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Dynamic Analysis of the Load Lifting Mechanisms

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This paper deals with the problem of dynamic behaviour of load lifting mechanism (such as elevators). In the case of considerable lifting heights, high velocity devices are applied, with the purpose of shortening cycle duration and increasing the capacity. In such case, the standard procedure of dynamic analysis is not applicable. In the paper, the procedure of establishing the appropriate dynamic model and corresponding equations is proposed. It enables the analysis of the relevant influences, such as variation of the rope free length, slipping of the elastic rope over the drum or pulley and damping due to the rope internal friction.

Keywords: elevators, pulleys, dynamic models, rope length variation, motion stability analysis

0 INTRODUCTION

In the most types of load lifting mechanisms with considerable lifting heights operating under usual conditions, the steel wire-rope and the driving drum (in cranes) or the driving pulley (in elevators) are applied.

The paper deals with the influence of these components upon the dynamic behaviour of the lifting mechanism, while other influences were left out of the scope of the research. These influences have already been investigated in details, e.g. dynamic phenomena in the gear box, in [5], the dynamic characteristics of the driving electromotor, in [12], etc. The research is based on the system with many degrees of freedom, also used in [13]. The analysis of the lifting mechanism with relatively low lifting velocity is usually performed by using dynamic models based on the longitudinal vibrations of elastic, homogeneous bar of a constant length, with or without mass, corresponding to certain boundary conditions, e.g. in [10]. The stability of beam acceleration and the effect of various stiffness values have been analysed in [6] and [11].

Boundary condition at the lower rope end is defined by the load mass and at the upper rope end by the moment of inertia of all driving mechanism components and by the excitation force corresponding to the mechanical characteristic of the driving electromotor, both reduced to the shaft of the driving drum or pulley. But, in the case of higher lifting velocity, (e.g. in the case of fast passenger elevator, mine shaft winding system, harbour crane, etc.) the numerous influences upon the dynamic behaviour of the lifting mechanism are also to be taken into account, such as:

- rope length variation during load lifting and lowering,
- steel wire-rope slipping over the driving pulley or drum,
- mechanical properties of the rope (such as elasticity modulus, rope internal friction, rope design, etc.),
- influence of dynamic processes on the incoming rope side upon the dynamic processes on the outgoing rope side in the systems with driving pulley,
- mechanical characteristic of the driving motor,
- influence of friction processes between the elevator car guiding shoes and rails.

Basic influence of the rope length variation appears directly through the variation of the basic dynamic parameter – stiffness of the rope free length. There is a considerable possibility that in the case of realistic devices with higher velocities and minor energy losses due to internal friction in elastic elements, the variable stiffness of the rope free length could induce the occurrence of parametric vibrations.

1 EQUATIONS OF LOAD LIFTING SYSTEM

A load lifting system with the driving pulley is presented in Fig. 1. It represents a more complex system for dynamic analysis in comparison with the system which contains a drum as a driving element.

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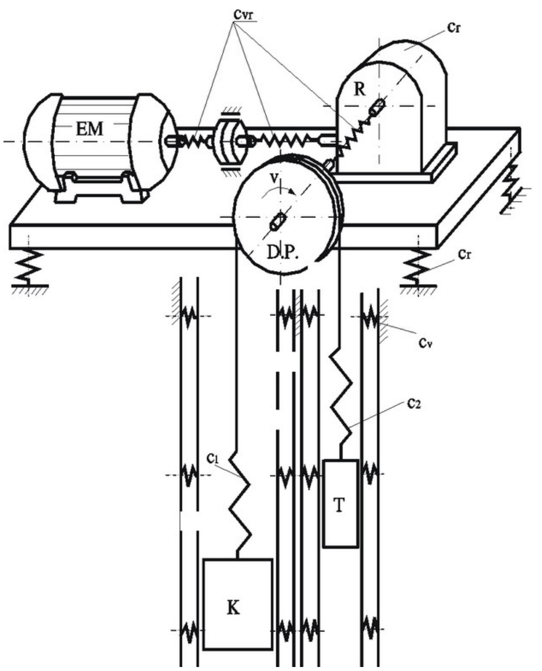

Fig.1. *Elevator driving mechanism*

The influence of the elasticity of shafts, gears, etc. can be neglected in comparison with the steel wire-rope elasticity. This enables the establishment of a simplified dynamic model of the driving mechanism, as shown in Fig. 2. The model takes into account the influence of the rope free length variations on both the incoming and outgoing pulley sides.

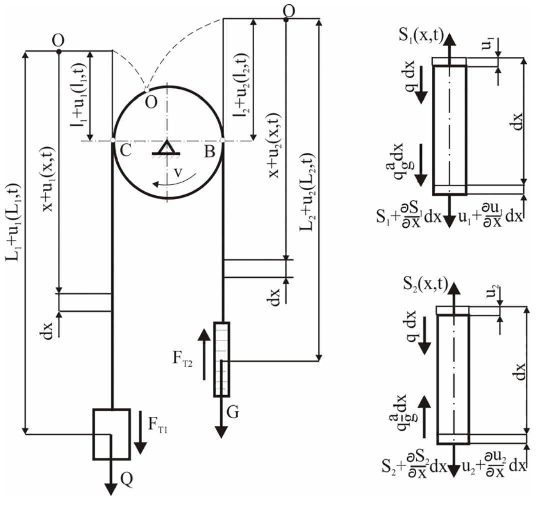

Fig.2. *Dynamic model*

The variations in rope free length (shortening or lengthening) directly affect the rope stiffness, and therefore also the dynamic behaviour of the rope, which is of significant importance in the systems with high lifting velocities. The influence of elastic rope slipping over the driving pulley is considered through the adequate boundary conditions.

According to [9] and Fig. 2, on the basis of the equilibrium conditions for the elementary rope length and for the driving pulley, the following system of equations can be established:

$$\frac{q}{g}\cdot\frac{\partial^2 u_1(x,t)}{\partial t^2} = E\cdot A\cdot\frac{\partial^2}{\partial x^2}\cdot\left[u_1(x,t)+b\cdot\frac{\partial u_1(x,t)}{\partial t}\right] + q\cdot(1\pm\frac{a}{g}) \tag{1}$$

$$\frac{q}{g}\cdot\frac{\partial^2 u_2(x,t)}{\partial t^2} = E\cdot A\cdot\frac{\partial^2}{\partial x^2}\cdot\left[u_2(x,t)+b\cdot\frac{\partial u_2(x,t)}{\partial t}\right] + q\cdot(1\pm\frac{a}{g}) \tag{2}$$

$$M_m = \frac{R}{i\cdot\eta}\cdot E\cdot A\cdot\frac{\partial}{\partial x} \left\{u_1(l_1,t)-u_2(l_2,t)+\frac{\partial}{\partial t}\left[u_1(l_1,t)-u_2(l_2,t)\right]-J_r\cdot\frac{a\cdot i}{\eta}\right\} \tag{3}$$

with:

u_1, u_2 - rope elastic deformations, m
E - elasticity modulus, Pa
A - rope cross-section, m^2
a - driving mechanism acceleration, ms^{-2}
M_m - driving motor torque, Nm
i - gear ratio,
η - driving mechanism efficiency,
J_r - moment of inertia of rotating masses, reduced to the pulley shaft, kgm^2
R - driving pulley radius, m
q - rope weight per meter, Nm^{-1}
g - gravity acceleration, ms^{-2}

In the presented Equations (1) to (3), the wire-rope is considered to be a viscous-elastic body (Kelvin's model), so the internal force can be defined as:

$$S(x,t) = E\cdot A\cdot\frac{\partial}{\partial x}\left[u(x,t)+b\cdot\frac{\partial u(x,t)}{\partial t}\right] \tag{4}$$

According to [2], dependence of the damping coefficient upon the rope stress can be adopted as:

$$b = \left(0.5 + \frac{2300}{350 + \sigma}\right) \cdot 10^{-4}, \text{ s}$$

with:

σ - rope stress, MPa

In the course of solving the Equation system (1) to (3), it is necessary to define the electro-mechanical model of the driving motor, or its mechanical characteristic in the form $M_m = f(v)$.

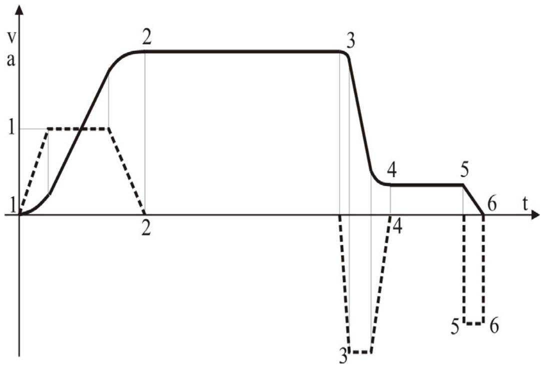

Fig. 3. *Diagram of velocity and acceleration for the elevator with double-speed motor*

For the passenger elevators, the parameters defining the "driving comfort" are specified, limiting the values of maximum acceleration to: a = 1.4 ms^{-2} and its time derivate to: $(da/dt)_{max}$ = 1.0 ms^{-3}, [7]. This makes it possible to simplify the analysis by excluding the Equation (3) from the previous system. Diagram of velocity and acceleration for the elevator with double-speed motor is presented in Fig. 3.

When considering the whole driving, the analysis becomes very complex. Therefore, the paper presents only the analysis of dynamic behaviour concerning the rope incoming side of the driving pulley. In this case, the further analysis is to be carried out only with respect to the Equation (1).

2 BOUNDARY CONDITIONS FOR THE ROPE INCOMING SIDE OF THE DRIVING PULLEY

In Fig. 4, the characteristic parameters satisfying the boundary conditions for the rope incoming side on the driving pulley or drum are presented.

Boundary conditions at point C, where the rope makes the first contact with the pulley, are dependent on whether the winding is with or without rope slipping.

The change of force in the wound rope part can be maintained only if this change is smaller than the adhesive force making it possible. Fig. 4.d shows different cases of force distribution over the wound rope length as a function of elevator velocity, [8].

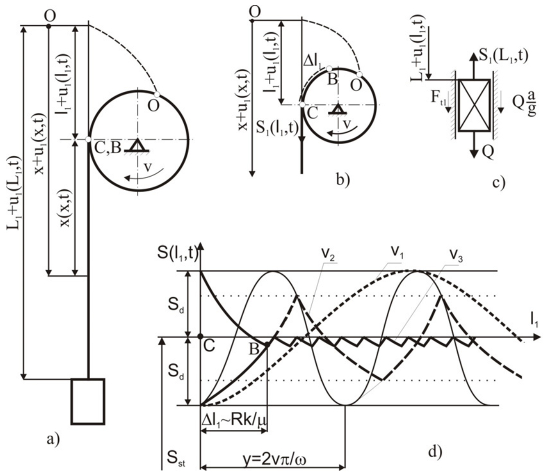

Fig. 4. *Boundary conditions for:*
a) the pulley without slipping,
b) the pulley with slipping,
c) the elevator car (or counterweight),
d) the distribution of forces and slipping over the wound rope length $v_1 > v_2 > v_3$

The limiting value of the rope winding velocity is determined from the condition:

$$\frac{\partial S(l_1, t)}{\partial x} \leq \frac{\partial S_{gr}}{\partial x}$$

with:

$$S(l_1, t) = S_{st} \cdot \left(1 + k \cdot \sin \frac{2 \cdot \pi}{v \cdot T} \cdot x\right)$$

- force in the wound rope part, no slipping,

$$S_{gr} = S_{st} \cdot e^{\frac{\mu}{R} \cdot (x - l_1)}$$

- force limit value on the basis of friction, On the basis of the previous expressions, the limit slipping velocity is:

$$v_s = \frac{R \cdot \omega}{\mu} \cdot k \tag{5}$$

with:

ω - vibration frequency of rope incoming side, s^{-1}

k - dynamic factor for elevators, k ≈ 0.1 to 0.3

μ - friction coefficient between rope and pulley,

S_{st} - rope static load at the incoming point C, N

T - period of vibration cycle, s^{-1}

If the elevator speed v exceeds the limit value v_s (the case v_1), Fig. 4.d, there occurs no rope slipping in the point C, so the force distribution over wound rope part remains the same. If the winding velocity is lower than v_{gr}, the force distribution over wound rope section has the form according to the dashed line v_2. For very low lifting velocities, the force over the wound rope part is practically equal to the static load (line v_3).

Average values for rope slipping displacement and velocity on the pulley incoming side are as follows:

$$\Delta \overline{l}_1 \cong \frac{2 \cdot R \cdot k}{\mu \cdot \pi} \tag{6}$$

$$\Delta \dot{\overline{l}}_1 \cong \frac{2 \cdot R \cdot k \cdot \omega}{\mu \cdot \pi} \tag{7}$$

Boundary condition for rope incoming point on the pulley without slipping, Fig. 4.a (point C), is given in the form:

$$u_1(l_1,t) = \int_0^t \frac{\partial u_1(l_1,t)}{\partial x}\left(\frac{dl_1}{dt}\right) \cdot dt \tag{8}$$

with:

l_1 - wound rope length, m

$dl_1 / dt = v$ - winding velocity, ms^{-1}

If the rope slipping is taken into account too, Fig. 4.b, the boundary condition is given in the form:

$$u_1(l_1,t) = \int_0^t \frac{\partial u_1(l_1,t)}{\partial x}\left(\frac{dl_1}{dt} - \frac{d}{dt}\left(\Delta l_1\right)\right) \cdot dt + \\ + \Delta l_1 \cdot \frac{\partial u_1(l_1,t)}{\partial x} \tag{9}$$

Boundary condition for the connection point of the rope with the elevator car or counterweight, when the friction forces between the sliding shoes and guiding rails are neglected, Fig. 4.c, is given as:

$$Q = E \cdot A \cdot \frac{\partial}{\partial x}\left(u_1(L_1,t) + b \cdot \frac{\partial u_1(L_1,t)}{\partial t}\right) + \\ + \left(\frac{Q}{g} \cdot \frac{\partial^2 u_1(L_1,t)}{\partial t^2} - a\right) \tag{10}$$

with:

Q - weight of the car with load passengers, N

The non-integral boundary condition (8) or (9) makes it impossible to solve partial differential Equation (1). Hence, the solution can be sought through by establishing the integral equations, which comprise both the differential equations and the appropriate boundary conditions.

3 THE INTEGRAL EQUATIONS AND ESTIMATION OF CRITICAL VELOCITY

A weightless string loaded at point M with the force F_i is shown in Fig. 5. The magnitude of rope point displacement (without slipping over the driving pulley) within the range $y < x$ shows linear growth from zero to the boundary value $u(y)$, while the displacement beneath the point M is of a steady value and equals to the boundary value $u(y)$.

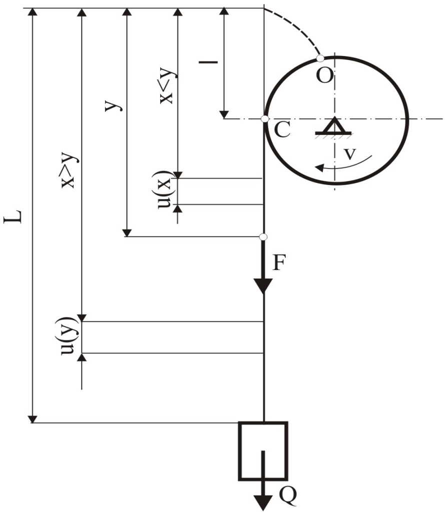

Fig.5. *Deformation of the rope*

According to this, for an elementary force (F_i =1), deformation can be defined, e.g. [3], as:

$$K(x,y)=\begin{cases}\dfrac{x-l}{E\cdot A} & \text{for } y>x\\ \dfrac{y-l}{E\cdot A} & \text{for } y>x\end{cases}$$

and displacement as $u(x) = K(x, y, l)\cdot F_i$.

In case of several forces acting in points at $x = y_i$, the displacement has the following form:

$$u(x)=\sum_{i=1}^{n} K(x,y_i,l)\cdot F_i$$

For the load case with evenly distributed rope load mass, the expression takes the form:

$$u(x)=\int_l^L K(x,y,l)\cdot q\cdot dy$$

After applying this procedure to the differential Equation (1), replacing floating argument with (y), multiplying it with the function $K(x, y, l_1) = K$ and performing necessary mathematical transformations, e.g. [1] and [4], the rope deformation becomes as follows:

$$\begin{aligned}u_1(x,t)=&-\int_{l_1}^{L_1} K\cdot q(y)\cdot\left[\frac{\partial^2 u_1}{\partial t^2}-g-a\right]\cdot dy+\\&+\int_0^t \frac{\partial u_1(l_1,t)}{\partial x}\left[\left(\frac{dl_1}{dt}\right)-\frac{d}{dt}(\Delta l_1)\right]\cdot dt+\\&+\Delta l\cdot\frac{\partial u_1(l_1,t)}{\partial x}+E\cdot A\int_{l_1}^{L_1} K\cdot\frac{\partial^3 u_1}{\partial s^2\cdot\partial t}\cdot ds-\\&-b\cdot E\cdot A\cdot\frac{\partial^2 u_1(L_1,t)}{\partial x\cdot\partial t}\cdot K(x,L_1,l_1)\end{aligned}\tag{11}$$

Applying the method of particular integrals, e.g. [10], given as:

$$U_1(x,t)=X_1(x)\cdot T_1(t)$$

and considering only the first mode of vibrations, form of which - in the case of a rope with the weight at its lower end - can be adopted as the straight line, i.e. $X_1(x)=x-l_1$, the simple differential equation in the following form is obtained:

$$\ddot{T}_1+\omega_1^2\cdot T_1=\varepsilon\cdot R(l_1,v)\cdot\dot{T}_1 \tag{12}$$

with:

$$\omega_1^2(l_1)=\frac{g\cdot E\cdot A}{\left[Q+\dfrac{q\cdot(L_1-l_1)}{3}\right]\cdot(L_1-l_1)}$$

rope fundamental frequency, s^{-1}

$$R(l_1,v)=\frac{1}{L_1-l_1}\cdot\left\{v-b\cdot\frac{E\cdot A\cdot g}{\varepsilon\cdot\left[Q+\dfrac{q\cdot(L_1-l_1)}{3}\right]}\right\}$$

$$\varepsilon=\frac{v}{L_1\cdot\omega_1(l_1)}.$$

Differential Equation (12) describes nonlinear vibrations with viscous friction when the parameters are "slow-time functions", with the solution, according to [10], in the form:

$$T_1(t)=a_1(t)\cdot\cos\theta(t)$$

whereby, after some mathematical transformations, the obtained values are:

$$a_1(t)=h_{10}\cdot\left(\frac{L_1-l_{10}}{L_1-l_1}\right)^{0.25}\cdot e^m$$

$$\theta_1(t)=\int_0^t \omega_1(l)\cdot dt+\theta_{10}$$

with:

$$h_{10}^0\cong-\frac{Q}{E\cdot A}\cdot\frac{a}{g}$$

amplitude value of the function,

l_{10} - length of wound rope at t=0, m

$$m=\left[\frac{1}{2}\cdot\frac{d}{dt}\cdot(\Delta l_1)\cdot\int_0^t\frac{dt}{L_1-l_1}-\frac{b}{2}\cdot\int_0^t\omega_1^2(l)\cdot dt\right]$$

After substituting previously mentioned values into Equation (9), and after performing mathematical transformations, the equation obtaines the form:

$$u_1(x,t)=(x-l_1)\cdot a_{10}\cdot\left(\frac{L_1-l_{10}}{L_1-l_1}\right)^{0.25}\cdot$$
$$\cdot e^{m}\cdot\cos\left[\int_0^t\omega_1(l)dt\right]+$$
$$+\left[Q+\frac{q\cdot(2\cdot L_1+x+l_1)}{2}\right]\cdot$$
$$\cdot\left(1+\frac{a}{g}\right)\cdot\frac{x-l_1}{E\cdot A}+u_1\left(l_1,t\right) \quad (13)$$

On the basis of Equations (4) and (13), the force in the rope is expressed as:

$$S_1\left(x,t\right)=E\cdot A\cdot a_1\left(t\right)\cdot\cos\theta_1\left(t\right)+$$
$$+\left[Q+q\cdot\left(L_1-x\right)\right]\cdot\left(1+\frac{a}{g}\right) \quad (14)$$

Analyzing Equation (14), it can be concluded that, during the rope free length decreasing phase (load lifting), under certain conditions, its deformation can be increased, causing a permanent growth of rope dynamic load. Such phenomenon, usually described as unstable lifting, appears in the case that $\frac{da_1\left(t\right)}{dt}\geq 0$. On the basis of this condition, the critical lifting velocity can be determined as:

$$v_c=\begin{cases}v_c^o & v\geq v_S\\ v_c^o+v_S & v<v_S\end{cases} \quad (15)$$

The critical lifting velocity in the case without rope slipping over the pulley at the incoming point C is:

$$v_c^o=\frac{2\cdot b\cdot g\cdot E\cdot A}{Q+\frac{q\cdot\left(L_1-l_1\right)}{3}} \quad (16)$$

with:

$$v_c^o\approx 2\cdot b\cdot\frac{q\cdot E\cdot A}{Q} \quad (17)$$

for:

$$Q>>q\cdot\left(L_1-l_1\right)$$

Limit slipping velocity, according to (5), is:

$$v_S\left(l_1\right)=\frac{k\cdot R}{\mu}\cdot\sqrt{\frac{v_c^o}{2\cdot b\cdot\left(L_1-l_1\right)}} \quad (18)$$

i.e.:

$$v_S\left(l_1\right)\approx\frac{k\cdot R}{\mu}\cdot\sqrt{\frac{q\cdot E\cdot A}{Q\cdot\left(L_1-l_1\right)}} \quad (19)$$

If the elevator velocity exceeds v_c in the lifting phase, the dynamic load will increase (unstable movement). Under these conditions two cases are possible:

- If the rope slipping at the incoming point is prevented (pulleys with special clamps for rope gripping), the unprevented increase of the dynamic load appears in the lifting phase,
- At standard pulleys, the increased dynamic load also appears in the lifting phase, but only until the moment when the slipping begins in the rope incoming point, $v<v_S$. During the rest of the lifting process, the load decreases, i.e. the stable movement occurs.

The value of critical velocity depends on the rope damping characteristics (due to internal friction), slipping conditions in the rope incoming point (C) and the basic characteristics of the elevator. When elevators with high load capacity, high velocity and lifting height (fast passenger elevators and mine elevators) are concerned, the lifting velocity can exceed the critical velocity, making it necessary to check the stability of the lifting process already in the design phase.

4 CONCLUSION

For driving mechanisms used for vertical load lifting (elevators and cranes), it is necessary to provide adequate conditions for the correct dynamic analysis of their parts' behavior and especially for the rope as the basic element, already in the design phase. Due to a significant influence of rope free length changes, due to its slipping over the driving pulley and for the reason of its mechanical characteristics, it is impossible to apply a classical dynamic model of longitudinal oscillations for homogenous stick of a constant length, especially by the elevators with large velocities and large lifting heights (express and mine elevators). The introduced dynamic model for elevator driving mechanism provides the analysis of relevant parameters and a base for the assessment of lifting process stability through the critical velocity level, defined with the help of expressions (15) to (19).

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