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Computational problems with the tetraquark X(3872)

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Constituent quark models predict a weak binding of the two heavy mesons D and D^* into tetraquark [1]. In Ref. [1] the wave function was expanded in terms of Gaussians of different sets of Jaccobi coordinates. The structure of this state turned out to be molecular with the mean distance of 2 fm between the two heavy c quarks.

It is tempting to use the same method for the $D + \bar{D}^*$ tetraquarks speculating that the recently discovered X(3872) resonance [2] which lies just above the $D\bar{D}^*$ threshold at 3.872 MeV might be interpreted as the weakly bound state of these two heavy mesons, in analogy to the DD^* molecular state.

However, the situation in the $D\bar{D}^*$ systems is rather different. The calculation poses serious computational difficulties which are a real challenge!

1. *Large isospin violation*
   The channel
   $D^0 + \bar{D}^{*0} = 3871.2$ MeV is **open**, and
   $D^+ + D^{*-} = 3879.3$ MeV is **closed**.
   Therefore an isospin violating term should be added to the interaction. Most model parameters should be refitted to account well for the important isospin multiplets. This would require a careful understanding of isospin multiplets. For a reliable prediction, the effective quark-quark interaction should give a unified description of light and heavy mesons and baryon (at least in ground and some low-lying states). For that reason we used in the study of the DD^* tetraquark the nonrelativistic constituent quark model with the Bhaduri or Grenoble (AL1) gluon exchange interactions since they at least partially satisfy these criteria. However, they do not contain an isospin breaking term and it would be a major effort to refit them for our delicate purpose.
2. *Open channels*
   In addition to $D^0 + \bar{D}^{*0}$ there are also two more open channels:
   $J/\psi + \rho = 3867$ MeV is **open**,
   $J/\psi + \omega = 3879$ MeV is **closed**,
   $J/\psi + \eta = 3644$ MeV is **open**.
   The last one is especially important since this is the channel in which the X(3872) state was discovered [2]. Therefore a coupled channel calculation is needed.

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3. *Low-lying open channel*
   Such a coupled channel calculation would be very difficult since the last open channel is way below the threshold. Therefore the relative wavefunction oscillates strongly and needs a large and reliable basis for expansion. The usual expansion in Gaussians with different width or different position or in harmonic oscillator function would require a large basis and high precision.

A promising approximation seems to be available, to artificially close the low-lying decay channel, assuming that it does not influence much the resonance near the threshold. The simplest way to do this is to restrict the four-body basis to singlet colour meson-singlet colour meson states. However, in this subspace no $\mathtt{color} \cdot \mathtt{color}$ interaction between (anti)quark of the first D meson and the (anti)quark of the second $\bar{D}$ meson is present, thus some additional interaction would be needed to produce the binding between two heavy mesons, e.g. a three-body interaction or an instanton generated interaction between light quarks. To produce binding, these additional interactions should be quite strong or long-ranged. Such modifications of the Bhaduri or AL1 constituent quark model would then require the refitting model parameters. If, on the other hand, the binding of D and $\bar{D}^*$ is a result of a significant admixture of the color octet-octet configuration as in the case of the DD^* tetraquark [1], where the small component of the atomic-like configuration (similar to Λ_b with cc diquark instead of b) produces just enough attractive force between heavy mesons to bind them, then one must solve the Hamiltonian in full color space. Since the total mass of the two mesons is so close to the energy of the newly discovered state in the charmonium spectra this calls for very accurate calculations where also the presence of the $J/\psi + \eta$ open channel must be taken into the account.

Other tricks to artificially close the low-lying decay channel seem risky due to its low energy. A simple variational calculation with partially restricted Hilbert space is likely to give a random resonance energy anywhere in between the $D + \bar{D}^*$ threshold and the asymptotic $J/\psi + \eta$ value, depending how far one goes with the optimisation. In [3] where this four quark system was essentially artificially enclosed inside the harmonic oscillator, they obtained strong binding more than 400 MeV below the D and $\bar{D}^*$ threshold but still almost 200 MeV above the energy of the free $J/\psi + \eta$.

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