Chinese Research on Mathematical Logic and the Foundations of Mathematics

*WANG Hongguang**
*DU Guoping***

Abstract

This paper outlines the Chinese research on mathematical logic and the foundations of mathematics. Firstly, it presents the introduction and spread of mathematical logic in China, especially the teaching and translation of mathematical logic initiated by Bertrand Russell's lectures in the country. Secondly, it outlines the Chinese research on mathematical logic after the founding of the People's Republic of China. The research in this period experienced a short revival under the criticism of the Soviet Union, explorations under the heavy influence of the Cultural Revolution, and the vigorous development of mathematical logic teaching and research after the period of "Reform and Opening Up" that started in the late 1970s, and the full integration of Chinese mathematical logic research into the international academic circle in the new century after 2000. In the third part, it focuses on the unique and original results of the Chinese mathematical logic research teams from the following three aspects: medium logic, lattice implication algebras and their lattice-valued systems of logic, and Chinese notation of logical constants, which can be used as a substantive supplement to the relevant literature on the history of mathematical logic in China. The last part is a reflection on the shortcomings of contemporary Chinese research on mathematical logic and the foundations of mathematics.

Keywords: Chinese logical research, mathematical logic, medium logic, lattice implication algebra, Chinese notation

Kitajske raziskave na področju matematične logike in osnov matematike

Izvleček

Članek povzema kitajske raziskave na področju matematične logike in osnov matematike. V uvodnem delu ponuja vpogled v predstavitev in širjenje matematične logike na Kitajskem. Še posebej pa se posveča poučevanju in prevajanju matematične logike, ki so ju

* WANG Hongguang, Postdoctoral researcher, Department of Philosophy, Peking University, Beijing, China.
Email address: hgwang@pku.edu.cn

** DU Guoping, Institute of Philosophy, Chinese Academy of Social Science (CAS), Beijing, China; School of Philosophy, Sichuan Normal University, Chengdu, China.
Email address: dgpnju@163.com

spodbudila predavanja Bertranda Russella na Kitajskem. Nadalje povzema raziskave matematične logike po ustanovitvi Ljudske republike Kitajske. Področje je v tem času doživelo hitri razcvet pod vplivom kritik matematične logike v Sovjetski zvezi, temu je sledilo raziskovanje, na katerega je močno vplivala kulturna revolucija, živahen razvoj poučevanja in preučevanja pa sta se začela konec 70. let po obdobju »reform in odpiranja svetu« in se po letu 2000 nadaljevala v popolno vključitev kitajskih raziskav na področju matematične logike v mednarodne akademske kroge. Tretji del članka se osredotoča na edinstvene in izvirne dosežke kitajskih skupin, ki se posvečajo raziskavam matematične logike, in sicer s stališč: medialne logike, mrežne implikacijske algebre in njihovih mrežno-vrednostnih sistemov logike ter kitajske notacije logičnih konstant, ki jo lahko uporabljamo kot konkretno dopolnilo k relevantni literaturi o zgodovini matematične logike na Kitajskem. Zadnji del članka predstavlja razmislek o pomanjkljivostih v sodobnih kitajskih raziskavah matematične logike in osnov matematike.

Ključne besede: kitajske logične raziskave, matematična logika, medialna logika, algebra mrežne implikacije, kitajska notacija

Introduction

The present paper is an overview of the Chinese research on mathematical logic and the foundations of mathematics. It mainly consists of four parts: The first part briefly presents the introduction and spread of mathematical logic in China, especially the teaching and translation of mathematical logic during the Republic of China (ROC) period initiated by Bertrand Russell's lectures in the country. The second part is an outline of Chinese research on mathematical logic after the founding of People's Republic of China (PRC). The research in this period had experienced a short revival under the criticism of the Soviet Union, explorations under the heavy influence of the Cultural Revolution (*Wenhua da geming* 文化大革命), and the vigorous development of mathematical logic teaching and research after the "Reform and Opening Up" (*Gaige Kaifang* 改革开放) that began in the late 1970s, and the full integration of Chinese mathematical logic research into the international academic circle in the new century after 2000. The third part focuses on the unique and original results of the Chinese mathematical logic research teams from the following three aspects: medium logic, lattice implication algebras and their lattice-valued systems of logic, and the Chinese notation of logical constants. The last part is a reflection on the shortcomings of contemporary Chinese research on mathematical logic and the foundations of mathematics.

There have been several earlier works on the history of mathematical logic in China. Lin Xiashui 林夏水 and Zhang Shangshui 张尚水 (1983) and Song Wenjian

宋文坚 (2000) all outline the contributions of Jin Yuelin 金岳霖and others to the introduction of mathematical logic into China since the 1920s, as well as the work of Chinese scholars in the fields of logical calculus, set theory, recursion theory, modal theory and proof theory. Chen Bo 陈波 (2019) gives a comprehensive introduction to the international publications and the main research progress of Chinese logic scholars in the fields of the history of logic (especially the history of Chinese logic), inductive and probabilistic logic, natural language logic, philosophy of logic, informal logic and critical thinking, legal logic, and so on, the establishment and rapid development of logic research institutions in the PRC is also introduced. Su Rina's 苏日娜 doctoral dissertation (2020), "History of Mathematical Logic in China (1920–1966)" (*Shuli luoji zai Zhongguo de fazhanshi yanjiu* (1920-1966) 数理逻辑在中国的发展史研究 (1920–1966)) presents the introduction of mathematical logic by Chinese scholars in the first half of the 20th century, reviews and summarizes the history and characteristics of mathematical logic during its initial foundation in China (1920–1949) and during the founding and development of the "new China" (1949–1966). Jan Vrhovski's paper (2021a) examines the work of Jin Yuelin and others in the Department of Philosophy of Tsinghua University, and the characteristics and main progress of the teaching and research of mathematical logic in this context from 1926 to 1945 are summarized. Du Guoping and Wang Hongguang (2020) also provide an overview of the introduction and research on logic in mainland China from the ROC to PRC, in particular, the achievements of contemporary Chinese mainland scholars in the fields of mathematical logic (modal logic, recursion theory, set theory, formalized methods and automatic reasoning, etc.) as well as philosophical logic (modal logic, many-valued logic, lattice-valued logic based on lattice implication algebras, paraconsistent logic, etc.).

This paper mainly focuses on the research in the period after the founding of PRC, especially the original work introduced in the third part, which can be used as a substantive supplement to the above-mentioned relevant literature.

The Beginning of Mathematical Logic in China

The science of logic was first introduced to China in the late Ming dynasty (early 17th century). Among the earliest works was the book *Mingli tan* 名理探 (*De logica*) translated by Li Zhizao 李之藻 and Francisco Furtado. This book was a translation of teaching materials on logic used by the members of the Jesuit order at Coimbra University in Portugal. Its original title was *In Universam Dialecticam Aristotelis Stagiritae*. By the early 20th century, scholars like Yan Fu 严复, Wang

Guowei 王国维, Hu Maoru 胡茂如 and others, one after another, translated the most important works on Western logic into Chinese.

In 1920, the renowned British scholar Bertrand Russell was invited to lecture in China for one year, in the framework of which he carried out five major series of lectures at Peking University, one of which was a series of lectures on "Mathematical Logic" (*Shuli luoji* 数理逻辑). Subsequently, in October 1921, the anthology *Five Great Lectures by Bertrand Russell* (*Luosu wu da yanjiang* 罗素五大演讲) was published by the New Knowledge Publishing House of Peking University, a publication which marked the start of dissemination of mathematical logic in China. In 1922, Fu Zhongsun 傅种孙 and Zhang Bangming 张邦铭 translated and published the book *Introduction to Mathematical Philosophy* (*Luosu suanli zhexue* 罗素算理哲学) by Russell. Later, some scholars gradually introduced mathematical logic into Chinese academic world: Zhang Shenfu 张申府 introduced the notion of mathematical logic related to Russell's philosophy[1] and his *Principia Mathematica*, Tang Zaozhen 汤璪真 as a mathematician introduced set theory in the context of mathematics, and Zhu Gongjin 朱公瑾 introduced Hilbert's conception of "symbolic logic". In 1926, Jin Yuelin started a course on logic at Tsinghua University in Beijing, in the framework of which he also taught content related to mathematical logic. One year later, in 1927, Wang Dianji 汪奠基 published a book entitled *Logic and Mathematical Logic* (*Luoji he shuxue luoji lun* 逻辑和数学逻辑论), which is the first monograph as a systematic introduction on mathematical logic and its history in China. In 1935, Tsinghua University published Jin Yuelin's textbook *Logic* (*Luoji* 逻辑), in which he provided an overview of Russell's systems of mathematical logic. Xiao Wencan 肖文灿 published a series of articles on set theory during 1933–1934, which were later collected and published in a volume entitled *A Primer on Set Theory* (*Jihelun chubu* 集合论初步) by The Commercial Press in 1939. From the early 20th century on, Chinese scholars like Yu Dawei 俞大维, Shen Youding 沈有鼎, Wang Xianjun 王宪钧, Hu Shihua 胡世华, Mo Shaokui 莫绍揆 (Moh Shaw-kwei) and others in succession travelled abroad to study mathematical logic at foreign universities, and later also returned to China. In this way, mathematical logic in China underwent a gradual development (also see Lin and Zhang 1983 for the development of mathematical logic in the Republican period).

Because research on the foundations of mathematics and mathematical logic is inextricably linked, in the following discussion it will be referred to research on mathematical logic.

1 For a detailed analysis of Zhang's critical introduction of Russell's logic, see Vrhovski (2021b, 229ff).

An Outline of Chinese Studies of Mathematical Logic

Mathematical Logic in the Foundation Period of the People's Republic of China (PRC)

In the time of foundation of the PRC, Chinese research on mathematical logic was influenced by the Soviet criticism of mathematical logic. However, at a dinner which took place in 1956, Mao Zedong told Jin Yuelin that mathematical logic was important, and needed to be taken care of. After it received this support from Mao, Chinese research into mathematical logic gradually became more active.

In this early period Shen Youding published two important articles, in 1953 and 1955, in which he investigated the paradox of the class of all grounded classes and semantical paradoxes, respectively. In 1950, Mo Shaokui constructed two new logical systems that could effectively prevent "paradoxes of implication". In 1954, Mo proved that in a many-valued system of logic, if we do not apply any restrictions on the use of principle of comprehension, by the same token we can also construct a theory of paradox analogous to that existing in two-valued logic. In 1957, after a one-year long campaign by the members of the IMCAS (Institute of Mathematics at Chinese Academy of Science), mathematical logic returned to Chinese universities. Hu Shihua conducted valuable research on recursive functions and recursive structures in the field of recursion theory around 1960s. He defined a kind of kernel function class in a very concise but powerful way and used it to construct a universal algorithm for normal algorithms and universal computers, like a Turing machine. He also extended the theory of recursive functions on natural number sets to sets of formulas, and established a computability theory, that is, the theory of recursive algorithms (Hu 1960a; 1960b; Hu and Lu 1960). In 1963, the 3rd National Experience-Sharing Conference on Computer Technology was convened in Xi'an. At the conference a special group for mathematical logic was organized, which was presided over by Hu Shihua. This was the first nationwide conference on mathematical logic held in China. Contributions submitted and presented at the conference involved topics such as many-valued logics, theory of algorithms, proof theory, the foundations of mathematics and the theory of automatization.

The Period of Twists and Turns (1966–1976)

Although in the ten years of the Cultural Revolution (*Wenhua da geming* 文化大革命) Chinese research on mathematical logic was greatly influenced by the related political developments, many Chinese researchers on mathematical logic still managed to work despite the adversities of the time and carry on with their studies and research in the field. Subsequently, in the year 1972, the American-Chinese logician Wang Hao returned to China and was received by Prime Minister Zhou Enlai. After that, Wang kept returning to China for several more times to present scientific reports to the Chinese scientific community. His visits and lectures brought a fresh wind into the Chinese academic world of mathematical logic, which gave an enormous boost to scholars in the field. However, under the heavy influence of the contemporary political circumstances, in this period Chinese scholars' research achievements in the field of mathematical logic were still rather limited.

The Last Quarter of the 20th Century (1977–1999)

After the conclusion of the Cultural Revolution, Chinese scientific research gradually returned to normal. In 1977, Wang Hao returned to China once again to deliver a series of six scientific reports which were later collated and published under the title *Popular Lectures on Mathematical Logic* (*Shuli luoji tongsu jianghua* 数理逻辑通俗讲话; Wang 1981). These lectures were of enormous help to Chinese scholars by enabling them to obtain a timely understanding of the current developmental situation in the international research on mathematical logic.

It was especially after China's 1978 reforms and opening up to the world, when the long-suppressed research enthusiasm of Chinese intellectuals experienced an unprecedented growth, and Chinese science finally obtained a series of significant results in mathematical logic. Thus, for instance, in the two years of 1979 and 1980 alone, Mo Shaokui published six scientific articles related to set theory and the theory of recursion, Zhang Jinwen 张锦文 published seven scientific articles about axiomatic set theory of weak predicate calculus and non-standard analysis, and Hong Jiawei 洪家威 published two articles on computational complexity. In the same period, some high-quality articles were even published in renowned international scientific periodicals. These included works covering Luo Libo's 罗里波 (also known as Lo Libo) achievements in model theory and decidability of free groups, published in the prestigious Western periodical *The Journal of Symbolic Logic* (Lo 1983a; 1983b), as well as Hong Jiawei's results on the theory of computational complexity, published in various international journals (Hong 1982a; 1982b; 1984).

Following 1978, Chinese universities and research institutes started recruiting graduate students in mathematical logic. These developments caused Chinese mathematical logic to enter a stage of overall and comprehensive development, which saw the emergence of an uninterrupted series of high-standard research achievements in the field. These included works like Wang Shiqiang's 王世强 article on the elementary concepts, methodology and theorems of lattice-valued model theory (Wang 1986); Feng Qi's 冯琦 study on the hierarchy of Ramsey cardinals (Feng 1990); Yi Bo's 伊波 and Xu Jiafu's 徐家福 article on analogy calculus (Yi and Xu 1993); Li Wei's 李未 theory of the limits of formal theory of sequencing—open logic (Li 1993); and Ying Mingsheng's 应明生 article on a logical system for approximate reasoning (Ying 1994).

The New Century (2000—)

Since the begin of the new century, China's community of independently educated as well as foreign-educated researchers in the field of mathematical logic has been constantly expanding. At present, Chinese scholars who are engaged in research of mathematical logic have already become an internationally influential and significant group of researchers, while Chinese research in the field has been completely integrated into the international developments in mathematical logic.

At the same time, a great number of Chinese scholars, such as Ding Decheng 丁德成, Feng Qi and others, have been actively engaged in the international circles of mathematical logicians. More specifically, in the recent years Ying Mingsheng's monograph *Topology in Process Calculus: Approximate Correctness and Infinite Evolution of Concurrent Programs* has been published by the Springer publishing house. Moreover, the Chinese logician Zhang Yi 张羿 assumed the role of the editor-in-chief of the international journal *Logic and Algebra*, while Zhao Xishun 赵希顺 was named a member of the editorial board of the international *Journal of Satisfiability, Modeling and Computation*.

Apart from this, in the last two decades a great number of international conferences related to mathematical logic were initiated and convened in China, such as a conference on the theory and application of models of computation, and a 2008 conference on computability, complexity and randomness.

At the same time, China's achievements in the international academic world of mathematical logic are also expanding, to a degree that it's impossible to offer a complete listing of these results here.

At present, there already exist close to 100 different Chinese textbooks on mathematical logic, while at the same time a considerable amount of the latest teaching materials on mathematical logic from the rest of the world is being continuously translated into Chinese. Courses on mathematical logic are offered at departments of philosophy and computer science at many Chinese universities, while at several comprehensive universities they also educate master's and doctoral students specialized in the field of mathematical logic. At relevant universities or research institutes undergraduate students are generally taught propositional calculus and first-order predicate calculus, while at the level of graduate studies they are taught set theory and modal logic. Finally at the level of doctoral studies they are taught subjects such as model theory, proof theory, theory of recursion and so on. Quite a few research institutions even make direct use of well-established foreign textbooks on mathematical logic.

A Few Comparatively Central Topics

Chinese research into mathematical logic consists of a few academic groups which have, focusing on a specific topic in one or the other domains of study, created comparatively central original achievements, rich in distinguishing features and qualities. These are summarized as follows in the following subsections.

Medium Logic

Chinese studies on medium logic were established in the 1980s as a result of the long-term cooperation between Zhu Wujia 朱梧槚 and Xiao Xi'an 肖奚安 (Zhu and Xiao 1984), while in the last four decades a group of young and middle-aged scholars engaged in research on the topic. Nowadays, medium logic has already evolved into a theory of logic all aspects of which, from its theory to application, are extremely rich in content.

The fundamental idea which gave rise to the establishment of medium logic was the so-called "intermediate principle". From Aristotle onwards, a distinction has been made between intermediate opposite opposition and non-intermediate contradictory opposition. The principle of non-intermediacy posits that all antinomies are non-intermediate contradictory oppositions, while the principle of intermediacy maintains that not all antinomies are non-intermediate contradictory oppositions. The principle of intermediacy recognizes that under certain circumstances there exists the state of "both A and B". Its philosophical basis rest on the intermediary state of transition that abounds in the process of transformation between two sides of an antinomy; its real basis, on the other hand, is in the intermediary states of various kinds of objective existence, such as, for example, dusk, which represents the intermediate state in the change of daytime into night-time, or the condition of being middle-aged, which is the intermediary stage between youth and old age, or semiconductors, which represent the intermediaries between conductors and isolators.

In medium logic we use P and $╕P$ to express contrary antithetical notions, while the symbol $\sim$ is used to designate a fuzzy negator, which is to be interpreted and read as "partially". If an object x satisfies $\sim P(x) \wedge \sim ╕P(x)$, i.e. that it partially possesses the property P and at the same time also possesses the property $╕P$, then x is referred to as the intermediary object of opposite antinomy $(P, ╕P)$.

The system of medium propositional logic MP consists of two single-variable conjunctions: $╕$ (opposite negator), $\sim$ (intermediary); and one binary conjunction $\rightarrow$ (implication). It further defines the single variable conjunction: $\neg A =_{df} A \rightarrow \sim A$.

The inference rules of MP consist of:

$(\in)$ $A_1, A_2, \ldots, A_n \vdash A_i \ (i=1, 2, \ldots, n)$;

(τ) If $\Gamma \vdash \Delta$, $\Delta \vdash A$, then $\Gamma \vdash A$;

$(\neg)$ If $\Gamma, \neg A \vdash B$, $\Gamma, \neg A \vdash \neg B$, then $\Gamma \vdash A$;

$(\rightarrow_-)$ $A \rightarrow B, A \vdash B$, $A \rightarrow B, \sim A \vdash B$;

$(\rightarrow_+)$ If $\Gamma, A \vdash B$, and $\Gamma, \sim A \vdash B$, then $\Gamma \vdash A \rightarrow B$;

(Y) $A \dashv\vdash \neg╕A, \neg\sim A$;

$(Y\sim)$ $\sim A \dashv\vdash \neg╕A, \neg A$;

$(Y╕)$ $╕A \dashv\vdash \neg A, \neg\sim A$;

$(╕╕_+)$ $A \vdash ╕╕A$;

$(╕╕_-)$ $╕╕A \vdash A$;

$(╕_\rightarrow)$ $A, ╕B \dashv\vdash ╕(A \rightarrow B)$;

$(\sim\sim)$ $A \rightarrow A \dashv\vdash \sim\sim A$.

The system of medium propositional logic MP* represents the system of medium propositional logic MP expanded by the binary connective "$\prec$", which is called a "truth degree operator" and read as "the degree of truth-value is not stronger than". In addition, it is also enlarged by the following three inference rules:

$(\prec)$ $A \prec B \dashv\vdash (A \rightarrow B) \vee (\sim A \wedge \sim B)$

$(\sim\prec)$ $\sim(A \prec B) \dashv\vdash (\sim A \wedge ╕B) \vee (A \wedge \sim B)$

$(╕\prec)$ $╕(A \prec B) \dashv\vdash A \wedge ╕B$

The elementary semantic of conjunctions in the system of medium propositional logic MP* is:

A	$\sim A$	$\urcorner A$	$\neg A$
0	1	2	2
1	2	1	2
2	1	0	1

$A \rightarrow B$ (rows: A; columns: B)	0	1	2
0	2	2	2
1	1	1	2
2	0	1	2

A□B (rows: A; columns: B)	0	1	2
0	2	2	2
1	1	2	2
2	0	1	2

Apart from the above-described system of medium propositional logic MP*, the system of medium logical calculus ML also includes the system of medium propositional logic MP, the systems of medium predicate logic MF and MF*, systems of medium predicate logic with identity ME and ME* (Xiao and Zhu 1985a–1985e; Zhu and Xiao 1985a; 1985b).

Today, medium logic has already developed into a very broad field of research, which consists mainly of the following already established and advanced research contents or directions:

(1) Medium system of algebra (Wu and Pan 1990);

(2) Medium system of modal logic (Zhang and Zhu 1995, etc.);

(3) Medium system of axiomatic set theory (Zhu and Xiao 1988, etc.);

(4) Medium proof theory (Zou 1988);

(5) Medium theory of forcing (Zhu et al. 1996);

(6) Medium system of reasoning with incomplete information (Deng 1994);

(7) Medium programming language MILL and its interpretation system (Song and Zhu 1994);

(8) Medium systems of theory and practice of automatic reasoning (Zhang and Zhu 1994a–1994c, etc.);

(9) Numeralization of medium truth-degree operators and their applications in computers (Hong et al. 2006; Hong et al. 2007).

Lattice Implication Algebras and Their Lattice-Valued Systems of Logic

Nonclassical logic constitutes one of the foundations of artificial intelligence. The lattice-valued system of logic based on lattice implication algebra is a kind of nonclassical logic. Since 1993, Xu Yang 徐扬 and other Chinese logicians have been conducting research on lattice implication algebras, lattice-valued systems of logic based on lattice implication algebras, and imprecise inference and automatic inference based on these systems of logic.

Lattice Implication Algebra

To set up a new system of logic, Xu Yang proposed a lattice implication algebra (Xu 1993), which represented a kind of nonclassical logical algebra combining lattices and implication algebra.

Definition 1. Lattice implication algebra is an algebraic system $\mathcal{L} = (L, \vee, \wedge, ', \rightarrow, O, I)$, where

(1) $(L, \vee, \wedge, O, I)$ is a bounded lattice, while O and I represent its least and greatest elements, respectively;

(2) $' : L \rightarrow L$ is an inverted order involutory mapping;

(3) $\rightarrow: L \times L \rightarrow L$ is a binary operation and for any $x, y, z \in L$, there exist

① $x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)$;

② $x \rightarrow x = I$;

③ $x \rightarrow y = y' \rightarrow x'$;

④ If $x \rightarrow y = y \rightarrow x = I$, then $x = y$;

⑤ $(x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x$;

⑥ $(x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z)$;

⑦ $(x \wedge y) \rightarrow z = (x \rightarrow z) \vee (y \rightarrow z)$.

All lattice implication algebras make up a proper class, which possesses many favourable properties, such as:

(1) $(L, \vee, \wedge)$ is a distributive lattice;

(2) $x \leq y$ iff $x \rightarrow y = I$;

(3) $x \rightarrow O = x', I \rightarrow x = x$;

(4) If $x \leq y$, then $z \rightarrow x \leq z \rightarrow y, y \rightarrow z \leq x \rightarrow z$;

(5) $x \to y \geq x' \vee y$;

(6) If $\forall x \in L, x \vee x' = I$, then $(L, \vee, \wedge, ')$ is a Boolean algebra;

(7) $(x \to y) \vee (y \to x) = I$;

(8) $(x \to y) \to y = x \vee y$;

(9) $z \to (x \to y) \geq (z \to x) \to (z \to y)$;

(10) $(x \to y) \to (z \to y) = (y \to x) \to (z \to x)$;

(11) $x \vee y = I$ iff $x \to y = y$.

If L is a finite chain, then there exists in L only one such implication →, so that it can become lattice implication algebra. Finite lattice implication algebra can be decomposed into a Cartesian product of finite chains. Over the interval [0, 1] we can define infinitely many kinds of lattice inference algebras.

A subset J of lattice inference algebra L is called its filter. If $I \in J$ and at the same time $x, x \to y \in J$, there are $y \in J$. A filter of L can be generated from a subset of L. It can also be used to define implicative filters, generated filters, prime filters, ultra filters, I-filters, associative filters (Jun 2001), fantastic filters, involution filters, obstinate filters and so on. Filters of lattice implication algebra can be mutually defined with congruence modulo relations. Apart from that, there also exist many results in filter-related methods of fuzzification, dual structures of filters (LI-ideal, ILI-ideal (Liu et al. 2003), WLI-ideal), constitutive categories of lattice inference algebras, and relations between lattice and other non-classical logical algebras.

Lattice-Valued Systems of Logic Based on Lattice Inference Algebras (Xu 1993)

Based on lattice inference algebra, Xu Yang and others established a lattice-valued system of propositional logic *LP(X)* with lattice inference algebra as its truth-value range (Qin and Xu 1994; Xu and Qin 1993) and lattice-valued first-order system of logic *LF(X)* (Xu et al. 1997), and on the basis of these further established a lattice-valued system of propositional logic L_{vpl} (Xu et al. 1999) and lattice-valued first-order system of logic L_{vfl} (Xu et al. 2000). For these systems of logic, they further researched their semantics, grammatical structures, and correlation properties, and presented their reliability, completeness, compatibility, deduction theorems as well as other important conclusions. Below, we will describe the main concepts and conclusions using the example of L_{vpl}.

The formulae of L_{vpl} are based on propositional variables and constant formulae by means of logical connectives, while the set of all its formulae F_p also constitutes the lattice inference algebra (briefly referred to as F_p). An assignment is a homomorphic mapping of F_p onto a lattice inference algebra. n-valued inference rules have the form (r_n, t_n), in which r_n is a n-valued partial operation on F_p, and t_n is an n-valued truth-value operation within $\mathcal{L}$. The domain of r_n is marked as $D_n(r_n)$, whereas R_n is subset of the set of all rules of n-valued inferce, $R = \bigcup_{n=0}^{+\infty} R_n$. L-type of fuzzy power set defined over F_p is $F_L(F_p)$.

Definition 2. Let X be any element of $F_L(F_p)$, let (r, t) be n-valued inference rule in R_n, and let a be any random element in the valuation field L.

(1) In $D_n(r)$, if, $X \circ r \supseteq \alpha \otimes (t \circ \prod^{n} X)$ then we refer to X of (r, t) as α - I type closed.

(2) In $D_n(r)$, if, $X \circ r \supseteq t \circ \prod^{n} (\alpha \otimes X)$ then we refer to in X of (r, t) as α - II-type closed.

If for any rule (r, t) in R, X of (r, t) is α - I (α - II) type closed, then we call X of R as α - I (α - II) type closed.

Definition 3. Let $T \subseteq F_L(F_p)$, and α represent any element in the valuation field L. If for any element T in T, T of R is α - i type closed, then R of T is α - I type reliable, in which i = I, II.

Definition 4. Let X be any element in $F_L(F_p)$, $T \subseteq F_L(F_p)$, let p be a formula in F_p, and let α, β, θ be any element in the valuation field L.

(1) The semantic of definition of X entails two different forms of p:

① $C_T^X(p) = \wedge_{T \in T}(\wedge_{q \in F_p}(X(q) \to T(q)) \to T(p))$

② $C^{\beta, X}_{(C_T^\phi, R(\alpha - i))}(p) = \wedge\{(Y(p) \mid Y \supseteq \beta \otimes (C_T^\phi \cup X)$, Y of R is α-i type closed$\}$, i = I, II.

(2) If we map P^{I} (P^{II}): $(n) \longrightarrow F_p \times L$ ($(n)=\{1, 2, \ldots, n\}$)

$i \mid\to (p_i, \theta_i)$

fulfilling the following conditions:

① $(p_n, \theta_n) = (p, \theta)$

② $\theta_i = \beta \otimes C_T^\phi(p_i)$ or

③ $\theta_i = \beta \otimes X(p_i)$ or

④ there exist $i_1, \ldots, i_k \leq i$ and the rule (r, t) of R_k, so that if we make

$(p_i,\theta_i)=(r(p_{i_1},...,p_{i_k}),\alpha\otimes t(\theta_{i_1},...,\theta_{i_k}))$ ($(p_i,\theta_i)=(r(p_{i_1},...,p_{i_k}),t(\alpha\otimes\theta_{i_1},...,\alpha\otimes\theta_{i_k}))$), then we call $(P^i, (n), X, (p, \theta) - (\alpha, \beta))$ to be a (α, α) - I type proof of value degree θ from X to p, i = I, II.

Theorem 1. (Reliability – completeness) If truth-value operations of R satisfy bounded semi-continuity, then for any formula p and i = I, II, there is

$$C_T^{\beta\otimes X}(p)=C^{\beta,X}_{(C^{\phi}_{T,R(\alpha-i)})}(p)=\vee\left\{\theta \mid there\ exist\ (P^i,(n),X,(p,\theta)-(\alpha,\beta))\right\}.$$

Definition 5. Let δ be any element in the valuation field L, let T be an element of $\mathcal{T}$, let p be a formula in F_p. If $T(p') = (T(p))'$, $X(p) \to T(p) \geq \delta$, then we say that T satifies X in the type δ - i, at the same time we also call X as satisfiable of the type δ - i, i = I, II.

Definition 6. Let τ be any element in the valuation field L. If

$$\vee\{C^{\beta,X}_{(C^{\phi}_{T,R(\alpha-i)})}(p)\otimes C^{\beta,X}_{(C^{\phi}_{T,R(\alpha-i)})}(p') \mid p\in F_p\}\leq t\ ,$$

then we call X as τ' - I type compatible of $(\alpha, \alpha, \mathcal{T})$, i = I, II.

Theorem 2. (Compatibility) If X is satisfiable of the type δ - i, then X of $(\alpha, \alpha, \mathcal{T})$ is compatible of type $\delta \otimes \delta$ - i, i = I, II.

Theorem 3. (Deductive theorem) Let (r_2^0, t_2) be an inference rule if $\mathcal{R}$, let p and q be any two formulae within F_p, and σ and θ be any elements within the valuation field L.

(1) If i = I, $C_T^{\beta\otimes X}(p\to q)\geq\sigma$, then $C_T^{\beta\otimes(X\cup\{\theta/p\})}(q)\geq\alpha\otimes t_2(\beta\otimes\theta,\sigma)$;

(2) If i = II, $C_T^{\beta\otimes X}(p\to q)\geq\sigma$, then $C_T^{\beta\otimes(X\cup\{\theta/p\})}(q)\geq t_2(\alpha\otimes\beta\otimes\theta,\alpha\otimes\sigma)$;

(3) If $T_h\subseteq T_H$, then $C_{T_h}^{\beta\otimes(X\cup\{\theta/p\})}(q)\leq C_{T_h}^{\beta\otimes X}(p\to q)$.

Where, $r_2^0(p, p\to q) = q$, $T_H = \{T \mid T$ is a homomorphic mapping of F_p into $L\}$.

Basing their work in lattice inference algebra, they also researched the corresponding imprecise inference (Xu et al. 2000) and resolution automatic reasoning (Xu et al. 2000; Xu et al. 2001; Xu et al. 2003; Xu et al. 2011) of the lattice-valued system of logic.

Chinese Notation

An appropriate symbolic notation can enable us to express logical thought in a clearer and more efficient manner, and subsequently to construct tools of inference. When it comes to logical constants (propositional connectives, quantifiers, modal operators etc.) in formal languages in particular, which constitute the core content of research on inference, constructing appropriate methods of symbolic notation can enable clearer and more precise expression, and subsequently also the expression of inference rules for logical constants.

The Chinese method of notation represents a kind of symbolic notation for logical constants as proposed by Du Guoping 杜国平 and others. In this kind of notation, by using only a pair of parentheses "( )" we can express each and every kind of logical constant (Du 2019a, 2022).

The most commonly used notation methods for logical constants include the infix expression method, Polish notation and reverse Polish notation.

The infix expression method places binary propositional conjunctions "disjunction", "conjunction", "entailment" and "equality" between their two linking symbols p and q, so that corresponding expressions are formed as "$p \vee q$", "$p \wedge q$", "$p \rightarrow q$" and "$p \leftrightarrow q$". In that regard, the infix method is identical to the common use of mathematical symbols +, -, ×, and ÷. When formulae become complex enough, the infix method must draw support from symbols such as parenthesis and others to express the priority of different combinations between symbols, all in order to avoid ambiguity. For example, with the use of parenthesis, the formulae $(p \vee q) \rightarrow r$ and $p \vee (q \rightarrow r)$ are able to express different meanings.

The Polish notation is an independent form of symbolic notation invented by the Polish logician Jan Łukasiewicz, which uses different capital letters to express logical connectives, placing these connectives in front of the propositions which they are connecting. For this reason, this notation method is also known as prefix expression method. Its concrete working method resides in using expressions like "*Np*", "*Cpq*", "*Kpq*", "*Apq*", "*Epq*" to express "negation", "entailment", "conjunction", "disjunction" and "equality", respectively (Łukasiewicz 1966, 22–30). One of the special advantages of Polish notation resides in the fact that it does not have to use parenthesis nor is it able to produce ambiguities. Its expressive efficiency is higher than that of the infix expression method.

The reverse Polish notation is also referred to as the suffix expression method. Its working method is similar to that of Polish notation, with the only difference being that the connective is placed behind the proposition that it is connecting.

Chinese notation is different from the above three notations. In contrast to the Polish notation, it uses other kinds of symbols, while it only uses a pair of parentheses to express each and every kind of logical constant. Thus, for example, it treats parenthesis "( )" as a ternary symbol. "(*ABCx*)" can thus be used to express all propositional connectives and the quantifiers ∀ or ∃. Furthermore, parenthesis

"( )" can also be regarded as a quaternary symbol. Thus, by using "(*ABCxD*)" we can express all propositional connectives, quantifiers $\forall$ or $\exists$, and modal operators $\Box$ or $\Diamond$.

For example, if we define (*ABCx*) as $[\neg A\vee\neg B]\wedge\forall x[B\rightarrow C]$, then (*ABBx*) is $[\neg A\vee\neg B]\wedge\forall x[B\rightarrow B]$. This is further equivalent to $[\neg A\vee\neg B]$, which is one of Sheffer functions, and can therefore define all propositional connectives including negation "¬", disjunction "∨", etc. In addition to that, $(C\neg CCx)$ is $[\neg C\vee\neg\neg C]\wedge\forall x\,[\neg C\rightarrow C]$, which is further equivalent to $\forall\, x\, C$, which in this way defines the universal quantifier.

Secondly, if (*ABCxD*) is defined as $[\neg A\wedge\neg B]\wedge\exists x[B\rightarrow C]\wedge\Box\,[C\rightarrow D]$, then (*ABBxB*) is $[\neg A\wedge\neg B]\wedge\exists x[B\rightarrow B]\wedge\Box[B\rightarrow B]$, this is equivalent to $[\neg A\wedge\neg B]$, which is a Sheffer function, as a result of which it can be used to give the definition of all propositional connectives, including the negation "¬", conjunction "∧", and so on. Aside from that, $(C\neg CCxC)$ is equal to $[\neg C\wedge\neg\neg C]\wedge\exists x[\neg C\rightarrow C]\wedge\Box[C\rightarrow C]$. This is equivalent to $\exists\, x\, C$, by which we have defined the existential quantifier. Moreover, $(C\neg C\neg CxC)$ is $[\neg C\wedge\neg\neg C]\wedge\exists x[\neg C\rightarrow\neg C]\wedge\Box[\neg C\rightarrow C]$, which is equivalent to $\Box C$, by which a definition was given for the "necessary" modal operator (Du 2019b; 2019c; 2020; 2021a; 2021b).

Chinese notation is inspired by Sheffer functions and the related ideas by Zhang Qingyu 张清宇. Sheffer functions employ a simple symbol, | or ↓, to denote the logical functions $\neg C\vee\neg D$ or $\neg C\wedge\neg D$, which specifies the propositional connectives as one symbol (Mendelson 2015, S21–23). On the basis of Sheffer functions, we have presented our specifications of the common logical constants. Apart from that, Chinese notation method is also greatly inspired by Zhang Qingyu's proposal of not using propositional connectives and instead using only parentheses and the nullary connective "T" to express the ideas of propositional connectives and quantifiers (Zhang 1995; 1996; 1997).

Chinese notation is an integral notation method. Because the left and right parentheses are used in pairs, their scope is clearly defined. Within themselves both parentheses have the capacity to express logical constants as well as the capacity to express the linking priority of symbols.

Some Reflections on Past Developments

Today, the development undergone by Chinese mathematical logic and foundations of mathematics in the last 100 years has already become an important and integral part of Chinese research on foundational theories. In its major developmental plans for science and technology, such as "Outline of the 14th Five-Year Plan for the Development of National Economic and Social Development and 2035 Long-Term Objectives of the People's Republic of China", the Chinese state places significant emphasis on research into foundational theories, which also includes mathematical logic, and has put forward numerous major research topics which are related to mathematical logic. In that way, Chinese research on mathematical logic has entered a time of favourable circumstances and great opportunities in the context of national strategic development.

On the other hand, looking back at the developmental trajectory of the past 100 years, Chinese mathematical logic encountered the following problems which are worth taking into further consideration:

(1) One of the comparatively central problems is the still pending progress in establishing influential scientific institutions. Thus, institutions like the Research Laboratory for Mathematical Logic at the Chinese Academy of Sciences, the Department of Mathematics at Nanjing University as well as the Institute of Logic and Cognition at Sun Yat-sen University have been in the past or still are China's most important research institutes for research into mathematical logic. If China wants to become a technological and scientific superpower, then it must accelerate the move to make mathematical logic an integral part of national research into foundation theories, by founding more internationally influential centres of scientific research that would specialize in this important scientific discipline.

(2) The problem of the relatively diffuse nature of research areas, and the still pending strengthening of scientific teams focusing on specific fields of research. As a field of theoretical research, mathematical logic is still basically in the state of having to struggle on its own. While Chinese mathematical logic is characterized by relatively focused research based on teacher-student relationships, Chinese academia still has not seen the formation of a group of experts that would garner international acclaim and influence in one specific area of such studies.

(3) Individual scholars tend to struggle on their own, and the number of academic exchanges is still insufficient. Therefore, China needs to work at establishing its own internationally influential scientific journal for mathematical logic, in order to advance and increase the academic exchanges among Chinese researchers in the field.

(4) Exchanges with the international academic world also need to be increased. China requires a substantial increase in the organization of international conferences related to mathematical logic, so as to boost young scholars' engagement in international academic exchanges.

(5) The problem of insufficient interdisciplinary research in science. Because of the disparities that exist among the humanities, natural sciences, and technology, different research groups for mathematical logic were formed in the areas of philosophy, mathematics, and computer science and artificial intelligence. Since these three groups of researchers still lack mutual exchanges and cooperation, there is an urgent need to address this issue in order to give rise to an atmosphere where the humanistic direction of research would receive equal attention as the scientific research in the field.

(6) The lack of significant, original results. Looking at the overall state of Chinese research on mathematical logic, we can notice that there is a profusion of results in secondary, follow-up research but at the same time, a great scarcity of ground-breaking original results, especially major, internationally leading scientific achievements. However, we are firmly convinced that soon after the promulgation of China's strategy emphasizing interdisciplinary scientific research, this situation will greatly improve!

The first draft of the manuscript was translated into English by Jan Vrhovski.

References

Chen, Bo 陈波. 2019. "Gaige Kaifang yilai de Zhongguo luojixue 改革开放以来的中国逻辑学 (Chinese Logic Since the Reform and Opening-up)." *Chongqing Ligong Daxue Xuebao* (*Shehui Kexue*) 重庆理工大学学报(社会科学) 33 (5): 7–19.

Deng, Guocai 邓国彩. 1994. "Zhongjie luoji lilun ji jiyu zhongjie luoji de buwanquan xinxi tuili xitong ISML 中介逻辑理论及基于中介逻辑的不完全信息推理系统ISML (Medium Logical Theory and A System of Reasoning with Incomplete Information ISML Based on Medium Logic)." MA Thesis, Nanjing University of Aeronautics and Astronautics.

Du, Guoping 杜国平. 2019a. "Guanyu 'buyong lianjiexi de luoji xitong' de zhuji 关于 '不用联结词的逻辑系统' 的注记 (Annotations on 'Logical

Systems without Connectives').” *Chongqing ligong daxue xuebao (shehui kexue)* 重庆理工大学学报(社会科学) 4: 7–12.

———. 2019b. “Buyong lianjieci de ‘she… qu…’ xing ziran tuiyan xitong 不用联结词的 ‘舍……取……’ 型自然推演系统 (A “not…but…”-Type Natural Deduction System Without Connectives).” *Hunan keji daxue xuebao* (*Shehui kexue ban*) 湖南科技大学学报 (社会科学版) 3: 21–24.

———. 2019c. “Jiyu kuohao biaoshifa de yijie luoji xitong 基于括号表示法的一阶逻辑系统 (First-order Logic System Based on Parenthesis Notation).” *Anhui daxue xuebao (zhexue shehui kexue ban)* 安徽大学学报(哲学社会科学版) 3: 35–41.

———. 2020. “Jiyu kuohao biaoshifa de 4-jia luoji ziran yanyi xitong 基于括号表示法的4 值逻辑自然演绎系统 (Study of Four-valued Logic-driven Natural Inference System Based on Parenthesis Notation).” *Hubei daxue xuebao (zhexue shehui kexue ban)* 湖北大学学报 (哲学社会科学版) 2: 36–49+167.

———. 2021a. “Heshe xitong jiqi dingli de nengxing zhengming 合舍系统及其定理的能行证明 (The Effective Proof of the System of Joint Denial and Its Theorems).” *Chongqing ligong daxue xuebao (shehui kexue)* 重庆理工大学学报 (社会科学) 6: 53–61.

———. 2021b. “‘Buke dejian’-xing mingti luoji ziran yanyi xitong ‘不可得兼’ 型命题逻辑自然推演系统 (The Natural Deduction System of Propositional Logic).” *Guangxi daxue xuebao* (*zhexue shehui kexue ban*) 广西大学学报 (哲学社会科学版) 3: 52–56+78.

———. 2022. “Parenthesis Notation.” *Journal of Research in Philosophy and History,* 5(1): 44–51.

Du, Guoping, and Wang Hongguang. 2020. “Logic Studies in Mainland China.” In *Dao Companion to Chinese Philosophy of Logic*, edited by Fung Y. M., 511–24. Springer.

Feng, Qi. 1990. “A Hierarchy of Ramsey Cardinals.” *Ann. Pure Appl. Logic* 49 (3): 257–77.

Hong, Jiawei. 1982a. “On Similarity and Duality of Computation.” *Inform. and Control* 62 (2–3): 109–28.

———. 1982b. “On some deterministic space complexity problems.” *SIAM J. Comput* 11 (3): 591–601.

———. 1984. “A Trade-off Theorem for Space and Reversal.” *Theoret. Comput Sci* 32 (1–2): 221–24.

Hong, Long, Xiao Xian, and Zhu Wujia. 2006. “Measure of Truth Scale and its Application (Ⅰ).” *Chinese Journal of Computers* 29 (12): 2186–94.

Hong, Long, Xiao Xian, and Zhu Wujia. 2007. “Measure of Truth Scale and its Application (Ⅱ).” *Chinese Journal of Computers* 30 (9): 1551–58.

Hu, Shihua 胡世华. 1960a. "Digui suanfa—digui suanfalun I 递归算法-递归算法论I (Recursive Algorithm—Theory of Recursive Algorithm (I)." *Shuxue xuebao* 数学学报 10 (1): 66–88.

———. 1960b. "Digui hanshu de fanshi—digui suanfalun III 递归函数的范式-递归算法论III (Normal Form of Recursive Algorithm—Theory of Recursive Algorithm (III)." *Shuxue Xuebao* 数学学报 10 (1): 89–97.

Hu, Shihua 胡世华, and Lu Zhongwan 陆中万. 1960. "Hehanshu—digui suanfalun II 核函数-递归算法论II (Kernel Functions—Theory of Recursive Algorithm (II)." *Shuxue Xuebao* 数学学报 10 (1): 98–103.

Jin, Yuelin 金岳霖. 1935. *Luoji* 逻辑 (*Logic*). Beijing: Guoli Qinghua daxue.

Jun, Y. B. 2001. "Fuzzy Positive Implicative and Fuzzy Associative Filters of Lattice Implication Algebras." *Fuzzy Sets and Systems* 121: 353–57.

Li, Wei. 1993. "An Open Logic System." *Sci. China Ser. A* 36 (3): 362–75.

Li, Zhizao 李之藻, and Francisco Furtado [Fu Fanji 傅汎際]. 1965 (1631/1639). *Mingli tan* 名理探 (*De Logica*), 2 vols. Reprint. Taipei.

Lin, Xiashui 林夏水, and Zhang Shangshui 张尚水. 1983. "Shuliluoji zai Zhongguo 数理逻辑在中国 (Mathematical Logic in China)." *Ziran kexueshi yanjiu* 自然科学史研究 2 (2): 175–82.

Liu, Y. L., Liu S. Y., XuY., and Qin K. Y. 2003. "ILI-ideals and Prime LI-ideals in Lattice Implication Algebras." *Information Sciences* 155: 157–75.

Lo, Libo. 1983a. "On the Number of Countable Homogeneous Models." *The Journal of Symbolic Logic* 48 (3): 539–41.

———. 1983b. "The T-theory for Free Groups is Undecidable." *The Journal of Symbolic Logic* 48 (3): 700–3.

Łukasiewicz, Jan. 1966. *Elements of Mathematical Logic*. Oxford: Pergamon Press.

Mendelson, Elliott. 2015. *Introduction to Mathematical Logic*. Boca Raton: CRC Press.

Moh, Shaw-kwei. 1950. "The Deduction Theorems of Two New Logical Systems." *Methodos* 2: 56–75.

———. 1954. "Logical Paradoxes for Many-Valued Systems." *The Journal of Symbolic Logic* 19: 37–40.

Qin, K. Y., and Xu Y. 1994. "Lattice-valued Proposition Logic (II)." *Journal of Southwest Jiaotong University* 1: 22–27.

Russell, Bertrand [Luosu 罗素]. 1921. *Luosu wu da yanjiang* 罗素五大演讲 (*Five Great Lectures by Bertrand Russell*). Beijing: Beijing daxue Xinzhi shushe.

———. 1922. *Luosu shuli zhexue* 罗素数理哲学 (*Russell's Mathematical Philosophy*). Translated by Fu Zhongsun 傅種孙, and Zhang Bangming 张邦铭. Shanghai: Shangwu yinshuguan.

Shen, Yuding. 1953. "Paradox of the Class of All Grounded Classes." *The Journal of Symbolic Logic* 18 (2): 114.

Shen, Yuding. 1955. "Two Semantical Paradox." *The Journal of Symbolic Logic* 20 (2): 119–20.

Song, Wenjian 宋文坚. 2000. "Zhongguo shuli luoji bashinian 中国数理逻辑八十年 (Mathematical Logic in China Eighty Years)." *Beijing Hangkong Hangtian Daxue Xuebao* (*Shehui Kexueban*) 北京航空航天大学学报(社会科学版) 13 (1): 12–17.

Song, Yunbo 宋云波, and Zhu Wujia 朱梧槚. 1994. "Zhongjie luoji chengxu sheji yuyan MILL jiqi jieshi xitong 中介逻辑程序设计语言MILL及其解释系统 (Medium Programming Language MILL and its Interpretation System)." In *Chengxu sheji yuyan yanjiu yu fazhan huiyi lunwenji* 程序设计语言研究与发展会议论文集 (*Proceedings of the Conference on Research and Development of Computer Languages*), Nanjing, February 1994, 80–81.

Su, Rina 苏日娜. 2020. "Shuli luoji zai Zhongguo de fazhanshi yanjiu (1920–1966) 数理逻辑在中国的发展史研究(1920–1966) (History of Mathematical Logic in China (1920–1966))." PhD diss., Inner Mongolia Normal University.

Vrhovski, Jan. 2021a. "'Qinghua School of Logic': Mathematical Logic at Qinghua University in Peking, 1926–1945." *History and Philosophy of Logic* 42 (3): 247–61.

———. 2021b. "Shadowlands of Objectivism and Comprehensiveness: An Introduction to Key Concepts in Zhang Shenfu's Philosophical Thought (1919–1948)." *Asian Studies* 9 (1): 227–62. https://doi.org/10.4312/as.2021.9.1.227-262.

Wang, Dianji 汪奠基. 1927. *Luoji yu shuxue luoji lun* 逻辑与数学逻辑论 (*Logic and Mathematical Logic*). Shanghai: Shangwu yinshuguan.

Wang, Hao 王浩. 1981. *Shuli luoji tongsu jianghua* 数理逻辑通俗讲话 (*Popular Lectures on Mathematical Logic*). Beijing: Kexue chubanshe.

Wang, Shiqiang. 1986. "Some Studies on Lattice-Valued Model Theory." In *Advances in Science of China. Mathematics* 2 edited by Chaohao Gu, and Yuan Wang, 73–78. New York: John Wiley & Sons.

Wu, Wangming 吴望名, and Pan Yin 潘吟. 1990. "Zhongjie daishu 中介代数 (Medium Algebras)." *Shuxue Yanjiu yu Pinglun: Yingwenban* 数学研究与评论：英文版 2: 265–70.

Xiao, Xi'an 肖奚安, and Zhu Wujia 朱梧槚. 1985a. "Zhongjie luoji de mingti yansuan xitong 中介逻辑的命题演算系统（Ⅰ）(A System of Propositional Calculus for Medium Logic I)." *Ziran zazhi* 自然杂志 4: 315–16.

———. 1985b. "Zhongjie luoji de mingti yansuan xitong 中介逻辑的命题演算系统（Ⅱ）(A System of Propositional Calculus for Medium Logic II)." *Ziran zazhi* 自然杂志 5: 394–95.

———. 1985c. "Zhongjie luoji de mingti yansuan xitong 中介逻辑的命题演算系统（Ⅲ）(A System of Propositional Calculus for Medium Logic III)." *Ziran zazhi* 自然杂志 6: 473.

———. 1985d. "Zhongjie luoji mingti yansuan de kuozhang 中介逻辑命题演算的扩张（II）(The Expansion of Propositional Calculus of Medium Logic II)." *Ziran zazhi* 自然杂志 10: 761.

———. 1985e. "Zhongjie luoji de daidengci de weici yansuan xitong 中介逻辑的谓词演算系统（II）(Medium Logical System of Predicate Calculus II)." *Ziran zazhi* 自然杂志 8: 601.

Xiao, Wencan 肖文灿. 1939. *Jihelun chubu* 集合论初步 (*A Primer in Set Theory*). Shanghai: Shangwu yinshuguan.

Xu, Yang 徐扬. 1993. "Ge-yunhan daishu 格蕴涵代数 (Lattice Implication Algebra)." *Xinan jiaotong daxue xuebao* 西南交通大学学报 1: 20–27.

Xu, Y., and Qin K. Y. 1993. "Lattice-Valued Propositional Logic (I)." *Xinan jiaotong daxue xuebao* 西南交通大学学报 2: 123–28.

Xu, Y., Qin K. Y., and Song Z. M. 1997. "Syntax of Lattice-Valued First-Order Logic FM." *Chinese Science Bulletin* 42 (16): 1337–40.

Xu, Y., Qin K. Y., Liu J., and Song Z. M. 1999. "L-Valued Propositional Logic L_{vpl}." *Information Sciences* 114: 205–35.

Xu, Y., Ruan D., Kerre E. E., and Liu J. 2000. "Resolution Principle Based on Lattice-Valued Propositional Logic LP(X)." *Information Sciences* 130: 195–223.

Xu, Y., Liu J., Song Z. M., and Qin K. Y. 2000. "On Semantics of L-Valued First-Order Logic L_{vfl}." *International Journal of General System* 29 (1): 53–79.

Xu, Y., Ruan D., and Liu J. 2000. "Approximate Reasoning Based on Lattice-Valued Propositional Logic. Fuzzy If-Then Rules in Computational Intelligence." In *Fuzzy Sets Theory and Applications*, edited by Ruan D., Kerre E. E., 81–105. Kluwer Academic Publishers.

Xu, Y., Ruan D., Kerre E. E., and Liu J. 2001. "Resolution Principle Based on First-Order Lattice-Valued Logic LF(X)." *Information Sciences* 132: 221–39.

Xu, Y., Ruan D., Qin K. Y., and Liu J. 2003. *Lattice-Valued Logic*. Springer.

Yi, Bo 伊波, and Xu Jiafu 徐家福. 1993. "Analogy Calculus." *Theoret. Comput. Sci* 113 (2): 211–30.

Ying, Mingsheng. 1994. "A Logic for Approximate Reasoning." *The Journal of Symbolic Logic* 59 (3): 830–37.

———. 2001. *Topology in Process Calculus: Approximate Correctness and Infinite Evolution of Concurrent Programs.* New York: Springer.

Zhang, Dongmo 张东摩, and Zhu Wujia 朱梧槚. 1994a. "Zhongjie zidong tuili de lilun yu shixian (I) – zhongjie mingti luoji de biaotuiyan xitong 中介自动推理的理论与实现（I）——中介命题逻辑的表推演系统 (The Theory and Practice of Mendium Automatic Reasoning (I) – Tableau Inference of Medium System of Propositional Logic)." *Moshi shibie yu rengong zhineng* 模式识别与人工智能 7 (2): 87–93.

———. 1994b. "Zhongjie zidong tuili de lilun yu shixian (II) – zhongjie weici luoji de biaotuiyan xitong 中介自动推理的理论与实现（ＩＩ）——中介谓词逻辑的表推演系统 (The Theory and Practice of Mendium Automatic Reasoning (II)—Tableau Inference of Medium System of Predicate Logic)." *Moshi shibie yu rengong zhineng* 模式识别与人工智能 7 (2): 175–80.

———. 1994c. "Zhongjie zidong tuili de lilun yu shixian (III) – zhongjie luoji dingli zhengmingqi 中介自动推理的理论与实现 （Ｉ II)——中介逻辑定理证明器 (The Theory and Practice of Mendium Automatic Reasoning (III)—A Theorem Proover for Medium Logic)." *Moshi shibie yu rengong zhineng* 模式识别与人工智能 7 (4): 263–68.

Zhang, Dongmo 张东摩, and Gong Ningsheng 宫宁生. 1995. "Zhongjie zidong tuili de lilun yu shixian (IV) – yilei jiyu zhongjie luoji de motai luoji xitong 中介自动推理的理论与实现(ＩＶ)——一类基于中介逻辑的模态逻辑系统 (The Theory and Practice of Mendium Automatic Reasoning (IV)—A Type of System of Modal Logic Based on Medium Logic)." *Moshi shibie yu rengong zhineng* 模式识别与人工智能 8 (1): 6–13.

Zhang, Dongmo 张东摩, and Zhu Wujia 朱梧槚. 1995. "Zhongjie zidong tuili de lilun yu shixian (V) – Zhongjie motai luoji MK de biao-tuiyan xitong 中介自动推理的理论与实现（Ｖ）——中介模态逻辑MK的表推演系统 (The Theory and Practice of Mendium Automatic Reasoning (V)—Medium System of Modal Logic MK's System of Tableau Inference)." *Moshi shibie yu rengong zhineng* 模式识别与人工智能 8 (2): 114–20.

Zhang, Qingyu 张清宇. 1995. "Buyong lianjieci de jingdian mingti luoji xitong 不用联接词的经典命题逻辑系统 (A System of Classical Propositional Logic Without Connectives)." *Zhexue yanjiu* 哲学研究 5: 40–47.

———. 1996. "Buyong lianjieci ke liangci de yijie luoji xitong 不用联接词和量词的一阶逻辑系统 (System of First-order Logic Without Connectives and Quantifiers)." *Zhexue yanjiu* 哲学研究 5: 72–79.

———. 1997. *Zhexue luoji yanjiu* 哲学逻辑研究 (*Studies in Philosophical Logic*). Beijing: Shehui kexue wenxian chubanshe.

Zhu, Chaohui 朱朝晖, Shi Qingsheng 施庆生, and Zhu Wujia 朱梧槚. 1996. "Chengxu jiannaji: you libo miaoshu de zhongjie luoji chengxu yuyi 程序兼纳集：由力迫描述的中介逻辑程序语义 (Sequence-Ordered Generic Sets: Describing the Sequence-Semantics of Medium Logic by Means of Forcing)." *Zhongguo kexue* (*E ji*) 中国科学（E辑） 6: 567–73.

Zhu, Wujia 朱梧槚, and Xiao Xi'an 肖奚安. 1984. "Shuxue jichu yu mohu shuxue jichu 数学基础与模糊数学基础 (Foundations of Mathematics and Foundations of Fuzzy Mathematics)." *Ziran zazhi* 自然杂志 1: 723–800.

———. 1985a. “Zhongjie luoji mingti yansuan de kuozhang 中介逻辑命题演算的扩张（Ⅰ) (The Expansion of Propositional Calculus of Medium Logic I).” *Ziran zazhi* 自然杂志 9: 681.

———. 1985b. “Zhongjie luoji de daidengci de weici yansuan xitong 中介逻辑的谓词演算系统（I） (Medium Logical System of Predicate Calculus I). *Ziran zazhi* 自然杂志 7: 540.

———. 1988. “Zhongjie gongli jihelun xitong MS 中介公理集合论系统MS (Medium System of Axiomatic Set Theory MS).” *Science in China Series A-Mathematics, Physics, Astronomy & Technological Science (in Chinese)* 18 (2): 113–23.

Zou, Jing 邹晶. 1988. “Daidengci de zhongjie weici luoji xitong ME* yuyi jieshi ji” kekaoxing, wanbeixing 带等词的中介谓词逻辑系统ME*语义解释及可靠性、完备性 (The Semantic Interpretation, Reliability and Completeness of Medium Predicate Logic System with Identity ME*).” *Kexue tongbao* 科学通报 33 (13): 961–63.