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On automorphisms of Haar graphs of abelian groups

Ted Dobson*

FAMNIT and IAM, University of Primorska, Muzejski trg 2, 6000 Koper, Slovenia

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Abstract

Let G be a group and $S \subseteq G$. In this paper, a Haar graph of G with connection set S has vertex set $\mathbb{Z}_2 \times G$ and edge set $\{(0,g)(1,gs) : g \in G \text{ and } s \in S\}$. Haar graphs are then natural bipartite analogues of Cayley digraphs, and are also called BiCayley graphs. We first examine the relationship between the automorphism group of the Cayley digraph of G with connection set S and the Haar graph of G with connection set S. We establish that the automorphism group of a Haar graph contains a natural subgroup isomorphic to the automorphism group of the corresponding Cayley digraph. In the case where G is abelian, we show there are exactly four situations in which the automorphism group of the Haar graph can be larger than the natural subgroup corresponding to the automorphism group of the Cayley digraph together with a specific involution, and analyze the full automorphism group in each of these cases. As an application, we show that all s-transitive Cayley graphs of generalized dihedral groups have a quasiprimitive automorphism group, can be constructed from digraphs of smaller order, or are Haar graphs of abelian groups whose automorphism groups have a particular permutation group theoretic property.

Keywords: Groups, graphs.

Math. Subj. Class.: 05C15, 05C10

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E-mail address: ted.dobson@upr.si (Ted Dobson)

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Avtomorfizmi Haarovih grafov abelskih grup

Ted Dobson*

FAMNIT and IAM, University of Primorska, Muzejski trg 2, 6000 Koper, Slovenia

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Povzetek

Naj bo G grupa in $S \subseteq G$. V tem članku je Haarov graf grupe G, ki pripada množici povezav S, graf, katerega množica vozlišč je $\mathbb{Z}_2 \times G$, množica povezav pa je $\{(0,g)(1,gs) : g \in G \text{ and } s \in S\}$. Haarovi grafi so naravni dvodelni analogi Cayleyjevih digrafov; imenujejo jih tudi biCayleyjevi grafi. Najprej pregledamo odnos med grupo avtomofizmov Cayleyjevega digrafa grupe G s povezavno množico S in Haarovega grafa grupe G s povezavno množico S. Ugotovimo, da grupa avtomorfizmov Haarovega grafa vsebuje naravno podgrupo, izmorfno grupi avtomorfizmov ustreznega Cayleyjevega digrafa. V primeru, da je grupa G abelska, pokažemo, da obstajajo natanko štiri situacije, v katerih je grupa avtomorfizmov Haarovega grafa lahko večja od naravne podgrupe, ki ustreza grupi avtomorfizmov Cayleyjevega digrafa skupaj s specifično involucijo; v vsakem od teh primerov analiziramo polno grupo avtomorfizmov. Na osnovi tega pokažemo, da za vse s-tranzitivne Cayleyjeve grafe posplošenih diedrskih grup velja, da imajo kvaziprimitivno grupo avtomorfizmov, da se dajo konstruirati iz digrafov manjšega reda, ali pa gre za Haarove grafe abelskih grup, katerih grupe avtomorfizmov imajo določeno posebno lastnost permutacijskih grup.

Ključne besede: Grupe, grafi.

Math. Subj. Class.: 05C15, 05C10

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E-poštni naslov: ted.dobson@upr.si (Ted Dobson)