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The enigmatic $\Delta(1600)$ resonance

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Abstract. Our recently proposed model of the $\Delta(1600)$ resonance, in which the dominant component is a quasi-bound state of the $\Delta(1232)$ and the pion, is confronted with a similar model of the $N^*(1440)$ resonance as its counterpart in the P11 partial wave. We stress an essentially different mechanism responsible for generating the two resonances.

The two low-lying resonances in the P11 and P33 partial waves, the Roper resonance ($N^*(1440)$) and the $\Delta(1600)$ resonance, have been attracting special attention due to their relatively low masses compared to the prediction of the quark model in which they figure as the first radial excitations in the respective channel, and have been considered as candidates for dynamically generated resonances. In order to understand the mechanism of their formation we study these two resonances in a chiral quark model, which may produce either a genuine resonance by exciting the quark core, or a dynamically generated resonance involving a baryon-meson quasi-bound state. We use a coupled channel approach involving the πN, $\pi\Delta$, and σN channels which — based on our previous experience — dominate the intermediate energy regime in the P11 and P33 partial waves. The Cloudy Bag Model (CBM) is used to fix the quark-pion vertices while the s-wave σ-baryon vertex is introduced phenomenologically with the coupling constant g_σ as a free parameter. Labeling the channels by α, β, γ, the Lippmann-Schwinger equation for the meson amplitude $\chi_{\alpha\gamma}$ for the process $\gamma \to \alpha$ can be cast in the form

$$\chi_{\alpha\gamma}(k_\alpha, k_\gamma) = \mathcal{K}_{\alpha\gamma}(k_\alpha, k_\gamma) + \sum_\beta \int dk \, \frac{\mathcal{K}_{\alpha\beta}(k_\alpha, k)\chi_{\beta\gamma}(k, k_\gamma)}{\omega(k) + E_\beta(k) - W} \,. \tag{1}$$

The half-on-shell pion amplitude consists of the resonant and non-resonant part,

$$\chi_{\alpha\gamma}(k, k_\gamma) = c_{\gamma R}\mathcal{V}_{\alpha R}(k) + \mathcal{D}_{\alpha\gamma}(k, k_\gamma)\,, \tag{2}$$

with the non-resonant part $\mathcal{D}_{\alpha\gamma}(k, k_\gamma)$ satisfying the same Lippmann-Schwinger equation, while the dressed vertex $\mathcal{V}_{\alpha R}(k)$ satisties the Lippmann-Schwinger equation with the same kernel and the bare vertex for the non-homogeneous part. Approximating the kernel $\mathcal{K}$ by a separable form, the integral equations reduce to a system of linear equations which can be solved exactly. The resulting amplitude is proportional to the K matrix which, in turn, determines the scattering T matrix. The Laurent-Pietarinen expansion is finally used to extract the information about the S-matrix poles in the complex energy plane.

The formation of the Roper resonance (N*(1440)) is studied in Ref. [1], confronting two mechanisms for resonance formation: the explicit inclusion of a resonant three-quark state in which one quark is promoted to the 2s state, and the dynamical generation in the absence of the resonant state. In both cases the nucleon pole is explicitly included. While the p-wave πN interaction is repulsive in the P11 channel, the s-wave σN interaction is attractive, and is able to support a (quasi) bound state for sufficiently strong g_σ. The resulting mass of the resonance is close to the PDG value in a relatively wide interval of g_σ, while its width is smaller than the PDG value and drops with increasing g_σ. Including a three-quark resonant state, the mass of the resonance remains almost the same, while its width increases and comes very close to its PDG value (see Table III in [1]). The result is rather insensitive to the mass of the three-quark resonant state, which allows us to use a value around 2 GeV, in agreement with the quark-model ordering of the 2s and 1p states, as well as with the recent results of the lattice calculations [2,3] which have not found a sizable three-quark component below ~1.7 GeV. We conclude that while the mass of the S-matrix pole is determined by the dynamically generated state, its width and modulus are strongly influenced by the three-quark resonant state. This conclusion is further supported by a smooth evolution of the S-matrix pole in the complex energy plane as the coupling of the σ as well as of the pion to the quark core is gradually increased on (see Fig. 1). Starting with two bare masses of 1750 MeV and 2000 MeV, both curves end up almost at the same point with the mass and width consistent with the PDG values.

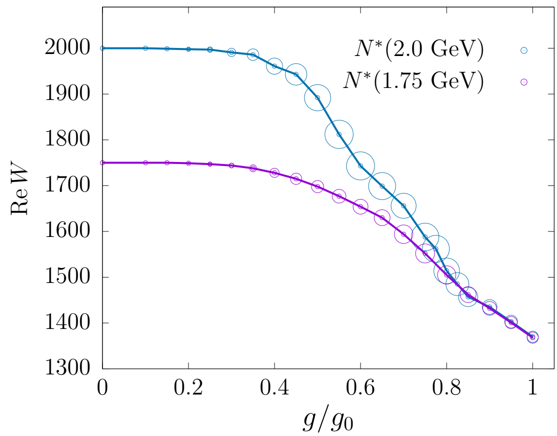


Fig. 1. Evolution of the N*(1440) mass (Re W) and the width (proportional to the radius of the circle) as a function of the interaction strength for two bare masses of the three-quark configuration, 1750 MeV and 2000 MeV; g/g_0 denotes the reduction factor, equal for each coupling constant. The radius at $g/g_0 = 1$ corresponds to Im $W = 180$ MeV.

Though we might expect that, because of apparently the same three-quark configuration, the situation with the Δ(1600) is similar to that with the N*(1440) resonance, this is not the case. One important difference is the nature of the p-wave πN interaction which is attractive in the P33 partial wave, in contrast to its repulsive character in the P11, P13, and P31 waves. Furthermore, the analog of the σN system, the σΔ(1232) system, turns out to make a sizable contribution to the scattering amplitude only above 1700 MeV, and hence the σ plays a minor role in the formation of the Δ(1600) resonance. In [5] we therefore consider only the πN and the πΔ channels. Since the πN coupling constant is fixed by the behavior of the scattering amplitudes near the threshold, the only free parameter in the underlying model (CBM) is the bag radius R which is inversely proportional to the cutoff energy; for the value of $R = 0.8$ fm, leading to the most consistent results for the nucleon as well as for the low lying resonances, it corresponds to ≈ 550 MeV.

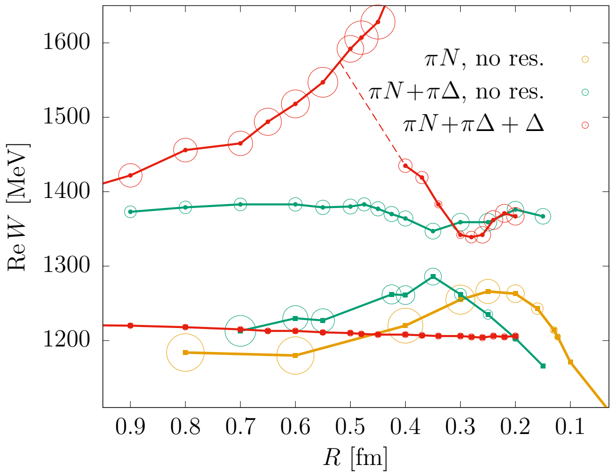


Fig. 2. Evolution of the poles as a function of the bag radius in the P33 partial wave in three different approximations: (i) including only the nucleon and the pion (orange curve and circles), (ii) including the nucleon and the Δ but without a resonant state (green), (iii) with the Δ resonant states (red). The width of the resonance $-2\mathrm{Im}\,W$ is proportional to the radius of the circle.

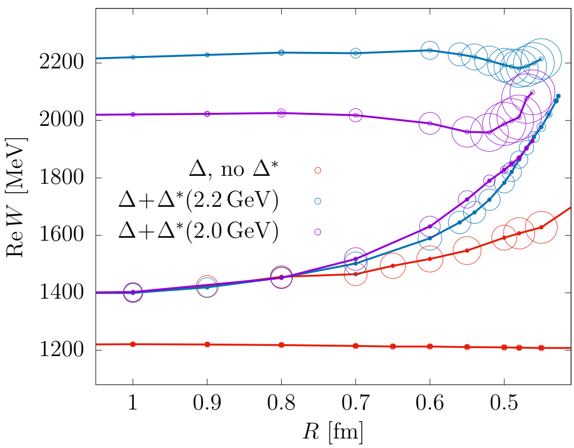
Already a few years after the discovery of the Δ(1232) resonance, it was conjectured that this resonance arises as a consequence of the attraction in the πN system at sufficiently strong cutoff [6]. In our model we do observe a resonance in the πN system manifesting itself as a pole in the complex energy plane at a mass around 1200 MeV, with a width that decreases with increasing interaction strength (decreasing R) (orange curve in Fig. 2). For $R = 0.123$ fm the mass and the width reach the values which agree well with the PDG values, and for $R = 0.050$ fm the system becomes bound. We next include the Δ (in addition to the nucleon) as the u-channel exchange particle in the kernel, and solve (1) for the nonresonant amplitude $\mathcal{D}$. Besides the pole at around 1200 MeV another pole slightly below 1400 MeV emerges (green curves in Fig. 2). The second pole is dominated by the πΔ configuration and can be interpreted as a progenitor of the Δ(1600) resonance.

We next include a three-quark state corresponding to the Δ(1232) in the s-channel and fix its bare mass such that the resulting Breit-Wigner mass (i.e., the zero of Re T) appears at 1232 MeV. With decreasing R the resonant state mixes more and more strongly with the lower dynamically generated state, forming the physical Δ(1232). The latter component dominates below $R = 0.2$ fm, nonetheless, the mass and the width of the resonance pole remain constant (red curves in Fig. 2) and stay close to the PDG value. The upper dynamically generated resonance is pushed toward a slightly higher mass and acquires a larger width. In the physically sensible region around $R \approx 0.8$ fm, the mass and the width come close to the PDG values for the Δ(1600) resonance. The attribution of this pole to the

$\Delta(1600)$ resonance is, however, not justified for smaller R, where its mass keeps increasing, and, in addition, another branch emerges, approaching the upper dynamically generated resonance.

We finally add a bare $(1s)^2(2s)$ configuration representing the first radial excitation of the $\Delta(1232)$. In the harmonic oscillator model, its mass is expected to lie ~ 1 GeV above the $(1s)^3$ configuration, so we fix its (bare) mass at 2.2 GeV, while its coupling is taken from the CBM. Apart from the two resonances discussed above, the third resonance emerges with a mass (Re W) close to the bare value. Increasing the strength of the interaction (decreasing R) we notice that it stays almost constant and — at least in the physically relevant regime of R's — well separated from the other two resonances.

Fig. 3. Evolution of the poles in the model including two resonant states with the second state at the bare mass of 2.2 GeV (blue curves) and at 2.0 GeV (violet), respectively, compared with the model involving Δ alone (red, the same curve as in Fig. 2).



We can therefore conclude that the radially excited quark state plays a very minor role in the formation of the $\Delta(1600)$ resonance, which in our model turns out to be primarily a quasi-bound state of $\Delta(1232)$ and the pion. This mechanism is therefore fundamentally different from that responsible for the formation of the $N^*(1440)$ resonance, discussed above, and originates in the different nature of the pion interaction in the two partial waves.

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