku najprej obravnavamo teoretični izračun minimalne gostote lidarskih točk, ki je potrebna za zajem topografskih podatkov v največjih merilih. Za ta namen smo uporabili teorem vzorčenja. Ker pa so topografski objekti in pojavi, ki so predstavljeni na topografskih kartah in v topografskih bazah, velikokrat pod vegetacijo (ceste, vodna telesa itd.), moramo poznati tudi delež prodiranja laserskih žarkov skozi vegetacijo za območje, kjer bomo zajemali topografske podatke. V raziskavi smo na testnem primeru na območju mesta Nova Gorica izračunali delež prodiranja laserskih žarkov za štiri različne vegetacijske tipe: redko mediteransko vegetacijo, gost termofilen listnati gozd, mešano vegetacijo (travniki, sadovnjaki in gozd) in pozidano območje. S povezavo teoretične minimalne gostote lidarskih točk in deleža prodiranja smo določili minimalno gostoto lidarskih točk za potrebe zajema podatkov na topografskih kartah največjih meril oziroma v topografskih bazah primerljive podrobnosti (od 1 : 1000 do 1 : 10.000).

KLJUČNE BESEDE

zračno lasersko skeniranje, lidar, topografija, gostota lidarskih točk, teorem vzorčenja

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1 INTRODUCTION

Lately the technology of laser scanning has importantly affected the principles of spatial acquisition of topographic and other physical data about the environment (Shan and Toth, 2009). Laser scanning is in the widest sense divided into airborne laser scanning, terrestrial laser scanning and short range laser scanning (Kraus, 2007). For laser scanning frequently the synonym expression LiDAR survey or short LiDAR is used. The term derives from the English description of the technology, i.e. light detection and ranging or LiDAR. The paper deals with airborne laser scanning, for which the short term LiDAR is used.

The main results of airborne LiDAR survey are clouds of georeferenced points containing data on the reflection order and the intensity of the returned pulse. To collect or later cartographically present topographic contents from the LiDAR point cloud, the recognition of individual objects and phenomena and the definition of the edges between them (i.e. edges defined by buildings, roads, etc.) are required. The precision in the definition of edges depends among others also on the LiDAR point density per surface unit. The paper describes the research, where the optimum LiDAR point density was defined theoretically and empirically for the purposes of topographic mapping of the largest scales.

Although today the data are acquired and processed in the digital form, the same graphical scales as those in the analogue time of cartography are still applicable for the definition of the level of accuracy and the details of the data base. The map scale thus defines the geometrical accuracy of data, based on which also the necessary positional accuracy can be determined.

For clarity reasons, the following definitions of several terms used in the paper are given. Positional accuracy of spatial data is an element of quality that defines the level of agreement between the survey results and the actual value. The term precision stands for the level of the scatter in results around the mean value and it is normally expressed with standard deviation. Accurate survey is at the same time true as well as precise.

Overall positional accuracy of data in the map is the result of survey accuracy and the accuracy of drawing the data in the map. Overall positional accuracy of data can be named geometrical accuracy of the map as well (Maling, 1989).

Figure 1. Final product determines the parameters of LiDAR survey.

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Preliminary (a priori) positional accuracy of LiDAR survey can be determined before the survey based on a simplified error model (Triglav-Čekada et al., 2009) (Figure 1). On the other hand, empirical (a posteriori) positional accuracy can be determined by taking into account rigorous error models (Schenk, 2001; Beinat and Crosilla, 2002; Friess, 2006; Skaloud and Lichti, 2006) or by conducting subsequent control survey (Crosilla et al., 2005). For the positional accuracy of LiDAR points before transformations among different coordinate systems, a general estimation of planimetric accuracy in the size of several decimetres and of height accuracy of about 1 decimetre can be given, as described by Maas (2003). When using low-flying (500 m - 800 m) LiDAR surveying systems with more precise inertial navigation systems (INS), the average planimetric accuracy can increase up to 1 decimetre (Triglav-Čekada et. al, 2009).

When focusing on data mapping, it appears that the smaller is the map scale, the larger is the influence of the survey accuracy, in our case LiDAR survey, on the total positional (geometrical) accuracy of the points in the map. This fact is the consequence of cartographic generalisation, due to which the positional accuracy of the presented data decreases compared to the originally acquired data.

The geometrical accuracy of a map is limited with the graphical accuracy in the particular map scale. In the analogue time of cartography the smallest line thickness that can still be drawn on a map of a certain scale represented the graphical accuracy of a map. At the same time this is the dimension of the smallest objects that can still be presented in a map of a certain scale, or the smallest allowable distance between two graphical elements of map contents. These conditions of graphical element resolution, such as point size, line thickness, distances between lines, etc., are the main factors limiting the geometrical accuracy, especially for map scales up to 1 : 10,000, while for smaller scales the main influence comes once again from cartographic generalisation (simplification of lines, presentation of conditional signs, movements, etc.). The kind of LiDAR survey that still allows the mapping of previously defined geometrical accuracy or minimum size of acquired objects depends on the LiDAR point density per surface unit. The minimum LiDAR point density defines the possibility of recognising the shape of the object, determining the accuracy of the recognised shape and the possibility of separating two close-by objects from each other. These parameters also determine the completeness of the final product. The minimum LiDAR point density also influences the thematic accuracy, since the shape of the object tells a lot about the object class (e.g. distinguishing between buildings and pavements).

Before ordering a LiDAR survey it is very important to know the necessary LiDAR point density per surface unit, since larger LiDAR point density per surface unit also means longer duration of the survey and consequently higher price. The purpose of the present paper is to determine the optimum laser point density for topographic mapping of the largest scales (from 1 : 1000 to 1 : 10,000) or for establishing equivalent topographic bases of large details, based on theoretical assumptions and performed empirical research.

Initially, the smallest still satisfactory theoretical LiDAR point density per surface unit will be determined by taking into account the geometrical accuracy of a map. In practice, the theoretical still satisfactory laser point density shall be increased, since the objects to be recorded from Geodetski vestnik 54/3 (2010)

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laser data are frequently covered by various types of vegetation. This is especially true when microtopography (detailed digital relief models), covered by thick forest, is to be recorded. In such cases LiDAR survey shall be used with considerably larger LiDAR point density per surface unit (Reutebuch et al., 2003). The percentage of laser beams penetrating to a specific height class (e.g. ground), is defined with the rate of laser beam penetration. The rate of laser beam penetration depends on the vegetation type (vegetation density and sort). Vegetation sorts also determine when to expect larger rate of single reflections (reflections only from tree tops), which can help define the vegetation sorts (Moffiet et al., 2005). When the rate of laser beam penetration through vegetation is very small, also the automatic classification of the LiDAR point cloud to height classes becomes unreliable (Forlani and Nardinocchi, 2007).

In a test LiDAR point cloud with the average density between 15 and 20 points/ m^2 the rate of penetration for four different vegetation types was defined. By taking into account the theoretically smallest LiDAR point densities compared to the geometrical accuracy of a product and to the rate of beam penetration, the research defined the optimum laser point density for topographic maps of the scales from 1 : 1000 to 1 : 10,000 or for the bases of comparable detail.

2 TOPOGRAPHIC MAPS OF THE LARGEST SCALES IN SLOVENIA

For local spatial planning (municipal spatial plans and municipal detailed spatial plans) local communities (municipalities) in Slovenia mainly use maps of the scales from 1 : 1000 to 1 : 10,000. With the Spatial Planning Act (Official Gazette of RS 33/2007) geodetic maps started to prevail as primary topographic base for spatial planning, thus replacing the previously used topographic and combined cadastral maps. Geodetic map is a presentation of physical structures and phenomena on the earth surface and below it in reduced scale according to topographic rules. The presented contents, their completeness, detail and precision depend on the purpose of use of geodetic map (Rules on geodetic map, 2004). In geodetic maps the topography is generally presented with the following layers: roads, railways, water bodies (streams, rivers and lakes), borders of different vegetation types, buildings, use of urban space (industrial, parking, other), contour lines and other.

In accordance with the Rules on the Symbols on Basic Topographic Maps (1982) the required graphical accuracy of the basic topographic maps in Slovenia was 0.13 mm. With the adoption of the Rules on geodetic map (2004) these rules are no longer valid, but the value of the required graphical accuracy can be used as limit value when determining the geometrical accuracy. Generally the graphical accuracy for different cartographic scales shall be better than 0.25 mm, with the average value of 0.2 mm (Maling, 1989). For example, in the scale of 1 : 1000 this average value of 0.2 mm represents 0.2 m in the field.

In Slovenia, the data for the elaboration and maintenance of topographical maps of the largest scales (scales 1 : 5000 and smaller) and of topographical contents of geodetic maps (scale 1 : 5000) are mainly acquired by photogrammetric stereorestitution (three-dimensional acquisition) from stereopairs of the Cyclical aerial survey (CAS) of Slovenia or vectorisation from orthophoto (two-dimensional acquisition). Maps of the scales less than 1 : 5000 are mainly

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made by using terrestrial surveys. The LiDAR technology allows the acquisition of topographic data with great details and high accuracy. Thus it would be reasonable for Slovenia to include LiDAR data in the national system of acquisition and maintenance of topographic data. Figure 2A shows the diagram of the current methodology for the elaboration of geodetic maps of the scale 1 : 5000, and Figure 2B shows our proposal that also includes data from LiDAR survey as the source of data.

Figure 2. Methodology of elaborating geodetic maps of the scale 1 : 5000: A) existing, B) proposed.

We propose the following two options for the acquisition of topographic data:

- LiDAR data shall be used as the basic source for the acquisition of topographic data and shall thus replace the acquisition from CAS stereopairs.
- For data acquisition the combination of LiDAR data and orthophoto, made from simultaneous survey, shall be used, considering at the same time the height LiDAR data. The use of orthophoto simplifies the identification and distinction of objects in the LiDAR point cloud.

The LiDAR technology could also be included in the acquisition of land cadastre data by using the LiDAR data for recording the field situation and to control the correctness of the land cadastre presentation. Nevertheless, field work could not be completely avoided, since boundary stones should be signalised before the LiDAR survey to enable reliable identification in the point cloud (Triglav Čekada, 2010).

3 THEORETICALLY THE SMALLEST LASER POINT DENSITY

To determine theoretically the smallest laser point density that allows the recognition of topographic objects from LiDAR data, the sampling theorem will be used, known also as the Nyquist-Shannon theorem (Göpfert, 1987; Kraus, 2007). It originates from the information theory. We could also use some other theories, such as Modulation Transfer Function (MTF), which describes the efficiency of object reproduction to an image or a film (Atkins, 2007), or the scale-space theory, which describes the image in different scales through single-parameter family of smoothed images (Ali, 2009).

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The sampling theorem defines the procedure of sampling analogue (continuous) signal into discrete (step) form, while preserving important information throughout the transformation. Normally this principle is used to study analogue signals, such as audio and video records, but also the physical (topographic) reality can be studied in a similar way. The Nyquist-Shannon theorem says that the sampling (digitalisation) frequency of signal (f_s) shall be at least twice the highest frequency of the original (analogue) signal (*f max*), in order to allow satisfactory reconstruction of the original record from the sampled record (Equation 1). The minimum frequency for still satisfactory sampling is also called the Nyquist frequency $(f_N = 2 f_{max})$ according to its author Harry Nyquist, who first discovered the rules and published them in 1928. In 1949 the theorem was formally confirmed and extended by Claude E. Shannon.

Figure 3. An example of signal sampling (the original signal is represented with full line, the sampled signal with dotted line).

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The easiest way to illustrate the principle of the sampling theorem is to use an example. Figure 3 presents continuous signal and the sampling of this signal with different frequencies. In Figure 3A this phenomenon is discretized with the sampling frequency, which is triple value of the Nyquist frequency. In Figure 3B the phenomenon is discretized with the Nyquist frequency, and in Figure 3C with one third of the Nyquist frequency. It can be noticed that changes in the signal can be detected by frequencies equal to or larger than the Nyquist frequency (cases A and B). If frequencies larger than the Nyquist frequency were used, even more details of the signal could be recognised (case A).

Figure 4 shows how the sampling principle can be applied in digital images with grid form and defined spatial resolution (in two dimensions of the space). If the smallest size of the object in nature, to be presented in an image, equals 1 m, it has to be photographed with spatial pixel resolution of the most 0.5 m in nature. As already mentioned, the smallest dimension of the object to allow representation in a map is defined with the geometrical accuracy of the map. In order to achieve the graphical accuracy of 0.2 mm in the scale of 1 : 5000, which represents the geometrical accuracy of the map of 1 m, the basic source for the acquisition shall be aerial image with the spatial resolution of the most 0.5 m.

Figure 4. Two-dimensional principle of the Nyquist frequency, applied to a digital image.

The same principle can be used for the LiDAR data. In the theoretical derivation it is first assumed that all LiDAR points reach the ground, that they are not dispersed through various heights, and that they are equally distributed (LiDAR points in the cloud are actually randomly distributed, but their average density per space unit can be calculated). To satisfy the geometrical accuracy of 1 m, the largest still acceptable distance between two points according to the Nyquist frequency has to be determined as 0.5 m, which is achieved with the density of 4 LiDAR points/ m^2 (Figure 5A). As can be seen in Figure 5, for the correct calculation of the point density the corner points shall be distributed among four squares of size 1 m^2 and the edge points between two squares of size 1 m^2 .

Theoretically the smallest LiDAR point density per surface unit ρ_t expressed in the number of points/ $m²$ and the geometrical accuracy GA of the map (defined in metres) are connected by the following equation (2):

$$
\rho_t = \frac{1}{\left(GA/2\right)^2} \tag{2}
$$

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For example: with Equation 2 the smallest theoretical density for the graphical accuracy of 0.13 mm in the map scale $1:5000$ (rounded) 10 LiDAR points/m² (Figure 5B) is calculated. For the map scale 1:1000 with the geometrical accuracy of 0.2 m (Figure 5C), the minimum necessary density of 100 LiDAR points/ m^2 is obtained. For such point densities LiDAR becomes uneconomical, which is why we propose for these cases to replace it with terrestrial laser scanning, wherever practicable (by oblique survey).

Figure 5. Two-dimensional principle of the Nyquist frequency, applied to LiDAR points.

With Equation 2 also the smallest LiDAR point density that allows the presentation of the object of the smallest dimensions in a map of a certain scale can be defined. In Equation 2 the geometrical accuracy GA is replaced with the smallest dimension still representable on the map.

In Figure 6 different densities of LiDAR data are presented as example, together with digital model of relief and three-dimensional models of single family houses. In Figure 6A with the original density of 20 points/ $m²$, specific patterns can be seen in the point cloud, representing medium high vegetation. Figure 6B shows the density of 10 points/ m^2 . Figure 6C, which shows a point cloud reduced to the density of 1 point/ m^2 , there appears a problem. The roof edges cannot be determined from such thin point cloud. The presented three-dimensional buildings were made from the original point cloud with the density of 20 points/ m^2 .

Figure 6. Examples of different LiDAR point densities: A) 20 points/m2, B) 10 points/m2 and C) 1 point/m2.

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4 DEFINITION OF LIDAR POINT DENSITY IN THE AREAS UNDER VEGETATION

In order to capture the details of objects hidden under vegetation (e.g. buildings under trees, roads in forests), the above defined theoretically the smallest LiDAR point density must be additionally increased. One of the possibilities is to order beside LiDAR survey also the required point density under vegetation for each vegetation type separately. However, for the planning of LiDAR survey it is easier to order the average LiDAR point density, as this allows easier control after the survey. Knowing the rate of laser beam penetration to the ground, also the smallest necessary and from the economical aspect optimum LiDAR point density is determined.

4.1 Test case

The test was performed in a LiDAR point cloud covering 20 ha, located in the area of Nova Gorica. For the acquisition the instrument ALTM3100 was used. The LiDAR survey was carried out in the beginning of the leaf season in early April 2006. We used the laser pulse frequency of 100 kHz and the average flight height of 1000 m above ground. The used INS system had the frequency of 200 Hz and the INS angle precision of 0.02° for the roll and pitch angles and 0.04° for the heading angle. The average density of the LiDAR point cloud was between 15 and 20 points/m².

Using the software TerraScan, the point cloud was classified semiautomatically according to the height changes and the laser beam echo order into the following classes: ground (last echo), low vegetation (before last echo, height-wise closest to the class ground), medium vegetation (vegetation lower than 2 m), high vegetation (vegetation between 2 m and 30 m), buildings and errors.

4.2 Rate of laser beam penetration through vegetation

The rate of penetration was studied according to four vegetation types (Figure 7):

(1) scarce Mediterranean vegetation: bushes, thin deciduous trees; prevailing types: holm oak (Quercus ilex), pubescent oak (Quercus pubescens), hop hornbeam (Ostrya carpinifolia),

(2) thermophilic forests of mixed deciduous trees with thick complex of tree vegetation; prevailing types: mountain oak (Quercus petraea), field maple (Acer campestre), acacia (Robinia pseudoacacia), white birch (Carpinus betulus),

(3) mixed vegetation: meadows, orchards and forest,

(4) built-up areas: buildings with included decorative sorts of shrubs and tree nurseries.

For the first two vegetation types Figure 7 also shows the cross-section of the LiDAR point cloud, where it can be noticed that the vegetation types are well distinguishable.

For each vegetation type we calculated the total average LiDAR point density per surface unit and the average point densities in different classified laser point classes for 10 test areas. Each test area was 100 m x 100 m large. For the first three vegetation types the classified classes *buildings* and *errors* were combined in the class *other*. In this class the estimation of the share Mihaela Triglav Čekada, Fabio Crosilla, Mojca Kosmatin Fras - TTHEORETICAL LIDAR POINT DENSITY FOR TOPOGRAPHIC MAPPING IN THE LARGEST SCALES

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of points which cannot be classified to any other class is given. For the fourth vegetation type (built-up areas) the class *buildings* becomes important, so we kept it as a separate class.

Since the used point cloud at the area of Nova Gorica is not of uniform point density per surface unit, the average measured point density was expressed in the relative value of the rate of penetration compared to the total point density. The total point density, which represents 100%, is the sum of all relative values of individual classified classes at the same test area. The relative rates of the laser beam penetration through vegetation are presented in Table 1, and the original point densities are described in Triglav-Čekada (2009).

veg.	ground	low	medium.	high	buildings	other
type		veg.	veg.	veg.		
(1)	9±3	11 ± 4	$25+4$	$40+12$		15±9
(2)	4 ± 2	$2+1$	$9 + 4$	$73+9$		$12+9$
(3)	$20 + 9$	$13+5$	$17 + 4$	31 ± 14		$19+10$
(4)	19±5	$15 + 4$	$19+3$	$18+3$	16+4	$13+5$

Table 1. Rate of laser beam penetration through vegetation with standard deviation expressed as % of all points per surface unit.

Figure 7. Typical cases of studied vegetation types.

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Most of the detail that can be presented in topographic maps can be acquired from the classes *ground* and *low vegetation* (roads, paths, railways, water bodies, building shapes, relief shapes). For this reason these two classes are shown in Table 1 as bold. In the derivation of the final optimum point density the classes *ground* and *low vegetation* were combined in the class *combined ground* (Table 2). For the vegetation type (3), mixed vegetation, 33 % of the points and for vegetation type (4), built-up area, 34 % of the points are in the class *combined ground*. For vegetation type (1), thin Mediterranean vegetation, 20 % of the points are in the class *combined ground*. The lowest penetration rate (6 %) in the class *combined ground* can be noticed in vegetation type (2), thermophilic forests of mixed deciduous trees, which was also expected.

4.3 Calculation of minimum LiDAR point density by taking into account the rate of laser beam penetration

The minimum laser point density per surface unit ρ_o can be calculated based on theoretically the smallest point density ρ_t and the rate of laser beam penetration through vegetation PR for the class *combined ground*, assuming that all points of one vegetation class are located at the same height. Thus, the optimum LiDAR point density is:

$$
\rho_o = \frac{\rho_i \cdot 100}{PR} = \frac{1}{(GA/2)^2} \cdot \frac{100}{PR}
$$
\n(3)

Table 2 presents the minimum laser point densities for the four tested vegetation types and two scales of topographic maps. The second column shows the penetration rates for the classified class *combined ground*. To calculate the optimum density, the graphical accuracy of 0.2 mm was used. For the topographic map in the scale of 1 : 5000 this graphical accuracy represents the geometrical accuracy (GA) of 1 m, which gives theoretically the smallest density ρ_t equalling 4 points/ m^2 . On the other hand, by using the penetration rate (Equation 3), for built-up area, mixed vegetation and thin Mediterranean vegetation the optimum point densities between 12 and 20 points/ $m²$ are obtained. Unfortunately, the majority of providers of LiDAR survey data consider the density of 20 points/ $m²$ as above-standard product, which also costs more.

The optimum point density in the scale of 1 : 1000 would grow from theoretically the smallest point density ρ_t of 100 points/m² to even larger unpractical value that would make it economically completely unjustified. For example, for the vegetation types mixed vegetation and built-up area the optimum point density is 300 points/ $m²$. The largest LiDAR point density in Slovenia ordered within public tenders has been in the last few years around 30 points/ m^2 , used for making detailed flood studies. The use of LiDAR would be most economical for topographic mapping in the scale of 1:10,000 and smaller scales, since we need relatively low minimum point density.

Table 2. Proposed minimum LiDAR data density as the number of points/m2.

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Large differences in the rate of laser beam penetration for different vegetation types confirm the thesis that the calculation of optimum point density should not neglect the rate of laser beam penetration. The difference is especially noticeable between thermophilic forests of mixed deciduous trees, where the class *combined ground* achieves only 6 % of all LiDAR points, and built-up area, where 34 % LiDAR points achieve the same class.

5 CONCLUSION

Before ordering LiDAR survey, it is sensible to define the necessary LiDAR point density per surface unit necessary for the particular purpose of the use of LiDAR data. From the aspect of economy, the smallest still acceptable point density is the best, since the price of LiDAR survey generally increases with the LiDAR point density. The paper is limited to the study of data acquisition for the purpose of topographic mapping of the largest scales (from 1 : 1000 to 1 : 10,000). The acquired data shall above all agree with the geometrical accuracy of the topographic map of a certain scale or the positional accuracy of the topographic base of comparable details. To define theoretically the smallest laser point density that allows the acquisition of topographic contents, the Nyquist-Shannon sampling theorem was reasonably used and the equation for its calculation was given. On an example, the smallest theoretical LiDAR point density was calculated for maps of scales 1 : 5000 and 1 : 1000. For map of scale 1 : 5000 the minimum density of 10 LiDAR points/m2 , and for map of scale 1 : 1000 100 LiDAR points/ $m²$ are calculated. The latter value is very high and economically unjustified. However, since topographic elements (relief, roads, water bodies, etc.) lie often below vegetation, the theoretical LiDAR point density was extended also by taking into account the rate of laser beam penetration below the vegetation. In the test case in the area of Nova Gorica we defined the penetration rate for four vegetation types in leaf season: scarce Mediterranean vegetation, thick thermophilic deciduous trees, mixed vegetation and built-up area. Based on the obtained results of the empirical research, a proposal for the optimum LiDAR point density (for all vegetation types except thick thermophilic forest) was given as: between 12 and 20 points/ m^2 for the map of scale 1 : 5000 and between 3 and 5 points/ m^2 for the map of scale 1 : 10,000. For the total area of Slovenia, of which more than 50 % is covered by forests, the use of LiDAR to obtain the topography below forests would be economically justified only for the scale 1 : 10,000 or smaller scales, however, assuming that only LiDAR data would be used. With further research the rate of laser beam penetration for the most frequent vegetation habitat types (Jogan et al., 2004) for leaf and non-leaf seasons for the whole area of Slovenia could be defined in more detail from the existing LiDAR data, which would allow the optimisation of the planning and ordering procedure of new LiDAR data for the purposes of national projects.

The optimum LiDAR point density defined in this paper allows survey planning for the needs of topographic mapping or establishing topographic bases of comparable details. The presented methodology can also be applied for other spatial databases used for local spatial planning. In this way the necessary LiDAR point density for Slovenia could also be defined for the following national bases: Building cadastre and some layers of the Cadastre of public infrastructure.

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Literature and sources:

Atkins, B. (2007). Modulation Transfer Function – what is it and why does it matter? http://photo.net/learn/optics/mtf/ (20. 2. 2010)

Ali, T.A. (2010). Building of robust multi-scale representations of LIDAR-based digital terrain model based on scalespace theory. Optics and Laser in Engineering, 48:3, 316-319.

Beinat, A., Crosilla, F. (2002). A generalized stochastic model for the optimal global registration of lidar range images. ISPRS Commission III Symposium 2002 "Photogrammetric Computer Vision", 9 - 13 September 2002, Graz, Austria.

Crosilla, F., Beinat, A., Visintini, D., Fico, B., Sossai, E. (2005). Likelihood and Accuracy Analysis of 3D Building Models from Airborne Laser Data. Proceedings of Italy-Canada 2005 Workshop on "3D Digital Imaging & Modelling: Application of heritage, industry, medicine & land", May 17-18, 2005, Padova, Italy.

Forlani, G., Nardinocchi, C. (2007). Adaptive filtering of aerial laser scanning data. ISPRS Workshop on Laser Scanning 2007 and SilviLaser 2007, Espoo, September 12-14, Finland.

Friess, P. 2006. Toward a rigorous methodology for airborne laser mapping. Proceedings of International Calibration and Orientation Workshop EuroCOW, 25-27 January 2006, Castelldefels, Spain.

Göpfert, W. (1987). Raumbezogene Informationssysteme. Karlsruhe: Wichmann Verlag.

Jogan, N., Kaligarič, M., Leskovar, I., Seliškar, A., Dobravec, J. (2004). Habitatni tipi Slovenije HTS 2004, ARSO, Ljubljana. http://www.arso.gov.si/narava/poro%C4%8Dila%20in%20publikacije/HabitatniTipiSlovenije2004.pdf (20. 2. 2010)

Kraus, K. (2007). Photogrammetry: Geometry from Images and Laser Scans, 2nd edition. Walter de Gruyter.

Maling, D.H. (1989). Measurements from maps: principles and methods of cartometry. Oxford: Pergamon press.

Maas, H.-G., (2003). Planimetric and height accuracy of airborne laserscanner data: User requirements and system performance. Photogrammetric Week (ur. D. Fritsch), Proceedings 49. Wichmann Verlag.

Moffiet, T., Mengersen, K., Witte, C., King, R., Denham, R. (2005). Airborne laser scanning: Exploratory data analysis indicates potential variables for classification of individual trees of forest stands according to species. ISPRS Journal of Photogrammetry & Remote Sensing, 59, 289-309.

Pravilnik o znakih za temeljne topografske načrte, (1982). Ljubljana.

Pravilnik o geodetskem načrtu. Ur.l. RS, št. 40/2004.

Reutebuch, S.E., McGaughey, R.J., Andersen, H-E., Carson, W.W. (2003). Accuracy of a high-resolution lidar terrain model under a conifer forest canopy, Can. J. Remote Sensing, 29, 5, 527-535.

Schenk, T. (2001). Modeling and analyzing systematic errors in airborne laser scanners. Technical Notes in Photogrametry N° 19, Department of Civil and Environmental Engineering and Geodetic Science, The Ohio State University, Columbus.

Shan, J., Toth, C.K. (2009). Topographic laser ranging and scanning. Taylor & Francis Group.

Skaloud, J., Lichti, D. (2006). Rigorous approach to bore-sight self-calibration in airborne laser scanning. ISPRS Journal of Photogrammetry & Remote Sensing, 61, 47-59.

Triglav-Čekada, M., Crosilla, F., Kosmatin-Fras, M. (2009). A simplified analytical model for a-priori lidar pointpositioning error estimation and a review of lidar error sources. Photogrammetric Engineering and Remote Sensing, 75:12, 1425-1439.

Triglav-Čekada, M. (2009). Optimization of the data processing methodology and accuracy analysis of airborne

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laser scanning data applied for local spatial planning. Doktorska disertacija. Ljubljana: Fakulteta za gradbeništvo in

Triglav Čekada, M. (2010). Zračno lasersko skeniranje in nepremičninske evidence. Geodetski vestnik, 54 (2), 181-194.

Zakon o prostorskem načrtovanju, (2007). ZPNačrt, UL RS 33/2007.

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