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Numerical Simulation on Mixed Convection in a Porous Medium Heated by a Vertical Cylinder

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Abstract

Numerical simulation has been performed for mixed convection of heated vertical cylinder with a constant temperature in a saturated porous medium subjected to lateral flow. Four different geometric aspect ratios 5,10,25 and 50 were considered, which give ratios of axial length to diameter. Based on dimensional analysis and nonlinear regression, correlations for the Nusselt number against the Rayleigh and the Peclet numbers have been obtained. It is shown that the average Nusselt number (Nu) is a function of the Rayleigh number (Ra_d) and the Peclet number (Pe): $Nu = 3.1\sqrt{Pe}$ for $Ra_d / Pe < 1$, and $\frac{Nu}{Pe^{1/2}} = 3.1 + 0.4\left(\frac{Ra_d}{Pe} - 1\right)^{1/2}$ for $Ra_d / Pe \geq 1$. As compared with $Nu = 3.1\sqrt{Pe}$ for the case of horizontal forced flow around a vertical cylinder, the results of this investigation indicates that heat transfer can be remarkable enhanced by natural convection when forced convection is weak.

INTRODUCTION

Convective heat transfer and fluid flow around vertical cylinder in porous media has recently received considerable attention in geophysical and engineering applications. Such applications include geothermal systems, chemical catalytic reactor, packed sphere beds, grain storage and thermal insulation engineering. However, most existing fundamental studies have focused on natural convection, or forced convection, while a very few have been reported for mixed convection despite its equal importance in many situations. Mixed convection about a vertical cylinder subjected to horizontal flow has not been investigated extensively [1].

Cheng [2] provided an extensive review of the literature on natural convection heat transfer in fluid saturated porous media He also investigated mixed convection from a horizontal circular cylinder in a saturated porous medium with boundary layer approximation[3]. In connecting with the forced convection by lateral flow, the analysis was made by Kimura [4] to cylinders of elliptic cross sections with integral methods. In his paper, Kimura provided the average Nusselt number for the steady state by integral solution with the formulation of $Nu = 3.1\sqrt{Pe}$, where the characteristic length in Nu is taken as the circumferential length πd . In addition, Romero [5] presented solutions for the temperature field due to Darcy flow past a slender body with a prescribed flux distribution embedded in a saturated porous medium. For the natural convection Sano et al. [6] studied the convective flows around a sphere embedded in a porous medium at small Rayleigh number and obtained asymptotic solutions for the transient and steady-state temperature distribution and flow pattern around the sphere. Campos et al. [7] studied the natural convection in the annulus of cylinder filled with porous medium, which has the heated inner wall and the cooled outer wall. However, pure natural or force convection rarely occurs in reality. In low Reynolds number flow conditions, heat transfer mechanisms of both forced convection and natural convection play a vital role and both mechanisms have to be properly accounted for.

Nomenclature

Roman symbols

c_p	specific heat	[J/(kg K)]
d	diameter of cylinder	[m]
d_p	diameter of the solid particle	[m]
g	gravitational acceleration	[m/s^2]
K	permeability	[m^2]
k	thermal conductivity	[W/(m K)]
L	length of cylinder	[m]
Nu	Average Nusselt number	
p	pressure	[Pa]
Pe	Peclet number	
$\overline{q}''$	average heat flux	[W/m^2]
Ra_d	Rayleigh number	
T	temperature	[K]
V	velocity vector	[m/s]
u,v,w	velocity components in the x,y,z system of coordinates	[m/s]
x,y,z	Cartesian coordinates	[m]

Greek symbols

▪ ż	Thermal diffusivity of porous medium ($= k_m/(\rho c_p)_f$)	[m^2/s]
▪ Ŕ	coefficient of thermal expansion	[K^{-1}]
ϕ	porosity of porous medium	
▪ Ď	density	[kg/m^3]
▪ Ę	dynamic viscosity	[Pa s]
▪ Ë	kinematic viscosity	[m^2/s]

Subscripts and superscripts

f	fluid
m	effective
s	solid
w	wall
0	inlet

Recently, numerical results for mixed convection around vertical cylinder by using two-dimensional numerical model have been reported [8,9]. In these research works, fluid flows along the axis of the cylinder are considered so that the problem is axisymmetric. As far as we know, there are very few papers on the problems of mixed convection of a vertical heated cylinder subjected to lateral flow in a saturated porous medium. The work by Ingham and Pop [10] is the only one that dealt with the similar problem. However, they assumed a partially heated infinite vertical cylinder in order to make the mathematical formulations easier. They also relied on the boundary layer approximations, which naturally limit the validity of their results in certain parametric ranges.

In the present work, three-dimensional numerical calculation was performed to simulate the mixed convection in a porous medium heated by vertical cylinder. From a parametric study, average Nusselt numbers were obtained and effects of ratio of length to diameter of cylinder, temperature differences, permeability of porous medium and lateral flow velocity on them were investigated. Results obtained from the simulation for two extreme cases, namely pure natural and forced convection, were compared with those in the literatures in order to test our code.

Governing equations and numerical methods

The analysis is based on solving Darcy's equations for three-dimensional flow around a vertical cylinder placed in porous medium. The vertical cylinder has a constant surface temperature (T_W). Fluid flows around the cylinder axis with a uniform velocity profile (V_0) and a constant fluid temperature (T_0). It is assumed that the flow is steady, laminar, three-dimensional and incompressible. The radiation and thermal dissipation effect are neglected. In addition, permeability and thermal conductivity of porous medium are homogeneous and isotropic, and it is in local thermal equilibrium with the fluid. It is assumed that the thermophysical properties of the fluid are independent of temperature except for the density in the buoyancy term, that is, the Boussinesq approximation is invoked.

The conservation equations for mass, momentum, energy in the fluid region are:

$$\nabla \cdot V = 0 \tag{1}$$

$$V = -\frac{K}{\mu}\nabla(p + \rho g) \tag{2}$$

$$(V \cdot \nabla)T = \alpha \nabla^2 T \tag{3}$$

where α is defined by $\alpha = k_m/(\rho c_p)_f$,and ρ and c_p denote the density and specific heat of the fluid, respectively. The effective thermal conductivity of the porous medium k_m is given by

$$k_m = k_s(1-\phi) + k_f \phi \tag{4}$$

With k_s and k_f representing the thermal conductivity of solid and liquid phase, respectively, and ϕ representing the porosity of the porous medium. At the same time, the permeability K is given by

$$K = \frac{d_p^2 \phi^3}{180 \cdot (1-\phi)^2} \tag{5}$$

With d_p representing the sphere's diameter of porous medium.

In addition, some nondimensional numbers are defined as follows, Peclect number $Pe = V_0 d / \alpha$; Average Nusselt number $Nu = \bar{q}'' \pi d / [(T_w - T_0) k_m]$; Rayleigh number $Ra_d = g\beta K d (T_w - T_0)/(\alpha \upsilon)$. Note that the Nusselt number is defined by πd . The boundary conditions are summarized in Table .1.

Table 1 The boundary conditions of numerical mode

Boundary type	Inflow	Top (Bottom)	Front (Back)	Outflow	Cylinder Wall
Velocity	$u = U_0$ $v = 0$ $w = 0$	$\partial u / \partial z = 0$ $\partial v / \partial z = 0$ $w = 0$	$\partial u / \partial y = 0$ $v = 0$ $\partial w / \partial y = 0$	Zero gradient	slip
Tempe-rature	$T = T_0$	$\partial T / \partial z = 0$	$\partial T / \partial y = 0$	Zero gradient	$T = T_w$

All the above calculations were performed using the FLUENT code. The governing Eqs. (1)-(3) were discretized using a finite volume method and the SIMPLE algorithm was used to solve the equations. A numerical scheme with non-uniform unstructured grids generated by GAMBIT software was applied to the present physical system. In order to obtain grid independent results, different finest grid spaces adjacent to the wall of size 0.005m, 0.002m and 0.001m are tested. It is proved that the difference between the results predicted based on 0.002m and 0.001m, the finest grid adjacent to the wall, are insignificant (<1%). Thus the finest grid, of size 0.002m and located adjacent to the wall, is chosen to get more detailed situation of the flow field.

Results and discussions

Numerical results compared with analytical solutions.

In order to confirm the validity of the numerical model, the numerical results were compared with analytical solution for forced convection and natural convection in a porous medium heated by a vertical cylinder, respectively. The average Nusselt numbers are in fairly good agreement with the data from References [4,11], as shown in Fig.1 and Fig.2.

Effect of Rayleigh number

Average Nusselt numbers for *L/d*=10 and eight different Rayleigh numbers are shown in Fig.3. Also shown on this graph is Nusselt number for the forced. It has shown that *Nu* is the function of *Ra* and *Pe*. Nu will increase with rising of Ra. When *Pe* is more than 8, *Nu* variations show a forced convection characteristic. However, just after the onset of the buoyancy drive secondary flow. This is a result of the buoyancy forces that become strong enough to destabilize the lateral flow. Moreover, an increase in the Nusselt number with the Rayleigh number is observed.

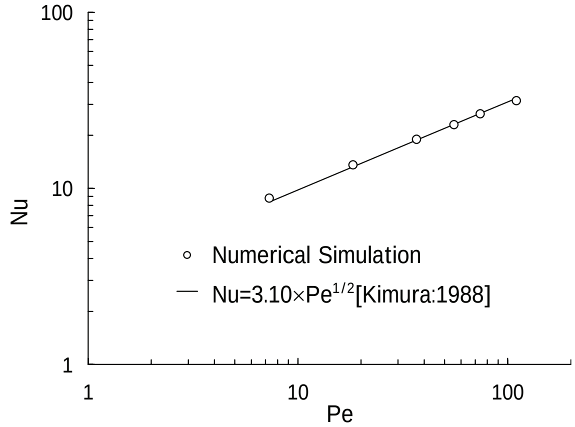

Fig.1. Calculation compared with analytical solution for Forced convection.

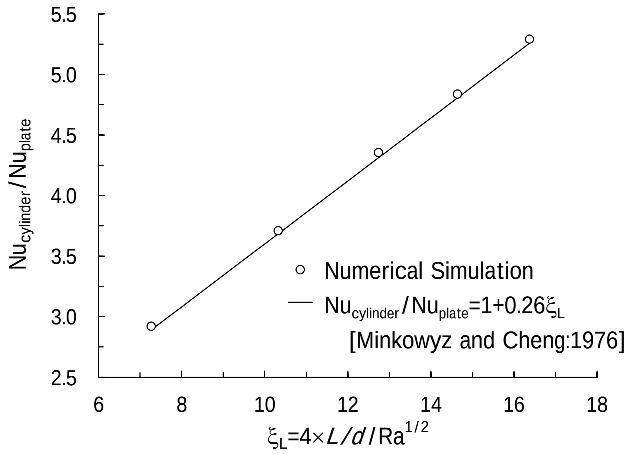

Fig.2. Calculation compared with analytical solution for natural convection(*Ra* is defined by the height).

The region with the secondary flow is described as the mixed convection region. As can be seen from Fig.3, with increase in Ra the point where *Nu* merges with forced convection occurs is shifted towards large *Pe*. Therefore, for large Rayleigh number the Nusselt numbers depart from the forced convection values even at large *Pe*. Even at the smallest Rayleigh number of 0.0119 the secondary flow effects are sufficiently large so that the variation of the Nusselt number is far away from that for forced convection predicted with the formulation of $Nu = 3.1\sqrt{Pe}$. For the largest Rayleigh number the secondary flow effects are largest. Consequently, the larger the Rayleigh number, the larger the mixed convection dominated region, and the larger heat transfer enhancement results.

Correlation formulas for mixed convection

Heat transfer results for mixed convection can be best presented in terms of two governing parameters, $Nu / Pe^{1/2}$ and Ra / Pe (Fig.4). As observed in all cases studied, the good collapse of the data points suggests that the transition of heat transfer processes occurring around vertical cylinder are fairly gradual.

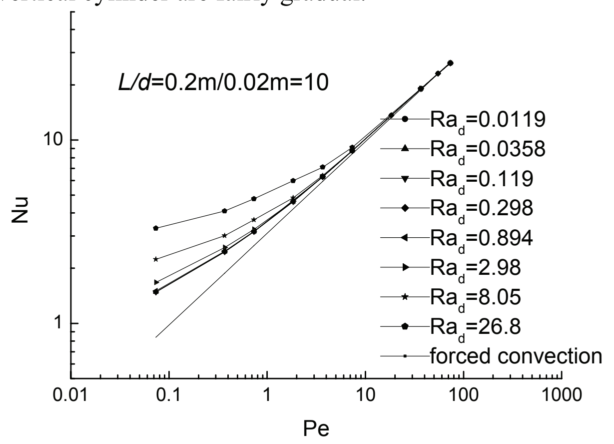

Fig.3. Average Nusselt number with Pe. (*L/d*=10)

The ratio Ra_d / Pe can be used to indicate the relative strengths of the two modes of convection in a mixed convection environment. It has shown the buoyancy effects become noticeable when Ra_d / Pe approaches unity. For Ra_d / Pe <<1, the forced convection component controls the heat transfer processes, while Ra_d / Pe>>1, buoyancy effects predominate. For forced convection, the correlation of heat transfer results based on numerical data is given by $Nu = 3.1\sqrt{Pe}$, which is in very good agreement with the results of Ref. [4]. For mixed convection, a least-squares fit of the data provided the following correlation:

$$\frac{Nu}{Pe^{1/2}} = 3.1 + 0.4\left(\frac{Ra_d}{Pe} - 1\right)^{1/2} \tag{6}$$

That is also indicated as a solid line in Fig.4. When the Rayleigh number is increased to large values, one would expect that the effect of forced convection would become negligible in comparison with the effect of natural convection, and consequently the Nusselt number should become nearly independent of the Peclet number. Accordingly, the power of correlation has been used as 1/2, which is consistent with the natural convection results. Eq.(6) reflects correctly the above nature of mixed convection heat transfer.

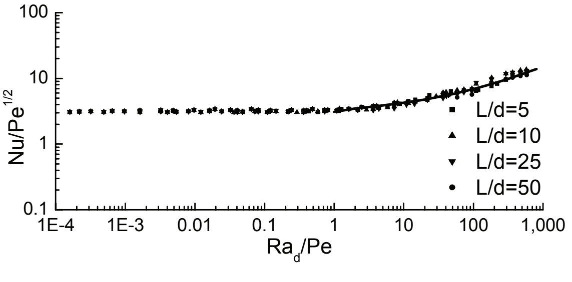

Fig.4. Heat transfer rate in the mixed convection

Conclusions

Mixed convection from a heated vertical cylinder in a saturated porous medium subjected to lateral flow has been examined numerically. It has been shown that the average heat transfer rate from a vertical cylinder is described by the following equations:

$$Nu = 3.1\sqrt{Pe} \quad \text{for } Ra_d / Pe < 1$$

and

$$\frac{Nu}{Pe^{1/2}} = 3.1 + 0.4\left(\frac{Ra_d}{Pe} - 1\right)^{1/2} \quad \text{for } Ra_d / Pe \geq 1 .$$

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