An Extended TOPSIS Method for Multiple Attribute Group Decision Making Based on Generalized Interval-valued Trapezoidal Fuzzy Numbers

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An Extended TOPSIS Method deals with multiple attribute group decision making problems in which the attribute values and weights take the form of the generalized interval-valued trapezoidal fuzzy numbers (GIVTFN). First, some properties are defined, such as the concept and the relational calculation rules of GIVTFN, the distance and its characteristics of GIVTFN, and the method which can transform the linguistic terms into GIVTFN. Second, the normalization method of GIVTFN is illustrated, and an extended TOPSIS method based on the GIVTFN is presented in detail. The order of the alternatives is ranked based on the relative closeness coefficient of TOPSIS. Finally, an illustrate example is given to show the effectiveness of this method and this decision making steps.

Povzetek: Z izboljšano metodo TOPSIS so dosegli boljše rezultate pri odločanju z mnogoterimi atributi.

1 Introduction

Multiple attribute decision making (MADM) is an important part of modern decision science. It has been extensively applied to various areas, such as society, economics, management, military and engineering technology. For example, the investment decision-making, project evaluation, the economic evaluation, the personnel evaluation etc. Since the object things are fuzzy, uncertainty and human thinking is ambiguous, the majority of the multi-attribute decision-making is uncertain and ambiguous, which is called the fuzzy multiple attribute decision-making (FMADM). Since Bellmanhe and Zadeh [1] initially proposed the basic model of fuzzy decision making based on the theory of fuzzy mathematics, FMADM has been receiving more and more attentions. Many achievements have been made on FMADM problems [2-5,7-21].

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is proposed by Hwang and Yoon [6], and it is a popular approach to MCDM problems. The basic principle is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. In the TOPSIS, the performance ratings and the weights of the criteria are given as crisp values. In many cases, crisp data are inadequate to model real life situations. Jahanshahloo et al [7] extends the TOPSIS method to the fuzzy decision making situations by considering interval numbers and defining crisp Euclidean distance between two interval numbers. Wang and Elhag[8] proposes a fuzzy TOPSIS method based on alpha level sets and presents a nonlinear programming (NLP) solution procedure by considering triangular fuzzy numbers. Liu and Zeng [9] proposes a new TOPSIS method to deal with the fuzzy multiple attribute group decision making problem based on the expected value operator of the trapezoidal fuzzy number when the fuzzy decision matrixes and the weights of the decision attributes and decision makers are all given by the trapezoidal fuzzy number. Tsaur et al. [10] convert the fuzzy MCDM problem into a crisp one via centroid defuzzification and then solve the non-fuzzy MCDM problem by the TOPSIS method. Chu and Lin [11] changed the fuzzy MCDM problem into a crisp one. Differing from the others, they first derive the membership functions of all the weighted ratings in a weighted normalized decision matrix and then convert them to crisp values by defuzzifying and then use TOPSIS method to solve this problem.

The concept of the interval-valued fuzzy set is initially proposed by Gorzlczany[12]and Turksen[13]. Some researchers focused on this research topic of interval-valued fuzzy numbers [12-18] in resent years, because the interval-valued fuzzy numbers are more general and better to express fuzzy information. Wang and Li [14-15] defined the expansion operation of the interval-valued fuzzy numbers, and proposed the concept and properties of the similarity coefficient based on the interval-valued fuzzy numbers. Hong and Lee [16] proposed the distance of the interval-valued fuzzy numbers. Ashtiani,et al[17] proposed definition of the interval-valued triangular fuzzy numbers and presented the extended TOPSIS group decision making method for the interval-valued triangular fuzzy numbers. Wei and Chen[18]proposed similarity measures between the generalized interval-valued trapezoidal fuzzy numbers (GIVTFN) for risk analysis. This paper proposed an extended TOPSIS Method to solve the multiple attribute group decision making problems of which the attribute weights and values are given with the form of GIVTFN.

In this paper we develop an extended TOPSIS method for multiple attribute group decision making based on the generalized interval-valued trapezoidal fuzzy numbers by defining the distance of GIVTFN. The remaining of this study is organized as follows. In the next section, we will briefly introduce the basic concept and the operation rules of the GIVTFN, and define the distance of GIVTFN. Section 3 describes the extended TOPSIS method to solve the multiple attribute group decision making problems by using GIVTFN. Section 4 gives a numerical example to explain validity of the decision-making steps and the method. The study is concluded in Section 5.

2 The basic concept of the interval-valued trapezoidal fuzzy numbers

2.1 The generalized trapezoidal fuzzy numbers

(1) The concept of the generalized trapezoidal fuzzy numbers

Definition 1 [19]: The generalized trapezoidal fuzzy numbers can be defined as a vector $\tilde{A}=(a_1,a_2,a_3,a_4;w_{\tilde{A}})$ (as shown in Fig1), and the membership function $a(x):R\to[0,1]$ is defined as follows:

$$a(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}\times w_{\tilde{A}}, & x\in(a_1,a_2)\\ w_{\tilde{A}}, & x\in(a_2,a_3)\\ \dfrac{x-a_4}{a_3-a_4}\times w_{\tilde{A}}, & x\in(a_3,a_4)\\ 0, & x\in(-\infty,a_1)\cup(a_4,\infty)\end{cases}$$

(1)

where $a_1\le a_2\le a_3\le a_4$ and $w_{\tilde{A}}\in[0,1]$.

The elements of the generalized trapezoidal fuzzy numbers $x\in R$ are real numbers, and its membership function $a(x)$ is the regularly and continuous convex function, showing the membership degree to the fuzzy sets. If $-1\le a_1\le a_2\le a_3\le a_4\le 1$, then $\tilde{A}$ is called the normalized trapezoidal fuzzy number. Especially, if $w_{\tilde{A}}=1$, then $\tilde{A}$ is called the trapezoidal fuzzy number (a_1,a_2,a_3,a_4); if $a_1<a_2=a_3<a_4$, then $\tilde{A}$ is reduced to a triangular fuzzy number. If $a_1=a_2=a_3=a_4$, then $\tilde{A}$ is reduced to a real number.

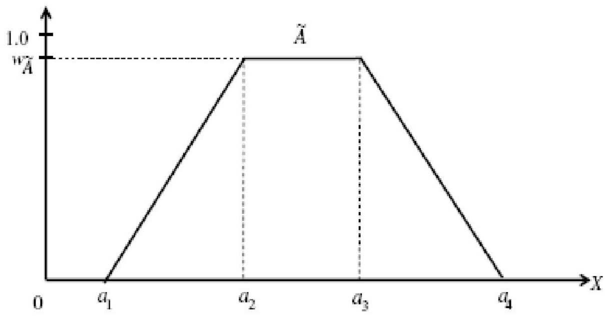

Figure 1: The generalized trapezoidal fuzzy number $\tilde{A}$.

(2) The operation of the generalized trapezoidal fuzzy number

Suppose that $\tilde{a}=(a_1,a_2,a_3,a_4;w_{\tilde{a}})$, $\tilde{b}=(b_1,b_2,b_3,b_4;w_{\tilde{b}})$ are the generalized trapezoidal fuzzy numbers, then the operational rules of the generalized trapezoidal fuzzy number are shown as follows:[20]

(i)
$$\tilde{a}\oplus\tilde{b}=(a_1,a_2,a_3,a_4;w_{\tilde{a}})\oplus(b_1,b_2,b_3,b_4;w_{\tilde{b}})=(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4;\min(w_{\tilde{a}},w_{\tilde{b}}))$$
(2)

(ii)
$$\tilde{a}-\tilde{b}=(a_1,a_2,a_3,a_4;w_{\tilde{a}})-(b_1,b_2,b_3,b_4;w_{\tilde{b}})=(a_1-b_4,a_2-b_3,a_3-b_2,a_4-b_1;\min(w_{\tilde{a}},w_{\tilde{b}}))$$
(3)

(iii)
$$\tilde{a}\otimes\tilde{b}=(a_1,a_2,a_3,a_4;w_{\tilde{a}})\otimes(b_1,b_2,b_3,b_4;w_{\tilde{b}})=(a,b,c,d;\min(w_{\tilde{a}},w_{\tilde{b}}))$$
(4)

where

$a=\min(a_1\times b_1,a_1\times b_4,a_4\times b_1,a_4\times b_4)$,

$b=\min(a_2\times b_2,a_2\times b_3,a_3\times b_2,a_3\times b_3)$

$c=\max(a_2\times b_2,a_2\times b_3,a_3\times b_2,a_3\times b_3)$,

$d=\max(a_1\times b_1,a_1\times b_4,a_4\times b_1,a_4\times b_4)$

If $a_1,a_2,a_3,a_4,b_1,b_2,b_3,b_4$ are the positive numbers, then

$$\tilde{a}\otimes\tilde{b}=(a_1\times b_1,a_2\times b_2,a_3\times b_3,a_4\times b_4;\min(w_{\tilde{a}},w_{\tilde{b}}))$$

(iv)
$$\tilde{a}/\tilde{b}=(a_1,a_2,a_3,a_4;w_{\tilde{a}})/(b_1,b_2,b_3,b_4;w_{\tilde{b}})=(a_1/b_4,a_2/b_3,a_3/b_2,a_4/b_1;\min(w_{\tilde{a}},w_{\tilde{b}}))$$
(5)

(3) The center of the gravity (COG) point of the generalized trapezoidal fuzzy numbers

Chen and Chen [21] proposed the concept of the COG point of the generalized trapezoidal fuzzy numbers, and suppose that the COG point of the generalized trapezoidal fuzzy numbers $\tilde{a}=(a_1,a_2,a_3,a_4;w_{\tilde{a}})$ is $(x_{\tilde{a}},y_{\tilde{a}})$, then [21]:

$$\begin{cases} y_{\tilde{a}}=\begin{cases} \dfrac{w_{\tilde{a}}\times\left(\dfrac{a_3-a_2}{a_4-a_1}+2\right)}{6} & if\ \ a_1\neq a_4 \\ w_{\tilde{a}}/2 & if\ \ a_1=a_4 \end{cases} \\ x_{\tilde{a}}=\dfrac{y_{\tilde{a}}\times(a_2+a_3)+(a_1+a_4)\times(w_{\tilde{a}}-y_{\tilde{a}})}{2\times w_{\tilde{a}}} \end{cases} \qquad (6)$$

2.2 The interval-valued trapezoidal fuzzy numbers

(1)The interval-valued trapezoidal fuzzy numbers [18]

Wang and Li [15] proposed the interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}}=\left[\tilde{\tilde{A}}^L,\tilde{\tilde{A}}^U\right]=\left[(a_1^L,a_2^L,a_3^L,a_4^L;w_{\tilde{\tilde{A}}^L}),(a_1^U,a_2^U,a_3^U,a_4^U;w_{\tilde{\tilde{A}}^U})\right]$ shown in Fig. 2, where $0\le a_1^L\le a_2^L\le a_3^L\le a_4^L\le 1$, $0\le a_1^U\le a_2^U\le a_3^U\le a_4^U\le 1$, $0\le w_{\tilde{\tilde{A}}^L}\le w_{\tilde{\tilde{A}}^U}\le 1$ and $\tilde{\tilde{A}}^L\subset\tilde{\tilde{A}}^U$. In Fig. 2, we can conclude that the interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}}$ are consist of the lower values of the interval-valued trapezoidal fuzzy number $\tilde{\tilde{A}}^L$ and the upper values of the interval-valued trapezoidal fuzzy number $\tilde{\tilde{A}}^U$. (Shown in Fig. 2)

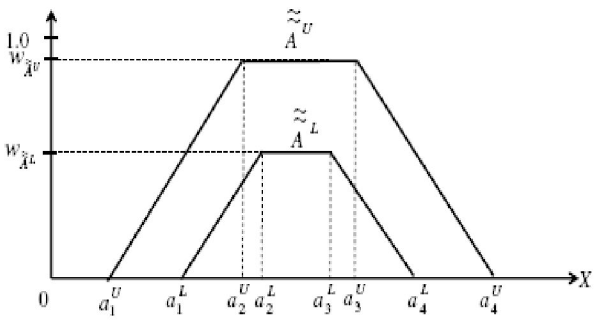

Figure 2: the interval-valued trapezoidal fuzzy numbers.

(2) The operation of the interval-valued trapezoidal fuzzy numbers [18]

Suppose that

$\tilde{\tilde{A}}=\left[\tilde{\tilde{A}}^L,\tilde{\tilde{A}}^U\right]=\left[(a_1^L,a_2^L,a_3^L,a_4^L;w_{\tilde{\tilde{A}}^L}),(a_1^U,a_2^U,a_3^U,a_4^U;w_{\tilde{\tilde{A}}^U})\right]$,

$\tilde{\tilde{B}}=\left[\tilde{\tilde{B}}^L,\tilde{\tilde{B}}^U\right]=\left[(b_1^L,b_2^L,b_3^L,b_4^L;w_{\tilde{\tilde{B}}^L}),(b_1^U,b_2^U,b_3^U,b_4^U;w_{\tilde{\tilde{B}}^U})\right]$

are the two interval-valued trapezoidal fuzzy numbers, where $0\le a_1^L\le a_2^L\le a_3^L\le a_4^L\le 1$, $0\le a_1^U\le a_2^U\le a_3^U\le a_4^U\le 1$, $0\le w_{\tilde{\tilde{A}}^L}\le w_{\tilde{\tilde{A}}^U}\le 1$, $\tilde{\tilde{A}}^L\subset\tilde{\tilde{A}}^U$, $0\le b_1^L\le b_2^L\le b_3^L\le b_4^L\le 1$, $0\le b_1^U\le b_2^U\le b_3^U\le b_4^U\le 1$, $0\le w_{\tilde{\tilde{B}}^L}\le w_{\tilde{\tilde{B}}^U}\le 1$, $w_{\tilde{\tilde{B}}^L}\subset w_{\tilde{\tilde{B}}^U}$. Then the operation is shown as follows:

(i) The sum of two interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}}$, $\tilde{\tilde{B}}$:

$$\begin{aligned}\tilde{\tilde{A}}\oplus\tilde{\tilde{B}}&=\left[(a_1^L,a_2^L,a_3^L,a_4^L;w_{\tilde{\tilde{A}}^L}),(a_1^U,a_2^U,a_3^U,a_4^U;w_{\tilde{\tilde{A}}^U})\right]\\&\oplus\left[(b_1^L,b_2^L,b_3^L,b_4^L;w_{\tilde{\tilde{B}}^L}),(b_1^U,b_2^U,b_3^U,b_4^U;w_{\tilde{\tilde{B}}^U})\right]\\&=\left[(a_1^L+b_1^L,a_2^L+b_2^L,a_3^L+b_3^L,a_4^L+b_4^L;\min(w_{\tilde{\tilde{A}}^L},w_{\tilde{\tilde{B}}^L})),\right.\\&\left.(a_1^U+b_1^U,a_2^U+b_2^U,a_3^U+b_3^U,a_4^U+b_4^U;\min(w_{\tilde{\tilde{A}}^U},w_{\tilde{\tilde{B}}^U}))\right]\end{aligned} \qquad (7)$$

(ii) The difference of two interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}}$, $\tilde{\tilde{B}}$

$$\begin{aligned}\tilde{\tilde{A}}-\tilde{\tilde{B}}&=\left[(a_1^L,a_2^L,a_3^L,a_4^L;w_{\tilde{\tilde{A}}^L}),(a_1^U,a_2^U,a_3^U,a_4^U;w_{\tilde{\tilde{A}}^U})\right]\\&-\left[(b_1^L,b_2^L,b_3^L,b_4^L;w_{\tilde{\tilde{B}}^L}),(b_1^U,b_2^U,b_3^U,b_4^U;w_{\tilde{\tilde{B}}^U})\right]\\&=\left[(a_1^L-b_4^L,a_2^L-b_3^L,a_3^L-b_2^L,a_4^L-b_1^L;\min(w_{\tilde{\tilde{A}}^L},w_{\tilde{\tilde{B}}^L})),\right.\\&\left.(a_1^U-b_4^U,a_2^U-b_3^U,a_3^U-b_2^U,a_4^U-b_1^U;\min(w_{\tilde{\tilde{A}}^U},w_{\tilde{\tilde{B}}^U}))\right]\end{aligned}$$

(8)

(iii) The product of two interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}}$, $\tilde{\tilde{B}}$

$$\begin{aligned}\tilde{\tilde{A}}\otimes\tilde{\tilde{B}}&=\left[(a_1^L,a_2^L,a_3^L,a_4^L;w_{\tilde{\tilde{A}}^L}),(a_1^U,a_2^U,a_3^U,a_4^U;w_{\tilde{\tilde{A}}^U})\right]\\&\otimes\left[(b_1^L,b_2^L,b_3^L,b_4^L;w_{\tilde{\tilde{B}}^L}),(b_1^U,b_2^U,b_3^U,b_4^U;w_{\tilde{\tilde{B}}^U})\right]\\&=\left[(a_1^L\times b_1^L,a_2^L\times b_2^L,a_3^L\times b_3^L,a_4^L\times b_4^L;\min(w_{\tilde{\tilde{A}}^L},w_{\tilde{\tilde{B}}^L})),\right.\\&\left.(a_1^U\times b_1^U,a_2^U\times b_2^U,a_3^U\times b_3^U,a_4^U\times b_4^U;\min(w_{\tilde{\tilde{A}}^U},w_{\tilde{\tilde{B}}^U}))\right]\end{aligned} \qquad (9)$$

(iv) The quotient of two interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}}$, $\tilde{\tilde{B}}$:

$$\begin{aligned}\tilde{\tilde{A}}/\tilde{\tilde{B}}&=\left[(a_1^L,a_2^L,a_3^L,a_4^L;w_{\tilde{\tilde{A}}^L}),(a_1^U,a_2^U,a_3^U,a_4^U;w_{\tilde{\tilde{A}}^U})\right]\\&/\left[(b_1^L,b_2^L,b_3^L,b_4^L;w_{\tilde{\tilde{B}}^L}),(b_1^U,b_2^U,b_3^U,b_4^U;w_{\tilde{\tilde{B}}^U})\right]\\&=\left[(a_1^L/b_4^L,a_2^L/b_3^L,a_3^L/b_2^L,a_4^L/b_1^L;\min(w_{\tilde{\tilde{A}}^L},w_{\tilde{\tilde{B}}^L})),\right.\\&\left.(a_1^U/b_4^U,a_2^U/b_3^U,a_3^U/b_2^U,a_4^U/b_1^U;\min(w_{\tilde{\tilde{A}}^U},w_{\tilde{\tilde{B}}^U}))\right]\end{aligned} \qquad (10)$$

(v) The product between an interval-valued trapezoidal fuzzy number $\tilde{\tilde{A}}$ and a constant number λ ($\lambda>0$):

$$\lambda\tilde{\tilde{A}}=\lambda\times\left[(a_1^L,a_2^L,a_3^L,a_4^L;w_{\tilde{\tilde{A}}^L}),(a_1^U,a_2^U,a_3^U,a_4^U;w_{\tilde{\tilde{A}}^U})\right] \\ =\left[(\lambda a_1^L,\lambda a_2^L,\lambda a_3^L,\lambda a_4^L;w_{\tilde{\tilde{A}}^L}),(\lambda a_1^U,\lambda a_2^U,\lambda a_3^U,\lambda a_4^U;w_{\tilde{\tilde{A}}^U})\right] \quad (11)$$

2.3 The distance of the interval-valued trapezoidal fuzzy numbers

Suppose that

$$\tilde{\tilde{A}}=\left[\tilde{\tilde{A}}^L,\tilde{\tilde{A}}^U\right]=\left[(a_1^L,a_2^L,a_3^L,a_4^L;w_{\tilde{\tilde{A}}^L}),(a_1^U,a_2^U,a_3^U,a_4^U;w_{\tilde{\tilde{A}}^U})\right],$$

$$\tilde{\tilde{B}}=\left[\tilde{\tilde{B}}^L,\tilde{\tilde{B}}^U\right]=\left[(b_1^L,b_2^L,b_3^L,b_4^L;w_{\tilde{\tilde{B}}^L}),(b_1^U,b_2^U,b_3^U,b_4^U;w_{\tilde{\tilde{B}}^U})\right]$$

are any two generalized trapezoidal fuzzy numbers, then the distance of two interval-valued trapezoidal fuzzy numbers($\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$) is calculated as follows:

(1)Utilize the formula (6) to calculate the coordinate of the COG point $\left(x_{\tilde{\tilde{A}}^L},y_{\tilde{\tilde{A}}^L}\right),\left(x_{\tilde{\tilde{A}}^U},y_{\tilde{\tilde{A}}^U}\right),\left(x_{\tilde{\tilde{B}}^L},y_{\tilde{\tilde{B}}^L}\right),\left(x_{\tilde{\tilde{B}}^U},y_{\tilde{\tilde{B}}^U}\right)$ which belongs to the generalized trapezoidal fuzzy numbers $\tilde{\tilde{A}}^L,\tilde{\tilde{A}}^U,\tilde{\tilde{B}}^L,\tilde{\tilde{B}}^U$ respectively.

(2) The distance of two interval-valued trapezoidal fuzzy numbers is:

$$d(\tilde{\tilde{A}},\tilde{\tilde{B}})=\sqrt{\left((y_{\tilde{\tilde{A}}^L}-y_{\tilde{\tilde{B}}^L})^2+(x_{\tilde{\tilde{A}}^L}-x_{\tilde{\tilde{B}}^L})^2+(y_{\tilde{\tilde{A}}^U}-y_{\tilde{\tilde{B}}^U})^2+(x_{\tilde{\tilde{A}}^U}-x_{\tilde{\tilde{B}}^U})^2\right)\Big/4} \quad (12)$$

where $d(\tilde{\tilde{A}},\tilde{\tilde{B}})$ satisfies the following properties:

(i) if $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ are the normalized interval-valued trapezoidal fuzzy numbers, then $0\le d(\tilde{\tilde{A}},\tilde{\tilde{B}})\le 1$

(ii) $\tilde{\tilde{A}}=\tilde{\tilde{B}}\Leftrightarrow d(\tilde{\tilde{A}},\tilde{\tilde{B}})=0$

(iii) $d(\tilde{\tilde{A}},\tilde{\tilde{B}})=d(\tilde{\tilde{B}},\tilde{\tilde{A}})$

(iv) $d(\tilde{\tilde{A}},\tilde{\tilde{C}})+d(\tilde{\tilde{C}},\tilde{\tilde{B}})\ge d(\tilde{\tilde{A}},\tilde{\tilde{B}})$

Obviously, the properties (i) and (iii) are satisfied.

For the properties (ii), if $\tilde{\tilde{A}}=\tilde{\tilde{B}}$, then $d(\tilde{\tilde{A}},\tilde{\tilde{B}})=0$. If $d(\tilde{\tilde{A}},\tilde{\tilde{B}})=0$, then the COG of $\tilde{\tilde{A}}$ is equal to $\tilde{\tilde{B}}$'s, so we approximately believe that $\tilde{\tilde{A}}=\tilde{\tilde{B}}$.

For properties (iv): In order to simplify the expression in distance formula, we suppose that

$\alpha_1=x_{\tilde{\tilde{A}}^L},\alpha_2=x_{\tilde{\tilde{B}}^L},\alpha_3=x_{\tilde{\tilde{C}}^L},\beta_1=y_{\tilde{\tilde{A}}^L},\beta_2=y_{\tilde{\tilde{B}}^L},\beta_3=y_{\tilde{\tilde{C}}^L},\gamma_1=x_{\tilde{\tilde{A}}^U},\gamma_2=x_{\tilde{\tilde{B}}^U},\gamma_3=x_{\tilde{\tilde{C}}^U},\eta_1=y_{\tilde{\tilde{A}}^U},\eta_2=y_{\tilde{\tilde{B}}^U},\eta_3=y_{\tilde{\tilde{C}}^U}$.

$$d(\tilde{\tilde{A}},\tilde{\tilde{B}})+d(\tilde{\tilde{B}},\tilde{\tilde{C}})\ge d(\tilde{\tilde{A}},\tilde{\tilde{C}})\Leftrightarrow\left(d(\tilde{\tilde{A}},\tilde{\tilde{B}})+d(\tilde{\tilde{B}},\tilde{\tilde{C}})\right)^2\ge d^2(\tilde{\tilde{A}},\tilde{\tilde{C}})$$

$$\Leftrightarrow\left(d(\tilde{\tilde{A}},\tilde{\tilde{B}})+d(\tilde{\tilde{B}},\tilde{\tilde{C}})\right)^2-d^2(\tilde{\tilde{A}},\tilde{\tilde{C}})\ge 0$$

$$\Leftrightarrow(\alpha_1-\alpha_2)^2+(\beta_1-\beta_2)^2+(\gamma_1-\gamma_2)^2+(\eta_1-\eta_2)^2+(\alpha_2-\alpha_3)^2+(\beta_2-\beta_3)^2+(\gamma_2-\gamma_3)^2+(\eta_2-\eta_3)^2 \\ +2\sqrt{\left[(\alpha_1-\alpha_2)^2+(\beta_1-\beta_2)^2+(\gamma_1-\gamma_2)^2+(\eta_1-\eta_2)^2\right]\times\left[(\alpha_2-\alpha_3)^2+(\beta_2-\beta_3)^2+(\gamma_2-\gamma_3)^2+(\eta_2-\eta_3)^2\right]} \\ -\left[(\alpha_1-\alpha_3)^2+(\beta_1-\beta_3)^2+(\gamma_1-\gamma_3)^2+(\eta_1-\eta_3)^2\right]\ge 0$$

Suppose that

$$dA=(\alpha_1-\alpha_2)^2+(\beta_1-\beta_2)^2+(\gamma_1-\gamma_2)^2+(\eta_1-\eta_2)^2+(\alpha_2-\alpha_3)^2+(\beta_2-\beta_3)^2+(\gamma_2-\gamma_3)^2+(\eta_2-\eta_3)^2 \\ -\left[(\alpha_1-\alpha_3)^2+(\beta_1-\beta_3)^2+(\gamma_1-\gamma_3)^2+(\eta_1-\eta_3)^2\right]$$

$$dB=2\sqrt{\left[(\alpha_1-\alpha_2)^2+(\beta_1-\beta_2)^2+(\gamma_1-\gamma_2)^2+(\eta_1-\eta_2)^2\right]\times\left[(\alpha_2-\alpha_3)^2+(\beta_2-\beta_3)^2+(\gamma_2-\gamma_3)^2+(\eta_2-\eta_3)^2\right]}$$

$$\because\ dA=(\alpha_1-\alpha_2)^2+(\beta_1-\beta_2)^2+(\gamma_1-\gamma_2)^2+(\eta_1-\eta_2)^2+(\alpha_2-\alpha_3)^2+(\beta_2-\beta_3)^2+(\gamma_2-\gamma_3)^2+(\eta_2-\eta_3)^2 \\ -\left[(\alpha_1-\alpha_3)^2+(\beta_1-\beta_3)^2+(\gamma_1-\gamma_3)^2+(\eta_1-\eta_3)^2\right] \\ =2(\alpha_1-\alpha_2)(\alpha_3-\alpha_2)+2(\beta_1-\beta_2)(\beta_3-\beta_2)+2(\gamma_1-\gamma_2)(\gamma_3-\gamma_2)+2(\eta_1-\eta_2)(\eta_3-\eta_2)$$

$$\because\ dB=2\sqrt{\left[(\alpha_1-\alpha_2)^2+(\beta_1-\beta_2)^2+(\gamma_1-\gamma_2)^2+(\eta_1-\eta_2)^2\right]\times\left[(\alpha_2-\alpha_3)^2+(\beta_2-\beta_3)^2+(\gamma_2-\gamma_3)^2+(\eta_2-\eta_3)^2\right]} \\ \ge 2\sqrt{\left[(\alpha_1-\alpha_2)(\alpha_2-\alpha_3)+(\beta_1-\beta_2)(\beta_2-\beta_3)\right]^2+\left[(\gamma_1-\gamma_2)(\gamma_2-\gamma_3)+(\eta_1-\eta_2)(\eta_2-\eta_3)\right]^2+dC} \\ =2\left[(\alpha_1-\alpha_2)(\alpha_2-\alpha_3)+(\beta_1-\beta_2)(\beta_2-\beta_3)+(\gamma_1-\gamma_2)(\gamma_2-\gamma_3)+(\eta_1-\eta_2)(\eta_2-\eta_3)\right]$$

where

$$dC = 2[(\alpha_1-\alpha_2)(\alpha_2-\alpha_3)(\gamma_1-\gamma_2)(\gamma_2-\gamma_3)+(\beta_1-\beta_2)(\beta_2-\beta_3)(\gamma_1-\gamma_2)(\gamma_2-\gamma_3)$$
$$+(\eta_1-\eta_2)(\eta_2-\eta_3)(\alpha_1-\alpha_2)(\alpha_2-\alpha_3)+(\eta_1-\eta_2)(\eta_2-\eta_3)(\beta_1-\beta_2)(\beta_2-\beta_3)]$$

$$\therefore dA+dB\ge 0 \Leftrightarrow \left(d(\tilde{\tilde{A}},\tilde{\tilde{B}})+d(\tilde{\tilde{B}},\tilde{\tilde{C}})\right)^2-d^2(\tilde{\tilde{A}},\tilde{\tilde{C}})\ge 0 \Leftrightarrow d(\tilde{\tilde{A}},\tilde{\tilde{B}})+d(\tilde{\tilde{B}},\tilde{\tilde{C}})\ge d(\tilde{\tilde{A}},\tilde{\tilde{C}})$$

2.4 Utilize the interval-valued trapezoidal fuzzy numbers to represent the linguistic terms

In the real decision making process, it is difficult to adopt the form of generalized interval-valued trapezoidal fuzzy numbers to give the attribute values and weights directly by the decision makers. So we usually adopt the form of linguistic terms. Wei and Chen [18] utilizes the interval-valued trapezoidal fuzzy numbers to represent the 9-member linguistic terms. (shown in Table 1)

Table 1: A 9-member interval linguistic term set.

linguistic terms (the attribute values)	linguistic terms (weights)	generalized interval-valued trapezoidal fuzzy numbers
Absolutely-poor(AP)	Absolutely-low(AL)	[(0.00,0.00,0.00,0.00;0.8), (0.00,0.00,0.00,0.00;1.0)]
Very-poor(VP)	Very-low (VL)	[(0.00,0.00,0.02,0.07;0.8), (0.00,0.00,0.02,0.07;1.0)]
poor (P)	low (L)	[(0.04,0.10,0.18,0.23;0.8), (0.04,0.10,0.18,0.23;1.0)]
Medium-poor(MP)	Medium-low(ML)	[(0.17,0.22,0.36,0.42;0.8), (0.17,0.22,0.36,0.42;1.0)]
Medium (F)	Medium (M)	[(0.32,0.41,0.58,0.65;0.8), (0.32,0.41,0.58,0.65;1.0)]
Medium-good(MG)	Medium-high(MH)	[(0.58,0.63,0.80,0.86;0.8), (0.58,0.63,0.80,0.86;1.0)]
good(G)	high(H)	[(0.72,0.78,0.92,0.97;0.8), (0.72,0.78,0.92,0.97;1.0)]
Very-good(VG)	very-high (VH)	[(0.93,0.98,1.00,1.00;0.8), (0.93,0.98,1.00,1.00;1.0)]
Absolutely-good(AG)	Absolutely-high (AH)	[(1.00,1.00,1.00,1.00;0.8), (1.00,1.00,1.00,1.00;1.0)]

3 Group decision making method

3.1 The description of the decision making problems

Let $E=\{e_1,e_2,\cdots,e_q\}$ be the set of decision makers in the group decision making, and $A=\{A_1,A_2,\cdots,A_m\}$ be the set of alternatives, and $C=\{C_1,C_2,\cdots,C_n\}$ be the set of attributes. Suppose that $\tilde{\tilde{a}}_{ijk}=\left[(a^L_{ijk1},a^L_{ijk2},a^L_{ijk3},a^L_{ijk4};w^L_{ijk}),(a^U_{ijk1},a^U_{ijk2},a^U_{ijk3},a^U_{ijk4};w^U_{ijk})\right]$ is the attribute value given by the decision maker e_k, where $\tilde{\tilde{a}}_{ijk}$ is an interval-valued trapezoidal fuzzy number for the alternative A_i with respect to the attribute C_j, and $\tilde{\tilde{\omega}}_{kj}=\left[(\omega^L_{kj1},\omega^L_{kj2},\omega^L_{kj3},\omega^L_{kj4};\eta^L_{kj}),(\omega^U_{kj1},\omega^U_{kj2},\omega^U_{kj3},\omega^U_{kj4};\eta^U_{kj})\right]$ is the attribute weight given by the decision maker e_k, where $\tilde{\tilde{\omega}}_{kj}$ is also an interval-valued trapezoidal fuzzy number. Let $\lambda=(\lambda_1,\lambda_2,\cdots,\lambda_q)$ be the weight vector of decision makers, where λ_k is a real number, and $\sum_{k=1}^{q}\lambda_k=1$. Then we use the attribute weights, the decision makers' weights, and the attribute values to rank the order of the alternatives.

3.2 Normalize the decision-making information

we need normalize the decision-making information, in order to eliminate the impact of different physical dimension to the decision-making result. Consider that there are generally benefit attributes (I_1) and cost attributes (I_2). The normalizing method is shown as follows:

For benefit attributes, where $m_{jk}=\max_i(a^U_{ijk4})$.

$$\tilde{\tilde{x}}_{ijk}=\left[(x^L_{ijk1},x^L_{ijk2},x^L_{ijk3},x^L_{ijk4};w^L_{ijk}),(x^U_{ijk1},x^U_{ijk2},x^U_{ijk3},x^U_{ijk4};w^U_{ijk})\right]$$
$$=\left[\left(\frac{a^L_{ijk1}}{m_{jk}},\frac{a^L_{ijk2}}{m_{jk}},\frac{a^L_{ijk3}}{m_{jk}},\frac{a^L_{ijk4}}{m_{jk}};w^L_{ijk}\right),\left(\frac{a^U_{ijk1}}{m_{jk}},\frac{a^U_{ijk2}}{m_{jk}},\frac{a^U_{ijk3}}{m_{jk}},\frac{a^U_{ijk4}}{m_{jk}};w^U_{ijk}\right)\right] \quad (13)$$

For cost attributes, where $n_{jk}=\min_i(a^L_{ijk1})$.

$$\tilde{\tilde{x}}_{ijk} = \left[(x^L_{ijk1}, x^L_{ijk2}, x^L_{ijk3}, x^L_{ijk4}; w^L_{ijk}), (x^U_{ijk1}, x^U_{ijk2}, x^U_{ijk3}, x^U_{ijk4}; w^U_{ijk}) \right]$$
$$= \left[\left((\frac{n_{jk}}{d^L_{ijk1}}, \frac{n_{jk}}{d^L_{ijk2}}, \frac{n_{jk}}{d^L_{ijk3}}, \frac{n_{jk}}{d^L_{ijk4}}; w^L_{ijk}), (\frac{n_{jk}}{d^U_{ijk1}}, \frac{n_{jk}}{d^U_{ijk2}}, \frac{n_{jk}}{d^U_{ijk3}}, \frac{n_{jk}}{d^U_{ijk4}}; w^U_{ijk}) \right) \right] \quad (14)$$

3.3 Combine the evaluation information of each decision maker

We can get the collective attribute values and weights, according to the different projects' attribute values and weights which were given by different experts under different attribute.

The combining steps are shown as follows:

$$\tilde{\tilde{x}}_{ij} = \left[(x^L_{ij1}, x^L_{ij2}, x^L_{ij3}, x^L_{ij4}; w^L_{ij}), (x^U_{ij1}, x^U_{ij2}, x^U_{ij3}, x^U_{ij4}; w^U_{ij}) \right]$$
$$= \sum_{k=1}^{q} \left(\lambda_k \tilde{\tilde{x}}_{ijk} \right) = \sum_{k=1}^{q} \left\{ \lambda_k \times \left[(x^L_{ijk1}, x^L_{ijk2}, x^L_{ijk3}, x^L_{ijk4}; w^L_{ijk}), (x^U_{ijk1}, x^U_{ijk2}, x^U_{ijk3}, x^U_{ijk4}; w^U_{ijk}) \right] \right\} \quad (15)$$
$$= \begin{bmatrix} \left(\sum_{k=1}^{q} \left(\lambda_k x^L_{ijk1} \right), \sum_{k=1}^{q} \left(\lambda_k x^L_{ijk2} \right), \sum_{k=1}^{q} \left(\lambda_k x^L_{ijk3} \right), \sum_{k=1}^{q} \left(\lambda_k x^L_{ijk4} \right); \min_k (w^L_{ijk}) \right), \\ \left(\sum_{k=1}^{q} \left(\lambda_k x^U_{ijk1} \right), \sum_{k=1}^{q} \left(\lambda_k x^U_{ijk2} \right), \sum_{k=1}^{q} \left(\lambda_k x^U_{ijk3} \right), \sum_{k=1}^{q} \left(\lambda_k x^U_{ijk4} \right); \min_k (w^U_{ijk}) \right) \end{bmatrix}$$

$$\tilde{\tilde{\omega}}_j = \left[(\omega^L_{j1}, \omega^L_{j2}, \omega^L_{j3}, \omega^L_{j4}; \eta^L_j), (\omega^U_{j1}, \omega^U_{j2}, \omega^U_{j3}, \omega^U_{j4}; \eta^U_j) \right]$$
$$= \sum_{k=1}^{q} \left(\lambda_k \times \tilde{\tilde{\omega}}_{kj} \right) = \sum_{k=1}^{q} \left(\lambda_k \times \left[(\omega^L_{kj1}, \omega^L_{kj2}, \omega^L_{kj3}, \omega^L_{kj4}; \eta^L_{kj}), (\omega^U_{kj1}, \omega^U_{kj2}, \omega^U_{kj3}, \omega^U_{kj4}; \eta^U_{kj}) \right] \right) \quad (16)$$
$$= \begin{bmatrix} \left(\sum_{k=1}^{q} \left(\lambda_k \omega^L_{kj1} \right), \sum_{k=1}^{q} \left(\lambda_k \omega^L_{kj2} \right), \sum_{k=1}^{q} \left(\lambda_k \omega^L_{kj3} \right), \sum_{k=1}^{q} \left(\lambda_k \omega^L_{kj4} \right); \min_k (\eta^L_{kj}) \right), \\ \left(\sum_{k=1}^{q} \left(\lambda_k \omega^U_{kj1} \right), \sum_{k=1}^{q} \left(\lambda_k \omega^U_{kj2} \right), \sum_{k=1}^{q} \left(\lambda_k \omega^U_{kj3} \right), \sum_{k=1}^{q} \left(\lambda_k \omega^U_{kj4} \right); \min_k (\eta^U_{kj}) \right) \end{bmatrix}$$

3.4 Construct the weighted matrix

Let $\tilde{\tilde{V}} = \left[\tilde{\tilde{v}}_{ij} \right]_{m \times n}$ be the weighted matrix, then:

$$\tilde{\tilde{v}}_{ij} = \left[(v^L_{ij1}, v^L_{ij2}, v^L_{ij3}, v^L_{ij4}; \varpi^L_{ij}), (v^U_{ij1}, v^U_{ij2}, v^U_{ij3}, v^U_{ij4}; \varpi^U_{ij}) \right] = \tilde{\tilde{x}}_{ij} \otimes \tilde{\tilde{\omega}}_j$$
$$= \begin{bmatrix} (x^L_{ij1}\omega^L_{j1}, x^L_{ij2}\omega^L_{j2}, x^L_{ij3}\omega^L_{j3}, x^L_{ij4}\omega^L_{j4}; \min(w^L_{ij}, \eta^L_j)), \\ (x^U_{ij1}\omega^U_{j1}, x^U_{ij2}\omega^U_{j2}, x^U_{ij3}\omega^U_{j3}, x^U_{ij4}\omega^U_{j4}; \min(w^U_{ij}, \eta^U_j)) \end{bmatrix} \quad (17)$$

3.5 The extended TOPSIS decision making method

(1) Determine the positive ideal solution and the negative ideal solution of the evaluation objects

Suppose that the positive ideal solution and the negative ideal solution are $\tilde{\tilde{V}}^+ = \left[\tilde{\tilde{v}}^+_j \right]_{1 \times n}, \tilde{\tilde{V}}^- = \left[\tilde{\tilde{v}}^-_j \right]_{1 \times n}$, then :

$$\tilde{\tilde{v}}^+_j = \left[(v^{L+}_{j1}, v^{L+}_{j2}, v^{L+}_{j3}, v^{L+}_{j4}; \varpi^{L+}_j), (v^{U+}_{j1}, v^{U+}_{j2}, v^{U+}_{j3}, v^{U+}_{j4}; \varpi^{U+}_j) \right]$$
$$= \begin{bmatrix} (\max_i (v^L_{ij1}), \max_i (v^L_{ij2}), \max_i (v^L_{ij3}), \max_i (v^L_{ij4}); \max_i (\varpi^L_{ij})), \\ (\max_i (v^U_{ij1}), \max_i (v^U_{ij2}), \max_i (v^U_{ij3}), \max_i (v^U_{ij4}); \max_i (\varpi^U_{ij})) \end{bmatrix} \quad (18)$$

$$\tilde{\tilde{v}}_j^- = \left[(v_{j1}^{L-}, v_{j2}^{L-}, v_{j3}^{L-}, v_{j4}^{L-}; \varpi_j^{L-}), (v_{j1}^{U-}, v_{j2}^{U-}, v_{j3}^{U-}, v_{j4}^{U-}; \varpi_j^{U-})\right]$$
$$= \begin{bmatrix} (\min_i(v_{ij1}^L), \min_i(v_{ij2}^L), \min_i(v_{ij3}^L), \min_i(v_{ij4}^L); \min_i(\varpi_{ij}^L)), \\ (\min_i(v_{ij1}^U), \min_i(v_{ij2}^U), \min_i(v_{ij3}^U), \min_i(v_{ij4}^U); \min_i(\varpi_{ij}^U)) \end{bmatrix} \quad (19)$$

(2) Calculate the weighted matrix and the COG of each attributes with respect to the positive ideal solution and the negative ideal solution

Based on the formula (6), we can calculate the COG $\left[(y,x)_v\right]_{4\times5}$ of each element in the weighted matrix and the barycentric coordinates $\left[(y,x)_{V^+}\right]_5$ and $\left[(y,x)_{V^-}\right]_5$ of each element with respect to the positive ideal solution and the negative ideal solution.

(3) Calculate the weighted distance between each project A_i and the ideal solution $\tilde{\tilde{V}}^+$ and negative ideal solution $\tilde{\tilde{V}}^-$:

$$d_i^+ = \sqrt{\sum_{j=1}^{n}\left[d(\tilde{\tilde{v}}_j^+, \tilde{\tilde{v}}_{ij})\right]^2} \quad (20)$$

$$d_i^- = \sqrt{\sum_{j=1}^{n}\left[d(\tilde{\tilde{v}}_j^-, \tilde{\tilde{v}}_{ij})\right]^2} \quad (21)$$

where $i = 1,2\cdots m, j = 1,2,\cdots n$.

(4) Calculate the relative closeness coefficient C_i :

$$C_i = \frac{d_i^-}{d_i^+ + d_i^-} \quad (22)$$

(5) Rank the alternatives

We rank each alternative, based on the relative closeness coefficient. The bigger the relative closeness coefficient is, the better the alternative is, vice versa.

4 Illustrative example

Suppose that a Telecommunication Company intends to choose a manager for R&D department from four volunteers named A1, A2, A3 and A4. The decision making committee assesses the four concerned volunteers based on five attributes: (1) the proficiency in identifying research areas (C1), (2) the proficiency in administration (C2), (3) the personality (C3), (4) the past experience (C4) and (5) the self-confidence (C5). The number of the committee members is three, labeled as DM1, DM2 and DM3, respectively, and the weight vector of the decision makers is $\lambda = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. Each decision maker utilizes the linguistic terms to assess the importance of each attribute, and the evaluation information of four volunteers is shown in Tables2, 3, 4 and 5, respectively [17].

Table 2: The attribute weights given by three DMs.

	c_1	c_2	c_3	c_4	c_5
DM1	VH	H	H	VH	M
DM2	VH	H	MH	H	MH
DM3	VH	MH	MH	VH	M

Table 3: The evaluation information given by DM1.

	c_1	c_2	c_3	c_4	c_5
a_1	VG	VG	VG	VG	VG
a_2	G	VG	VG	VG	MG
a_3	VG	MG	G	G	G
a_4	G	F	F	G	MG

Table 4: The evaluation information given by DM2.

	c_1	c_2	c_3	c_4	c_5
a_1	G	MG	G	G	VG
a_2	G	VG	VG	VG	MG
a_3	G	G	MG	VG	G
a_4	VG	F	MG	F	G

Table 5: The evaluation information given by DM3.

	c_1	c_2	c_3	c_4	c_5
a_1	MG	F	G	VG	VG
a_2	MG	MG	G	MG	G
a_3	VG	VG	VG	VG	MG
a_4	MG	VG	MG	VG	F

The decision making steps are shown as follows:

(1) Convert the linguistic terms into the interval-valued trapezoidal fuzzy numbers, and then get:

$$\left[\tilde{\tilde{\omega}}_{kj}\right]_{3\times 5} = \begin{bmatrix} [(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.0)], \\ [(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.0)], \\ [(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.0)], \\ [(0.72,0.78,0.92,0.97;0.8),(0.72,0.78,0.92,0.97;1.0)], \\ [(0.72,0.78,0.92,0.97;0.8),(0.72,0.78,0.92,0.97;1.0)], \\ [(0.58,0.63,0.80,0.86;0.8),(0.58,0.63,0.80,0.86;1.0)], \\ [(0.72,0.78,0.92,0.97;0.8),(0.72,0.78,0.92,0.97;1.0)], \\ [(0.58,0.63,0.80,0.86;0.8),(0.58,0.63,0.80,0.86;1.0)], \\ [(0.58,0.63,0.80,0.86;0.8),(0.58,0.63,0.80,0.86;1.0)], \\ [(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.0)], \\ [(0.72,0.78,0.92,0.97;0.8),(0.72,0.78,0.92,0.97;1.0)], \\ [(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.0)], \\ [(0.32,0.41,0.58,0.65;0.8),(0.32,0.41,0.58,0.65;1.0)] \\ [(0.58,0.63,0.80,0.86;0.8),(0.58,0.63,0.80,0.86;1.0)] \\ [(0.32,0.41,0.58,0.65;0.8),(0.32,0.41,0.58,0.65;1.0)] \end{bmatrix}$$

$$\left[\tilde{\tilde{a}}_{ij1}\right]_{4\times 5} = \begin{bmatrix} [(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.00)], \\ [(0.72,0.78,0.92,0.97;0.8),(0.72,0.78,0.92,0.97;1.00)], \\ [(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.00)], \\ [(0.72,0.78,0.92,0.97;0.8),(0.72,0.78,0.92,0.97;1.00)], \\ [(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.00)], \\ [(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.00)], \\ [(0.58,0.63,0.80,0.86;0.8),(0.58,0.63,0.80,0.86;1.00)], \\ [(0.32,0.41,0.58,0.65;0.8),(0.32,0.41,0.58,0.65;1.00)], \\ [(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.00)], \\ [(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.00)], \\ [(0.72,0.78,0.92,0.97;0.8),(0.72,0.78,0.92,0.97;1.00)], \\ [(0.32,0.41,0.58,0.65;0.8),(0.32,0.41,0.58,0.65;1.00)], \\ [(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.00)], \\ [(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.00)], \\ [(0.72,0.78,0.92,0.97;0.8),(0.72,0.78,0.92,0.97;1.00)], \\ [(0.72,0.78,0.92,0.97;0.8),(0.72,0.78,0.92,0.97;1.00)], \\ [(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00,1.0)] \\ [(0.58,0.63,0.80,0.86;0.8),(0.58,0.63,0.80,0.86,1.0)] \\ [(0.72,0.78,0.92,0.97;0.8),(0.72,0.78,0.92,0.97,1.0)] \\ [(0.58,0.63,0.80,0.86;0.8),(0.58,0.63,0.80,0.86,1.0)] \end{bmatrix}$$

$$\left[\tilde{\tilde{a}}_{ij2}\right]_{4\times 5} = \begin{bmatrix} [(0.72,0.78,0.92,0.97;0.8),(0.72,0.78,0.92,0.97;1.00)], \\ [(0.72,0.78,0.92,0.97;0.8),(0.72,0.78,0.92,0.97;1.00)], \\ [(0.72,0.78,0.92,0.97;0.8),(0.72,0.78,0.92,0.97;1.00)], \\ [(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.00)], \\ [(0.58,0.63,0.80,0.86;0.8),(0.58,0.63,0.80,0.86;1.00)], \\ [(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.00)], \\ [(0.72,0.78,0.92,0.97;0.8),(0.72,0.78,0.92,0.97;1.00)], \\ [(0.32,0.41,0.58,0.65;0.8),(0.32,0.41,0.58,0.65;1.00)], \\ [(0.72,0.78,0.92,0.97;0.8),(0.72,0.78,0.92,0.97;1.00)], \\ [(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.00)], \\ [(0.58,0.63,0.80,0.86;0.8),(0.58,0.63,0.80,0.86;1.00)], \\ [(0.58,0.63,0.80,0.86;0.8),(0.58,0.63,0.80,0.86;1.00)], \\ [(0.72,0.78,0.92,0.97;0.8),(0.72,0.78,0.92,0.97;1.00)], \\ [(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.00)], \\ [(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.00)], \\ [(0.32,0.41,0.58,0.65;0.8),(0.32,0.41,0.58,0.65;1.00)], \\ [(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00,1.0)] \\ [(0.58,0.63,0.80,0.86;0.8),(0.58,0.63,0.80,0.86,1.0)] \\ [(0.72,0.78,0.92,0.97;0.8),(0.72,0.78,0.92,0.97,1.0)] \\ [(0.72,0.78,0.92,0.97;0.8),(0.72,0.78,0.92,0.97,1.0)] \end{bmatrix}$$

$$\left[\tilde{\tilde{a}}_{ij3}\right]_{4\times 5} = \left[\begin{array}{l}
[(0.58,0.63,0.80,0.86;0.8),(0.58,0.63,0.80,0.86;1.00)], \\
[(0.58,0.63,0.80,0.86;0.8),(0.58,0.63,0.80,0.86;1.00)], \\
[(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.00)], \\
[(0.58,0.63,0.80,0.86;0.8),(0.58,0.63,0.80,0.86;1.00)], \\
[(0.32,0.41,0.58,0.65;0.8),(0.32,0.41,0.58,0.65;1.00)], \\
[(0.58,0.63,0.80,0.86;0.8),(0.58,0.63,0.80,0.86;1.00)], \\
[(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.00)], \\
[(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.00)], \\
[(0.72,0.78,0.92,0.97;0.8),(0.72,0.78,0.92,0.97;1.00)], \\
[(0.72,0.78,0.92,0.97;0.8),(0.72,0.78,0.92,0.97;1.00)], \\
[(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.00)], \\
[(0.58,0.63,0.80,0.86;0.8),(0.58,0.63,0.80,0.86;1.00)], \\
[(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.00)], \\
[(0.58,0.63,0.80,0.86;0.8),(0.58,0.63,0.80,0.86;1.00)], \\
[(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.00)], \\
[(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00;1.00)], \\
[(0.93,0.98,1.00,1.00;0.8),(0.93,0.98,1.00,1.00,1.0)] \\
[(0.72,0.78,0.92,0.97;0.8),(0.72,0.78,0.92,0.97,1.0)] \\
[(0.58,0.63,0.80,0.86;0.8),(0.58,0.63,0.80,0.86,1.0)] \\
[(0.32,0.41,0.58,0.65;0.8),(0.32,0.41,0.58,0.65,1.0)]
\end{array}\right]$$

(2) Combine the individual preferences in order to obtain a collective preference value of each alternative:

$$\left[\tilde{\tilde{x}}_{ij}\right]_{4\times 5} = \left[\begin{array}{l}
[(0.743,0.797,0.907,0.943;0.800),(0.743,0.797,0.907,0.943;1.000)], \\
[(0.673,0.730,0.880,0.933;0.800),(0.673,0.730,0.880,0.933;1.000)], \\
[(0.860,0.913,0.973,0.990;0.800),(0.860,0.913,0.973,0.990;1.000)], \\
[(0.743,0.797,0.907,0.943;0.800),(0.743,0.797,0.907,0.943;1.000)], \\
[(0.610,0.673,0.793,0.837;0.800),(0.610,0.673,0.793,0.837;1.000)], \\
[(0.813,0.863,0.933,0.953;0.800),(0.813,0.863,0.933,0.953;1.000)], \\
[(0.743,0.797,0.907,0.943;0.800),(0.743,0.797,0.907,0.943;1.000)], \\
[(0.523,0.600,0.720,0.767;0.800),(0.523,0.600,0.720,0.767;1.000)], \\
[(0.790,0.847,0.947,0.980;0.800),(0.790,0.847,0.947,0.980;1.000)], \\
[(0.860,0.913,0.973,0.990;0.800),(0.860,0.913,0.973,0.990;1.000)], \\
[(0.743,0.797,0.907,0.943;0.800),(0.743,0.797,0.907,0.943;1.000)], \\
[(0.493,0.557,0.727,0.790;0.800),(0.493,0.557,0.727,0.790;1.000)], \\
[(0.860,0.913,0.973,0.990;0.800),(0.860,0.913,0.973,0.990;1.000)], \\
[(0.813,0.863,0.933,0.953;0.800),(0.813,0.863,0.933,0.953;1.000)], \\
[(0.860,0.913,0.973,0.990;0.800),(0.860,0.913,0.973,0.990;1.000)], \\
[(0.657,0.723,0.833,0.873;0.800),(0.657,0.723,0.833,0.873;1.000)], \\
[(0.930,0.980,1.000,1.000;0.800),(0.930,0.980,1.000,1.000;1.000)] \\
[(0.627,0.680,0.840,0.897;0.800),(0.627,0.680,0.840,0.897;1.000)] \\
[(0.673,0.730,0.880,0.933;0.800),(0.673,0.730,0.880,0.933;1.000)] \\
[(0.540,0.607,0.767,0.827;0.800),(0.540,0.607,0.767,0.827;1.000)]
\end{array}\right]$$

$$\left[\tilde{\tilde{\omega}}_{j}\right]_{5} = \left[\begin{array}{l}
[(0.930,0.980,1.000,1.000;0.800),(0.930,0.980,1.000,1.000;1.000)], \\
[(0.627,0.680,0.840,0.897;0.800),(0.627,0.680,0.840,0.897;1.000)], \\
[(0.627,0.680,0.840,0.897;0.800),(0.627,0.680,0.840,0.897;1.000)], \\
[(0.930,0.980,1.000,1.000;0.800),(0.930,0.980,1.000,1.000;1.000)], \\
[(0.320,0.410,0.580,0.650;0.800),(0.320,0.410,0.580,0.650;1.000)]
\end{array}\right]$$

(3)Calculate the weighted decision making matrix:

$$
\left[\tilde{\tilde{v}}_{ij}\right]_{4\times5} =
\begin{bmatrix}
[(0.691,0.781,0.907,0.943;0.800),(0.691,0.781,0.907,0.943;1.000)], \\
[(0.626,0.715,0.880,0.933;0.800),(0.626,0.715,0.880,0.933;1.000)], \\
[(0.800,0.895,0.973,0.990;0.800),(0.800,0.895,0.973,0.990;1.000)], \\
[(0.691,0.781,0.907,0.943;0.800),(0.691,0.781,0.907,0.943;1.000)], \\
[(0.382,0.458,0.666,0.750;0.800),(0.382,0.458,0.666,0.750;1.000)], \\
[(0.510,0.587,0.784,0.855;0.800),(0.510,0.587,0.784,0.855;1.000)], \\
[(0.466,0.542,0.762,0.846;0.800),(0.466,0.542,0.762,0.846;1.000)], \\
[(0.328,0.408,0.605,0.687;0.800),(0.328,0.408,0.605,0.687;1.000)], \\
[(0.495,0.576,0.795,0.879;0.800),(0.495,0.576,0.795,0.879;1.000)], \\
[(0.539,0.621,0.818,0.888;0.800),(0.539,0.621,0.818,0.888;1.000)], \\
[(0.466,0.542,0.762,0.846;0.800),(0.466,0.542,0.762,0.846;1.000)], \\
[(0.309,0.379,0.610,0.708;0.800),(0.309,0.379,0.610,0.708;1.000)], \\
[(0.800,0.895,0.973,0.990;0.800),(0.800,0.895,0.973,0.990;1.000)], \\
[(0.756,0.846,0.933,0.953;0.800),(0.756,0.846,0.933,0.953;1.000)], \\
[(0.800,0.895,0.973,0.990;0.800),(0.800,0.895,0.973,0.990;1.000)], \\
[(0.611,0.709,0.833,0.873;0.800),(0.611,0.709,0.833,0.873;1.000)], \\
[(0.298,0.402,0.580,0.650;0.800),(0.298,0.402,0.580,0.650;1.000)] \\
[(0.201,0.279,0.487,0.583;0.800),(0.201,0.279,0.487,0.583;1.000)] \\
[(0.215,0.299,0.510,0.607;0.800),(0.215,0.299,0.510,0.607;1.000)] \\
[(0.173,0.249,0.445,0.537;0.800),(0.173,0.249,0.445,0.537;1.000)]
\end{bmatrix}
$$

(4) Determine the positive ideal solution and the negative ideal solution :

$$
\tilde{\tilde{V}}^{+} = \begin{array}{l}
[[(0.800,0.895,0.973,0.990;0.800),(0.800,0.895,0.973,0.990;1.000)], \\
[(0.510,0.587,0.784,0.855;0.800),(0.510,0.587,0.784,0.855;1.000)], \\
[(0.539,0.621,0.818,0.888;0.800),(0.539,0.621,0.818,0.888;1.000)], \\
[(0.800,0.895,0.973,0.990;0.800),(0.800,0.895,0.973,0.990;1.000)], \\
[(0.298,0.402,0.580,0.650;0.800),(0.298,0.402,0.580,0.650;1.000)]]
\end{array}
$$

$$
\tilde{\tilde{V}}^{-} = \begin{array}{l}
[[(0.626,0.715,0.880,0.933;0.800),(0.626,0.715,0.880,0.933;1.000)], \\
[(0.328,0.408,0.605,0.687;0.800),(0.328,0.408,0.605,0.687;1.000)], \\
[(0.309,0.379,0.610,0.708;0.800),(0.309,0.379,0.610,0.708;1.000)], \\
[(0.611,0.709,0.833,0.873;0.800),(0.611,0.709,0.833,0.873;1.000)], \\
[(0.173,0.249,0.445,0.537;0.800),(0.173,0.249,0.445,0.537;1.000)]]
\end{array}
$$

(5) Calculate the weighted matrix and the COG of each attributes with respect to the positive ideal solution and the negative ideal solution (y, x) :

$$
\left[(y,x)_v\right]_{4\times5} =
\begin{bmatrix}
[(0.3333,0.8283),(0.4166,0.8283)],[(0.3422,0.5645),(0.4278,0.5645)],[(0.3429,0.6863), \\
[(0.3381,0.7873),(0.4227,0.7873)],[(0.3427,0.6837),(0.4284,0.6837)],[(0.3418,0.7159), \\
[(0.3215,0.9107),(0.4019,0.9107)],[(0.3438,0.6540),(0.4298,0.6540)],[(0.3438,0.6540), \\
[(0.3333,0.8283),(0.4166,0.8283)],[(0.3397,0.5071),(0.4246,0.5071)],[(0.3441,0.5026), \\
(0.4287,0.6863)],[(0.3215,0.9107),(0.4019,0.9107)],[(0.3341,0.4809),(0.4176,0.4809)] \\
(0.4273,0.7159)],[(0.3258,0.8691),(0.4072,0.8691)],[(0.3393,0.3880),(0.4242,0.3880)] \\
(0.4298,0.6540)],[(0.3215,0.9107),(0.4019,0.9107)],[(0.3386,0.4084),(0.4233,0.4084)] \\
(0.4301,0.5026)],[(0.3299,0.7540),(0.4123,0.7540)],[(0.3383,0.3515),(0.4229,0.3515)]
\end{bmatrix}
$$

$$
\left[(y,x)_{V^{+}}\right]_5 = \begin{array}{l}
[[(0.3215,0.9107),(0.4019,0.9107),(0.3427,0.6837)],[(0.4284,0.6837),(0.3418,0.7159), \\
(0.4273,0.7159),(0.3215,0.9107)],[(0.4019,0.9107),(0.3341,0.4809),(0.4176,0.4809)]]
\end{array}
$$

$$
\left[(y,x)_{V^{-}}\right]_5 = \begin{array}{l}
[[(0.3381,0.7873),(0.4227,0.7873)],[(0.3397,0.5071),(0.4246,0.5071)],[(0.3441,0.5026), \\
(0.4301,0.5026)],[(0.3299,0.7540),(0.4123,0.7540)],[(0.3383,0.3515),(0.4229,0.3515)]]
\end{array}
$$

(6) Calculate the weighted distance between each project A_i and the ideal solution and negative ideal solution:

$$d^{+} = (0.1050\ 0.1140\ 0.0707\ 0.2500)$$

$$d^{-} = (0.2002\ 0.2136\ 0.2097\ 0.0292)$$

(7) Calculate the relative closeness coefficient:

$$C = (0.6560\ 0.6520\ 0.7479\ 0.1046\ )$$

(8) Rank the alternatives:

Based on relative closeness coefficient, we can rank the order of each alternatives: $a_3 \succ a_1 \succ a_2 \succ a_4$.

(9) Analysis:

In this example, our approach produces the same ranking as the literature [17], which proves the approach presented in this paper is effective. Comparing with the literatures [7-11], all of them utilize the TOPSIS method to deal with the decision making problems under the fuzzy information environment. The method in this paper, however, can deal with the more complex decision making problems under the generalized interval-valued trapezoidal fuzzy information environment. Comparing with the literature [17], in the fuzzy information, this method solves the FMADA problem based on the generalized interval-valued trapezoidal fuzzy information, and the literature [17] solves the FMADA problem based on the interval-valued triangular fuzzy numbers. In decision making method, literature [17] firstly uses the lower limits and the upper limits of the interval-valued triangular fuzzy numbers to calculate the relative closeness coefficient based on the TOPSIS method, then it uses the mean closeness coefficient to rank the order of the alternatives, so this method is not considering the interval-valued triangular fuzzy numbers as a whole; in this study, we proposed the extended TOPSIS based on the definition of the distance and the comparison method between the generalized interval-valued trapezoidal fuzzy numbers. Comparing with the literature [18], both of them are the decision making problems based on the generalized interval-valued trapezoidal fuzzy numbers. The literature [18] ranks the order of the alternatives based on the similarity which is hard to calculate. The method proposed in this paper, however, is easy to calculate the similarity.

5 Conclusion

Fuzzy multiple attribute decision making (FMADM) is wildly used in various areas. The interval-valued trapezoidal fuzzy numbers can be precisely express the attribute values and weights of the decision making process. This study proposes an extended TOPSIS method for solving the MADM problems which the attribute weights and values are given with the form of GIVTFN and the decision making steps. This method is simple and easy to understand. This method constantly enriches and develops the theory and method of FMADM, and it proposes a new idea for solving the FMADM problems.

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