On Vega's Work in Physics and Astronomy

O Vegovem delu s področja fizike in astronomije

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Abstract

J. Vega tackled in separate booklets some problems in physics and astronomy he had encountered working on his monumental Lectures in Mathematics. *Three of these booklets, devoted to the form of the rotating earth, to the linear central motion and the planets' masses, are briefly considered.*

Povzetek

J. Vega je v ločenih snopičih obravnaval nekatere probleme iz fizike in astronomije, na katere je naletel med pisanjem svojega monumentalnega dela Matematična predavanja. *Na kratko so obravnavani trije snopiči, posvečeni obliki vrteče se Zemlje, linearnemu centralnemu gibanju in masam planetov.*

Jurij (George) Vega devoted a substantial part of his *Lectures in Mathematics* to topics in the domain of physics and astronomy. Apparently, he understood physics and astronomy as applied mathematics. In the third volume of his *Lectures* in 1788 he considered "the mechanics of solid bodies" including the motion of projectiles and planets and in the fourth volume in 1800 "basic hydrodynamics, aerostatics, hydraulics, and the resistance of solid bodies in fluids." He also dwelt on some of the problems encountered in separate booklets:

i.) *Mathematical reflections on the form of a solid sphere uniformly rotating around a fixed axis and the consequences of this assumption for astronomy, geography, and mechanics with respect to our earth's spheroid* (1798),

ii.) *An attempt to disclose a mystery in the well-known doctrine of universal gravitation* (1800), and

iii.) *Research on the calculation of masses of heavenly bodies and of the sun from their average distances and orbital periods* (1801).

He also discussed the metric system and wrote in favour of it and its acceptance in the Habsburg Monarchy (1801, 1802, 1803, 1804).

Let us briefly discuss these three booklets.

Booklet I

Starting with a rotating sphere, Vega first studied the resultant of the weight and the inertial force on a test body on its surface. Demanding that the surface of the earth be perpendicular to this resultant, for the cross section of the earth he obtained an ellipse with the oblateness:

$$\frac{A-a}{A} = 1 - \sqrt{1 - \frac{4\pi^2 r}{T^2 g}} \approx \frac{2\pi^2 r}{T^2 g} = \frac{1}{579}. \tag{1}$$

A and a are the equatorial and polar radius of the earth, respectively, T the rotation period, i.e. the sidereal day, and g the acceleration of gravity at the surface of the earth. In passing Vega mentioned that according to "the fundamental equations of hydrostatics" a more complicated curve is obtained. The hydrostatic pressure is obtained by multiplying the acceleration of gravity and the distance from the center. If this product remains constant, the oblateness

$$\frac{A-a}{A} = \frac{4\pi^2 r}{T^2 g} = \frac{2}{579} \approx \frac{1}{289} \tag{2}$$

follows. Of course, this is only an approximation because the acceleration depends on the distance from the center, whereas its value on the surface was taken.

According to Newton, the acceleration of gravity at the equator is less than at the pole for two reasons: the earth is rotating, and the point at the equator is farther away from the center. We see that in Vega's model they contribute equal amounts. The obtained oblateness did not differ much from the present value of 1/298.25. However, Vega did not follow this line of thought, but returned to the smaller value of the oblateness and made calculations with it and with a third value, 1/415. Apparently, he was not aware of the work of Alexis Claude Clairaut of 1743, even if he may have known the oblateness of 1/217 obtained by Pierre Bouguer and Pierre-Louis Moreau de Maupertuis in 1738.

Booklet II

Consider a point mass at rest attracted by a point central body with a very great mass. As the point mass is released at the origin it begins to fall towards the central body, which is at rest at b. Newton's second law gives the velocity of the point mass:

$$v = \dot{x} = \left(\frac{2gr_E^2}{b}\right)^{1/2} \left(\frac{x}{b-x}\right)^{1/2}. \tag{3}$$

Thereby g is the acceleration of gravity on the surface of the earth and r_E the radius of the earth. The point mass reaches coordinate x at time:

$$t = \left(\frac{b^3}{2r_E^2 g}\right)^{1/2} \int_0^{x/b} \frac{(1-\xi)^{1/2} d\xi}{\xi^{1/2}} =$$

$$= \left(\frac{b^3}{2r_E^2 g}\right)^{1/2} \left[\left(\frac{x}{b}\right)^{1/2} \left(1-\frac{x}{b}\right)^{1/2} + \arcsin\left(\frac{x}{b}\right)^{1/2}\right]. \tag{4}$$

Vega knew this integral well because he used it as an example in his *Lectures on Mathematics.* According to equation (4) for $x = b$ the point mass encounters the central body at time $\pi(b^3/2r_E^2 g)$. However, Vega was upset by observing that thereby the velocity of the point mass increases towards infinity.

Thus, the question arose whether the point mass can pass the central body or not. Here Vega was using a model, as we do today, in which the dimensions of both bodies are reduced to zero so that the bodies do not collide. At first Vega thought that the point mass could not pass the central body. In despair he was even prepared to accept that in this exceptional case the relation $\lim x^{-1}|_{x\to 0} \to \infty$ does not hold. In the spring of 1788 he also uphold this point of view in the third volume of his *Lectures.*

Later on he had second thoughts and derived equations for attractive forces proportional to $1/r^n$ with $n = 3, 4 \ldots$. The equations showed him that problems of the stated kind persist for even n only. This he did not accept and concluded that the point mass can pass the central body and is in fact oscillating around it. Only then he introduced a symmetric coordinate system with the origin at the central body. This he accomplished during the campaign against the Turks and published in an addendum to the third volume of the *Lectures* in 1790. It could have happened that Vega deliberately proceeded step by step in order to achieve a more dramatic tale.

Booklet III

In the third volume of his *Lectures in Mathematics* Vega derived the generalized third Kepler's law

$$\mathcal{G}(m_S + m_P) = \frac{4\pi^2 a_P^3}{T_P^2} \tag{5}$$

which was already known to Newton. m_S and m_P are the mass of the sun and the planet, respectively, a_P is the great half-axis of the elliptical orbit, and T_P the sidereal orbital period of the planet. Of course, the gravitational constant $\mathcal{G}$ and the mass of the earth m_E were not known prior to 1798. So Vega first used the equation for the earth:

$$\mathcal{G}(m_S + m_E) = \frac{4\pi^2 a_E^3}{T_E^2} \quad \text{and} \quad \frac{m_S}{m_E} = \frac{4\pi^2 a_E^3}{\mathcal{G} m_E T_E^2} - 1. \tag{6}$$

In this case $\mathcal{G} m_E = g r_E^2$ with the acceleration g on the surface of the earth and r_E the radius of the earth. Using the last equation, Vega could calculate the ratio of the masses of the sun and the earth. He obtained 339,676, the present value 332,946.043 being about 2% smaller. The generalization of Kepler's law – that is the motion of the sun – affects Vega's last digit.

Further on, rearranging equation (5), dividing it by m_E and using equation (6), Vega deduced the equation

$$\frac{m_P}{m_E} = \frac{4\pi^2 a_P^3}{\mathcal{G} m_E T_P^2} - \frac{m_S}{m_E} = \left(\frac{m_S}{m_E} + 1\right)\left(\frac{a_P}{a_E}\right)^3 \left(\frac{T_P}{T_E}\right)^{-2} - \frac{m_S}{m_E}. \tag{7}$$

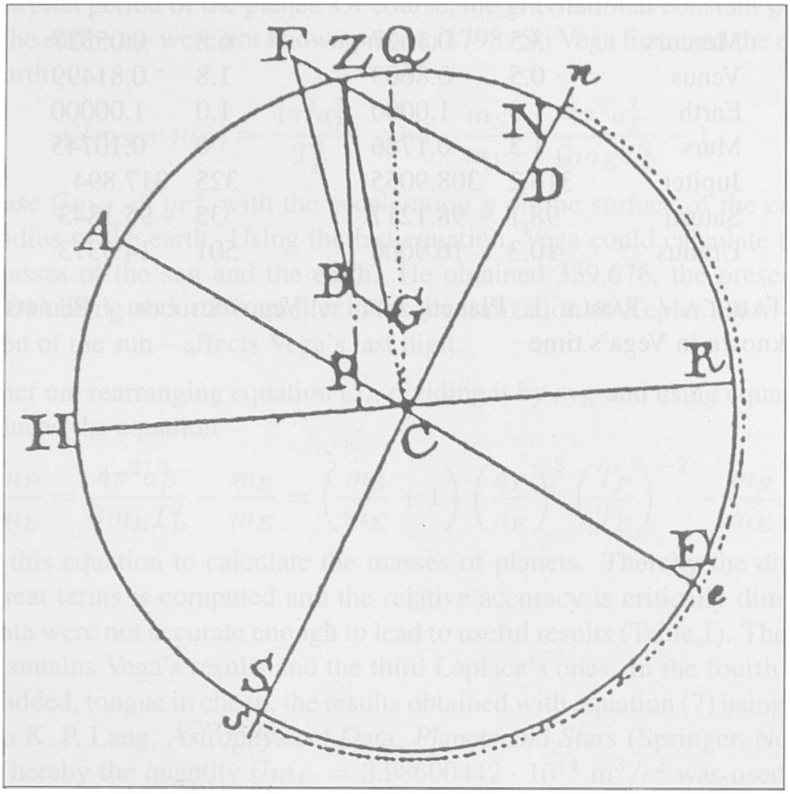
He used this equation to calculate the masses of planets. Thereby the difference of two great terms is computed and the relative accuracy is critically diminished. Vega's data were not accurate enough to lead to useful results (Table 1). The second column contains Vega's results and the third Laplace's ones. In the fourth column we have added, tongue in cheek, the results obtained with equation (7) using current data from K. P. Lang, *Astrophysical Data. Planets and Stars* (Springer, New York 1992). Thereby the quantity $\mathcal{G} m_E = 3.98600442 \cdot 10^{14}\ \mathrm{m^3/s^2}$ was used, which is known with better accuracy than the gravitational constant. In this way even with current data only Jupiter's mass can be estimated as $m_P/m_E = 325$. For other planets with much smaller mass no useful estimate can be obtained. The fifth column contains current data for the relative mass of the planets. The situation was somewhat more favourable in the case of the Jovian satellites, for which Vega calculated the ratio of the masses of the satellite and Jupiter. Masses of planets with satellites could be calculated by Kepler's third law using orbital data of the satellites. It is interesting to note that Laplace's and Vega's results for planets with satellites are more accurate than for other planets.

Other physicists at that time also did not take into account the accuracy of data. So for instance Lavoisier and Laplace in their *Report on Heat* in 1783 gave the specific heat of iron to 6 digits, whereas their heat of fusion for ice – they measured with an ice calorimeter – was about 6% too low. It was not customary to discuss the accuracy of the data and of the results.

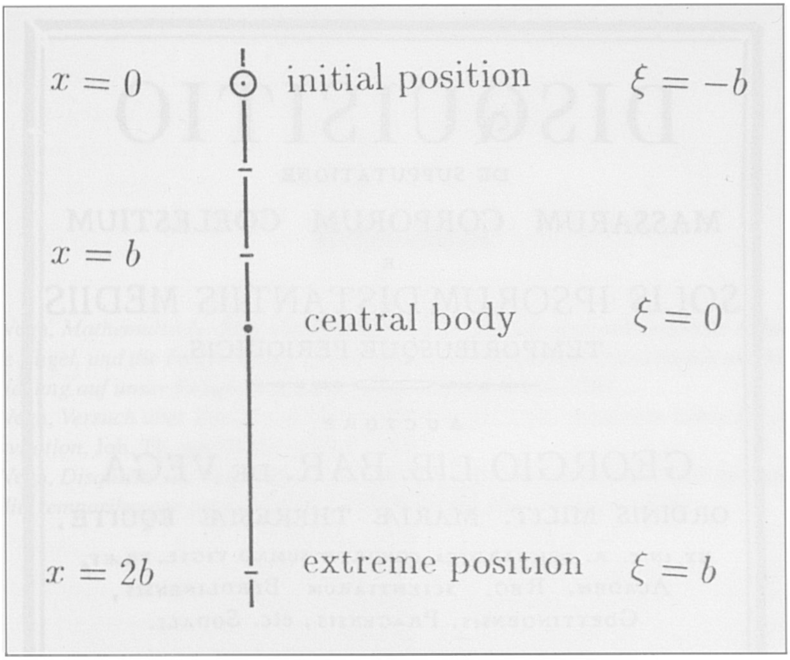
Planet	m_P/m_E Vega	m_P/m_E Laplace	m_P/m_E Equation (7)	m_P/m_E Present
Mercury	3.5	0.1627	6.8	0.05527
Venus	0.5	0.8603	1.8	0.81499
Earth	1.0	1.0000	1.0	1.00000
Mars	1.3	0.1786	0	0.10745
Jupiter	316.2	308.9055	325	317.894
Saturn	98.1	98.1217	35	95.1843
Uranus	10.3	16.9006	501	14.5373

TABELA / TABLE 1. Planeti znani v Vegovem času / Planets known in Vega's time

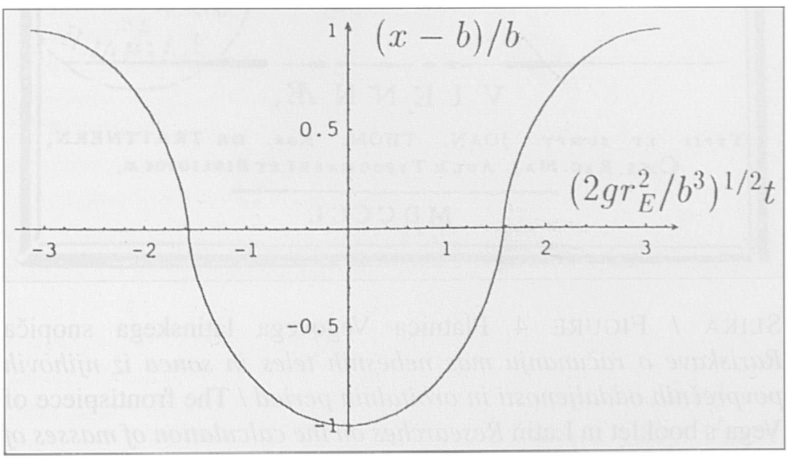
Figures



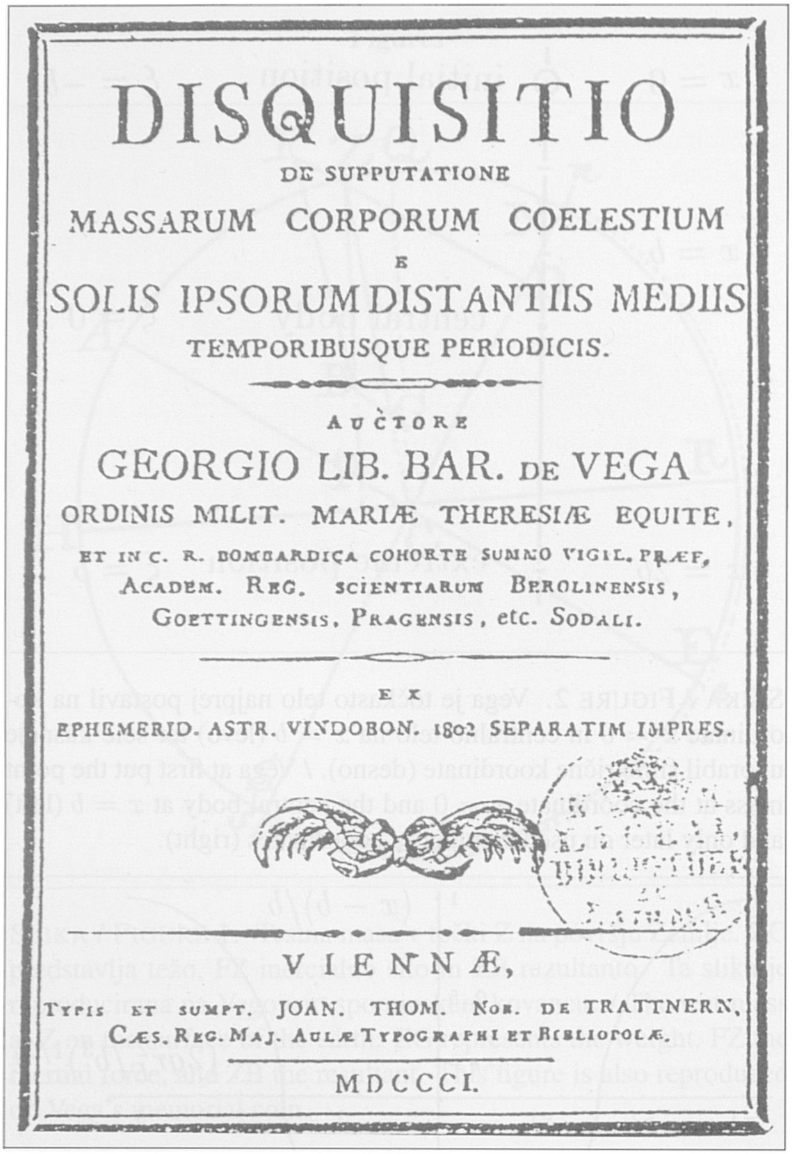
SLIKA / FIGURE 1. Testna masa v točki Z na površju Zemlje. ZG predstavlja težo, FZ inercialno silo in ZB rezultanto. Ta slika je reproducirana na Vegovem spominskem kovancu. / The test mass at Z on the surface of the earth: ZG represents the weight, FZ the inertial force, and ZB the resultant. This figure is also reproduced on Vega's memorial coin.



SLIKA / FIGURE 2. Vega je točkasto telo najprej postavil na koordinate $x = 0$ in centralno telo na $x = b$ (levo) ter šele kasneje uporabil simetrične koordinate (desno). / Vega at first put the point mass at the coordinate $x = 0$ and the central body at $x = b$ (left) and only later on used symmetric coordinates (right).



SLIKA / FIGURE 3. Linearno centralno gibanje po Vegovo: prikazana je odvisnost $(x - b)/b$ od $(2gr_E^2/b^3)^{1/2}t$. / Linear central motion according to Vega: the dependence of $(x - b)/b$ from $(2gr_E^2/b^3)^{1/2}t$ is shown.



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SLIKA / FIGURE 4. Platnica Vegovega latinskega snopiča *Raziskave o računanju mas nebesnih teles in sonca iz njihovih povprečnih oddaljenosti in orbitalnih period* / The frontispiece of Vega's booklet in Latin *Researches on the calculation of masses of heavenly bodies and of the sun from their average distances and orbital periods*

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[1] G. Vega, *Mathematische Betrachtungen über eine sich um eine unbewegliche Achse drehende feste Kugel, und die Folgen dieser Voraussetzung für Astronomie, Geographie und Mechanik, in Beziehung auf unser Erdsphäroid*, Beyer und Maring, Erfurt, 1898.

[2] G. Vega, *Versuch über Enthüllung eines Geheimnisses in der bekannten Lehre der allgemeinen Gravitation*, Joh. Th. von Trattner, Wien, 1800.

[3] G. Vega, *Disquisitio de supputatione massarum corporum coelestium e solis ipsorum distantiis mediis temporibusque periodicis*, Joan. Thom. de Trattner, Viennae, 1800.