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DETERMINATION OF THE CONDITIONS FOR THE EXISTENCE OF HIGHER-ORDER DIFFERENTIAL ELECTROMAGNETIC INVARIANTS

DOLOČITEV POGOJEV ZA OBSTOJ DIFERENCIALNIH ELEKTROMAGNETNIH INVARIANT VIŠJEGA REDA

Boris Nevzlin[1], Valentina Zagirnyak[ℜ], Veronika Zahorulko[2]

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Abstract

A four-element dipole representation by first-order electromagnetic invariants according to differential transformation and increments is well known. The paper deals with a most general description of the conditions of existence of an electromagnetic invariant for a four-element dipole with active-reactive components in a differential form and as increments of any order. It is shown analytically that invariants exist at mutual transformations of increments into differentials and differentials into increments.

ℜ Corresponding author: Valentina Zagirnyak, (Eng.), Tel.: +38 05366 36218, Fax: +38 05366 36000, Mailing address: Kremenchuk Mykhailo Ostrohradskyi National University, Manufacturing Engineering Department, Vul. Pershotravneva, 20, 39600, Kremenchuk, Ukraine, E-mail address: mzagirn@kdu.edu.ua

[1] Kremenchuk Mykhailo Ostrohradskyi National University, Electric machines and Apparatus Department, Vul. Pershotravneva, 20, 39600, Kremenchuk, Ukraine

[2] Kremenchuk Mykhailo Ostrohradskyi National University Electric machines and Apparatus Department, Vul. Pershotravneva, 20, 39600, Kremenchuk, Ukraine

Povzetek

Predstavitev dipola, sestavljenega iz štirih elementov, z elektromagnetno invarianto prvega reda je z vidika diferencialne transformacije in inkrementov že dobro poznana. Članek obravnava splošen opis pogojev za obstoj elektromagnetnih invariant dipola, sestavljenega iz štirih elementov, z delovno in jalovo komponento v diferencialni obliki in kot inkrementi poljubnega reda. Analitično je dokazano, da pri medsebojni transformaciji inkrementov v diference in diferenc v inkremente invariante obstajajo.

1 INTRODUCTION

A four-element dipole with active-reactive components (Fig. 1, a, b, c) is known [1, 2] to represent electromagnetic invariants according to differential transformation and increments that are of the form:

$$\frac{\frac{\partial C}{\partial \omega}}{\frac{\partial g}{\partial \omega}} = \frac{\partial C}{\partial g} = -\frac{C_1 + C_2}{g_1 + g_2}, \quad \frac{\frac{C_2 - C_1}{\omega_2 - \omega_1}}{\frac{g_2 - g_1}{\omega_2 - \omega_1}} = \frac{\Delta C}{\Delta g} = -\frac{C_1 + C_2}{g_1 + g_2}; \quad \frac{\frac{\partial \alpha}{\partial \omega}}{\frac{\partial R}{\partial \omega}} = \frac{\partial \alpha}{\partial R} = -\frac{\alpha_1 + \alpha_2}{R_1 + R_2},$$

$$\frac{\frac{\alpha_2 - \alpha_1}{\omega_2 - \omega_1}}{\frac{R_2 - R_1}{\omega_2 - \omega_1}} = \frac{\Delta \alpha}{\Delta R} = -\frac{\alpha_1 + \alpha_2}{R_1 + R_2}; \quad \frac{\frac{\partial L}{\partial \omega}}{\frac{\partial R}{\partial \omega}} = \frac{\partial L}{\partial R} = -\frac{L_1 + L_2}{R_1 + R_2}, \quad \frac{\frac{L_2 - L_1}{\omega_2 - \omega_1}}{\frac{R_2 - R_1}{\omega_2 - \omega_1}} = \frac{\Delta L}{\Delta R} = -\frac{L_1 + L_2}{R_1 + R_2}, \qquad (1)$$

where ω –arbitrary circular frequency; ω_2, ω_1 –circular frequencies and $\omega_2 > \omega_1$; $C_2, \alpha_2, L_2, g_2, R_2$ –values C, α, L, g, R at frequency ω_2; $C_1, \alpha_1, L_1, g_1, R_1$ –values C, α, L, g, R at frequency ω_1.

Conditions for invariants existence consist of, respectively:

$$C_1 g_2 - C_2 g_1 \neq 0, \quad \alpha_1 R_2 - \alpha_2 R_1 \neq 0, \quad L_1 R_2 - L_2 R_1 \neq 0. \qquad (2)$$

The same papers [1, 2] state that (1) there exist invariants not only according to frequency ω, but also according to order *n* of derivatives and increments:

$$\frac{\partial^n \tilde{N}}{\partial^n g} = \frac{\partial \tilde{N}}{\partial g} = -\frac{\tilde{N}_1 + \tilde{N}_2}{g_1 + g_2}, \quad \frac{\partial^n \alpha}{\partial^n R} = \frac{\partial \alpha}{\partial R} = -\frac{\alpha_1 + \alpha_2}{R_1 + R_2}, \quad \frac{\partial^n L}{\partial^n R} = \frac{\partial L}{\partial R} = -\frac{L_1 + L_2}{R_1 + R_2}, \qquad (3)$$

but conditions for their existence are not given.

The purpose of this paper: determination of conditions for the existence of higher order invariants.

2 MATERIAL AND RESULTS OF RESEARCH

Second derivatives with respect to *C* and *g*:

$$\frac{\partial^2 \tilde{N}}{\partial \omega^2} = \frac{-2(C_1 + C_2)(C_1 g_2 - C_2 g_1)^2 \left[(g_1 + g_2)^2 - 3\omega^2 (C_1 + C_2)^2 \right]}{\left[(g_1 + g_2)^2 + \omega^2 (C_1 + C_2)^2 \right]^3},$$

$$\frac{\partial^2 g}{\partial\omega^2}=\frac{2(g_1+g_2)(C_1g_2-C_2g_1)^2\left[(g_1+g_2)^2-3\omega^2(C_1+C_2)^2\right]}{\left[(g_1+g_2)^2+\omega^2(C_1+C_2)^2\right]^3}. \tag{4}$$

The second condition for the existence of a second-order differential invariant follows from (4):

$$g_1+g_2\neq\sqrt{3}\omega(C_1+C_2), \tag{5}$$

which is supplementary to (2).

Analogously, for a third-order differential invariant, the second condition for existence (the first one, as before, is (2)) is of the form:

$$g_1+g_2\neq\omega(C_1+C_2), \tag{6}$$

and for the fourth one:

$$(g_1+g_2)^4-5\omega^2(C_1+C_2)^2\left[2(g_1+g_2)^2-\omega^2(C_1+C_2)^2\right]\neq 0. \tag{7}$$

Condition (7), obviously, always exists at

$$\sqrt{2}(g_1+g_2)=\omega(C_1+C_2). \tag{8}$$

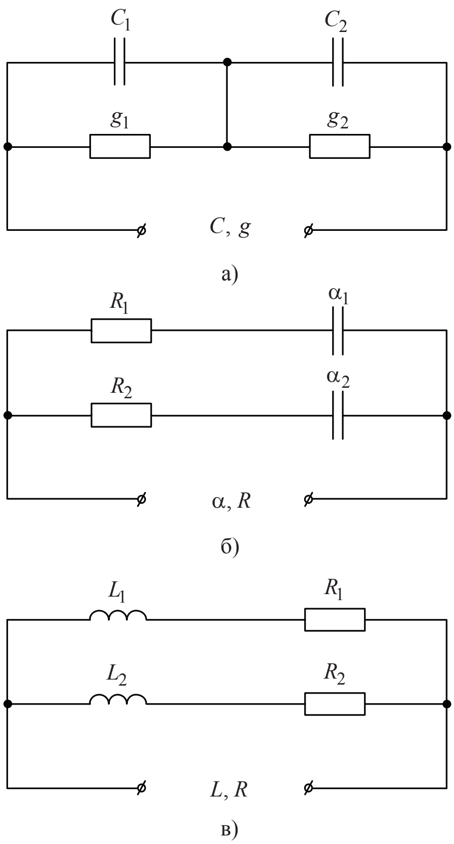

Figure 1: *Circuits of four-element dipoles: a) C_1, C_2 and C – condensers capacities and terminal capacitance of the circuit at arbitrary frequency, g_1, g_2 and g – the same concerning conductivity; b) α_1, α_2 and α – potential coefficients of condensers and overall potential*

coefficient of the circuit, R_1, R_2 and R – the same concerning resistances;
c) L_1, L_2, and L –branches inductances and overall inductance of the circuit,
R_1, R_2, and R – the same concerning resistances

Let us determine a derivative of the (*n*+1)-th order by a mathematical induction method, [3], in accordance with which a formula is considered true for any transformation, if it is proved that it, being true for *n*-th transformation, is also true for (*n*+1)-th transformation.
As invariant of the *n*-th order is of the form:

$$\frac{\partial^n \tilde{N}}{\partial^n g}=\frac{\dfrac{\partial^n \tilde{N}}{\partial \omega^n}}{\dfrac{\partial^n g}{\partial \omega^n}}=\frac{-\dfrac{(\tilde{N}_1+\tilde{N}_2)(\tilde{N}_1+\tilde{N}_2)^2(\tilde{N}_1 g_2-\tilde{N}_2 g_1)^2 f_n(C_1,C_2,g_1,g_2,\omega)}{\left[(g_1+g_2)^2+\omega^2(\tilde{N}_1+\tilde{N}_2)^2\right]^{n+1}}}{\dfrac{(g_1+g_2)(\tilde{N}_1+\tilde{N}_2)^2(\tilde{N}_1 g_2-\tilde{N}_2 g_1)^2 f_n(C_1,C_2,g_1,g_2,\omega)}{\left[(g_1+g_2)^2+\omega^2(\tilde{N}_1+\tilde{N}_2)^2\right]^{n+1}}}, \tag{9}$$

where $f_n(C_1,C_2,g_1,g_2,\omega)$ –the determined function for the derivative of the *n*-th order, e.g. for $n=3$ $f_n=-24\omega\left[(g_1+g_2)^2-\omega^2(C_1+C_2)^2\right]$.

Invariant of the (*n*+1)-th order is obtained:

$$\frac{\partial^{n+1}\tilde{N}}{\partial^{n+1}g}=\frac{\dfrac{\partial^{n+1}\tilde{N}}{\partial\omega^{n+1}}}{\dfrac{\partial^{n+1}g}{\partial\omega^{n+1}}}=\frac{-\dfrac{(\tilde{N}_1+\tilde{N}_2)(\tilde{N}_1+\tilde{N}_2)^2(\tilde{N}_1 g_2-\tilde{N}_2 g_1)^2\{\left[(g_1+g_2)^2+\omega^2(\tilde{N}_1+\tilde{N}_2)^2\right]\times}{\left[(g_1+g_2)^2+\omega^2(\tilde{N}_1+\tilde{N}_2)^2\right]^{n+2}}\to}{\dfrac{(g_1+g_2)(\tilde{N}_1+\tilde{N}_2)^2(\tilde{N}_1 g_2-\tilde{N}_2 g_1)^2\{\left[(g_1+g_2)^2+\omega^2(\tilde{N}_1+\tilde{N}_2)^2\right]\times}{\left[(g_1+g_2)^2+\omega^2(\tilde{N}_1+\tilde{N}_2)^2\right]^{n+2}}\to}\to$$

$$\to\frac{\to\dfrac{\times\{\left[(g_1+g_2)^2+\omega^2(\tilde{N}_1+\tilde{N}_2)^2\right]\dfrac{\partial}{\partial\omega}\left[fn(C_1,C_2,g_1,g_2,\omega)\right]-}{}\to}{\to\dfrac{\times\{\left[(g_1+g_2)^2+\omega^2(\tilde{N}_1+\tilde{N}_2)^2\right]\dfrac{\partial}{\partial\omega}\left[fn(C_1,C_2,g_1,g_2,\omega)\right]-}{}\to}\to$$

$$\to\frac{\to\dfrac{-2\omega(n+1)(\tilde{N}_1+\tilde{N}_2)^2 f_n(C_1,C_2,g_1,g_2,\omega)\}}{}}{\to\dfrac{-2\omega(n+1)(\tilde{N}_1+\tilde{N}_2)^2 f_n(C_1,C_2,g_1,g_2,\omega)\}}{}}=-\frac{\tilde{N}_1+\tilde{N}_2}{g_1+g_2}. \tag{10}$$

In this case, the condition for existence of (*n*+1)-th invariant, except (2), is of the form:

$$\left[(g_1+g_2)^2+\omega^2(\tilde{N}_1+\tilde{N}_2)^2\right]\frac{\partial}{\partial\omega}\left[f_n(C_1,C_2,g_1,g_2,\omega)\right]\neq$$
$$\neq 2(n+1)\omega(\tilde{N}_1+\tilde{N}_2)^2 f_n(C_1,C_2,g_1,g_2,\omega). \tag{11}$$

Thus, it can be stated that as a relation of any order derivatives of *C* and *g* with respect to frequency represents an invariant with respect to frequency, it is also an invariant with respect to the order of differential transformation with the condition for existence (2) for all invariants and (5) or (**6**), or (7)–(8) – for invariants of the determined orders (in a general form (**11**)).

Analogously to the circuits shown in Fig. 1, b, c, higher-order invariants are of the form (3) and, accordingly, conditions for their existence are of the form (except (2):

$$\omega(R_1+R_2)\neq\sqrt{3}(\alpha_1+\alpha_2), \tag{12}$$

$R_1+R_2\neq\sqrt{3}\omega(L_1+L_2)$ – second-order invariant,

$$\omega(R_1+R_2)\neq\alpha_1+\alpha_2, \tag{13}$$

$R_1+R_2\neq\sqrt{3}\omega(L_1+L_2)$ – third-order invariant,

$$\begin{gathered}\sqrt{2}\omega(R_1+R_2)\neq\alpha_1+\alpha_2,\ \sqrt{2}(R_1+R_2)\neq\omega(L_1+L_2)\ \text{or}\\ \omega^2(R_1+R_2)^4-5(\alpha_1+\alpha_2)^2\left[2(R_1+R_2)^2\omega^2-(\alpha_1+\alpha_2)^2\right]\neq 0,\end{gathered} \tag{14}$$

$(R_1+R_2)^4-5\omega^2(L_1+L_2)^2\left[2(R_1+R_2)^2-\omega^2(L_1+L_2)^2\right]\neq 0$ – fourth-order invariant.

In the general form, the condition for existence:

$$\begin{gathered}\left[\omega^2(R_1+R_2)^2+(\alpha_1+\alpha_2)^2\right]\frac{\partial}{\partial\omega}\left[f_n(\alpha_1,\alpha_2,R_1,R_2,\omega)\right]\neq\\ \neq 2\omega(n+1)(\alpha_1+\alpha_2)^2 f_n(\alpha_1,\alpha_2,R_1,R_2,\omega);\\ \left[(R_1+R_2)^2+\omega^2(L_1+L_2)^2\right]\frac{\partial}{\partial\omega}\left[f_n(L_1,L_2,R_1,R_2,\omega)\right]\neq\\ \neq 2\omega(n+1)(L_1+L_2)^2 f_n(L_1,L_2,R_1,R_2,\omega).\end{gathered} \tag{15}$$

Let us determine conditions for the existence of invariants according to increments. Second-order increment (otherwise –finite differences of the second order [4]) can be determined as:

$$\frac{\Delta^2\tilde{N}}{\Delta^2 g}=\frac{\dfrac{\Delta\tilde{N}_{\omega2}-\Delta\tilde{N}_{\omega1}}{\Delta\omega_2-\Delta\omega_1}}{\dfrac{\Delta g_{\omega2}-\Delta g_{\omega1}}{\Delta\omega_2-\Delta\omega_1}}=\frac{\dfrac{\tilde{N}_3-\tilde{N}_2-(\tilde{N}_2-\tilde{N}_1)}{\omega_3-\omega_2-(\omega_2-\omega_1)}}{\dfrac{g_3-g_2-(g_2-g_1)}{\omega_3-\omega_2-(\omega_2-\omega_1)}}=\frac{-\dfrac{(\tilde{N}_1+\tilde{N}_2)(\tilde{N}_1g_2-\tilde{N}_2g_1)^2(\omega_3-\omega_1)\times}{(\omega_3+\omega_1-2\omega_2)\times}\rightarrow}{\dfrac{(g_1+g_2)(\tilde{N}_1g_2-\tilde{N}_2g_1)^2(\omega_3-\omega_1)\times}{(\omega_3+\omega_1-2\omega_2)\times}\rightarrow}\rightarrow$$

$$\rightarrow\frac{\rightarrow\dfrac{\times\{(g_1+g_2)^2-(\tilde{N}_1+\tilde{N}_2)^2\left[\omega_2(\omega_3+\omega_1)+\omega_3\omega_1\right]\}}{\times\prod_{i=1}^{3}\left[(g_1+g_2)^2-\omega_i^2(\tilde{N}_1+\tilde{N}_2)^2\right]}}{\rightarrow\dfrac{\times\{(g_1+g_2)^2-(\tilde{N}_1+\tilde{N}_2)^2\left[\omega_2(\omega_3+\omega_1)+\omega_3\omega_1\right]\}}{\times\prod_{i=1}^{3}\left[(g_1+g_2)^2-\omega_i^2(\tilde{N}_1+\tilde{N}_2)^2\right]}}=\frac{\tilde{N}_1+\tilde{N}_2}{g_1+g_2}, \tag{16}$$

where $C_1, C_2, C_3, g_1, g_2, g_3$ –values *C* and *g*, respectively, at frequencies $\omega_1,\omega_2,\omega_3$.

The condition for existence in this case:

$$(g_1+g_2)^2\neq(\tilde{N}_1+\tilde{N}_2)^2\left[\omega_2(\omega_3+\omega_1)+\omega_3\omega_1\right]. \tag{17}$$

Analogously, for other invariants:

$$\frac{\Delta^2\alpha}{\Delta^2 R}=-\frac{\alpha_1+\alpha_2}{R_1+R_2};\qquad \frac{\Delta^2 L}{\Delta^2 R}=-\frac{L_1+L_2}{R_1+R_2}, \tag{18}$$

conditions for existence:

$$\left(\alpha_1+\alpha_2\right)^2 \neq \left(R_1+R_2\right)^2\left[\omega_2\left(\omega_3+\omega_1\right)+\omega_3\omega_1\right];$$
$$\left(R_1+R_2\right)^2 \neq \left(L_1+L_2\right)^2\left[\omega_2\left(\omega_3+\omega_1\right)+\omega_3\omega_1\right]. \quad (19)$$

Obviously, value $\omega_2\left(\omega_3+\omega_1\right)+\omega_3\omega_1$ at approximation $\Delta\omega \to 0$, where $\omega_3=\omega_2+\Delta\omega=\omega_1+2\Delta\omega$ tends to value $3\omega^2$ then conditions (17) and (19) turn into (5) and (12), which confirms the correctness of the performed transformations.

For increments of any *n*-th order, the condition for the existence of an invariant is of the form, e.g. with respect to *C* and *g*,

$$f_n\left(C_1, C_2, g_1, g_2, \omega_1 \div \omega_{n+1}\right) \neq 0. \quad (20)$$

It should be noted that invariants also exist in mutual transformations of increments into differentials and differentials into increments, i.e. (omitting lengthy intermediate transformations):

$$\frac{\Delta(\partial C)}{\Delta(g)}=\frac{\partial(\Delta C)}{\partial(\Delta g)}=-\frac{C_1+C_2}{g_1+g_2}; \quad \frac{\Delta(\partial\alpha)}{\Delta(\partial R)}=\frac{\partial(\Delta\alpha)}{\partial(\Delta R)}=-\frac{\alpha_1+\alpha_2}{R_1+R_2};$$
$$\frac{\Delta(\partial L)}{\Delta(\partial R)}=\frac{\partial(\Delta L)}{\partial(\Delta R)}=-\frac{L_1+L_2}{R_1+R_2}. \quad (21)$$

In this case, the condition for existence of an invariant e.g. with respect to *C* and *g* in transformations of increments into derivatives:

$$\left(g_1+g_2\right)^4 \neq 2\omega_1\omega_2\left(\tilde{N}_1+\tilde{N}_2\right)^2\left[\left(g_1+g_2\right)^2+\left(\omega_1^2+\omega_1\omega_2+\omega_2^2\right)\left(\tilde{N}_1+\tilde{N}_2\right)^2\right], \quad (22)$$

and transformation of derivatives into increments provide an invariant existing at any positive real values C_1, C_2, g_1, g_2 (meeting (2).

Analogous results also take place for circuits б,*R,* and *L*, *R*.

3 CONCLUSIONS

Conditions for the existence of invariants of a four-element dipole with active-reactive components at differential transformations and in increments of any order and also for mutual differential-difference transformations have been determined.

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