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Hybrid mesons spectroscopy

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Abstract. The hybrids are mesons with constituent gluonic components. The most commonly studied hybrids are composed of a quark, an antiquark and a gluon. These mesons are studied in a variational approach to QCD in the Coulomb gauge. Within the variational approach, a confining linear potential has been shown to emerge from the Dyson-Schwinger equations, at least at the hadronic scale. This potential has been used to first calculate the spectrum of the gluelump, which is an idealized system defined as gluonic excitations bounded to a static, localized color octet source (for example a very heavy quark and antiquark). The next step has been to introduce the quark-antiquark dynamics to calculate the spectrum of heavy hybrid mesons. Our results are in good agreement with the lattice data.

The hybrids are mesons with constituent gluonic components. The most commonly studied hybrids are composed of a quark, an antiquark and a gluon. These mesons are studied in a variational approach to QCD in the Coulomb gauge. This particular gauge has been chosen for its advantages, in particular in the Coulomb gauge the degrees of freedom are physical. This makes the QCD Hamiltonian close in spirit to quantum mechanical models of QCD, for example the constituent quark model, and particularly adapt to calculate the spectrum of particles. In our variational approach we used a variational gaussian vacuum on which the quasiparticle states were built. This variational approach to Coulomb gauge QCD has been developed as a method to introduce effective degrees of freedom (constituent gluons and quarks) in such a way that the connection to QCD is not destroyed. The vacuum expectation value (VEV) of the Coulomb gauge Hamiltonian was calculated and the variational principle applied, resulting in a set of four coupled Dyson-Schwinger equations.

$$\frac{1}{d(\mathbf{k})} = \frac{1}{g} - I_d(\mathbf{k}),\ f(\mathbf{k}) = 1 + I_f(\mathbf{k}),$$
$$\chi(\mathbf{k}) = I_\chi(\mathbf{k}),\ \omega_k^2 = \mathbf{k}^2 + \chi(\mathbf{k})^2 + I_\omega(\mathbf{k}) + I_\omega^0,$$

where $d(\mathbf{k})$ is called the ghost form factor, $f(\mathbf{k})$ is called Coulomb form factor, $\chi(\mathbf{k})$ curvature, ω_k is the gap function and the integral functions I are

$$
\begin{aligned}
I_d(\mathbf{k}) &= \frac{N_C}{2}\int \frac{d^3 q}{(2\pi)^3}(1-(\hat{k}\hat{q})^2)\frac{d(\mathbf{k}-\mathbf{q})}{(\mathbf{k}-\mathbf{q})^2\omega(\mathbf{q})},\\
I_f(\mathbf{k}) &= \frac{N_C}{2}\int \frac{d^3 q}{(2\pi)^3}(1-(\hat{k}\hat{q})^2)\frac{d(\mathbf{k}-\mathbf{q})f(\mathbf{k}-\mathbf{q})}{(\mathbf{k}-\mathbf{q})^2\omega(\mathbf{q})},\\
I_\chi(\mathbf{k}) &= \frac{N_C}{4}\int \frac{d^3 q}{(2\pi)^3}(1-(\hat{k}\hat{q})^2)\frac{d(\mathbf{k}-\mathbf{q})f(\mathbf{q})}{(\mathbf{k}-\mathbf{q})^2},\\
I^0_\omega &= \frac{N_C}{4}g^2\int \frac{d^3 q}{(2\pi)^3}\frac{3-(\hat{k}\hat{q})^2}{\omega(\mathbf{q})},\\
I_\omega(\mathbf{k}) &= \frac{N_C}{4}\int \frac{d^3 q}{(2\pi)^3}[1+(\hat{k}\hat{q})^2]\frac{d(\mathbf{k}-\mathbf{q})^2 f(\mathbf{k}-\mathbf{q})}{(\mathbf{k}-\mathbf{q})^2}\frac{\omega_k^2-[\omega(\mathbf{q})-\chi(\mathbf{q})+\chi(\mathbf{k})]^2}{\omega_k}.
\end{aligned}
$$

Unfortunately the VEVs cannot be calculated exactly, mainly due to the difficulty to deal with the Faddeev-Popov operator ($\nabla_i D_i$), and theoretical approximations, such as the rainbow ladder approximation, had to be adopted. This makes the model an effective one but with still strong connection to QCD. Within the variational approach and with the approximations used, a confining linear potential has been shown to emerge from the Dyson-Schwinger equations, at least at the hadronic scale.

$$V_{CL}(k) = V_C(k) + V_L(k),$$

where

$$V_C(k) = -\frac{4\pi\alpha(k)}{k^2},\quad \alpha(k) = \frac{4\pi Z}{\beta^{\frac{3}{2}}\log^{\frac{3}{2}}\left(\frac{k^2}{\Lambda^2_{QCD}}+c\right)},\quad V_L(k) = -\frac{8\pi k}{k^4}.$$

This potential has been used to first calculate the spectrum of the gluelump [1], which is an idealized system defined as gluonic excitations bounded to a static, localized color octet source (for example a very heavy quark and antiquark). The gluelump states could be classified according to the J^{PC} quantum numbers and we found that the ordering of the various spin-parity states matches those found in lattice computations. The absolute energy scale of these levels is set by the variational parameter. We were able to reproduce the lattice data and in particular the inversion of the gluelump unnatural parity state 1^{+-} below the natural parity state 1^{--} [1], which is typical of the lattice results. The next step has been to introduce the quark-antiquark dynamics to calculate the spectrum of heavy hybrid mesons [2]. The relation between heavy hybrids and gluelumps is very close: low hybrids are expected to be approximately classified by the product of gluelump and $\bar{Q}Q$ quantum numbers. The final and more important result is the good agreement found between our results and the lattice ones. Again we reproduced in particular the inversion of the 1^{--}, 0^{-+}, 1^{-+}, 2^{-+} hybrids (which correspond to the 1^{+-} gluelump) below the 1^{+-}, 0^{++}, 1^{++}, 2^{++} hybrids (which correspond to the 1^{--} gluelump) and we were able to explain the physical reasons of this inversion [2].

References

1. Peng Guo, Adam Szczepaniak, Giuseppe Galatà, Andrea Vassallo and Elena Santopinto, *Phys.Rev. D* **77** (2008) 056005.
2. Peng Guo, Adam Szczepaniak, Giuseppe Galatà, Andrea Vassallo and Elena Santopinto, *Phys.Rev. D* **78** (2008) 056003.