BLED WORKSHOPS
IN PHYSICS
VOL. 17, NO. 1
p. 50

Proceedings of the Mini-Workshop
Quarks, Hadrons, Matter
Bled, Slovenia, July 3 - 10, 2016

The equation of state in the quasispin Nambu–Jona-Lasinio model

Mitja Rosina[a,b]

[a]Faculty of Mathematics and Physics, University of Ljubljana, Jadranska 19, P.O. Box 2964, 1001 Ljubljana, Slovenia
[b]J. Stefan Institute, 1000 Ljubljana, Slovenia

Abstract. We study to what extent the soluble two-level Nambu–Jona-Lasino model can be applied to the study of the equation of state of quark matter. We have found that in the relation of energy versus temperature the phase transition does occur at a similar temperature as in Lattice QCD calculations.

1 Introduction

We have designed a simple model similar to the Nambu–Jona-Lasino model, in order to explore pedagogically [1,2] several phenomena and approximations similar to those in full Nambu–Jona-Lasinio or Lattice QCD. In our previous studies, it was very instructive to get consistent results with such a simple model for ground state (vacuum) properties such as the chiral condensate, as well as multipion energies, pion-pion scattering length, and sigma meson energy and width [3]. In this contribution we explore the application to the phase transition of quark matter.

In order to have a soluble two-level model with finite number of quarks, we make the following simplifications:

1. We assume a sharp 3-momentum cutoff $0 \leq |\mathbf{p}_i| \leq \Lambda$;
2. The space is restricted to a box of volume $\mathcal{V}$ with periodic boundary conditions. This gives a finite number of discrete momentum states, $\mathcal{N} = N_h N_c N_f \mathcal{V}\Lambda^3/6\pi^2$ occupied by N quarks. (N_h, N_c and N_f are the number of quark helicities, colours and flavours.)
3. We take an average value of kinetic energy for all momentum states: $|\mathbf{p}_i| \to P = \frac{3}{4}\Lambda$.
4. While in the NJL model the interaction conserves the sum of momenta of both quarks we assume that each quark conserves its momentum and only switches from the Dirac level to Fermi level.
5. Temporarily, we restrict to one flavour of quarks, $N_f = 1$.

We get a simplified NJL-like Hamiltonian

$$
\begin{aligned}
H = & \sum_{k=1}^{N}\Big(\gamma_5(k)h(k)P + m_0\beta(k)\Big) + \\
& -\frac{g}{2}\Big(\sum_{k=1}^{N}\beta(k)\sum_{l=1}^{N}\beta(l) + \sum_{k=1}^{N} i\beta(k)\gamma_5(k)\sum_{l=1}^{N} i\beta(l)\gamma_5(l)\Big) \quad .
\end{aligned}
$$

Here γ_5 and β are Dirac matrices, $h = \boldsymbol{\sigma}\cdot\mathbf{p}/|\mathbf{p}|$, m_0 is the bare quark mass and $g = 4G/\mathcal{V}$.

We introduce the quasispin operators which obey the spin commutation relations

$$
j_x = \frac{1}{2}\beta\ , \quad j_y = \frac{1}{2}i\beta\gamma_5\ , \quad j_z = \frac{1}{2}\gamma_5\ ,
$$

$$
R_\alpha = \sum_{k=1}^{N}\frac{1+h(k)}{2}j_\alpha(k)\ ,\ L_\alpha = \sum_{k=1}^{N}\frac{1-h(k)}{2}j_\alpha(k)\ ,\ J_\alpha = R_\alpha + L_\alpha = \sum_{k=1}^{N} j_\alpha(k)\,.
$$

The model Hamiltonian can then be written as

$$
H = 2P(R_z - L_z) + 2m_0 J_x - 2g(J_x^2 + J_y^2) \quad .
$$

The three model parameters $\Lambda = 648\,\mathrm{MeV}$, $G = 40.6\,\mathrm{MeV\,fm}^3$, $m_0 = 4.58\,\mathrm{MeV}$ have been fitted (in a Hartree-Fock + RPA approximation) to the observables

$$
\begin{aligned}
M &= \sqrt{\Big(E_g(N) - E_g(N-1)\Big)^2 - P^2} = 335\,\mathrm{MeV} \\
Q &= \langle g|\bar{\psi}\psi|g\rangle = \frac{1}{\mathcal{V}}\langle g|\sum_i \beta(i)|g\rangle = \frac{1}{\mathcal{V}}\langle g|J_x|g\rangle = 250^3\,\mathrm{MeV}^3 \\
m_\pi &= E_1(N) - E_g(N) = 138\,\mathrm{MeV}\,.
\end{aligned}
$$

The values of our model parameters are very close to those of full Nambu–Jona-Lasinio model used by the Coimbra group [4] and by Buballa [5].

2 The canonical ensemble

It is easy to evaluate the matrix elements of the quasispin Hamiltonian using the angular momentum algebra. If N is not too large the corresponding sparse matrix can be diagonalized using *Mathematica*. The eigenstates and eigenvalues $\epsilon(\nu)$ are labeled with the quasispin quantum numbers R and L corresponding to the operators $|\mathbf{R}|^2$ and $|\mathbf{L}|^2$ which commute with the Hamiltonian. The eigenvalues may be highly degenerate (degeneracy $D(\nu)$ is due to the permutation symmetry of different single particle labels $p(i)$). The ground state band with $R = L = N/4$ is nondegenerate and corresponds to the vacuum and multipion states.

To get the equation of state (energy versus temperature) we apply the canonical ensemble

$$E = \sum \epsilon(\nu)\, D(\nu) \exp(-\epsilon(\nu)/T) / \sum D(\nu) \exp(-\epsilon(\nu)/T)\,.$$

It is plotted in Fig.1 for two different values of quark numbers. As expected, the phase transition is sharper for the larger number of quarks. However, the values of energy are about the same. The model in its present form does not offer yet the thermodynamic limit in which the energy would be proportional to the volume. The reason is in the approximation 3 ($|\mathbf{p}_i| \to P = \frac{3}{4}\Lambda$) which makes lifting quarks to the upper level too expensive. Only collective excitations (multipion states, σ mesons etc.) contribute significantly to the equation of state, but they are roughly independent of the size of the normalization volume. Improvements to make a more flexible average of the kinetic energy are in progress.

In the graph for the Lattice QCD *asqtad* and *p4* refer to two different improved staggered fermion actions [6]. At low temperature the curve corresponds to the meson gas with 3 light degrees of freedom while at high temperatures it corresponds free gas of quarks and gluons (18 quarks + 18 antiquarks + 16 gluons = 52 massless degrees of freedom).

Note the difference in the vertical scale in our NJL curves (energy) as compared to the Lattice QCD curve (energy density/T^4). As mentioned before, we are not yet in the thermodynamic limit and it would not be meaningful to plot energy density. However, even in the simple model we get the temperature of the phase transition with approximately the same value and width.

3 The two-flavour case: the SU(4) algebra

In order to proceed to two flavours, a larger group than SU(2) would be needed. It is the O(3)⊗O(3)⊂O(5)⊂O(6) group (or equivalently SU(4) group) with fifteen generators

$$\tau^\alpha; \qquad \gamma_5\tau^\alpha; \qquad \beta, \quad i\beta\gamma_5\tau^\alpha; \qquad \gamma_5, \quad i\beta\gamma_5, \quad \beta\tau^\alpha,$$

where τ^α are isospin operators with $\alpha = 1, 2, 3$.

With these generators we can express the two-flavour Hamiltonian

$$H = \sum_{k=1}^{N} \Big(\gamma_5(k) h(k) P + m_0 \beta(k) \Big) + \\ - \frac{g}{2} \Big(\sum_{k=1}^{N} \beta(k) \sum_{l=1}^{N} \beta(l) + \sum_{k=1}^{N} i\beta(k)\gamma_5(k)\boldsymbol{\tau}(k) \cdot \sum_{l=1}^{N} i\beta(l)\gamma_5(l)\boldsymbol{\tau}(l) \Big) \quad .$$

Work is in progress.

We have discussed this SU(4) symmetry in 2009 but have not exploited it yet [2]. This symmetry has been recently widely popularized and applied by L.Glozman [7–11].

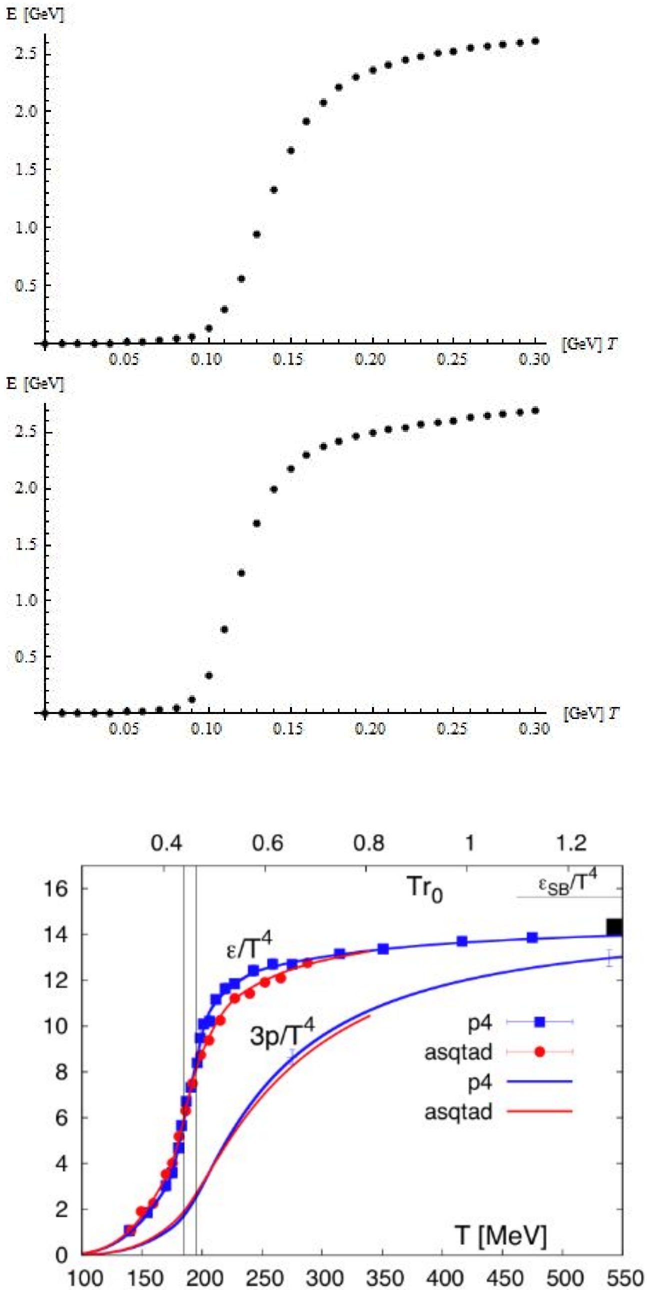

Fig. 1. Equation of state in the Quasispin NJL for N = 96 and N = 192, compared to the Lattice QCD [6]

References

1. B. T. Oblak and M. Rosina, Bled Workshops in Physics **7**, No. 1 (2006) 92; **8**, No. 1 (2007) 66; **9**, No. 1 (2008) 98 ; also available at http://www-f1.ijs.si/BledPub.
2. M.Rosina and B.T.Oblak, Few-Body Syst. **47** (2010) 117-123.
3. M. Rosina, Bled Workshops in Physics **16**, No. 1 (2015) 91.
4. M. Fiolhais, J. da Providência, M. Rosina and C. A. de Sousa, Phys. Rev. C **56** (1997) 3311.
5. M. Buballa, Phys. Reports **407** (2005) 205.
6. Bazavov et al., Phys.Rev.D **80**(2009)14504[arXiv:0903.4379].
7. L. Ya Glozman, arXiv:1407.2798v3 [hep-ph].
8. M. Denissenya, L. Ya Glozman, C.B. Lang, Phys.Rev.D **91**(2015)034505 [arXiv:1410.8751v2 [hep.lat]]; PoS(LATTICE 2014)068 [arXiv:1411.1578v1[hep-lat]].
9. L. Ya Glozman, M. Pak, arXiv:1504.02323v2 [hep-lat] .
10. M. Denissenya, L. Ya Glozman, M. Pak, Phys.Rev.D**92**(2015) 074508 [arXiv:1508.01413[hep-ph]]; arXiv:1505.03285v1[hep-lat]; arXiv:1510.09015[hep-lat].
11. L. Ya Glozman, arXiv:1608.07269 [hep-lat].