Attitudes to Risk and Roulette

Adi Schnytzer
Sara Westreich[1]

Abstract

We present an empirical framework for determining whether or not customers at the roulette wheel are risk averse or risk loving. Thus, we present a summary of the Aumann-Serrano (2007) risk index as generalized to allow for the presence of risk lovers by Schnytzer and Westreich (2010). We show that, for any gamble, whereas riskiness increases for gambles with positive expected return as the amount placed on a given gamble is increased, the opposite is the case for gambles with negative expected return. Since roulette involves binary gambles, we restrict our attention to such gambles exclusively and derive empirically testable hypotheses. In particular, we show that, all other things being equal, for gambles with a negative expected return, riskiness decreases as the size of the contingent payout increases. On the other hand, riskiness increases if the gamble has a positive expected return. We also prove that, for positive return gambles, riskiness increases, *ceteris paribus*, in the variance of the gamble while the reverse is true for gambles with negative expected returns. Finally, we apply these results to the specific gambles involved in American roulette and discuss how we might distinguish between casino visitors who are risk averse and those who are risk loving as well as those who may suffer from gambling addictions of one form or another.

[1] Dr. Adi Schnytzer is a professor at the Departments of Economics and Management, respectively, Bar Ilan University, Israel.

Dr. Sara Westreich is a member of the Interdisciplinary Department of the Social Sciences, Bar-Ilan University

Introduction

Who plays roulette in a casino? Since the expected return to playing is negative, the obvious answer would appear to be risk lovers. But this is not necessarily the case. Thus, a risk averse consumer may decide to set aside a given sum as a conceptual "entrance fee", enter the casino (where there is no entrance fee) and play with his entrance money either until he loses it all or until he decides to leave with money left over or even a profit, whichever occurs first. It has even been suggested by Mobilia (1993) [2], using a rational addiction framework, that such risk averse gamblers may even be addicted. Since Mobilia's model does not involve any explicit considerations of risk, we do not deal with the addiction issue here. In this paper, we present an empirical framework for determining whether or not customers at the roulette wheel are risk averse or risk loving.

We proceed as follows. In section 1, we present a summary of the Aumann-Serrano risk index (Aumann and Serrano (2007), hereafter [AS]), as generalized to allow for the presence of risk lovers by Schnytzer and Westreich (2010) (hereafter [SW]). We show that, for any gamble, whereas riskiness increases for gambles with positive expected return as the amount placed on a given gamble is increased, the opposite is the case for gambles with negative expected return. Since roulette involves binary gambles, we restrict our attention to such gambles exclusively and derive empirically testable hypotheses in section 2. In particular, we show that, all other things being equal, for gambles with a negative

[2] The model is based on Gary Becker and Kevin Murphy (1988). For other applications, see Chaloupka (1988, 1990a, 1990b) and Becker, Grossman, and Murphy (1990).

expected return, riskiness decreases as the size of the contingent payout increases. On the other hand, riskiness increases if the gamble has a positive expected return. We also prove that, for positive return gambles, riskiness increases, *ceteris paribus*, in the variance of the gamble while the reverse is true for gambles with negative expected returns. In section 3, we apply these results to the specific gambles involved in American roulette and discuss how we might distinguish between casino visitors who are risk averse and those who are risk loving as well as those who may suffer from gambling addictions of one form or another.

The Generalized Aumann and Serrano Index of Riskiness

Following [AS] and [SW] we outline the notion of a *generalized index of inherent riskiness,* with no a priori assumptions about attitudes toward risk. A *utility function* is a strictly monotonic twice continuously differentiable function u defined over the entire line. We normalize u so that

$$u(0) = 0 \quad \text{and} \quad u'(0) = 1$$

If u is concave then an agent with a utility function u is risk averse, while if u is convex, then an agent with a utility function u is risk lover.

The following definition is due to Arrow (1965 and 1971) and Pratt (1964):

Definition 1.1 *The coefficient of absolute risk of an agent i with utility function u_i and wealth w is given by:*

$$\rho_i(w) = \rho_i(w, u_i) = -u_i''(w)/u_i'(w)$$

Note $u_i(x)$ is concave in a neighborhood of w if and only if $\rho_i(w) > 0$, while if it is convex if and only if $\rho_i(w) < 0$.

Definition 1.2 *Call i at least risk averse or no more risk loving than j (written $i \trianglerighteq j$) if for all levels w_i and w_j of wealth, j accepts at w_j any gamble that i accepts at w_i. Call i more risk averse or less risk loving than j (written $i \triangleright j$) if $i \trianglerighteq j$ and $j \underline{)}\, i$.*[3]

We have:

Corollary 1.3 *Given agents i and j, then*

$$i \trianglerighteq j \Leftrightarrow r_i(w_i) \geq r_j(w_j)$$

for all w_i and w_j.

Definition 1.4 *An agent is said to have Constant Absolute Risk (CAR) utility function if his normalized utility function $u(x)$ is given by*

$$u_\alpha(x) = \begin{cases} \alpha^{-1}(1-e^{-\alpha x}), & \alpha \neq 0 \\ x & \alpha = 0 \end{cases}$$

If $\alpha > 0$ then the agent is risk-averse with a CARA utility function, while if $\alpha < 0$ then the agent is risk-loving with a CARL - Constant Absolute

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[3] Note that in [AS] the above is defined for risk averse agents only, and is denoted by ``i is at least as risk averse as j ''.

Risk-Loving - utility function . If $\alpha = 0$ then the agent is risk neutral. The notion of ``CAR'' is justified since for any α, the coefficient of absolute risk ρ defined in Def.1.1, satisfies $\rho(w) = \alpha$ for all w, that is, the Arrow-Pratt coefficient is a constant that does not depend on w.

Proposition 1.5 *An agent i has CAR utility function if and only if for any gamble g and any two wealth levels, i either accepts g at both wealth levels, or rejects g at both wealth levels.*

The next theorem appears in [SW] extending the original idea of [AS]. It verifies the existence of the general index for the following class of gambles. A gamble g is *gameable* if it results in possible losses and possible gains. If g has a continuous distribution function, then it is gameable if it is bounded from above and below, that is, its distribution function is truncated.

Theorem 1.6 [AS,SW] *Let g be a gameable gamble and let α be the unique nonzero root of the equation*

$$Ee^{-\alpha g} - 1 = 0$$

Then for any wealth, a person with utility function u_α is indifferent between taking and not taking g. In other words, the CAR utility function u_α satisfies for all x,

$$Eu_\alpha(g + x) = u_\alpha(x).$$

Moreover, α is positive (negative) if and only if Eg is positive (resp. negative).

Definition 1.7 *Given a gamble g, denote the number α obtained in Th.1.6 by the upper limit of taking g.*

The notation upper limit is justified by the following:

Theorem 1.8 *Let* α *be the upper limit of taking a gamble* g*. Then:*

1. If $Eg > 0$ *then all CARL accept* g *and a CARA person with a utility function* u_β *accepts* g *if and only if*

$$0 < \beta < \alpha$$

2. If $Eg < 0$ *then all CARA reject* g *and a CARL person with a utility function* u_β *accepts* g *if and only if*

$$\beta < \alpha < 0$$

3. If $E(g) = 0$ *the all CARA people reject* g *while all CARL people accept* g*.*

We propose here the following general index of inherent riskiness. Given a gamble g and its upper limit α define its index Q(g) by:

$$Q(g) = e^{-\alpha}$$

Theorem 1.8 and the fact that Q is a monotonic decreasing function of α, imply that:

Corollary 1.9 An increase in riskiness corresponds to a decrease in the set of constant risk-attitude agents that will accept the gamble.

Caution: The corollary above does not say that constant risk-attitude agents prefer less risky gambles. It says that they are more likely to accept them.

It is straightforward to check the following properties:

Corollary 1.10 *The generalized index* $Q(g)$ *given in (6) satisfies:*

1. $Q(g) > 0$ for all g.
2. If $Eg > 0$ then $Q(g) < 1$ and if $Eg < 0$ then $Q(g) > 1$.

When $Eg = 0$ then $Q(g) = 1$.

3. $Q(Ng) = Q(g)^{1/N}$. In particular

$$Q(-g) = Q(g)^{-1}$$

Remark 1.11 *Unlike the case of the [AS]- index, homogeneity of degree* 1 *does not hold. However, when* $E(g) > 0$ *then it is replaced by (increasing) monotonicity. This follows since in this case* $Q(g) < 1$, *hence if* $t < 1$ *then* $Q(tg) = (Q(q))^{1/t} < Q(g)$, *while if* $t > 1$ *then* $(Q(q))^{1/t} > Q(g)$. *This is no longer true for gambles with negative positive return. If* $E(g) < 0$ *then* $Q(g) > 1$ *and Q is monotonically decreasing with respect to multiplication by t. This follows by the same argument as above, with the reverse inequalities.*

Put simply, the remark says that, for a risk averse person, the greater the stake the riskier the gamble, whereas for a risk lover the more money invested in a particular gamble, the less the risk! Following Cor. 1.9, consider the suggested index of riskiness as the opposite to the number of constant risk attitude gamblers who will accept it. Now, the intuition for the risk averse person is straight-forward: placing more money in situation of risk is undesirable since the marginal utility of money is falling and this kind of individual wants to sleep at night. So, as the amount at stake rises, the riskiness rises and there are fewer constant risk attitude risk averse gamblers who will accept it.

For the risk lover, on the other hand, the marginal utility of money is rising. Thus, the more money he stands to win, *ceteris paribus*, the better of he is. Besides which, the risk lover gets utility from the adrenalin rush that accompanies gambling. Accordingly, as the amount waged on a given gamble increases, there will be more constant risk attitude risk loving gamblers who will accept it. In other words, the gamble is less risky.

Binary Gambles

In this section we further turn to a discussion of specific properties of the index of inherent risk as it applies to binary gambles. For this case, we prove that our index is a monotonic function of Var(g), which is increasing for gambles with Eg>0 and decreasing otherwise.

Let g be a gamble that results in a gain of M with probability p and a loss of L with probability $q = 1 - p$. We assume M and L are positive real numbers. Note that:

$$Eg = p(M + L) - L \qquad \sigma^2(g) = p(1 - p)(M + L)^2$$

In order to generate the empirically testable hypotheses discussed in the next section, we summarize partial relations between expected utilities, expectations of gambles, chances to win and riskiness. We start with expected utilities of Constant Absolute Risk *(*CAR) utility functions. Consider $Eu_a(g) = Eu_a(L, M, Eg)$ as a function of the independent variables L, M and Eg.

Proposition 2.1 *Assume g results in a gain of M with probability p and a loss of L otherwise. Let $u_\alpha(x) = \alpha^{-1}(1-e^{-\alpha x})$, $\alpha \neq 0,$ be a CAR utility function. Then:*

$$\alpha > 0 \text{ implies } \frac{\partial Eu_\alpha}{\partial M} < 0 \text{ and } \alpha < 0 \text{ implies } \frac{\partial Eu_\alpha}{\partial M} > 0.$$

Proof. By (1) we have

$$p = \frac{Eg + L}{M + L} .$$

Hence

$$Eu_\alpha(g) = \alpha^{-1}(1 - pe^{-\alpha M} - (1-p)e^{\alpha L}) = \alpha^{-1}(1 - \frac{Eg+L}{M+L}(e^{-\alpha M} - e^{\alpha L}) - e^{\alpha L})$$

A straightforward computation gives:

$$\frac{\partial Eu_\alpha}{\partial M} = \frac{e^{-\alpha M}\alpha^{-1}p}{L+M}\left(1 + \alpha(L+M) - e^{\alpha(L+M)}\right)$$

We claim that $f(\alpha) = 1 + \alpha(L+M) - e^{\alpha(L+M)}$ is negative for all $\alpha \neq 0$. Indeed,

$$f'(\alpha) = L + M - (L+M)e^{\alpha(L+M)} = (L+M)(1 - e^{\alpha(L+M)})$$

If $\alpha > 0$ then $f'(\alpha) < 0$ while if $\alpha < 0$ then $f'(\alpha) > 0.$
Since $f(0) = f'(0) = 0$, our claim follows.

Since $Eu_\alpha(g) = f(\alpha)$ multiplied by a positive value, the desired result follows. QED

We consider now how Q = Q(g) is related to the other variables . Following Th.1.6 we need to solve $Ee^{-\alpha g}-1=0$. That is:

$$0=pe^{-\alpha M}+qe^{\alpha L}-1$$

The following is quite intuitive.

Proposition 2.2 *Let g be a gamble that results in a gain M with probability p and a loss L otherwise. Consider Q(g) as a function of the independent variables L, M and Eg. Then we have:*

If $Eg<0$ then $\frac{\partial Q(g)}{\partial M}<0$ and if $Eg>0$ then $\frac{\partial Q(g)}{\partial M}>0$. Finally, if $Eg=0$ then $\frac{\partial Q(g)}{\partial M}=0$.

Proof. Assume $M_1<M_2$. Let g_1 be the gamble resulting in M_1 and g_2 resulting in M_2. Let α_1 satisfies $Eu_{\alpha_1}(g_1)=0$. By Th.1.8, if $Eg<0$ then $\alpha_1<0$ and since $M_1<M_2$ it follows by Prop. 2.1 that $Eu_{\alpha_1}(g_1)<Eu_{\alpha_1}(g_2)$. Hence an agent with utility function u_{α_1} accepts g_2. This implies by Th.1.8 that $\alpha_1<\alpha_2$, where $\alpha_2<0$ is the upper limit of taking g_2. Since Q=e$^{-\alpha}$ we have $Q(g_1)>Q(g_2)$ and we are done. When $Eg>0$ then by $\alpha_1>0$, and by Prop. 2.1, $0=Eu_{\alpha_1}(g_1)>Eu_{\alpha_1}(g_2)$. Hence α_1 rejects g_2 and thus $\alpha_2<\alpha_1$ and $Q(g_1)<Q(g_2)$. If $Eg=0$ then Q(g) = 1 and the result follows. QED

For binary gambles, fixing Eg and increasing M, means increasing Vg=Var(g). Thus Prop.2.2 implies that for a given Eg>0, $\frac{\partial Q(g)}{\partial Vg} > 0$ and for a given Eg<0, $\frac{\partial Q(g)}{\partial Vg} < 0.$

Since $\frac{\partial Eu_\alpha}{\partial Q} = \frac{\partial Eu_\alpha}{\partial M}\frac{\partial M}{\partial Q}$ we have by Proposition 2.1 and 2.2 that:

Corollary 2.3 If Eg > 0 then for a risk lover $\frac{\partial Eu_\alpha}{\partial Q} > 0,$ and for a risk averse $\frac{\partial Eu_\alpha}{\partial Q} < 0.$

If Eg < 0 then for a risk lover $\frac{\partial Eu_\alpha}{\partial Q} < 0.$

Roulette

The casino game of roulette is probably the simplest practical example of the inherent risk index. In this case, every possible bet is a binary gamble where the return to a losing bet is always the outlay and both the probability of success and the concomitant payout are known. There is thus no uncertainty here, merely risk. Accordingly, roulette also provides the simplest case for a study of attitudes towards risk of casino gamblers. In the absence of data, we are restricted to proving some potentially interesting empirically testable hypotheses. We hope to be able to test these when/if data are forthcoming.

Table 1 provides complete details for the different kinds of bets available in the American version of the game[4].

Table 1: American Roulette

Bet name	Winning spaces	Payout M	Odds p=1/(odds+1)	Expected value (on a $1 bet) = Eg
0	0	35 to 1	37 to 1	−$0.053
00	00	35 to 1	37 to 1	−$0.053
Straight up	Any single number	35 to 1	37 to 1	−$0.053
Row 00	0, 00	17 to 1	18 to 1	−$0.053
Split	any two adjoining numbers vertical or horizontal	17 to 1	18 to 1	−$0.053
Trio	0, 1, 2 or 00, 2, 3	11 to 1	11.667 to 1	−$0.053
Street	any three numbers horizontal	11 to 1	11.667 to 1	−$0.053
Corner	any four adjoining numbers in a block	8 to 1	8.5 to 1	−$0.053
Six Line	any six numbers from two horizontal rows	5 to 1	5.33 to 1	−$0.053
1st Column	1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34	2 to 1	2.167 to 1	−$0.053
2nd Column	2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35	2 to 1	2.167 to 1	−$0.053
3rd Column	3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36	2 to 1	2.167 to 1	−$0.053
1st Dozen	1 through 12	2 to 1	2.167 to 1	−$0.053
2nd Dozen	13 through 24	2 to 1	2.167 to 1	−$0.053
3rd Dozen	25 through 36	2 to 1	2.167 to 1	−$0.053
Odd	1, 3, 5, ..., 35	1 to 1	1.111 to 1	−$0.053
Even	2, 4, 6, ..., 36	1 to 1	1.111 to 1	−$0.053
Red	1, 3, 5, 7, 9, 12, 14, 16, 18, 19, 21, 23, 25, 27, 30, 32, 34, 36	1 to 1	1.111 to 1	−$0.053
Black	2, 4, 6, 8, 10, 11, 13, 15, 17, 20, 22, 24, 26, 28, 29, 31, 33, 35	1 to 1	1.111 to 1	−$0.053
1 to 18	1, 2, 3, ..., 18	1 to 1	1.111 to 1	−$0.053
19 to 36	19, 20, 21, ..., 36	1 to 1	1.111 to 1	−$0.053
Five Number	0, 00, 1, 2, 3	6 to 1	6.6 to 1	−$0.079

[4] In the European version, the setup of the wheel is slightly different.

The initial bet is returned in addition to the mentioned payout. Note also that 0 and 00 are neither odd nor even in this game.

The crucial questions are: what kinds of gamblers play roulette and can we determine their attitudes to risk based on the kinds of bets they place? Are they all risk-lovers? Or perhaps some of them are people who pay a certain amount of money for fun, this being the amount they are willing to lose when gambling and which they view as an “entrance fee” or some such and then bet as risk-averse gamblers so that any losing bets provide zero utility while winning bets provide positive utility?

Indeed, according to the rational addiction model of Mobilia (1993), as farfetched as it may seem when simple intuition is applied, there may even be risk averse gamblers who are addicted! Thus, a rational risk averse gambler who obtains utility from the act of gambling (as he might from smoking a cigarette) may be shown to be rationally addicted if the quantity of gambling demanded today is a function of gambling in the future. But this requires the very strange assumption that such a gambler obtains actual (as distinct from positive expected) utility from even losing gambles. Finally, it should be stressed that attitude towards risk nowhere comes into the Mobilia model. On the other hand, her utility function adopted permits a far wider interpretation than our own.

Be all of this as it may, it seems clear that in principle there may be both risk lovers and risk averse gamblers to be seen in a casino (and among them will be those who are addicted and those who are not)[5]. Now, since our utility functions are static, we can shed no light on addiction

[5] We are unaware of any formal model explaining gambling addiction for risk lovers, but there seems no reason to rule out such a possibility a priori.

but we can generate some testable hypotheses regarding attitudes to risk.

The two different points of view yield different ways of calculating the index of riskiness. We can either assume that each gamble yields a possible loss of 1 and a possible gain of M. In this case only risk lovers bet. We will denote this gamble by g_1 and calculate Q_1 according to these assumptions.

To allow for risk averse players, let's assume that the gambler is ready to pay \$0.5 for the fun (his entrance fee). Let now g_2 be the gamble where one can either lose 0.5\$ or win M+0.5. From table I, it follows that the expected return for g_2 is:

$$E(g_2)=E(g_1+0.5)=0.447.$$

Let Q_2 be the corresponding index of risk. Note that the two indexes are different, and by the previous section, one is a monotonic decreasing function of M and the other is increasing.

We suggest that data on bets can shed light on gambler type. If most gamblers are risk averse who willingly spend some money on gambling for fun, they will choose the smaller M. If they are "big" risk lovers they will choose the greater M, but if they are "small" risk lovers they can choose other gambles.

Table 2: Two possible calculations for the Risk Index (Q)

Bet name	Payout = M	$Q_1(g)$	$Q_2(g)$
0	35 to 1	1.003065	0.959765
00	35 to 1	1.003065	0.959765
Straight up	35 to 1	1.003065	0.959765
Row 00	17 to 1	1.006318	0.919738
Split	17 to 1	1.006318	0.919738
Trio	11 to 1	1.00978	0.880007
Street	11 to 1	1.00978	0.880007
Corner	8 to 1	1.013457	0.840812
Six Line	5 to 1	1.02138	0.805094
1st Column	2 to 1	1.05467	0.76214
2nd Column	2 to 1	1.05467	0.76214
3rd Column	2 to 1	1.05467	0.76214
1st Dozen	2 to 1	1.05467	0.76214
2nd Dozen	2 to 1	1.05467	0.76214
3rd Dozen	2 to 1	1.05467	0.76214
Odd	1 to 1	1.111	0.538585
Even	1 to 1	1.111	0.538585
Red	1 to 1	1.111	0.538585
Black	1 to 1	1.111	0.538585
1 to 18	1 to 1	1.111	0.538585
19 to 36	1 to 1	1.111	0.538585
Five Number	6 to 1	1.027295	0.33569

Comments:

1. We have by Prop. 2.2, that $Eg < 0 \Rightarrow \frac{\partial Q(g)}{\partial M} < 0$. This is demonstrated in the table in the column of Q_{1}. The case when Eg > 0 is demonstrated in Q_2.

2. Based upon these observations, we would predict that if most players are "big" risk-lovers then more roulette players choose to play 35 to 1 gambles and fewest would chooses even money gambles. Unfortunately, we have no data that would permit us to test this hypothesis formally, but we have been told that the following holds in casinos operated by HIT in Slovenia and elsewhere in Southern Europe.[6] First, less than 5 percent of all gamblers play 2 to 1 or even money gambles. Second, in most instances there are multiple bets on one spin of the wheel. Thus, most of the gamblers choose 17 to 1 or 35 to 1 gambles, but most of the customers will cover, with such bets, approximately 12 of the available numbers (out of 37) on one roulette spin. Finally, following winning bets, gamblers will proceed to cover more numbers in a subsequent bet. There is no observable trend following losing bets.

[6] This information was provided by Igor Rus of HIT.

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