



ISSN 1855-931X

THE EFFECT OF SUBSIDIES ON CONSUMER'S SURPLUS

Caroline Buts and Marc Jegers¹

Abstract

The EU policy with regard to State aid is a regularly debated topic in multiple branches of the academic literature. Interest in the subject is growing and techniques used by the European Commission to assess its effects have evolved over time. However, too little is known about the economic impact of State aid measures. In addition, questions are being raised about the welfare standard used when evaluating aid proposals made by Member States. More specifically, a shift from a focus on producers and markets towards a broader welfare measure, including the effect on consumers, is called for. In response, this paper models the effect of subsidies on consumer's surplus. A duopoly in Bertrand competition is studied. The effect of granting a subsidy to either one or both duopolists is calculated and compared to a situation without aid. In both cases, consumer's surplus increases compared to the situation without aid. In this way, the effect on consumer's surplus can be quantified and should in a case analysis later be compared to effects on producer's surplus and government cost. The quantified effect on consumers can also be seen as a measure for how well a market failure is targeted, i.e. it allows us to measure the usefulness of an aid by looking at the value for the end-user of a product or service.

Key Words: State aid, welfare, consumer's surplus, microeconomics

Topic Groups: microeconomics, industrial organization, politics and business

JEL Classification: D12, D00, D04

1. INTRODUCTION

From time to time, governments intervene in the market in order to stimulate domestic industries or to correct a market failure. Especially within a single market-multi country economy such as the European Union, where goods and services can move freely between countries, this can have serious consequences for firms competing across borders, including wasteful spending when a subsidy race between governments arises. The notion of State aid was already embedded in the founding Treaties of the European Union. In a nutshell, any measure that grants an advantage to an undertaking, is selective, puts a burden on state

¹ Caroline Buts is a research fellow of the Research Foundations Flanders (FWO). Both authors are at the Vrije Universiteit Brussel, department of Applied Economics (APEC). Suggestions made by Gert Huybrechts are gratefully acknowledged.

resources, distorts competition and affects trade between Member States is forbidden, with some exceptions confirming the rule. The system applied by the EU to assess proposed State aid measures has been evolving since the beginning, with other approaches, better evaluation techniques and more transparency. As mentioned by Buelens et al. (2007), this kind of State aid control only exists in the EU and the EFTA. Moreover, from Blauburger (2009) we learn that EU State aid control even goes beyond the mere prohibition originally incorporated in the Treaties. The European Commission has successfully used it as a policy tool to shape or direct State aid policies of Member States by promoting the image of what they consider 'good' aid and at the same time discouraging 'bad' aid.

The extent of the legislative and guiding framework with regard to State aid that is currently in place within the European Union proves that it is considered important to constrain governments in their aid policies towards industry. Some basic rules such as the general prohibition and possible exceptions as well as an understanding of the concept of State aid are clear. However, there is still a substantial part to be studied with regard to the economic impact of State aid and the preferred manner of investigating a notified State aid as becomes clear from the questions raised amongst others by Buelens et al. (2007). Martin and Strasse (2005) make a plea for more attention towards the effects of State aid on consumer's surplus. The move towards the utilization of a social welfare standard in State aid analysis (instead of only looking at an effect on rival undertakings), proposed by Nitsche and Heidhues (2006), emphasises even more the relevance of knowledge about the effect of State aid on consumers, knowledge which is lacking. More generally, "the impact of public support policies remains seriously under-researched and more research is crucially needed to draw firm conclusions" (Buigues and Sekkat, 2011, p.1). With this paper, we try to fit one more piece into the puzzle by contributing to a better understanding of the effects of State aid, looking at changes in consumer's surplus caused by subsidies to firms. The findings are policy relevant, mainly in two ways. First, the effects on consumers can by means of the model be taken into account when evaluating State aid measures by comparing them to effects on producer's surplus and government cost under the condition that the necessary data are available or can be estimated. The model can be adapted to fit specific industries and aid measures. Second, the model studies changes in consumer's surplus implying that it can be used by policy makers and designers of aid measures as a means to calculate the impact of an aid on the end-users of a product or service.

The paper is organized as follows: the next section discusses literature on welfare effects of State aid as well as on competition effects. The third section builds on existing theory to further develop a model explaining the effect of State aid on consumers by looking at changes in consumer's surplus. We briefly discuss situations of monopoly and quasi-perfect competition and then move to a more frequently observed market structure, oligopoly. Here, we compare the situation where no aid is granted to the one where a subsidy is given to one or both firms of a duopoly. The fourth section concludes.

2. LITERATURE

Economic literature covers a variety of aspects with regard to State aid. Studies try to bring clarity over effectiveness, motives and effects on competition and welfare. The topic is studied in light of strategic trade policy, where State aid is considered to be avoided, or it can be seen as a part of competition between governments, to come to an efficient allocation of public resources. Often, aid for one specific objective is studied such as aid for rescue and restructuring, or aid for research and development. This tends to be the case in

effectiveness and efficiency studies. This is understandable as effectiveness is measured differently depending on the objective. Moreover, studies are mostly country and time specific, sometimes resulting in contradictory findings. This paper focuses on the welfare and competition effects of State aid and has a broader scope than just aid for one specific objective. We study one form of aid, i.e. subsidies, but do not specify the objective or industry. The general framework proposed in section 3 can in principle be adapted to fit any kind of subsidy or industry. The literature review below thus focuses on welfare effects of State aid in general. The other and more extensive lines of literature with regard to State aid fall outside the scope of this paper.

2.1. Discussion on the most appropriate welfare standard

Defining welfare effects of State aid requires first of all an agreement on the definition of the concept welfare itself. What exactly should be included in a welfare calculation is frequently a point of discussion. Depending on the policy, welfare analysis can study effects on consumers, but might also include changes in producer's surplus. This exactly pinpoints an important difference between methodologies of State aid control and that of other parts of competition policy. Indeed, in merger and antitrust policies consumer welfare is the primary focus when making a case analysis. State aid policy was designed to avoid wasteful subsidy races and is primarily concerned about competition distortions. Discussion is however ongoing to move towards a broader welfare standard for State aid evaluations. Martin and Strasse (2005) as well as Friederiszick et al. (2006) state that a proper welfare analysis should include effects on consumers. The aid granting authority of course looks at the interest of local producers. The European Commission focuses on the effect on foreign producers. This unilateral focus on producers asks for a countervailing focus on effects on consumers. Nitsche and Heidhues (2006) suggest a social welfare standard that includes also equity effects. Consumer's and producer's surpluses can be compared to the cost of the policy. The benefit of consumers may not always go together with the benefit of producers just as an intended national gain does not necessarily entail beneficial effects at the international scene. From Friederiszick et al. (2006) we learn that State aid can try to increase welfare mainly in two ways. It can target efficiency and aim to shift the welfare frontier outwards, or it can have equity rationales and thus cause shifts along the welfare frontier. A discussion or evaluation on efficiency is often easier than one on redistribution. Nevertheless, both are essential in a complete study of welfare.

2.2. Welfare effects of State aid

As mentioned, welfare studies focus on different types of aid. We try to provide an insightful and structured overview of this literature.

According to Fumagalli (2003), aid to attract foreign direct investment can have welfare increasing effects. This can only be the case when the loss of domestic firms due to increased competition does not outweigh the benefit of the investment that was attracted and the benefit stemming from increased competition itself. It will thus depend on the relative sizes of both effects whether the aid turns out to be beneficial or detrimental to (local) welfare. Chor (2009) also studies aid to attract investment, but differentiates between State aid targeted at variable costs and State aid targeted at fixed costs. He finds that small subsidies are welfare increasing for the host economy. Furthermore, this increase in welfare is larger when the subsidy aims to lower variable costs than when affecting fixed costs. While studying an integrated market, Collie (2000, b) explains that the effect of prohibiting State aid depends on the trade-off between an increased deadweight loss from oligopoly

power and a reduction of deadweight loss due to distortionary taxation (used to finance the aid). As the latter more than compensates the former, a reduction or prohibition of State aid will raise overall welfare. To that end, Collie pleads for a gradual reduction of State aid by setting ceilings and gradually lowering them. In a similar study on the WTO Agreement on Subsidies and Countervailing measures, Collie (2000, a) concludes that global welfare with a prohibition in place is higher compared to the Nash equilibrium where a given level of subsidies exists. Martin and Valbonesi (2008) study market dynamics of integration and find that the number of firms in equilibrium goes down with economic integration. This is due to increased efficiency as more firms compete. Governments then try to help national firms through this process. Subsidising is here an equilibrium outcome, but at the same time reduces welfare in the market concerned.

Bertsch et al. (2010) focus on a more indirect welfare effect of State aid. They show that providing rescue aid on a regular basis, in an industry with negative idiosyncratic shocks, makes it easier for firms to (tacitly) collude. Where this happens, State aid will always have a harmful impact on consumer welfare. Katsoulakos (2005) studies State aid specifically targeted to R&D and lists four components that need to be included when directly assessing welfare within one country: the effect on innovations, the effect on prices and outcome, other efficiency gains and other distortions. In addition, possible welfare effects as a result of a changed level of competition, changes in allocative efficiency and altered R&D performance can also be included. Also, a case study on Airbus by Neven and Seabright (1995) points to welfare losses related to State aid as the consumer benefit created by the entry of Airbus was smaller than the negative effects for the incumbent firm, Boeing.

Literature on effectiveness and efficiency of subsidies or other forms of State aid is closely related to the studies on welfare effects. Often both are discussed.

Next to the direct welfare effects, subsidies can also have indirect welfare effects when the programme turns out to be inefficient. Every public support policy contains an opportunity cost. Resources spent on wasteful subsidies could have been used for other projects that might have a positive impact on welfare. For this reason, it is interesting to study which percentage of subsidies was well spent. However, going into detail on this extensive line of literature would lead us too far. Nevertheless, some general remarks and conclusions are noteworthy. As mentioned above, these kind of efficiency studies are mostly objective-, time-, country-, and industry specific. Results are often contradictory. Overall, it can be concluded that local conditions play an important role in determining the real effect of an aid measure. Nevertheless, Buigues and Sekkat (2011) observe a trend. They distinguish between two types of policies to implement public support programmes: a decentralized policy versus a centralized policy. The former works considerably better than the latter. For support policies to have the best effects, it is thus important to have lower amounts of aid, but a focus on advise, partnerships and sharing of knowledge instead of high subsidies and a central approach. Moreover, a central approach makes it more difficult to have timely controls of the subsidy use in place in order to adjust or stop the projects that are not working.

Comino and Manenti (2005) compare several techniques to promote open source software and conclude that the use of subsidies is always welfare reducing, whereas other options such as mandated adoption may benefit welfare. This touches upon an important evaluation point, i.e. to take into account other options than subsidies. While evaluating a proposed State aid measure, it should always be checked whether there are no alternative policy

actions possible. These can then be compared both on the level of competition distortion and welfare effects.

3. MODELLING THE EFFECTS OF STATE AID

Our focus is on extending the understanding of the effects of State aid on consumer's surplus. The results are useful in the sense proposed by Corley and Gioa (2011): we aim to contribute for further theoretical model building but moreover proposes several practically relevant methods of calculation. As mentioned above, we concentrate on one specific form of State aid, namely subsidies to local firms. The reason for this is that most other forms of intervention can be modelled as a subsidy with equivalent effects. An important distinction has to be made between aid targeted at reducing variable costs and fixed costs reducing aid. The Commission's approach is more in favour to fixed cost aid. This was very obvious in the Ryanair case.² The aid granted for the opening up of new air routes was approved. However, the operational aid granting Ryan air favourable landing rights was prohibited. In fact, we observe most subsidies in the form of a lump sum.

3.1. Subsidies in monopolistic markets and perfectly competitive markets

In case of monopoly, a subsidy to the monopolist will not distort competition as there are no (potential) competitors. There can be an effect when there is a potential entrant. A subsidy to the incumbent firm can refrain the other one from entering as it cannot compete anymore with the subsidised firm. A subsidy to the entrant can bring another player into the market and thus increase competition. Of course, in both cases there can be other effects, such as on consumers and government.

In case of perfect competition, the competition effect depends on the design of the subsidy (Jegers and Buts, 2011). In case of a lump sum to one firm, there will normally not be an effect on the levels of price and output (as the fixed cost term disappears once the derivative of the profit function is taken). Here, the recipient firm will just cash in the subsidy, at least when one confines the analysis to be static. In a dynamic model, one might allow the subsidy to be that large that it enables the firm to invest considerably more than its competitors, in for example, production techniques or quality of the product. When this is the case, the firm will be able to produce more efficiently, will eventually decrease its costs or increase quality and will therefore gain the entire market as rational consumers will only buy from this firm. In case of a subsidy that reduces variable costs going to one of the firms, the effect will be the same: the firm will gain a competitive advantage over its rivals and will be able to monopolise the market (again assuming that consumers are rational).

3.2. Duopoly

We concentrate on the simplest oligopoly, a duopoly. Decisions are taken simultaneously, and are analysed in a Bertrand game. We study three situations. The first one is the base case where no subsidy is given. As a second step, we consider the case where a subsidy is awarded to one of the duopolists. In the third situation, the same subsidies are awarded to both firms.

3.2.1. Situation 1: no subsidy

We study a duopoly consisting of firm i and firm j . Both firms have the same cost function:

² Case C76/2002. Decision published in the Official Journal JOCE L/137/2004.

$$C(q_i) = f + gq_i + hq_i^2$$

They face the following demand functions:

$$q_i = A - Bp_i + \theta Bp_j$$

$$q_j = A - Bp_j + \theta Bp_i$$

$$(0 \leq \theta < 1)$$

As is shown by Jegers and Buts (2011), the Bertrand market equilibrium is reached for the following prices and quantities:

$$p_i = p_j = \frac{(A + (1 - \theta)B(g + 2hA))}{2(1 - \theta)B(1 + h(1 - \theta)B)}$$

and

$$q_i = q_j = \frac{(A - (1 - \theta)Bg)}{2(1 + h(1 - \theta)B)}$$

The consumer's surplus is:

$$CS_i = \frac{\left(\frac{A}{B} + \theta p_j - p_i\right)q_i}{2}$$

After substituting price and quantity and rewriting, we obtain the following consumer's surplus for one firm:

$$CS_i = \frac{(A - Bg(1 - \theta))^2}{8B(1 + hB(1 - \theta))^2}$$

So the total consumer's surplus in the duopoly is:

$$CS = \frac{(A - Bg(1 - \theta))^2}{4B(1 + hB(1 - \theta))^2}$$

3.2.2. Situation 2: subsidy to one firm

The government decides to grant a subsidy to firm i. This subsidy is assumed to change the recipient firm's cost function as follows:

$$C(q_i) = (f - \sigma_f) + (g - \sigma_g)q_i + hq_i^2$$

Firm j does not receive this aid and thus faces the same cost function as before.

Market equilibrium is obviously not affected by the fixed cost subsidy (Jegers and Buts, 2011):

$$p_{i,s} = \frac{\beta(\alpha - \sigma_g) + \alpha\gamma}{\beta^2 - \gamma^2}$$

$$p_{j,s} = \frac{\alpha\beta + \gamma(\alpha - \sigma_g)}{\beta^2 - \gamma^2}$$

(With: $\alpha = \frac{A}{B} + g + 2hA$, $\beta = 2(1 + hB)$ and $\gamma = \theta(1 + 2hB)$)

We calculate aggregate consumer's surplus to compare it with the situation where no aid is granted:

$$\begin{aligned} CS_s &= \frac{\left(\frac{A}{B} + \theta p_{j,s} - p_{i,s}\right)q_{i,s}}{2} + \frac{\left(\frac{A}{B} + \theta p_{i,s} - p_{j,s}\right)q_{j,s}}{2} \\ &= \frac{1}{2B(\beta^2 - \gamma^2)^2} \left(A\beta^2 - A\gamma^2 + \theta B\alpha\beta + \theta B\alpha\gamma - \theta B\gamma\sigma_g - B\alpha\beta \right. \\ &\quad \left. + B\beta\sigma_g - B\alpha\gamma \right)^2 + \frac{1}{2B(\beta^2 - \gamma^2)^2} \left(A\beta^2 - A\gamma^2 + \theta B\alpha\beta \right. \\ &\quad \left. - \theta B\beta\sigma_g + \theta B\alpha\gamma - B\alpha\beta - B\alpha\gamma + B\gamma\sigma_g \right)^2 \end{aligned}$$

Or with $\Lambda = A\beta^2 - A\gamma^2 + \theta B\alpha\beta + \theta B\alpha\gamma - B\alpha\beta - B\alpha\gamma$:

$$CS_s = \frac{(\Lambda - B\sigma_g(\theta\gamma - \beta))^2}{2B(\beta^2 - \gamma^2)^2} + \frac{(\Lambda - B\sigma_g(\theta\beta - \gamma))^2}{2B(\beta^2 - \gamma^2)^2}$$

Compared to the first situation, consumer's surplus has increased. The proof of this can be found in Appendix 2. Granting a variable cost subsidy to one duopolist thus increases consumer welfare judging from the increase of consumer's surplus. Granting a fixed cost subsidy has no effect (in a static sense) on consumer welfare.

3.2.3. Situation 3: same subsidies to both firms

As in situation 2, the subsidy is modelled to alter the cost function. However, at this point, both firms receive the same subsidy:

$$C(q_i) = (f - \sigma_f) + (g - \sigma_g)q_i + hq_i^2$$

$$C(q_j) = (f - \sigma_f) + (g - \sigma_g)q_j + hq_j^2$$

Again, we derive market equilibrium in a Bertrand game and find following prices and outputs:

$$p_{i,s,s} = p_{j,s,s} = \frac{(A + (1 - \theta)B(g - \sigma_g + 2hA))}{2(1 - \theta)B(1 + h(1 - \theta)B)}$$

and

$$q_{i,s,s} = q_{j,s,s} = \frac{1}{2} \frac{A - (1 - \theta)B(g - \sigma_g)}{1 + h(1 - \theta)B}$$

We calculate consumer's surplus for one firm:

$$CS_{i,s,s} = \frac{(A - Bg + B\sigma_g + Bg\theta - B\theta\sigma_g)^2}{8B(1 + hB(1 - \theta))^2}$$

Leading to the total consumer's surplus:

$$CS_{s,s} = \frac{(A - Bg + B\sigma_g + Bg\theta - B\theta\sigma_g)^2}{4B(1 + hB(1 - \theta))^2}$$

Total consumer's surplus is now higher compared to the situation where no subsidies are granted (Appendix 3). Granting the same variable cost subsidy to both duopolists thus increases consumer's surplus.

We can also compare consumer's surplus when aid is granted to both duopolists to consumer's surplus when aid is granted to one duopolist. This relationship depends on the parameters used. Numerical examples can be found both for which $CS_{s,s}$ is larger than CS_s and the other way around.

4. CONCLUSION

To date, there are clear gaps in the understanding of the effects of State aid. We contribute to a better comprehension of the mechanism behind subsidies by modelling its effects from a rather neglected point of view, namely consumers. As mentioned in the introduction, Martin and Strasse (2005) and Nitsche and Heidhues (2006) call for a broadening of the welfare standard used in evaluating State aid cases. More specifically, explicit attention should also be paid to the effects of an aid measure on consumer welfare. In response to this, we model the effects of subsidies, one specific but frequently used form of State aid, on consumer's surplus, contributing on a policy level to the development of more economic analysis when examining proposed State aid measures and adding to the academic literature on the economics of State aid. We compare situations where no aid is granted to those where a subsidy is granted to one firm and where equal subsidies are given to both firms in a duopoly. Consumer's surplus rises in both cases. This means that providing one or both firms of a duopoly with subsidies, improves consumer welfare (keeping to the assumptions of the model). Numerical examples can be used to show that the relation between consumer's surplus when granting State aid to one or both duopolists depends on the parameters used. To get a more complete picture when assessing a proposed State aid measure, we remind that it is of course important to weigh changes in consumer's surplus against government costs and the effects on producers.

Although being very theoretical, this paper is of high practical relevance. Over the past decade, economic analysis has gradually taken a more and more prominent role in State aid control as is clear from, for example, the State Aid Action Plan, the economic framework proposed by Friederiszick et al (2006), recent modernization initiatives and several guidelines that were issued for specific types of aid. Modelling the effects of subsidies on consumer's surplus constitutes one more step towards a broad economic analysis towards including all stakeholders. The model we propose is quite general, but can be fitted to several types of subsidies and industries. It is possible to distinguish between different types of subsidies by changing the impact on the cost function. Specific industries can be modelled by changing the cost and demand functions. This enables a quantification of welfare changes for consumers, which can be used in an analysis next to the effect on producers and government cost. When policy makers do not explicitly wish to include the effect on consumers, the model can be used for quantification of the correction of the targeted market failure. It measures to what extent the subsidy is able to improve the market failure by looking at the impact it had on the end-user of a product or service.

Future work calls for specific and more extended forms of this model to be able to more carefully construct criteria such as the de minimis rule, excluding low amounts of aid from notification. Also, several basic forms of the model could be designed to facilitate the use for those who evaluate cases as including more firms for example significantly complicates calculations. It is clear that this model thus provides a first insight into the effects of State aid on consumers with possibly high practical relevance in the future. However, more steps need to be taken in order to broaden the picture and to one day arrive at a better scientifically founded evaluation system of all State aid measures.

REFERENCES

- Bertsch, C., Calcagno, C. & Le Quement, M. (2010). State aid and tacit collusion, European University Institute, Working paper.
- Blauberger, M. (2009). Of 'good' and 'bad' subsidies: European State aid control through soft and hard law, *West European Politics*, 32: 719-737.
- Buelens, C., Garnier, G. & Meiklejohn, R. (2007). The economic analysis of State aid: Some open questions, European Economy Economic papers, no. 286.
- Buigues, P. A. & Sekkat, K. (2011). Public subsidies to business: an international comparison, *Journal of Industry, Competition and Trade*, 11: 1-24.
- Chor, D. (2009). Subsidies for FDI: Implications from a model with heterogeneous firms, *Journal of International Economics*, 78: 113-125.
- Collie, D. (2000, a). A rationale for the WTO prohibition of export subsidies: strategic export subsidies and world welfare, *Open Economies*, 11: 229-245.
- Collie, D. (2000, b). State aid in the European Union: the prohibition of State aid in an integrated market, *International Journal of Industrial Organization*, 18: 867-884.
- Comino, S. & Manenti, F. (2005). Government policies supporting open source software for the mass market, *Review of Industrial Organization*, 26: 217-240.
- Corley, K. G. & Gioia, D. A. (2011). Building a theory about theory building: What constitutes a theoretical contribution?, *Academy of Management Review*, 36: 12-32.
- Friederiszick, H., Röller, L. H. & Verouden, V. (2006). European State aid control: an economic framework, in Buccrossi P. (ed.), 2008, Handbook of antitrust economics, Cambridge, MIT Press, 625-671.
- Fumagalli, C. (2003). On the welfare effects of competition for foreign direct investments, *European Economic Review*, 47: 963-983.

- Jegers, M. & Buts, C. (2011). State aid and between-country competition in an economic union: a microeconomic analysis in: Dumont, M. & Gayp, G. (Eds.), 2011, *International business, not as usual*, Antwerpen, Garant, 81-94.
- Katsoulakos, Y. (2005). State aid to R&D and competition: an economic assessment methodology. Online: <http://www.intertic.org/Strategic%20Trade%20Papers/Katsoulakos!.pdf> (Consulted: 03/05/2011).
- Martin, S. & Strasse, C. (2005). La politique communautaire des aides d'Etat- est-elle une politique de concurrence? *Concurrences*, 3: 52-59.
- Martin, S. & Valbonesi, P. (2008). Equilibrium State aid in integrating markets, *The BE Journal of Economic Analysis and Policy*, 8: 1-37.
- Neven, D. & Seabright, P. (1995). European industrial policy: the Airbus case, *Economic Policy*, 21: 1-47.
- Nitsche, R. & Heidhues, P. (2006). Study on the methods to analyse the impact of State aid on competition, European Economy Economic papers, no. 244.

APPENDIX 1: COMPARING CONSUMER'S SURPLUS: NO AID VERSUS SUBSIDY TO ONE FIRM)

We compare consumer's surplus in a situation where no aid is granted to consumer's surplus in a situation where a subsidy is given to firm i and not to firm j:

$$CS = \frac{(A - Bg(1 - \theta))^2}{4B(1 + hB(1 - \theta))^2}$$

$$CS_s = \frac{(\Lambda - B\sigma_g(\theta\gamma - \beta))^2}{2B(\beta^2 - \gamma^2)^2} + \frac{(\Lambda - B\sigma_g(\theta\beta - \gamma))^2}{2B(\beta^2 - \gamma^2)^2}$$

with: $\Lambda = A\beta^2 - A\gamma^2 + \theta B\alpha\beta + \theta B\alpha\gamma - B\alpha\beta - B\alpha\gamma$, $\alpha = \frac{A}{B} + g + 2hA$, $\beta = 2(1 + hB)$ and $\gamma = \theta(1 + 2hB)$

The proof that CS_s is larger than CS consists of four steps. First, it is shown that $-B\sigma_g(\theta\gamma - \beta)$ is positive and that $-B\sigma_g(\theta\beta - \gamma)$ is negative. Then, these two terms are verified further to discover that the absolute value of the former is larger than the absolute value of the latter. So, what is added to Λ in the numerator of the first term of CS_s is larger than what is deducted from Λ in the second term of CS_s . Third, we prove that Λ as well as the denominator are positive. This implies that $CS_s > \frac{\Lambda^2}{B(\beta^2 - \gamma^2)^2} = CS_{s, ll}$ where $CS_{s, ll}$ is a lower limit of CS_s for $\sigma_g \rightarrow 0$. Fourth, we compare $CS_{s, ll}$ with CS and establish that $CS_{s, ll} \geq CS$, from which $CS_s > CS$.

Step 1: Proof: $-B\sigma_g(\theta\gamma - \beta) > 0$ and $-B\sigma_g(\theta\beta - \gamma) < 0$
 $\theta\gamma - \beta$ is negative as $\beta > \gamma$ and $0 \leq \theta < 1$. Therefore, $-B\sigma_g(\theta\gamma - \beta)$ is positive.

$\theta\beta - \gamma = \theta$ and is thus positive. Therefore, $-B\sigma_g(\theta\beta - \gamma)$ is negative.

Step 2: Proof: $-B\sigma_g(\theta\gamma - \beta) > B\sigma_g(\theta\beta - \gamma)$

As $\theta\beta - \gamma = \theta$, we have to compare with $-B\sigma_g(\theta\gamma - \beta)$ with $B\sigma_g\theta$.

Substituting the parameters in $-B\sigma_g(\theta\gamma - \beta)$ leads to: $2B\sigma_g - B\sigma_g\theta^2 + 2hB^2\sigma_g - 2hB^2\sigma_g\theta^2$

The difference between the first two terms is positive as well as the difference between the last two terms as $0 \leq \theta < 1$. Thus, if the difference between the first two terms is already larger than $B\sigma_g\theta$, the proof is established.

This is indeed the case: $2B\sigma_g - B\sigma_g\theta^2 = B\sigma_g(2 - \theta^2) > B\sigma_g > B\sigma_g\theta$ as $0 \leq \theta < 1$.

It thus follows that what is added to Λ in the numerator of the first term of CS_s is larger than what is deducted from Λ in the second term of CS_s . The denominator being positive, this implies that $CS_s > \frac{\Lambda^2}{B(\beta^2 - \gamma^2)^2}$ if $\Lambda > 0$, which will be proved in the following step.

Step 3: Proof: $\Lambda > 0$

$$\Lambda = A\beta^2 - A\gamma^2 + \theta B\alpha\beta + \theta B\alpha\gamma - B\alpha\beta - B\alpha\gamma$$

with $\alpha = \frac{A}{B} + g + 2 \cdot h \cdot A$, $\beta = 2 \cdot (1 + h \cdot B)$ and $\gamma = \theta \cdot (1 + 2 \cdot h \cdot B)$

Substituting the parameters leads, after some calculations, to:

$$\Lambda = 2A + 2hAB + A\theta + 2hAB\theta + Bg\theta + Bg\theta^2 + 2hB^2g\theta^2 - 2Bg - 2hB^2g$$

We know that $A - Bg + Bg\theta > 0$ as prices should be larger than the variable cost g , and for all practical purposes result in positive quantities traded.

$$A > Bg - Bg\theta$$

Then as

$$\Lambda > 2(Bg - Bg\theta) + 2h(Bg - Bg\theta)B + (Bg - Bg\theta)\theta + 2h(Bg - Bg\theta)B\theta + Bg\theta + Bg\theta^2 + 2hB^2g\theta^2 - 2Bg - 2hB^2g = 0$$

Step 4: Proof: $CS_{s,II} > C_s$

At last, we have to prove that $\frac{\Lambda^2}{B(\beta^2 - \gamma^2)^2} \geq \frac{(A - Bg(1 - \theta))^2}{4B(1 + hB(1 - \theta))^2}$

In order to compare the two, we substitute Λ , β and γ by their defining expressions to obtain

for the left hand side: $\frac{(A - Bg(1 - \theta))^2}{B(2\theta hB - 2hB + \theta - 2)^2}$

Now we can compare $\frac{(A - Bg(1 - \theta))^2}{4B(1 + hB(1 - \theta))^2}$ and $\frac{(A - Bg(1 - \theta))^2}{B(2\theta hB - 2hB + \theta - 2)^2}$

The numerators are equal. It thus suffices to compare the denominators (clearly positive):

$$4B(1 + hB(1 - \theta))^2 \text{ and } B(2\theta hB - 2hB + \theta - 2)^2$$

After some calculations we arrive at: $4B\theta - B\theta^2 + 4hB^2\theta - 4hB^2\theta^2 \geq 0$, as $0 \leq \theta < 1$

APPENDIX 2: COMPARING CONSUMER'S SURPLUS: NO AID VERSUS THE SAME SUBSIDY TO TWO FIRMS

We compare consumer's surplus in a situation where no aid is granted with consumer's surplus in a situation where an equal subsidy is given to both duopolists:

$$CS = \frac{(A - Bg(1 - \theta))^2}{4B(1 + hB(1 - \theta))^2}$$

$$CS_{s,s} = \frac{(A - Bg + B\sigma_g + Bg\theta - B\theta\sigma_g)^2}{4B(1 + hB(1 - \theta))^2}$$

We will establish that $CS_{s,s} > CS$

Proof:

The difference between these two levels of consumer's surplus stems from differences in the numerator: $B\sigma_g - B\theta\sigma_g$. This difference is positive as $0 \leq \theta < 1$. This means that if $A - Bg + Bg\theta$ is positive, consumer's surplus is higher in the situation a subsidy is given to both duopolists. This is indeed the case (Appendix 1, step3).