

The binary locating-dominating number of some convex polytopes

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Abstract

In this paper the binary locating-dominating number of convex polytopes is considered. The exact value is determined and proved for convex polytopes D_n and R_n'' , while for the convex polytopes R_n , Q_n and U_n a tight upper bound of the locating-dominating number is presented.

Keywords: Locating-dominating number, convex polytopes.

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1 Introduction

Let G be a simple connected undirected graph $G = (V, E)$, where V is a set of vertices, and E is a set of edges. The *open neighborhood* of a vertex $v \in V$ is $N_G(v) = \{u \in V \mid (u, v) \in E\}$ and the *closed neighborhood* is $N_G[v] = \{u \in V \mid (u, v) \in E\} \cup \{v\}$. We write $N(v)$ or $N[v]$ if the graph G is clear from the context [4]. For a graph $G = (V, E)$ a *dominating set* is a vertex set $D \subseteq V$ such that the union of the closed neighborhoods of the vertices in D is all of V ; that is, $\bigcup_{v \in D} N[v] = V$. Equivalently, each vertex not in D is adjacent to at least one vertex in D , e.g. for every vertex $v \in V \setminus D$, $N(v) \cap D \neq \emptyset$. The

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domination number of G , denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G .

The concept of a dominating set can also be studied through assigning a weight of 1 will be assigned to the vertices in D and a weight of 0 to the vertices of $V \setminus D$. In this case, D is a dominating set of G if for every vertex in G the sum of weights for closed neighborhoods is at least 1, i.e. $|N[v] \cap S| \geq 1$ for each $v \in V$. A dominating set $S \subseteq V$ is a *binary locating-dominating set* if for every two different vertices $u, v \in V \setminus S$ holds $N(u) \cap S \neq N(v) \cap S$ ([12]). The *binary locating-dominating number* of G , denoted by $\gamma_{l-d}(G)$, is the minimum cardinality of a binary locating-dominating set. In the sequel all terms about the locating-dominating number or set is denoted by binary locating-dominating number or set.

The article [11] studies the smallest cardinalities of locating-dominating codes on chains and cycles and the extreme values of the cardinality of a minimum r -identifying or r -locating-dominating code in any connected undirected graph G having a given number, n , of vertices is studied in [8]. For more information about these issues, see [3, 9, 14, 15, 21]. The authors of the papers [23, 24, 27] study the single-fault-tolerant locating-dominating sets and an open neighborhood locating-dominating sets in trees. More information on locating-dominating sets can be found in [12, 13, 15, 25].

The identifying code problem and binary locating-dominating problem are NP-hard in a general case [6, 7]: I. Charon et al. proved in [7] that, given a graph G and an integer k , the decision problem of the existence of an r -identifying code, or of an r -locating-dominating code, of size at most k in G , is NP-complete for any r .

The comprehensive list of papers related to identifying code and binary locating-dominating problems were given in [19].

The following theorem gives a tight lower bound of binary locating-dominating number on regular graphs:

Theorem 1.1 (Slater [26]). *If G is a regular graph of degree r , then*

$$\gamma_{l-d}(G) \geq \left\lceil \frac{2 \cdot |V(G)|}{r+3} \right\rceil.$$

Graphs of convex polytopes were introduced by Bača [1]. The classes of convex polytopes Q_n and R_n were introduced in [2]. The metric dimension of convex polytopes D_n , Q_n and R_n are equal to 3, as was proved in [16]. In [17] it was proven that metric dimension of convex polytopes S_n , T_n and U_n is also equal to 3. Minimal doubly resolving sets and the strong metric dimension of convex polytopes D_n and T_n are studied in [18]. M. Salman et al. [22] were considering three similar optimization problems: the fault-tolerant metric dimension problem, the local metric dimension problem and the strong metric dimension problem of two convex polytopes S_n and U_n .

2 A modified integer linear programming formulation

An integer linear programming (ILP) formulation of minimum identifying code problem was given in [5]. If S is an identifying set, then decision variables x_i are defined as:

$$x_i = \begin{cases} 1, & i \in S \\ 0, & i \notin S \end{cases}$$

Then, the ILP formulation of minimum identifying code problem from [5] is presented as follows:

$$\min \sum_{i \in V} x_i \tag{2.1}$$

subject to

$$\sum_{j \in N[i]} x_j \geq 1, \quad i \in V \tag{2.2}$$

$$\sum_{j \in N[i] \nabla N[k]} x_j \geq 1, \quad i, k \in V, i \neq k \tag{2.3}$$

$$x_i \in \{0, 1\}, \quad i \in V \tag{2.4}$$

The objective function (2.1) ensures that the identifying code set has a minimal cardinality, and constraints (2.2) defines S to be a dominating set. Identifying feature is represented by constraints (2.3) while the binary nature of decision variables x_i are given by constraints (2.4).

This formulation can not be directly used for the binary locating-dominating problem. Therefore, it needs to be adapted by changing constraints (2.3) into the constraints (2.5).

$$x_i + x_k + \sum_{j \in N(i) \nabla N(k)} x_j \geq 1, \quad i, k \in V, i \neq k \tag{2.5}$$

Constraints (2.3) and (2.5) are the same when vertices i and k are not neighbors, e.g. $N[i] \nabla N[k] = \{i, j\} \cup (N(i) \nabla N(k))$. The change between (2.3) and (2.5) is reflected only when vertices i and k are neighbors, i.e. $i \in N(k)$. Then, by constraints (2.5), at least one of vertices i, k or some $j \in N(i) \nabla N(k)$ must be in S . When i and k are not neighbors, then $N[i] \nabla N[k] = \{i, j\} \cup (N(i) \nabla N(k))$, so constraints (2.3) and (2.5) are equal.

In [28] it was noted that if $d(u, v) \geq 3$ then u, v has no neighbors in common, therefore, $N(u) \cap S \neq N(v) \cap S$ need not be checked for equivalence. This becomes computationally important for large graphs as it allows us to minimize the number of constraints generated by the locating requirement. Using this idea, constraints (2.5) would be further improved:

$$x_i + x_k + \sum_{j \in N(i) \nabla N(k)} x_j \geq 1, \quad i, k \in V, i \neq k, d(i, k) \leq 2 \tag{2.6}$$

The proposed formulation with a reduced number of constraints can be used to find the exact optimal values for problems of small dimensions. Moreover, as it can be seen from [10], ILP formulation can be tackled by efficient metaheuristic approaches for obtaining suboptimal solutions for large dimensions.

3 The exact values

3.1 Convex polytope D_n

The graph of convex polytope D_n , on Figure 1, was introduced in [16]. It consists of $2n$ 5-sided faces and a pair of n -sided faces. Mathematically, it has vertex set $V(D_n) = \{a_i, b_i, c_i, d_i \mid i = 0, 1, \dots, n - 1\}$ and edge set $E(D_n) = \{(a_i, a_{i+1}), (d_i, d_{i+1}), (a_i, b_i), (b_i, c_i), (c_i, d_i), (b_{i+1}, c_i) \mid i = 0, 1, \dots, n - 1\}$. Note that arithmetic in the subscripts is performed modulo n .

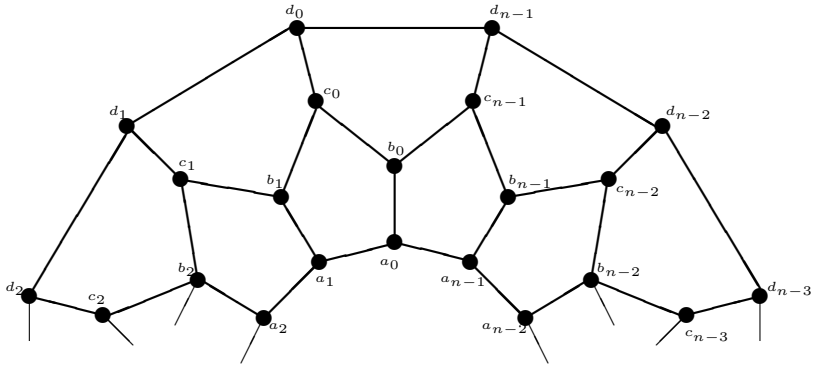


Figure 1: The graph of convex polytope D_n .

Table 1: Locating-dominating vertices in D_n .

n	$v \in V \setminus S$	$S \cap N[v]$	$v \in V \setminus S$	$S \cap N[v]$
$3k$	a_{3i} b_{3i+1} c_{3i} d_{3i}	$\{a_{3i+1}, b_{3i}\}$ $\{a_{3i+1}, c_{3i+1}\}$ $\{b_{3i}\}$ $\{d_{3(i-1)+2}\}$	a_{3i+2} b_{3i+2} c_{3i+2} d_{3i+1}	$\{a_{3i+1}\}$ $\{c_{3i+1}\}$ $\{b_{3(i+1)}, d_{3i+2}\}$ $\{c_{3i+1}, d_{3i+2}\}$
$3k + 1$	a_{3i} b_{3i+1} c_{3i} d_{3i+1} a_{3k} a_0	$\{a_{3(i-1)+2}, b_{3i}\}$ $\{c_{3i+1}\}$ $\{b_{3i}, d_{3i}\}$ $\{c_{3i+1}, d_{3i}\}$ $\{b_{3k}, a_{3(k-1)+2}\}$ $\{b_0\}$	a_{3i+1} b_{3i+2} c_{3i+2} d_{3i+2} c_{3k}	$\{a_{3i+2}\}$ $\{a_{3i+2}, c_{3i+1}\}$ $\{b_{3(i+1)}\}$ $\{d_{3(i+1)}\}$ $\{b_{3k}, d_{3k}\}$
$3k + 2$	a_{3i+1} b_{3i} c_{3i} d_{3i+1} b_{3k} a_{3k+1} d_{3k+1}	$\{a_{3i}, b_{3i+1}\}$ $\{a_{3i}, c_{3(i-1)+2}\}$ $\{b_{3i+1}, d_{3i}\}$ $\{d_{3i}\}$ $\{a_{3k}, c_{3(k-1)+2}\}$ $\{a_{3k}, b_{3k+1}\}$ $\{d_{3k}\}$	a_{3i+2} b_{3i+2} c_{3i+1} d_{3i+2} c_{3k} c_{3k+1} b_0	$\{a_{3(i+1)}\}$ $\{c_{3i+2}\}$ $\{b_{3i+1}\}$ $\{c_{3i+2}, d_{3(i+1)}\}$ $\{b_{3k+1}, d_{3k}\}$ $\{b_{3k+1}\}$ $\{a_0\}$

Theorem 3.1.

$$\gamma_{l-d}(D_n) = \left\lceil \frac{4 \cdot n}{3} \right\rceil.$$

Proof. Firstly, notice that D_n is a regular graph of degree 3, with $4n$ vertices. Then, by Theorem 1.1 it holds $\gamma_{l-d}(D_n) \geq \left\lceil \frac{2 \cdot 4 \cdot n}{3+3} \right\rceil = \left\lceil \frac{4 \cdot n}{3} \right\rceil$.

Let

$$S = \begin{cases} \{a_{3i+1}, b_{3i}, c_{3i+1}, d_{3i+2} \mid i = 0, \dots, k-1\}, & n = 3k \\ \{b_{3k}, d_{3k}\} \cup \{a_{3i+2}, b_{3i}, c_{3i+1}, d_{3i} \mid i = 0, \dots, k-1\}, & n = 3k+1 \\ \{a_{3k}, b_{3k+1}, d_{3k}\} \cup \{a_{3i}, b_{3i+1}, c_{3i+2}, d_{3i} \mid i = 0, \dots, k-1\}, & n = 3k+2 \end{cases}$$

Now, let us prove that S is a locating-dominating set of D_n . In order to do that, we need to consider three possible cases:

Case 1: $n = 3k$. As can be seen from Table 1, neighborhoods of all vertices in $V \setminus S$ and their intersections with set S are non-empty and distinct. Although some formulas for some intersections can be somewhat similar, they are distinct. For example, $S \cap N[a_{3i+2}] = \{a_{3(i+1)}\} \neq \{a_{3i+1}\} = S \cap N[b_{3i+1}]$, since indices $3(i+1) = 3i+3 \neq 3i+1$. Similarly, $S \cap N[d_{3i+1}] = \{c_{3i+2}, d_{3i}\} \neq \{c_{3i+2}, d_{3(i+1)}\} = S \cap N[d_{3i+2}]$;

Case 2: $n = 3k+1$. As in the previous case, once again, all intersections of neighborhoods $N[v]$ with set S , i.e. $S \cap N[v]$, are non-empty and distinct. This also can be seen from Table 1;

Case 3: $n = 3k+2$. As in both previous cases, once again, all intersections of neighborhoods $N[v]$ with set S , i.e. $S \cap N[v]$, are non-empty and distinct, which also can be seen from Table 1. □

3.2 Convex polytope R''_n

The graph of convex polytope R''_n on Figure 2 is introduced in [20]. It has vertex set $V = \{a_i, b_i, c_i, d_i, e_i, f_i \mid i = 0, \dots, n-1\}$ and edge set $E = \{(a_i, a_{i+1}), (a_i, b_i), (b_i, c_i), (b_{i+1}, c_i), (c_i, d_i), (d_i, e_i), (d_{i+1}, e_i), (e_i, f_i), (f_i, f_{i+1}) \mid i = 0, \dots, n-1\}$.

Theorem 3.2.

$$\gamma_{l-d}(R''_n) = 2 \cdot n.$$

Proof. It can be seen that R''_n is a regular graph of degree 3, with $6n$ vertices. Then, by Theorem 1.1 it holds $\gamma_{l-d}(R''_n) \geq \left\lceil \frac{2 \cdot 6 \cdot n}{3+3} \right\rceil = 2 \cdot n$. Now, let us prove that a set $S = \{b_i, e_i \mid i = 0, \dots, n-1\}$ is a binary locating-dominating set of R''_n . Indeed, it is easy to see that all intersections $S \cap N[a_i] = \{b_i\}$; $S \cap N[c_i] = \{b_i, b_{i+1}\}$; $S \cap N[d_i] = \{e_{i-1}, e_i\}$ and $S \cap N[f_i] = \{e_i\}$ are non-empty and distinct. Since S is a binary locating-dominating set of R''_n and $|S| = 2 \cdot n$ therefore, $\gamma_{l-d}(R''_n) \leq 2 \cdot n$. Due to the previously proved fact that $\gamma_{l-d}(R''_n) \geq 2 \cdot n$, it is proven that $\gamma_{l-d}(R''_n)$ is equal to $2 \cdot n$. □

4 The upper bounds

4.1 Convex polytope Q_n

The graph of convex polytope Q_n in Figure 3, is introduced in [2]. It has vertex set $V(Q_n) = \{a_i, b_i, c_i, d_i \mid i = 0, 1, \dots, n-1\}$ and edge set $E(Q_n) = \{(a_i, a_{i+1}), (b_i, b_{i+1}),$

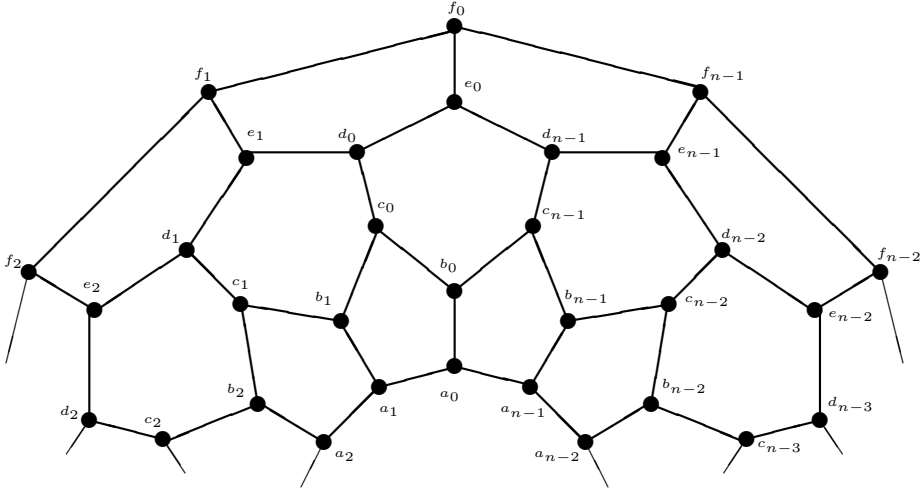


Figure 2: The graph of convex polytope R''_n .

$(d_i, d_{i+1}), (a_i, b_i), (b_i, c_i), (c_i, d_i), (b_{i+1}, c_i) \mid i = 0, 1, \dots, n - 1$. We call the cycle induced by set of vertices $\{a_0, a_1, \dots, a_{n-1}\}$ the inner cycle, the cycle induced by $\{d_0, d_1, \dots, d_{n-1}\}$ the outer cycle, and the middle cycle are induced by set of vertices $\{b_0, b_1, \dots, b_{n-1}\}$. This polytope consists of n 5-sided faces, n 4-sided faces and n triangles.

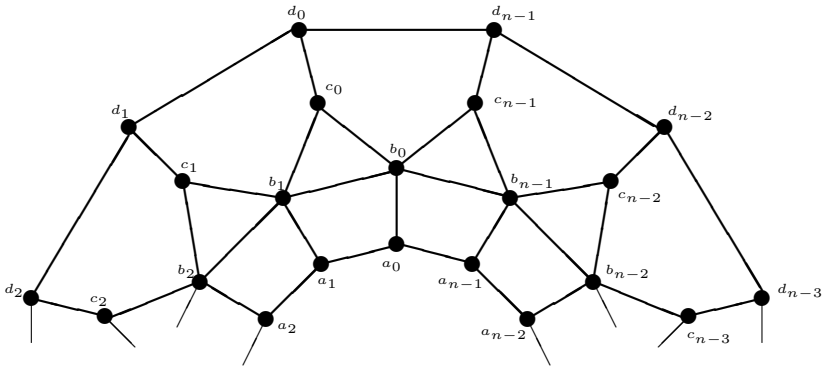


Figure 3: The graph of convex polytope Q_n .

Theorem 4.1.

$$\gamma_{l-d}(Q_n) \leq \left\lceil \frac{4 \cdot n}{3} \right\rceil,$$

and this bound is tight.

Table 2: Additional data for Q_n compared to D_n .

n	$v \in V \setminus S$	$S \cap N[v]$	$v \in V \setminus S$	$S \cap N[v]$
$3k$	b_{3i+1}	$\{a_{3i+1}, b_{3i}, c_{3i+1}\}$	b_{3i+2}	$\{b_{3(i+1)}, c_{3i+1}\}$
$3k + 1$	b_{3i+1}	$\{b_{3i}, c_{3i+1}\}$	b_{3i+2}	$\{a_{3i+2}, b_{3(i+1)}, c_{3i+1}\}$
$3k + 2$	b_{3i} b_{3k}	$\{a_{3i}, b_{3i+1}, c_{3(i-1)+2}\}$ $\{a_{3k}, b_{3k+1}, c_{3(k-1)+2}\}$	b_{3i+2} b_0	$\{b_{3i+1}, c_{3i+2}\}$ $\{a_0, b_1, b_{3k+1}\}$

Proof. Let

$$S = \begin{cases} \{a_{3i+1}, b_{3i}, c_{3i+1}, d_{3i+2} \mid i = 0, \dots, k - 1\}, & n = 3k \\ \{b_{3k}, d_{3k}\} \cup \{a_{3i+2}, b_{3i}, c_{3i+1}, d_{3i} \mid i = 0, \dots, k - 1\}, & n = 3k + 1 \\ \{a_{3k}, b_{3k+1}, d_{3k}\} \cup \{a_{3i}, b_{3i+1}, c_{3i+2}, d_{3i} \mid i = 0, \dots, k - 1\}, & n = 3k + 2 \end{cases}$$

Note that this set is the same as for convex polytopes D_n . This is not a surprise, since convex polytopes Q_n have only n additional edges $(b_i, b_{i+1}), i = 0, \dots, n - 1$ compared to D_n . Therefore, except vertices $b_i, i = 0, \dots, n - 1$, all neighborhoods of vertices in $V \setminus S$ and their intersection with set S are the same as in Table 1. Additional data is presented in Table 2.

As can be seen from Table 3 and additional data from Table 2, in all three cases, neighborhoods of all vertices in $V \setminus S$ and their intersection with set S are non-empty and distinct. Therefore set S is a locating-dominating set for Q_n . Since $|S| = \lceil \frac{4n}{3} \rceil$ therefore, $\gamma_{l-d}(Q_n) \leq \lceil \frac{4n}{3} \rceil$.

Using the CPLEX solver on the integer linear programming formulation (2.1), (2.2), (2.4), and (2.6) we have obtained optimal solutions: $\gamma_{l-d}(Q_5) = 7, \gamma_{l-d}(Q_6) = 8, \gamma_{l-d}(Q_7) = 10, \dots, \gamma_{l-d}(Q_{28}) = 38, \gamma_{l-d}(Q_{29}) = 39$ and $\gamma_{l-d}(Q_{30}) = 40$ which all match the proposed upper bound in this theorem. Therefore, the proposed upper bound is tight. □

4.2 Convex polytope R_n

The graph of convex polytope R_n , on Figure 4, has been introduced in [2]. It has vertex set $V = \{a_i, b_i, c_i \mid i = 0, \dots, n - 1\}$ and edge set $E = \{(a_i, a_{i+1}), (a_i, b_i), (a_{i+1}, b_i), (b_i, b_{i+1}), (b_i, c_i), (c_i, c_{i+1}) \mid i = 0, \dots, n - 1\}$. This graph consists of n 4-sided faces and $2n$ triangles.

Theorem 4.2.

$$\gamma_{l-d}(R_n) \leq n,$$

and this bound is tight.

Proof. Let $S = \{b_i \mid i = 0, \dots, n - 1\}$. It is easy to see that all intersections $S \cap N[a_i] = \{b_{i-1}, b_i\}$ and $S \cap N[c_i] = \{b_i\}$ are non-empty and distinct. Since S is a binary locating-dominating set of R_n and $|S| = n$ therefore, $\gamma_{l-d}(R_n) \leq n$.

Using the CPLEX solver on integer linear programming formulation (2.1), (2.2), (2.4), and (2.6), we have obtained optimal solutions. For $5 \leq n \leq 31, \gamma_{l-d}(R_n) = n$, which match the proposed upper bound in this theorem. Therefore, the proposed upper bound is tight. □

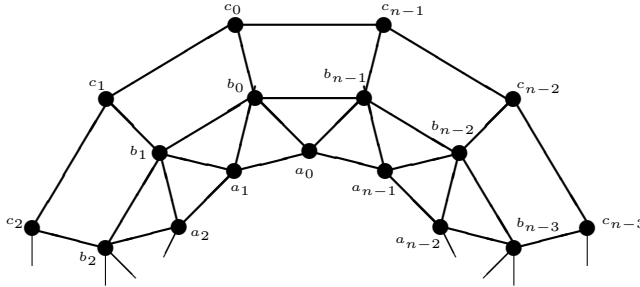


Figure 4: The graph of convex polytope R_n .

4.3 Convex polytope U_n

Mathematically, the graph of convex polytope U_n , on Figure 5, introduced in [17], has vertex set $V = \{a_i, b_i, c_i, d_i, e_i \mid i = 0, \dots, n - 1\}$ and edge set $E = \{(a_i, a_{i+1}), (a_i, b_i), (b_i, b_{i+1}), (b_i, c_i), (c_i, d_i), (c_{i+1}, d_i), (d_i, e_i), (e_i, e_{i+1}) \mid i = 0, \dots, n - 1\}$. This graph in Figure 5 has $2n$ 5-sided faces and n 4-sided faces.

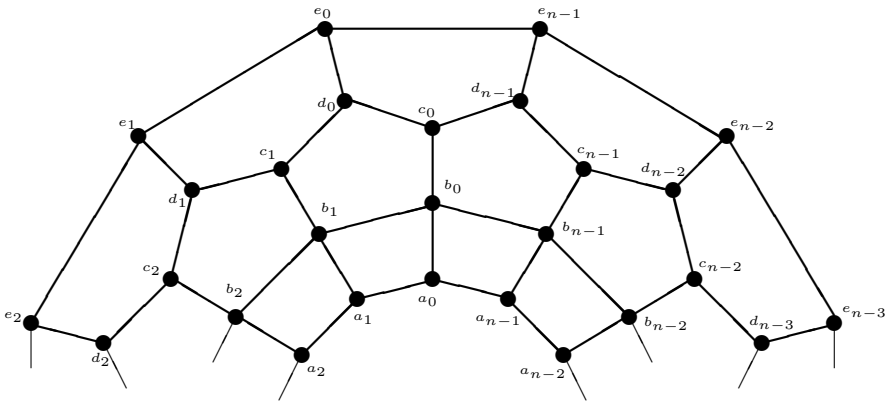


Figure 5: The graph of convex polytope U_n .

Theorem 4.3.

$$\gamma_{l-d}(U_n) \leq \left\lceil \frac{5 \cdot n}{3} \right\rceil,$$

and this bound is tight.

Proof. If $n = 3k$, let $S = \{a_{3i+1}, b_{3i}, c_{3i+1}, d_{3i+2}, e_{3i} \mid i = 0, \dots, k - 1\}$, if $n = 3k + 1$ let $S = \{a_{3k}, c_{3k}, e_{3k}\} \cup \{a_{3i}, b_{3i+2}, c_{3i}, d_{3i+1}, e_{3i} \mid i = 0, \dots, k - 1\}$ and if $n = 3k + 2$ let $S = \{a_{3k+1}, b_{3k}, c_{3k+1}, e_{3k+1}\} \cup \{a_{3i+1}, b_{3i}, c_{3i+1}, d_{3i+2}, e_{3i+1} \mid i = 0, \dots, k - 1\}$.

Now, let us prove that S is a locating-dominating set of U_n . In order to do that, as we did in proofs of previous Theorems, we need to consider three possible cases. As it can be seen from Table 3, in all three cases, neighborhoods of all vertices in $V \setminus S$ and their intersection with set S are non-empty and distinct.

Using the CPLEX solver on the integer linear programming formulation (2.1), (2.2), (2.4), and (2.6) we have obtained optimal solutions: $\gamma_{l-d}(U_5) = 9$, $\gamma_{l-d}(U_6) = 10$, $\gamma_{l-d}(U_7) = 12, \dots, \gamma_{l-d}(U_{22}) = 38$, $\gamma_{l-d}(U_{23}) = 39$ and $\gamma_{l-d}(U_{24}) = 40$ which all match the proposed upper bound in this theorem. Therefore, the proposed upper bound is tight. \square

Table 3: Locating-dominating vertices in U_n .

n	$v \in V \setminus S$	$S \cap N[v]$	$v \in V \setminus S$	$S \cap N[v]$
$3k$	a_{3i}	$\{a_{3i+1}, b_{3i}\}$	a_{3i+2}	$\{a_{3i+1}\}$
	b_{3i+1}	$\{a_{3i+1}, b_{3i}, c_{3i+1}\}$	b_{3i+2}	$\{b_{3(i+1)}\}$
	c_{3i}	$\{b_{3i}, d_{3(i-1)+2}\}$	c_{3i+2}	$\{d_{3i+2}\}$
	d_{3i}	$\{c_{3i+1}, e_{3i}\}$	d_{3i+1}	$\{c_{3i+1}\}$
	e_{3i+1}	$\{e_{3i}\}$	e_{3i+2}	$\{d_{3i+2}, e_{3(i+1)}\}$
$3k + 1$	a_{3i+1}	$\{a_{3i}\}$	a_{3i+2}	$\{a_{3(i+1)}, b_{3i+2}\}$
	b_{3i}	$\{a_{3i}, b_{3(i-1)+2}, c_{3i}\}$	b_{3i+1}	$\{b_{3i+2}\}$
	c_{3i+1}	$\{d_{3i+1}\}$	c_{3i+2}	$\{b_{3i+2}, d_{3i+1}\}$
	d_{3i}	$\{c_{3i}, e_{3i}\}$	d_{3i+2}	$\{c_{3(i+1)}\}$
	e_{3i+1}	$\{d_{3i+1}, e_{3i}\}$	e_{3i+2}	$\{e_{3(i+1)}\}$
	b_{3k} b_0	$\{a_{3k}, b_{3(k-1)+2}, c_{3k}\}$ $\{a_0, c_0\}$	d_{3k}	$\{c_{3k}, e_{3k}\}$
$3k + 2$	a_{3i}	$\{a_{3i+1}, b_{3i}\}$	a_{3i+2}	$\{a_{3i+1}\}$
	b_{3i+1}	$\{a_{3i+1}, b_{3i}, c_{3i+1}\}$	b_{3i+2}	$\{b_{3(i+1)}\}$
	c_{3i}	$\{b_{3i}, d_{3(i-1)+2}\}$	c_{3i+2}	$\{d_{3i+2}\}$
	d_{3i}	$\{c_{3i+1}\}$	d_{3i+1}	$\{c_{3i+1}, e_{3i+1}\}$
	e_{3i}	$\{e_{3i+1}\}$	e_{3i+2}	$\{d_{3i+2}, e_{3i+1}\}$
	a_{3k}	$\{a_{3k+1}, b_{3k}\}$	c_{3k}	$\{b_{3k}, d_{3(k-1)+2}\}$
	d_{3k}	$\{c_{3k+1}\}$	e_{3k}	$\{e_{3k+1}\}$
	b_{3k+1}	$\{a_{3k+1}, b_0, b_{3k}, c_{3k+1}\}$	d_{3k+1}	$\{c_{3k+1}, e_{3k+1}\}$
	a_0	$\{a_1, a_{3k+1}, b_0\}$	c_0	$\{b_0\}$
	e_0	$\{e_1, e_{3k+1}\}$		

5 Conclusions

In this paper, we are studying the locating-dominating sets and the binary locating-dominating number of some convex polytopes. We are dealing with some classes of convex polytopes by considering classes: D_n, R''_n, R_n, Q_n and U_n . For D_n and R''_n exact values are obtained and proved, while for R_n, Q_n and U_n tight upper bounds are given.

Future work can be directed towards determining a binary locating-dominating set of some other challenging classes of graphs. The other promising direction for future work is solving of some other similar graph problem on convex polytopes.

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