



## 5 On Emergent SUSY Gauge Theories

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**Abstract.** We present the basic features of emergent SUSY gauge theories where an emergence of gauge bosons as massless vector Nambu-Goldstone modes is triggered by the spontaneously broken supersymmetry rather than the physically manifested Lorentz violation. We start considering the supersymmetric QED model extended by an arbitrary polynomial potential of massive vector superfield that induces the spontaneous SUSY violation in the visible sector. As a consequence, a massless photon appears as a companion of a massless photino emerging as a goldstino in the tree approximation, and remains massless due to the simultaneously generated special gauge invariance. This invariance is only restricted by the supplemented vector field constraint invariant under supergauge transformations. Meanwhile, photino being mixed with another goldstino appearing from a spontaneous SUSY violation in the hidden sector largely turns into the light pseudo-goldstino. Such pseudo-goldstionic photinos considered in an extended supersymmetric Standard Model framework are of a special observational interest that, apart from some indication of the QED emergence nature, may appreciably extend the scope of SUSY breaking physics being actively studied in recent years.

**Povzetek.** Predstavim osnovne lastnosti umeritvenih teorij, "emergent supersymmetry", pri katerih postanejo brezmasni vektorski Nambu-Goldstonovi bozoni umeritvena polja, sproži pa njihov nastanek spontano zlomljena supersimetrija in ne kršitev Lorentzove invariance. Najprej predstavim supersimetrični model kvantne elektrodinamike, ki ga posplošim s tem, da dopustim za masivni vektorski superpotencial polinom poljubne stopnje. To polje sproži spontani zlom supersimetrije v opazljivem sektorju. Pojavita se brezmasni foton in njegov spremljevalec, prav tako brezmasni fotino. Fotino, na drevesnem nivoju je to brezmasni goldstino, ostane brezmasen tudi po kvantnih popravkih zaradi posebne spontano nastale umeritvene invariance. To invarianco omejuje samo dopolnjen pogoj na vektorsko polje, ki pa je invarianten na superumeritvene transformacije. Fotino postane lahki psevdo-goldstino, ko tvori superpozicijo s še enim goldstinom, ki se pojavi ob spontani zlomitvi supersimetrije v skritem sektorju. Ti psevdo-goldstonski fotini iz razširjenih supersimetričnih modelov Standardnega modela, so posebej zanimivi za meritve. Te lahko, poleg potrditve, da se kvantna elektrodinamika v teh teorijah pojavi spontano, podprejo supersimetrične teorije, ki so v zadnjih letih zelo popularne.

### 5.1 Introduction

It is long believed that spontaneous Lorentz invariance violation (SLIV) may lead to an emergence of massless Nambu-Goldstone modes [1] which are identified

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with photons and other gauge fields appearing in the Standard Model. This idea [2] supported by a close analogy with the dynamical origin of massless particle excitations for spontaneously broken internal symmetries has gained new impetus [3–7] in recent years.

In this connection, one important thing to notice is that, in contrast to the spontaneous violation of internal symmetries, SLIV seems not to necessarily imply a physical breakdown of Lorentz invariance. Rather, when appearing in a gauge theory framework, this may ultimately result in a noncovariant gauge choice in an otherwise gauge invariant and Lorentz invariant theory. In substance, the SLIV ansatz, due to which the vector field develops a vacuum expectation value (vev)

$$\langle A_\mu(x) \rangle = n_\mu M \quad (5.1)$$

(where  $n_\mu$  is a properly-oriented unit Lorentz vector,  $n^2 = n_\mu n^\mu = \pm 1$ , while  $M$  is the proposed SLIV scale), may itself be treated as a pure gauge transformation with a gauge function linear in coordinates,  $\omega(x) = n_\mu x^\mu M$ . From this viewpoint gauge invariance in QED leads to the conversion of SLIV into gauge degrees of freedom of the massless Goldstonic photon emerged.

A good example for such a kind of the “inactive” SLIV is provided by the nonlinearly realized Lorentz symmetry for underlying vector field  $A_\mu(x)$  through the length-fixing constraint

$$A_\mu A^\mu = n^2 M^2. \quad (5.2)$$

This constraint in the gauge invariant QED framework was first studied by Nambu a long ago [8], and in more detail in recent years [9–13]. The constraint (5.2) is in fact very similar to the constraint appearing in the nonlinear  $\sigma$ -model for pions [14],  $\sigma^2 + \pi^2 = f_\pi^2$ , where  $f_\pi$  is the pion decay constant. Rather than impose by postulate, the constraint (5.2) may be implemented into the standard QED Lagrangian  $L_{\text{QED}}$  through the invariant Lagrange multiplier term

$$L_{\text{tot}} = L_{\text{QED}} - \frac{\lambda}{2} (A_\mu A^\mu - n^2 M^2) \quad (5.3)$$

provided that initial values for all fields (and their momenta) involved are chosen so as to restrict the phase space to values with a vanishing multiplier function  $\lambda(x)$ ,  $\lambda = 0$ <sup>1</sup>.

One way or another, the constraint (5.2) means in essence that the vector field  $A_\mu$  develops the vev (5.1) and Lorentz symmetry  $SO(1, 3)$  breaks down to  $SO(3)$  or  $SO(1, 2)$  depending on whether the unit vector  $n_\mu$  is time-like ( $n^2 > 0$ ) or space-like ( $n^2 < 0$ ). The point, however, is that, in sharp contrast to the nonlinear  $\sigma$  model for pions, the nonlinear QED theory, due to gauge invariance in the starting Lagrangian  $L_{\text{QED}}$ , ensures that all the physical Lorentz violating effects turn out to be non-observable. Actually, as was shown in the tree [8] and one-loop approximations [9], the nonlinear constraint (5.2) implemented as a supplementary condition appears in essence as a possible gauge choice for the vector field  $A_\mu$ , while the  $S$ -matrix remains unaltered under such a gauge convention. So, as

<sup>1</sup> Otherwise, as was shown in [15] (see also [12]), it might be problematic to have the ghost-free QED model with a positive Hamiltonian.

generally expected, the inactive SLIV inspired by the length-fixing constraint (5.2), while producing an ordinary photon as a true Goldstonic vector boson ( $a_\mu$ )

$$A_\mu = a_\mu + n_\mu(M^2 - n^2 a^2)^{\frac{1}{2}}, \quad n_\mu a_\mu = 0 \quad (a^2 \equiv a_\mu a^\mu), \quad (5.4)$$

leaves physical Lorentz invariance intact<sup>2</sup>. Later similar result was also confirmed for spontaneously broken massive QED [10], non-Abelian theories [11] and tensor field gravity [13].

From this point of view, emergent gauge theories induced by the inactive SLIV mechanism are in fact indistinguishable from conventional gauge theories. Their Goldstonic nature could only be seen when taking the gauge condition of the length-fixing constraint type (5.2). Any other gauge, e.g. Coulomb gauge, is not in line with Goldstonic picture, since it breaks Lorentz invariance in an explicit rather than spontaneous way. As to an observational evidence in favor of emergent theories the only way for inactive SLIV to cause physical Lorentz violation would be if gauge invariance in these theories appeared slightly broken in an explicit, rather than spontaneous, way. Actually, such a gauge symmetry breaking, induced by some high-order operators, leads in the presence of SLIV to deformed dispersion relations for matter and gauge fields involved. This effect typically appears proportional to powers of the ratio  $M/M_p$ , so that for some high value of the SLIV scale  $M$  it may become physically observable even at low energies. Though one could speculate about some generically broken or partial gauge symmetry [16], this seems to be too high price for an actual Lorentz violation which may stem from SLIV<sup>3</sup>. And, what is more, is there really any strong theoretical reason left for the Lorentz invariance to be physically broken, if the Goldstonic gauge fields are anyway generated through the “safe” inactive SLIV models which recover conventional Lorentz invariance?

Nevertheless, it may turn out that SLIV is not the only reason why massless photons could dynamically appear, if spacetime symmetry is further enlarged. In this connection, special interest may be related to supersymmetry. Actually, as we try to show below, the situation is changed remarkably in the SUSY inspired emergent models which, in contrast to non-SUSY analogues, could naturally have some clear observational evidence. We argue that a generic source for massless photons may be spontaneously broken supersymmetry rather than physically

<sup>2</sup> Indeed, the nonlinear QED contains a plethora of Lorentz and CPT violating couplings when it is expressed in terms of the pure Goldstonic photon modes  $a_\mu$ . However, the contributions of all these couplings to physical processes completely cancel out among themselves.

<sup>3</sup> In this connection, the simplest possibility could be a conventional QED Lagrangian extended by the vector field potential energy terms,  $L = L_{\text{QED}} - \frac{\lambda}{4} (A_\mu A^\mu - n^2 M^2)^2$ , where  $\lambda$  is a coupling constant. This Lagrangian being sometimes referred to as the “bumblebee” model (see [7] and references therein) is in a sense a linear version of the nonlinear QED appearing in the limit  $\lambda \rightarrow \infty$ . Actually, both of models are physically equivalent in the infrared energy domain, where the Higgs mode is considered infinitely massive. However, as we see shortly, whereas the nonlinear QED model successfully matches supersymmetry, the “bumblebee” model cannot be conceptually realized in the SUSY context.

manifested spontaneous Lorentz violation [17]. Towards this end, we consider supersymmetric QED model extended by an arbitrary polynomial potential of massive vector superfield that induces the spontaneous SUSY violation<sup>4</sup>. As a consequence, a massless photon emerges as a companion of a massless photino being Goldstone fermion in the broken SUSY phase in the visible sector (section 2). Remarkably, this masslessness appearing at the tree level is further protected against radiative corrections by the simultaneously generated special gauge invariance. This invariance is only restricted by the supplemented vector field constraint invariant under supergauge transformations (section 3). Meanwhile, photino being mixed with another goldstino appearing from a spontaneous SUSY violation in the hidden sector largely turns into the light pseudo-goldstino whose physics seems to be of special interest (section 4). And finally, we conclude (section 5).

## 5.2 Extended supersymmetric QED

We now consider the supersymmetric QED extended by an arbitrary polynomial potential of a general vector superfield  $V(x, \theta, \bar{\theta})$  which in the standard parametrization [18] has a form

$$V(x, \theta, \bar{\theta}) = C(x) + i\theta\chi - i\bar{\theta}\bar{\chi} + \frac{i}{2}\theta\theta S - \frac{i}{2}\bar{\theta}\bar{\theta}S^* - \theta\sigma^\mu\bar{\theta}A_\mu + i\theta\theta\bar{\theta}\lambda' - i\bar{\theta}\bar{\theta}\theta\lambda' + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D', \quad (5.5)$$

where its vector field component  $A_\mu$  is usually associated with a photon. Note that, apart from the conventional photino field  $\lambda$  and the auxiliary  $D$  field, the superfield (5.5) contains in general the additional degrees of freedom in terms of the dynamical  $C$  and  $\chi$  fields and nondynamical complex scalar field  $S$  (we have used the brief notations,  $\lambda' = \lambda + \frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}$  and  $D' = D + \frac{1}{2}\partial^2 C$  with  $\sigma^\mu = (1, \vec{\sigma})$  and  $\bar{\sigma}^\mu = (1, -\vec{\sigma})$ ). The corresponding SUSY invariant Lagrangian may be written as

$$\mathcal{L} = \mathcal{L}_{\text{SQED}} + \sum_{n=1} b_n V^n|_D \quad (5.6)$$

where terms in this sum ( $b_n$  are some constants) for the vector superfield (5.5) are given through the  $V^n|_D$  expansions into the component fields. It can readily be checked that the first term in this expansion appears to be the known Fayet-Iliopoulos  $D$ -term, while other terms only contain bilinear, trilinear and quadrilinear combination of the superfield components  $A_\mu$ ,  $S$ ,  $\lambda$  and  $\chi$ , respectively<sup>5</sup>. Actually, there appear higher-degree terms for the scalar field component

<sup>4</sup> It is worth noting that all the basic arguments related to the present QED example can be then straightforwardly extended to the Standard Model.

<sup>5</sup> Note that all terms in the sum in (5.6) except Fayet-Iliopoulos  $D$ -term explicitly break gauge invariance which is then recovered for Goldstonic gauge modes. Without loss of generality, we may restrict ourselves to the third degree superfield polynomial in the Lagrangian  $\mathcal{L}$  (5.6) to eventually have a theory with dimensionless coupling constants for component fields. However, for completeness sake, it seems better to proceed with a general case.

$C(x)$  only. Expressing them all in terms of the  $C$  field polynomial

$$P(C) = \sum_{n=1}^n \frac{b_n}{2} C^{n-1}(x) \quad (5.7)$$

and its first three derivatives

$$P'_C \equiv \frac{\partial P}{\partial C}, \quad P''_C \equiv \frac{\partial^2 P}{\partial C^2}, \quad P'''_C \equiv \frac{\partial^3 P}{\partial C^3} \quad (5.8)$$

one has for the whole Lagrangian  $\mathcal{L}$

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + i\lambda\sigma^\mu\partial_\mu\bar{\lambda} + \frac{1}{2} D^2 \\ & + P\left(D + \frac{1}{2}\partial^2 C\right) + P'_C\left(\frac{1}{2}SS^* - \chi\lambda' - \bar{\chi}\bar{\lambda}' - \frac{1}{2}A_\mu A^\mu\right) \\ & + \frac{1}{2}P''_C\left(\frac{i}{2}\overline{\chi\chi}S - \frac{i}{2}\chi\chi S^* - \chi\sigma^\mu\bar{\chi}A_\mu\right) + \frac{1}{8}P'''_C(\chi\chi\overline{\chi\chi}). \end{aligned} \quad (5.9)$$

where, for more clarity, we still omitted matter superfields in the model reserving them for section 4. As one can see, extra degrees of freedom related to the  $C$  and  $\chi$  component fields in a general vector superfield  $V(x, \theta, \bar{\theta})$  appear through the potential terms in (5.9) rather than from the properly constructed supersymmetric field strengths, as is appeared for the vector field  $A_\mu$  and its gaugino companion  $\lambda$ .

Varying the Lagrangian  $\mathcal{L}$  with respect to the  $D$  field we come to

$$D = -P(C) \quad (5.10)$$

that finally gives the following potential energy for the field system considered

$$U(C) = \frac{1}{2}[P(C)]^2. \quad (5.11)$$

The potential (5.11) may lead to the spontaneous SUSY breaking in the visible sector provided that the polynomial  $P$  (5.7) has no real roots, while its first derivative has,

$$P \neq 0, \quad P'_C = 0. \quad (5.12)$$

This requires  $P(C)$  to be an even degree polynomial with properly chosen coefficients  $b_n$  in (5.7) that will force its derivative  $P'_C$  to have at least one root,  $C = C_0$ , in which the potential (5.11) is minimized and supersymmetry is spontaneously broken. As an immediate consequence, that one can readily see from the Lagrangian  $\mathcal{L}$  (5.9), a massless photino  $\lambda$  being Goldstone fermion in the broken SUSY phase make all the other component fields in the superfield  $V(x, \theta, \bar{\theta})$ , including the photon, to also become massless. However, the question then arises whether this masslessness of photon will be stable against radiative corrections since gauge invariance is explicitly broken in the Lagrangian (5.9). We show below that it may be the case if the vector superfield  $V(x, \theta, \bar{\theta})$  would appear to be properly constrained.

### 5.3 Constrained vector superfield

We have seen above that the vector field  $A_\mu$  may only appear with bilinear mass terms in the polynomially extended Lagrangian (5.9). Hence it follows that the ‘‘bumblebee’’ model mentioned above<sup>4</sup> with nontrivial vector field potential containing both a bilinear mass term and a quadrilinear stabilizing term can in no way be realized in the SUSY context. Meanwhile, the nonlinear QED model, as will become clear below, successfully matches supersymmetry.

Let us constrain our vector superfield  $V(x, \theta, \bar{\theta})$  by analogy with constrained vector field in the nonlinear QED model (see (5.3)). This can be done again through the invariant Lagrange multiplier term simply adding it to the above Lagrangian (5.6)

$$\mathcal{L}_{\text{tot}} = \mathcal{L} + \frac{1}{2} \Lambda (V - C_0)^2|_{\text{D}} \quad (5.13)$$

where  $\Lambda(x, \theta, \bar{\theta})$  is some auxiliary vector superfield, while  $C_0$  is the constant background value of the  $C$  field for which potential  $U$  (5.11) has the SUSY breaking minimum (5.12) in the visible sector.

We further find for the Lagrange multiplier term in (5.13) that (denoting  $\tilde{C} \equiv C - C_0$ )

$$\begin{aligned} \Lambda (V - C_0)^2|_{\text{D}} = & C_\lambda \left[ \tilde{C} D' + \left( \frac{1}{2} S S^* - \chi \lambda' - \bar{\chi} \bar{\lambda}' - \frac{1}{2} A_\mu A^\mu \right) \right] \\ & + \chi_\lambda \left[ 2 \tilde{C} \chi' + i(\chi S^* + i \sigma^\mu \bar{\chi} A_\mu) \right] + \bar{\chi}_\lambda \left[ 2 \tilde{C} \bar{\chi}' - i(\bar{\chi} S - i \chi \sigma^\mu A_\mu) \right] \\ & + \frac{1}{2} S_\lambda \left( \tilde{C} S^* + \frac{i}{2} \bar{\chi} \chi \right) + \frac{1}{2} S_\lambda^* \left( \tilde{C} S - \frac{i}{2} \chi \bar{\chi} \right) \\ & + 2 A_\lambda^\mu (\tilde{C} A_\mu - \chi \sigma_\mu \bar{\chi}) + 2 \lambda'_\lambda (\tilde{C} \chi) + 2 \bar{\lambda}'_\lambda (\tilde{C} \bar{\chi}) + \frac{1}{2} D'_\lambda \tilde{C}^2 \end{aligned} \quad (5.14)$$

where

$$C_\lambda, \chi_\lambda, S_\lambda, A_\lambda^\mu, \lambda'_\lambda = \lambda_\lambda + \frac{i}{2} \sigma^\mu \partial_\mu \bar{\chi}_\lambda, D'_\lambda = D_\lambda + \frac{1}{2} \partial^2 C_\lambda \quad (5.15)$$

are the component fields of the Lagrange multiplier superfield  $\Lambda(x, \theta, \bar{\theta})$  in the standard parametrization (5.5). Varying the Lagrangian (5.13) with respect to these fields and properly combining their equations of motion

$$\frac{\partial \mathcal{L}_{\text{tot}}}{\partial (C_\lambda, \chi_\lambda, S_\lambda, A_\lambda^\mu, \lambda_\lambda, D_\lambda)} = 0 \quad (5.16)$$

we find the constraints which put on the  $V$  superfield components

$$C = C_0, \quad \chi = 0, \quad A_\mu A^\mu = S S^*, \quad (5.17)$$

being solely determined by the spontaneous SUSY breaking in the visible sector (5.12)

$$P'_C|_{C=C_0} = 0. \quad (5.18)$$

Again, as before in non-SUSY case (5.3), we only take a solution with initial values for all fields (and their momenta) chosen so as to restrict the phase space to vanishing values of the multiplier component fields (5.15) that will provide a ghost-free theory with a positive Hamiltonian.

Now substituting the constraints (5.17, 5.18) into the total Lagrangian  $\mathcal{L}_{\text{tot}}$  (5.13, 5.9) we eventually come to the basic Lagrangian in the broken SUSY phase

$$\mathcal{L}_{\text{tot}}^{\text{br}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\lambda\sigma^{\mu}\partial_{\mu}\bar{\lambda} + \frac{1}{2}D^2 + P(C_0)D, \quad A_{\mu}A^{\mu} = SS^* \quad (5.19)$$

being supplemented by the vector field constraint, as indicated. So, for the constrained vector superfield,

$$\hat{V}(x, \theta, \bar{\theta}) = C_0 + \frac{i}{2}\theta\theta S - \frac{i}{2}\bar{\theta}\bar{\theta}S^* - \theta\sigma^{\mu}\bar{\theta}A_{\mu} + i\theta\theta\bar{\theta}\lambda - i\bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D, \quad (5.20)$$

we have the almost standard SUSY QED Lagrangian with the same states - photon, photino and an auxiliary scalar D field - in its gauge supermultiplet, while another auxiliary complex scalar field S gets only involved in the vector field constraint. The linear (Fayet-Iliopoulos) D-term with the effective coupling constant  $P(C_0)$  in (5.19) shows that the supersymmetry in the theory is spontaneously broken due to which the D field acquires the vev,  $D = -P(C_0)$ . Taking the nondynamical S field in the constraint (5.17) to be some constant background field (for a more formal discussion, see below) we come to the SLIV constraint (5.2) which we discussed above regarding an ordinary non-supersymmetric QED theory (sec.1). As is seen from this constraint in (5.19), one may only have a time-like SLIV in the SUSY framework but never a space-like one. There also may be a light-like SLIV, if the S field vanishes<sup>6</sup>. So, any possible choice for the S field corresponds to the particular gauge choice for the vector field  $A_{\mu}$  in an otherwise gauge invariant theory. Thus, a massless photon emerging first as a companion of a massless photino (being Goldstone fermion in the broken SUSY phase) remains massless due to this gauge invariance.

We conclude by showing that our extended Lagrangian  $\mathcal{L}_{\text{tot}}$  (5.13, 5.9), underlying the emergent QED model, is SUSY invariant, and also the constraints (5.17) on the field space appearing due to the Lagrange multiplier term in (5.13) are consistent with the supersymmetry. The first part of this assertion is somewhat immediate since the Lagrangian  $\mathcal{L}_{\text{tot}}$ , aside from the standard supersymmetric QED part  $\mathcal{L}_{\text{S QED}}$  (5.6), only contains D-terms of various vector superfield products. They are, by definition, invariant under conventional SUSY transformations [18] which for the component fields (5.5) of a general superfield  $V(x, \theta, \bar{\theta})$  (5.5) are

<sup>6</sup> Indeed, this case, first mentioned in [8], may also mean spontaneous Lorentz violation with a nonzero vev  $\langle A_{\mu} \rangle = (\widetilde{M}, 0, 0, \widetilde{M})$  and Goldstone modes  $A_{1,2}$  and  $(A_0 + A_3)/2 - \widetilde{M}$ . The "effective" Higgs mode  $(A_0 - A_3)/2$  can be then expressed through Goldstone modes so that the light-like condition  $A_{\mu}^2 = 0$  is satisfied.

written as

$$\begin{aligned}
 \delta_\xi C &= i\xi\chi - i\bar{\xi}\bar{\chi}, \quad \delta_\xi\chi = \xi S + \sigma^\mu\bar{\xi}(\partial_\mu C + iA_\mu), \quad \frac{1}{2}\delta_\xi S = \bar{\xi}\bar{\lambda} + \bar{\sigma}_\mu\partial^\mu\chi, \\
 \delta_\xi A_\mu &= \xi\partial_\mu\chi + \bar{\xi}\partial_\mu\bar{\chi} + i\xi\sigma_\mu\bar{\lambda} - i\lambda\sigma_\mu\bar{\xi}, \quad \delta_\xi\lambda = \frac{1}{2}\xi\sigma^\mu\bar{\sigma}^\nu F_{\mu\nu} + \xi D, \\
 \delta_\xi D &= -\xi\sigma^\mu\partial_\mu\bar{\lambda} + \bar{\xi}\sigma^\mu\partial_\mu\lambda.
 \end{aligned} \tag{5.21}$$

However, there may still be left a question whether the supersymmetry remains in force when the constraints (5.17) on the field space are “switched on” thus leading to the final Lagrangian  $\mathcal{L}_{\text{tot}}^{\text{br}}$  (5.19) in the broken SUSY phase with the both dynamical fields  $C$  and  $\chi$  eliminated. This Lagrangian appears similar to the standard supersymmetric QED taken in the Wess-Zumino gauge, except that the supersymmetry is spontaneously broken in our case. In the both cases the photon stress tensor  $F_{\mu\nu}$ , photino  $\lambda$  and nondynamical scalar  $D$  field form an irreducible representation of the supersymmetry algebra (the last two line in (5.21)). Nevertheless, any reduction of component fields in the vector superfield is not consistent in general with the linear superspace version of supersymmetry transformations, whether it be the Wess-Zumino gauge case or our constrained superfield (5.20). Indeed, a general SUSY transformation does not preserve the Wess-Zumino gauge: a vector superfield in this gauge acquires some extra terms when being SUSY transformed. The same occurs with our constrained superfield as well. The point, however, is that in the both cases a total supergauge transformation

$$V \rightarrow V + i(\Omega - \Omega^*), \tag{5.22}$$

where  $\Omega$  is a chiral superfield gauge transformation parameter, can always restore the superfield initial form. Actually, the only difference between these two cases is that whereas the Wess-Zumino supergauge leaves an ordinary gauge freedom untouched, in our case this gauge is unambiguously fixed in terms of the above vector field constraint (5.17). However, this constraint is valid under SUSY transformations provided that the scalar field components  $\varphi$  and  $F$  in the  $\Omega$  are properly chosen. Actually, the non-trivial part of the  $\hat{V}$  superfield transformation which can not be gauged away from the emergent theory (5.19) has the form

$$\hat{V} \rightarrow \hat{V} + i\theta\theta F - i\bar{\theta}\bar{\theta}F^* - 2\theta\sigma^\mu\bar{\theta}\partial_\mu\varphi. \tag{5.23}$$

according to which its vector and scalar field components transform as

$$A_\mu \rightarrow A'_\mu = A_\mu - \partial_\mu(2\varphi), \quad S \rightarrow S' = S + 2F. \tag{5.24}$$

It can be immediately seen that our basic Lagrangian  $\mathcal{L}_{\text{tot}}^{\text{br}}$  (5.19) being gauge invariant and containing no the scalar  $S$  field is automatically invariant under either of these two transformations individually. In contrast, the supplementary vector field constraint (5.17), though it is also turned out to be invariant under supergauge transformations (5.24), but only if they are made jointly. Indeed, for any choice of the scalar  $\varphi$  in (5.24) there can always be found such a scalar  $F$  (and vice versa) that the constraint remains invariant

$$A_\mu A^\mu = SS^* \rightarrow A'_\mu A'^\mu = S'S'^* \tag{5.25}$$



In other words, the vector field constraint is invariant under supergauge transformations (5.24) but not invariant under an ordinary gauge transformation. As a result, in contrast to the Wess-Zumino case, the supergauge fixing in our case will also lead to the ordinary gauge fixing. We will use this supergauge freedom to reduce the  $S$  field to some constant background value and find the final equation for the gauge function  $\varphi(x)$ . So, for the parameter field  $F$  chosen in such a way to have

$$S' = S + 2F = Me^{i\alpha(x)}, \quad (5.26)$$

where  $M$  is some constant mass parameter (and  $\alpha(x)$  is an arbitrary phase), we come in (5.25) to

$$(A_\mu - 2\partial_\mu\varphi)(A^\mu - 2\partial^\mu\varphi) = M^2. \quad (5.27)$$

that is precisely our old SLIV constraint (5.2) being varied by the gauge transformation (5.24). Recall that this constraint, as was thoroughly discussed in Introduction (sec.1), only fixes gauge (to which such a gauge function  $\varphi(x)$  has to satisfy), rather than physically breaks gauge invariance.

To summarize, it was shown that the spontaneous SUSY breaking constraints on the allowed configurations of the physical fields (5.17) in a general polynomially extended Lagrangian (5.13) are entirely consistent with the supersymmetry. In the broken SUSY phase one eventually comes to the standard SUSY QED type Lagrangian (5.19) being supplemented by the vector field constraint invariant under supergauge transformations. One might think that, unlike the gauge invariant linear (Fayet-Iliopoulos) superfield term, the quadratic and higher order superfield terms in the starting Lagrangian (5.13) would seem to break gauge invariance. However, this fear proved groundless. Actually, as was shown above in the section, this breaking amounts to the gauge fixing determined by the nonlinear vector field constraint mentioned above. It is worth noting that this constraint formally follows from the SUSY invariant Lagrange multiplier term in (5.13) for which is required the phase space to be restricted to vanishing values of all the multiplier component fields (5.15). The total vanishing of the multiplier superfield provides the SUSY invariance of such restrictions. Any non-zero multiplier component field left in the Lagrangian would immediately break supersymmetry and, even worse, would eventually lead to ghost modes in the theory and a Hamiltonian unbounded from below.

## 5.4 Spontaneous SUSY breaking in visible and hidden sectors: photino as pseudo-goldstino

Let us now turn to matter superfields which have not yet been included in the model. In their presence the spontaneous SUSY breaking in the visible sector, which fundamentally underlies our approach, might be phenomenologically ruled out by the well-known supertrace sum rule [18] for actual masses of quarks and leptons and their superpartners<sup>7</sup>. However, this sum rule is acceptably relaxed

<sup>7</sup> Note that an inclusion of direct soft mass terms for scalar superpartners in the model would mean in general that the visible SUSY sector is explicitly, rather than spontaneously,

when taking into account large radiative corrections to masses of supersymmetric particles that proposedly stem from the hidden sector. This is just what one may expect in conventional supersymmetric theories with the standard two-sector paradigm, according to which a hidden sector is largely responsible for SUSY breaking, and the visible sector feels this SUSY breaking indirectly via messenger fields [18]. In this way SUSY can indeed be spontaneously broken at the tree level as well that ultimately leads to a double spontaneous SUSY breaking pattern in the model considered.

We may suppose, just for uniformity, only D-term SUSY breaking both in visible and hidden sectors<sup>8</sup>. Properly, our supersymmetric QED model may be further extended by some extra local  $U'(1)$  symmetry which is proposed to be broken at very high energy scale  $M'$  (for some appropriate anomaly mediated scenario, see [19] and references therein). It is natural to think that due to the decoupling theorem all effects of the  $U'(1)$  are suppressed at energies  $E \ll M'$  by powers of  $1/M'$  and only the  $D'$ -term of the corresponding vector superfield  $V'(x, \theta, \bar{\theta})$  remains in essence when going down to low energies. Actually, this term with a proper choice of messenger fields and their couplings naturally provides the  $M_{\text{SUSY}}$  order contributions to masses of scalar superpartners.

As a result, the simplified picture discussed above (in sections 2 and 3) is properly changed: a strictly massless fermion eigenstate, the true goldstino  $\zeta_g$ , should now be some mix of the visible sector photino  $\lambda$  and the hidden sector goldstino  $\lambda'$

$$\zeta_g = \frac{\langle D \rangle \lambda + \langle D' \rangle \lambda'}{\sqrt{\langle D \rangle^2 + \langle D' \rangle^2}}. \quad (5.28)$$

where  $\langle D \rangle$  and  $\langle D' \rangle$  are the corresponding D-component vevs in the visible and hidden sectors, respectively. Another orthogonal combination of them may be referred to as the pseudo-goldstino  $\zeta_{pg}$ ,

$$\zeta_{pg} = \frac{\langle D' \rangle \lambda - \langle D \rangle \lambda'}{\sqrt{\langle D \rangle^2 + \langle D' \rangle^2}}. \quad (5.29)$$

In the supergravity context, the true goldstino  $\zeta_g$  is eaten through the super-Higgs mechanism to form the longitudinal component of the gravitino, while the pseudo-goldstino  $\zeta_{pg}$  gets some mass proportional to the gravitino mass from supergravity effects. Due to large soft masses required to be mediated, one may generally expect that SUSY is much stronger broken in the hidden sector than in the visible one,  $\langle D' \rangle \gg \langle D \rangle$ , that means in turn the pseudo-goldstino  $\zeta_{pg}$  is largely the photino  $\lambda$ ,

$$\zeta_{pg} \simeq \lambda. \quad (5.30)$$

These pseudo-goldstonic photinos seem to be of special observational interest in the model that, apart from some indication of the QED emergence nature, may

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broken that would immediately invalidate the whole idea of the massless photons as the zero Lorentzian modes triggered by the spontaneously broken supersymmetry.

<sup>8</sup> In general, both D- and F-type terms can be simultaneously used in the visible and hidden sectors (usually just F-term SUSY breaking is used in both sectors [18]).

shed light on SUSY breaking physics. The possibility that the supersymmetric Standard Model visible sector might also spontaneously break SUSY thus giving rise to some pseudo-goldstino state was also considered, though in a different context, in [20,21]. Normally, if the visible sector possesses the R-symmetry which is preserved in the course of the mediation, then the pseudo-goldstino mass is protected up to the supergravity effects which violate R-symmetry. As a result, the pseudo-goldstino mass appears proportional to the gravitino mass, and, eventually, the same region of parameter space simultaneously solves both gravitino and pseudo-goldstino overproduction problems in the early universe [21].

Apart from cosmological problems, many other sides of new physics related to pseudo-goldstinos appearing through the multiple SUSY breaking were also studied recently (see [20–22] and references therein). The point, however, is that there have been exclusively used non-vanishing F-terms as the only mechanism of the visible SUSY breaking in models considered. In this connection, our pseudo-goldstonic photinos solely caused by non-vanishing D-terms in the visible SUSY sector may lead to somewhat different observational consequences. One of the most serious differences belongs to Higgs boson decays provided that our QED model is further extended to supersymmetric Standard Model. For the cosmologically safe masses of pseudo-goldstino and gravitino ( $\lesssim 1\text{keV}$ , as typically follows from R-symmetric gauge mediation) these decays are appreciably modified. Actually, the dominant channel becomes the conversion of the Higgs boson (say, the lighter CP-even Higgs boson  $h^0$ ) into a conjugated pair of corresponding pseudo-goldstinos  $\phi_{pg}$  and  $\bar{\phi}_{pg}$  (being superpartners of pseudo-goldstinos  $\zeta_{pg}$  and  $\bar{\zeta}_{pg}$ , respectively),  $h^0 \rightarrow \phi_{pg} + \bar{\phi}_{pg}$ , once it is kinematically allowed. This means that the Higgs boson will dominantly decay invisibly for F-term SUSY breaking in a visible sector [21]. By contrast, for the D-term SUSY breaking case considered here the roles of pseudo-goldstino and pseudo-sgoldstino are just played by photino and photon, respectively, that could make the standard two-photon decay channel of the Higgs boson to be even somewhat enhanced. In the light of recent discovery of the Higgs-like state [23] just through its visible decay modes, the F-term SUSY breaking in the visible sector seems to be disfavored by data, while D-term SUSY breaking is not in trouble with them.

## 5.5 Concluding remarks

It is well known that spontaneous Lorentz violation in general vector field theories may lead to an appearance of massless Nambu-Goldstone modes which are identified with photons and other gauge fields in the Standard Model. Nonetheless, it may turn out that SLIV is not the only reason for emergent massless photons to appear, if spacetime symmetry is further enlarged. In this connection, a special link may be related to supersymmetry that we tried to argue here by the example of supersymmetric QED that can be then straightforwardly extended to the Standard Model.

The main conclusion which has appeared in the SUSY context is that spontaneous Lorentz violation caused by an arbitrary potential of vector superfield  $V(x, \theta, \bar{\theta})$  never goes any further than some noncovariant gauge constraint put on

its vector field component  $A_\mu(x)$  associated with a photon. This allows to think that physical Lorentz invariance is somewhat protected by SUSY, thus only admitting the “condensation” of the gauge degree of freedom in the vector field  $A_\mu$ . The point, however, is that even in this case when SLIV is “inactive” it inevitably leads to the generation of massless photons as vector Nambu-Goldstone modes provided that SUSY itself is spontaneously broken. In this sense, a generic trigger for massless photons to dynamically emerge happens to be spontaneously broken supersymmetry rather than physically manifested Lorentz noninvariance.

To see how this idea may work we considered supersymmetric QED model extended by an arbitrary polynomial potential of a general vector superfield that induces the spontaneous SUSY violation in the visible sector. In the broken SUSY phase one eventually comes to the standard SUSY QED type Lagrangian (5.19) being supplemented by the vector field constraint invariant under supergauge transformations. As result, a massless photon appears as a companion of a massless photino which emerges in fact as the Goldstone fermion state in the tree approximation. However, being mixed with another goldstino appearing from a spontaneous SUSY violation in the hidden sector this state largely turns into the light pseudo-goldstino. Remarkably, the photon masslessness appearing at the tree level is further protected against radiative corrections by the simultaneously generated special gauge invariance. This invariance is only restricted by the nonlinear gauge condition put on vector field values,  $A_\mu A^\mu = |S|^2$ , so that any possible choice for the nondynamical  $S$  field corresponds to the particular gauge choice for the vector field  $A_\mu$  in an otherwise gauge invariant theory. The point, however, is that this nonlinear gauge condition happens at the same time to be the SLIV type constraint which treats in turn the physical photon as the Lorentzian NG mode. So, figuratively speaking, the photon passes through three evolution stages being initially the massive vector field component of a general vector superfield (5.9), then the three-level massless companion of the Goldstonic photino in the broken SUSY stage (5.12) and finally the generically massless state as the emergent Lorentzian mode in the inactive SLIV stage (5.17).

As to pseudo-goldstonic photinos appeared in the model, they seem to be of special observational interest that, apart from some indication of the QED emergence nature, may appreciably extend the scope of SUSY breaking physics being actively discussed in recent years. In contrast to all previous considerations with non-vanishing F-terms as a mechanism of visible SUSY breaking, our pseudo-goldstonic photinos caused by non-vanishing D-terms in the visible SUSY sector will lead to somewhat different observational consequences. These and related points certainly deserve to be explored in greater detail.

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