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Pion electro-production in the Roper region: K-matrix approach*

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Abstract. We review a method to calculate the pion electro-production amplitude in a framework of a coupled channel formalism incorporating quasi-bound quark-model states and discuss the results for the M_{1-} and the S_{1-} amplitudes in the P11 partial wave obtained in the Cloudy Bag Model.

1 Introduction

The P11 Roper resonance $N(1440)$ is of particular interest among the low-lying nucleon excitations, not only because of its relatively low mass, but primarily because of the rather peculiar behavior of the scattering and electro-excitation amplitudes. This clearly indicates that the structure of the resonance can not be explained by a simple excitation of the quark core (like most of the other low-lying states). The mesons, in particular the pion and the σ-meson, play an important role, yet the question remains whether it is possible to explain the Roper solely in terms of the quark and meson degrees of freedom or exotic degrees of freedom have to be incorporated like the explicit gluons [1–3].

We have developed a general method to incorporate excited baryons represented as quasi-bound quark-model states into a coupled channel calculation using the K-matrix approach that can be applied to meson scattering as well as to electro and weak-production of mesons [4,5]. The method ensures unitarity through the symmetry of the K-matrix. It can be applied to a class of Hamiltonians with linear meson-baryon coupling, in which case it is possible to construct an exact expression for the T-matrix without explicitly specifying the form of the asymptotic states. The method is particularly suitable to investigate the interplay of different channels involving the low-lying isobars such as the Delta and the chiral mesons which we expect to play the dominant role in the dynamics of the Roper resonance.

* Talk delivered by B. Golli

In the next section we give a short review of the method and in the following section we discuss in more detail the proton and the neutron helicity amplitudes calculated in our model and confront them to the predictions of some phenomenological models.

2 A short overview of the K-matrix approach

We assume that in the energy range of the Roper resonance, the electro-production processes can be described in terms of the γN, πN, $\pi\Delta$ and σN channels. The latter two channels correspond to two-pion decay. The electro-production amplitude $\mathcal{M}_{\pi N}$ is obtained from the Heitler's equation

$$\mathcal{M}_{\pi N} = \mathcal{M}^{K}_{\pi N} + i\left[T_{\pi N\pi N}\mathcal{M}^{K}_{\pi N} + T_{\pi N\pi\Delta}\mathcal{M}^{K}_{\pi\Delta} + T_{\pi N\sigma N}\mathcal{M}^{K}_{\sigma N}\right], \tag{1}$$

where $T_{\pi N\, MB}$ are the T-matrices for the $M + B \to \pi + N$ processes (M is ether π or σ; B either N or Δ), while the amplitudes

$$\mathcal{M}^{K}_{MB} = -\frac{\mathcal{N}_\gamma}{\sqrt{k_0 k_\gamma}}\,\langle\Psi^{MB}(m_J m_I; k_0, l)|\tilde{V}^{\gamma}_{\mu}(\mathbf{k}_\gamma)|\Psi_N(m_s m_t)\rangle \tag{2}$$

are the matrix elements of the EM interaction $\tilde{V}^{\gamma}_{\mu}(\mathbf{k}_\gamma)$ between the nucleon ground state Ψ_N and the principal-value state of the MB system. Here k_γ is the photon 3-momentum, k_0 is the momentum and l the angular momentum of the outgoing pion, the m stand for the respective third components of the nucleon spin and isospin and those of the MB system, and $\mathcal{N}_\gamma = \sqrt{k_\gamma\omega_\gamma M_N/W}$. In (1) the T matrices and the M amplitudes involving the $\pi\Delta$ and the σN channels have been already averaged over the invariant masses of the respective hadron.

The principal value states $|\Psi^{MB}\rangle$ assume the form

$$|\Psi^{MB}\rangle = \mathcal{N}_{MB}\left\{[a^\dagger(k_M)|\widetilde{\Psi}_B\rangle]^{\frac{1}{2}\frac{1}{2}} + \sum_{\mathcal{R}} c^{MB}_{\mathcal{R}}|\Phi_{\mathcal{R}}\rangle\right.$$
$$\left. + \sum_{M'B'}\int \frac{dk\,\chi^{M'B'\,MB}(k)}{\omega_k + E_{B'}(k) - W}\,[a^\dagger(k)|\widetilde{\Psi}_{B'}\rangle]^{\frac{1}{2}\frac{1}{2}}\right\}. \tag{3}$$

The first term represents the free meson (π or σ) and the baryon (N or Δ) and defines the channel, the next term is the sum over *bare* tree-quark states $\Phi_{\mathcal{R}}$ involving different excitations of the quark core, the third term introduces meson clouds around different isobars, $E(k)$ is the energy of the recoiled baryon. The sum in the latter term includes also inelastic channels in which case the integration over the mass of the unstable intermediate hadrons (σ or Δ) is implied. The state $\widetilde{\Psi}_{B'}$ represents either the nucleon or the intermediate Δ; in the latter case it is normalized as $\langle\widetilde{\Psi}_\Delta(M'_\Delta)|\widetilde{\Psi}_\Delta(M_\Delta)\rangle = \delta(M_\Delta - M'_\Delta)$. The meson amplitudes $\chi^{M'B'\,MB}(k)$ are proportional to the (half) off-shell matrix elements of the K-matrix. From the variational principle for the K-matrix it is possible to derive an integral equation for the meson amplitudes which is equivalent to the Lippmann-Schwinger equation for the K-matrix. The resulting χ amplitude takes the form

$$\chi^{M'B'\,MB}(k) = -\sum_{\mathcal{R}} \widetilde{c}_{\mathcal{R}}^{MB} \widetilde{\mathcal{V}}_{B'\mathcal{R}}^{M'}(k) + \mathcal{D}^{M'B'\,MB}(k)\,, \tag{4}$$

where $\mathcal{D}^{M'B'\,MB}(k)$ originates in the non-resonant background processes while the first term represents the contribution of various resonances; in the P11 case these are the nucleon, the N(1440), the N(1710) ... Here

$$\widetilde{c}_{\mathcal{R}}^{MB} = \frac{\widetilde{\mathcal{V}}_{B\mathcal{R}}^{M}}{Z_{\mathcal{R}}(W)(W - M_{\mathcal{R}})}\,, \tag{5}$$

where $\widetilde{\mathcal{V}}_{B\mathcal{R}}^{M}$ is the dressed matrix element of the quark-meson interaction between the resonant state and the baryon state in the channel MB, and $Z_{\mathcal{R}}$ is the wave-function normalization. The physical resonant state $\mathcal{R}$ is a superposition of the bare 3-quark states $\Phi_{\mathcal{R}'}$, hence $\widetilde{\mathcal{V}}_{B\mathcal{R}}^{M} = \sum_{\mathcal{R}'} u_{\mathcal{R}\mathcal{R}'}\mathcal{V}_{B\mathcal{R}'}^{M}$, where $\mathcal{V}_{B\mathcal{R}'}^{M}$ are the matrix elements with the bare 3-quark states.

In the vicinity of a resonance, e.g. the Roper N(1440), the term in the sum (4) corresponding to this particular resonance dominates. The amplitudes as well as the channel state (3) can be split into the *resonant* contribution, proportional to the coefficient $\widetilde{c}_{\mathcal{R}}^{MB}$ corresponding to the chosen resonance, and the *background* contribution consisting of the non-resonant processes and the contribution from the resonances other than the chosen one. The principal-value state (3) can be cast in the form

$$|\Psi^{MB}\rangle = -K_{\pi N\,MB}\sqrt{\frac{k_0 W}{\pi^2\omega_0 E_N}}\,\frac{\sqrt{\mathcal{Z}_{\mathcal{R}}}}{\widetilde{\mathcal{V}}_{N\mathcal{R}}^{\pi}}\,|\widehat{\Psi}^{\mathrm{res}}\rangle + |\Psi^{MB\,(\mathrm{bkg})}\rangle, \tag{6}$$

where

$$\begin{aligned} |\widehat{\Psi}_{\mathcal{R}}^{\mathrm{res}}\rangle = \mathcal{Z}_{\mathcal{R}}^{-\frac{1}{2}}\Bigg[&\sum_{\mathcal{R}'} u_{\mathcal{R}\mathcal{R}'}(W)|\Phi_{\mathcal{R}'}\rangle - \int dk\,\frac{\widetilde{\mathcal{V}}_{N\mathcal{R}}^{\pi}(k)[a^\dagger(k)|\Psi_N\rangle]^{\frac{1}{2}\frac{1}{2}}}{\omega_k + E_N(k) - W} \\ &- \sum_{MB}\int dk\,\frac{\widetilde{\mathcal{V}}_{B\mathcal{R}}^{M}(k)[a^\dagger(k)|\widehat{\Psi}_B\rangle]^{\frac{1}{2}\frac{1}{2}}}{\omega_k + E_B(k) - W}\Bigg]\,, \end{aligned} \tag{7}$$

and $K_{\pi N\,MB}$ is the K-matrix element for the $M + B \to \pi + N$ process. The state (7) has a familiar interpretation of a 3-quark state dressed by a cloud of mesons. The resonant part of the electro-production amplitudes then reads

$$\mathcal{M}_{\pi N}^{(\mathrm{res})} = -\sqrt{\frac{\omega_\gamma E_N^\gamma}{\pi^2\omega_0 E_N}}\,\frac{\sqrt{Z_{\mathcal{R}}}}{\mathcal{V}_{N\mathcal{R}}}\,\langle\widehat{\Psi}_{\mathcal{R}}^{(\mathrm{res})}(W)|\tilde{V}^\gamma|\Psi_N\rangle\, T_{\pi N\pi N}\,, \tag{8}$$

while the background part satisfies

$$\mathcal{M}_{\pi N}^{(\mathrm{bkg})} = \mathcal{M}_{\pi N}^{K\,(\mathrm{bkg})} + i\left[T_{\pi N\pi N}\mathcal{M}_{\pi N}^{K\,(\mathrm{bkg})} + \overline{T}_{\pi N\pi\Delta}\overline{\mathcal{M}}_{\pi\Delta}^{K\,(\mathrm{bkg})} + \overline{T}_{\pi N\sigma N}\overline{\mathcal{M}}_{\sigma N}^{K\,(\mathrm{bkg})}\right], \tag{9}$$

where $\overline{T}$ and $\overline{\mathcal{M}}$ are the amplitudes averaged over the invariant masses of the intermediate hadron using the averaging procedure introduced in [4].

3 The helicity amplitudes

The electro-production amplitudes at the photon point (i.e. at $Q^2 = 0$) have been extensively discussed in [5]; here we discuss in more detail the helicity amplitudes for the proton and the neutron. In our approach, the transverse and the scalar helicity amplitude are defined as

$$A_{\frac{1}{2}} = -\xi_{\mathcal{R}} \, \langle \widehat{\Psi}^{\mathrm{res}}_{\mathcal{R}} (m_s' = \tfrac{1}{2}) | \tilde{V}^{M1} | \Psi_N (m_s = -\tfrac{1}{2}) \rangle \,, \tag{10}$$

$$S_{\frac{1}{2}} = -\xi_{\mathcal{R}} \, \langle \widehat{\Psi}^{\mathrm{res}}_{\mathcal{R}} (m_s' = \tfrac{1}{2}) | \tilde{V}^{C0} | \Psi_N (m_s = \tfrac{1}{2}) \rangle \,, \tag{11}$$

with $m_t' = m_t = \frac{1}{2}$ for the proton and $m_t' = m_t = -\frac{1}{2}$ for the neutron, and $\xi_{\mathcal{R}} = \mathrm{sign}(g_{\pi N \mathcal{R}}/g_{\pi NN})$. The relation to the electro-production amplitudes at the *pole* of the K-matrix, $W = M_{\mathcal{R}}$, is determined through (8). Using the expression for the elastic width of the resonance $\Gamma_{\pi N} = 2\pi\omega_0 E_N \mathcal{V}^{\pi}_{N\mathcal{R}}(k_0)^2/Z_{\mathcal{R}} k_0 W$ and the relation $\mathrm{Im} T_{\pi N \pi N} = \Gamma_{\pi N}/\Gamma$ (at $W = M_{\mathcal{R}}$) we find

$$\mathrm{Im}\,{}_{p,n}M^{\frac{1}{2}}_{1-} = -\xi_{\mathcal{R}} \sqrt{\frac{k_W M_N \Gamma_{\pi N}}{6\pi k_0 M_{\mathcal{R}} \Gamma^2}} \, A^{p,n}_{\frac{1}{2}} \,, \qquad \mathrm{Im}\,{}_{p,n}S^{\frac{1}{2}}_{1-} = \xi_{\mathcal{R}} \sqrt{\frac{k_W M_N \Gamma_{\pi N}}{3\pi k_0 M_{\mathcal{R}} \Gamma^2}} \, S^{p,n}_{\frac{1}{2}} \,. \tag{12}$$

We have performed the calculation of the electro-production amplitudes in the Cloudy Bag Model (CBM) with the same choice of parameters as in the calculation of the scattering amplitudes [4]. We use the same bag radius for the excited states as for the ground state.

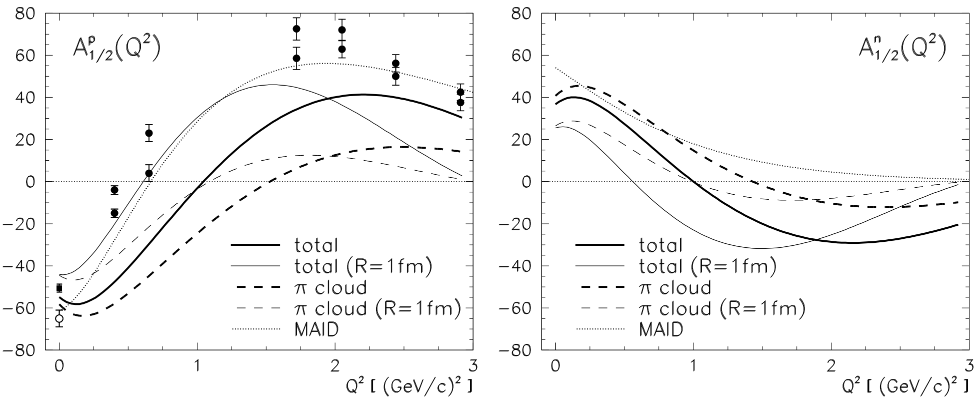

Fig. 1. Transverse helicity amplitudes for the proton (left panel) and the neutron (right panel) at the pole of the K matrix ($W = 1530$ MeV) for two values of the bag radius ($R = 0.83$ fm and $R = 1$ fm). The contribution of the meson cloud includes the $\gamma\pi\pi'$ interaction and the pion corrections to the $\gamma BB'$ vertex. Empty circle: PDG value [6]; full square and circles: analyses of newer JLab experiments. Two values at each $Q^2 \neq 0$ correspond to two different extraction approaches (see [7] for details). The MAID parametrization is given in [8].

The transverse helicity amplitudes for the proton and the neutron are displayed in Fig. 1 at the pole of the K-matrix where the relations (12) are fulfilled, and in Fig. 2 at the nominal mass of the Roper resonance. The difference between the calculated amplitudes in these two cases arises through the W dependence of the ground state admixture and of the pion amplitudes in (7). We reproduce the value at the photon point for the proton as well as for the neutron. These values are dominated by the pion cloud effects while the contribution from the bare quark core is almost negligible. At higher Q^2 the quark core contribution becomes stronger and positive for the proton (and negative for the neutron) while that of the pion cloud diminishes. As a result the amplitudes exhibit a zero crossing which is observed also in the experiment. At the smaller bag radius, the crossing occurs at somewhat higher Q^2 which may signify a too strong pion field.

Since no experimental data are available for the neutron for $Q^2 \neq 0$, we can compare our prediction only to the phenomenological expression of Ref. [8] where, however, no zero crossing occurs. In our model the zero crossing in the neutron case originates in the same mechanism as in the proton case; precise measurements of pion electro-production on the neutron may therefore provide a serious check for the proposed meson cloud picture.

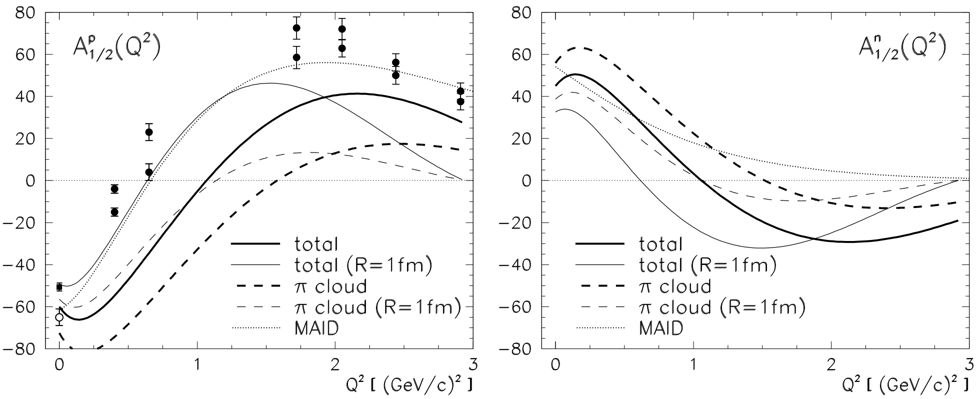

Fig. 2. Transverse helicity amplitudes at the nominal energy of the resonance (W = 1440 MeV). Notation as in Fig. 1.

The situation is rather controversial in the case of the scalar helicity amplitudes displayed in Fig. 3. Here we give the calculated values only at the pole of the K-matrix since the dependence on W is weak. The most striking feature is that the amplitudes cross zero while the experimental points – lacking values at low Q^2 – do not indicate this type of behavior. The zero crossing is again a consequence of the same mechanism as in the case of the transverse amplitudes. The MAID phenomenological analysis shows a rather sharp drop at $Q^2 \to 0$ though their amplitude does not cross zero. On the other hand, in this limit the imaginary part of the SAID partial wave analysis [9] does reach a negative value at $W = 1530$ MeV, supporting the possibility of a zero crossing also in this case. From this comparison we can conclude that it is again the pion cloud that governs the behaviour of the amplitudes at low Q^2; this effect is likely to be exaggerated in the present model.

The calculated scalar amplitudes for the neutron displayed in the right panel of Fig. 3 remain close to zero except at low Q^2. This result is less reliable because of a rather strong cancellation of different contributions; yet we do not see a mechanism in our model that could yield a relatively large amplitude of the MAID phenomenological analysis.

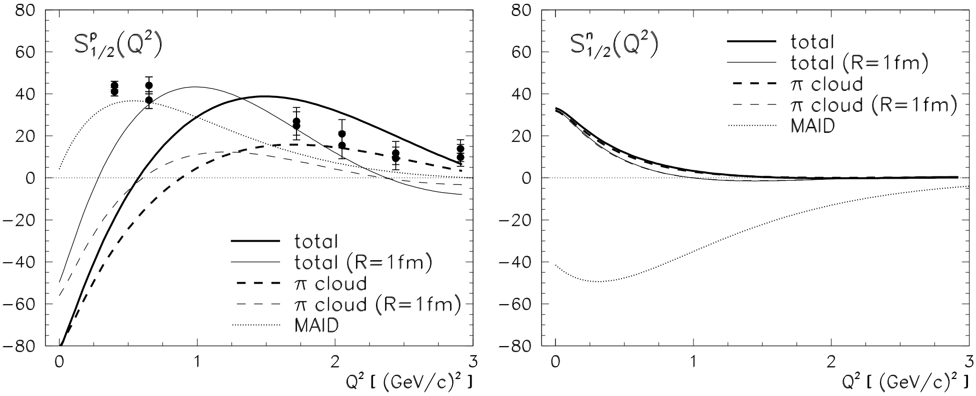

Fig. 3. Scalar helicity amplitudes at the pole of the K matrix ($W = 1530$ MeV). Notation as in Fig. 1.

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