BLED WORKSHOPS
IN PHYSICS
VOL. 14, NO. 1
p. 30

Proceedings of the Mini-Workshop
Looking into Hadrons
Bled, Slovenia, July 7 - 14, 2013

Quark-Pair Contributions to Weak Transition Form Factors of Heavy-Light Mesons⋆

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Abstract. Weak transition form factors of heavy-light mesons are discussed within a relativistic constituent-quark model employing the point-form of relativistic quantum mechanics. We study such form factors in the space- and the time-like momentum-transfer regions for heavy-light to heavy-light and heavy-light to light-light transitions. We investigate the influence of non-valence degrees-of-freedom which lead to, so called, "quark-pair contributions" in the weak transition current. To this aim the weak transition form factors are first calculated for space-like momentum transfers in a frame where quark-pair contributions are supposed to be suppressed. Analytical continuation to time-like momentum transfers and subsequent comparison with the time-like transition form factors obtained from a direct decay calculation gives us an estimate of the role quark-pair contributions may play for decay kinematics. A simple dynamical mechanism for quark-pair contributions based on the $^3P_0$ quark-pair creation model is suggested.

Our goal is to describe the electroweak structure of mesons within a constituent-quark model in a relativistically invariant way. To this aim we make use of the Bakamjian-Thomas construction [1, 2] and choose the point-form of relativistic dynamics [3]. The point form is characterized by the property that interaction terms enter all four components of the 4-momentum operator, whereas the generators of Lorentz transformations stay free of interactions. This makes it comparably simple to boost and rotate wave functions and add angular momenta. The essence of the Bakamjian-Thomas construction in point form is that the 4-momentum operator factorizes into an interaction-dependent mass operator and a free 4-velocity operator

$$\hat{P}^\mu = \hat{\mathcal{M}}\,\hat{V}^\mu_{\mathrm{free}} = \left(\hat{\mathcal{M}}_{\mathrm{free}} + \hat{\mathcal{M}}_{\mathrm{int}}\right)\hat{V}^\mu_{\mathrm{free}}\,. \tag{1}$$

The dynamics of the system is thus completely encoded in the mass operator.

Since we are dealing with processes during which the particle number is not necessarily conserved, we have to allow for particle creation and annihilation. This is accomplished by using a coupled-channel framework with a matrix mass operator $\hat{\mathcal{M}}$ that acts on the direct sum of the pertinent multiparticle Hilbert spaces. The diagonal entries $\hat{M}_i$ of this matrix mass operator are the sum of the

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relativistic kinetic energies of the particles in channel i. In addition, the $\hat{M}_i$ may contain instantaneous interactions between the particles, like the confinement potential between quark and antiquark. Off-diagonal entries of the matrix mass operator are vertex operators $\hat{K}_{i\to j}$ and $\hat{K}_{j\to i} = \hat{K}^\dagger_{i\to j}$ which describe the absorption and emission of particles and hence the transition from one channel to the other. The vertex interactions we use are derived from common field-theoretical interaction Lagrangean densities [4,5].

A most convenient basis to represent all these operators is formed by a complete set of velocity states [6]. A velocity state is a multiparticle momentum state in the rest frame

$$|\mathbf{k}_i, \mu_i\rangle \equiv |\mathbf{k}_1, \mu_1; \mathbf{k}_2, \mu_2; \ldots; \mathbf{k}_n, \mu_n\rangle, \text{ with } \sum_{i=1}^{N} \mathbf{k}_i = 0\,, \tag{2}$$

which is boosted to an overall 4-velocity V ($V_\mu V^\mu = 1$)

$$|V; \mathbf{k}_1, \mu_1; \mathbf{k}_2, \mu_2; \ldots; \mathbf{k}_n, \mu_n\rangle = \hat{U}_{B_c(V)} |\mathbf{k}_1, \mu_1; \mathbf{k}_2, \mu_2; \ldots; \mathbf{k}_n, \mu_n\rangle \tag{3}$$

by means of a rotationless boost $B_c(V)$. Using velocity states, matrix elements of vertex operators can be simply related to appropriate interaction Lagrangean densities [4,5]

$$\langle V'; \mathbf{k}'_j, \mu'_j | \hat{K} | V; \mathbf{k}_i, \mu_i\rangle \propto V^0\, \delta^3(\mathbf{V} - \mathbf{V}')\, \langle \mathbf{k}'_j, \mu'_j | \hat{\mathcal{L}}_{\text{int}}(0) | \mathbf{k}_i, \mu_i\rangle. \tag{4}$$

At this point it is worthwhile to remark that conservation of the overall 3-velocity at interaction vertices is a specific feature of the Bakamjian-Thomas construnction and does not hold, in general, for point-form quantum-field theories. It is this overall velocity-conserving delta function that leads to wrong cluster properties, an unwanted feature of the Bakamjian-Thomas construction which is observed in any form of relativistic dynamics and is not just specific to the point form [2]. The physical consequences of wrong cluster properties in our case are that the gauge-boson-hadron vertices, which we analyze to obtain the transition form factors, may not only depend on the momenta attached to the vertex, but also on the lepton momenta. Formally such wrong cluster properties could be cured by means of, so called, “packing operators” , but practically these are hard to construct. We will therefore adopt another strategy to end up with sensible results for the weak transition form factors.

Neutrino-Meson Scattering

Our first goal is to derive weak $B \to D$ transition form factors for space-like momentum transfers as they can, in principle, be measured in $\nu_e\, B^- \to e^- D^0$ scattering. In order to account for dynamical exchange of W-bosons we set up a 4-channel problem that includes all states which occur during such a scattering process if considered within a valence-quark picture (i.e. $|\nu_e, b, \bar{u}\rangle$, $|e, W^+, b, \bar{u}\rangle$, $|e, c, \bar{u}\rangle$, $|\nu_e, W^-, c, \bar{u}\rangle$). An instantaneos confinement potential between quark and antiquark is included in the diagonal entries of the matrix mass operator.

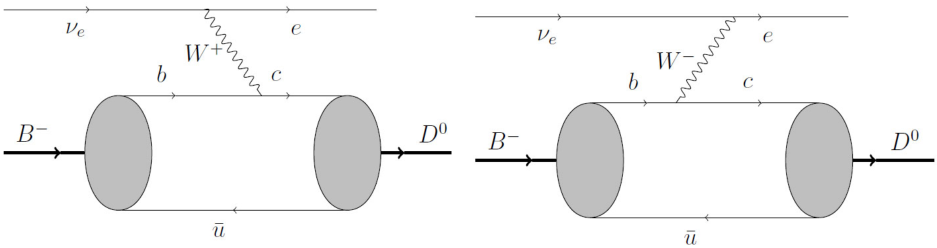

Fig. 1. Time-ordered contributions to the invariant 1W-exchange amplitude for $\nu_e\, B^- \to e^- D^0$ scattering.

Using perturbation theory for the weak coupling we calculate the invariant 1W-exchange amplitude for $\nu_e\, B^- \to e^- D^0$ scattering. It is the sum of two time-ordered contributions which are given by the (velocity-state) matrix elements

$$\Gamma_1 = \langle V'; \mathbf{k}'_D; \mathbf{k}'_e, \mu'_e | \hat{K}_W (m - \hat{M}_{\bar{u}bWe})^{-1} \hat{K}^\dagger_W | V; \mathbf{k}_B; \mathbf{k}_{\nu_e}, \mu_{\nu_e} \rangle \,,$$
$$\Gamma_2 = \langle V'; \mathbf{k}'_D; \mathbf{k}'_e, \mu'_e | \hat{K}_W (m - \hat{M}_{\bar{u}cW\nu_e})^{-1} \hat{K}^\dagger_W | V; \mathbf{k}_B; \mathbf{k}_{\nu_e}, \mu_{\nu_e} \rangle$$

that correspond to the graphs shown in Fig. 1. As expected, the sum of both contributions is proportional to the contraction of a lepton with a meson current times the covariant W-boson propagator. This allows us to identify the weak meson current in a unique way:

$$\tilde{J}^\nu_{B\to D}(\mathbf{k}'_D, \mathbf{k}_B) = \int d^3\tilde{k}'_{\bar{u}}\, f_{\text{kinem}} \sum_{\mu'_c\, \mu_b} \mathcal{W}_{\mu_b \mu'_c} \left[\bar{u}_{\mu'_c}(\mathbf{k}'_c) \gamma^\nu \frac{1-\gamma^5}{2} u_{\mu_b}(\mathbf{k}_b) \right]$$
$$\times \Psi^*_D(|\tilde{\mathbf{k}}'_{\bar{u}}|)\, \Psi_B(|\tilde{\mathbf{k}}_{\bar{u}}|)\,. \qquad (5)$$

Momenta with a tilde refer to the $Q\bar{u}$ rest frame ($Q = b, c$), whereas momenta without tilde rather refer to the $\nu_e b\bar{u}$ or $ec\bar{u}$ rest frame, respectively. Momenta with and without tilde are connected by means of Lorentz boosts which also give rise to the Wigner-rotation factor $\mathcal{W}_{\mu_b \mu'_c}$. f_{kinem} is an uniquely determined kinematical factor (see, e.g., Ref. [7]) and Ψ_D, Ψ_B are the D- and B-meson bound-state wave functions – in our case for simplicity pure s-wave. The expression within the square brackets represents the weak quark current.

Since we have worked with a velocity-state representation the bound-state current in Eq. (5) still does not transform like a 4-vector under Lorentz transformations, it rather transforms by a Wigner rotation. Only after going back to the physical particle momenta $p_B = B_c(V) k_B$ and $p'_D = B_c(V) k'_D$ we end up with a meson current that transforms like a 4-vector:

$$J^\mu(\mathbf{p}'_D, \mathbf{p}_B) := B_c(V)^\mu{}_\nu\, \tilde{J}^\nu_{B\to D}(\mathbf{k}'_D, \mathbf{k}_B)\,. \qquad (6)$$

Table 1. Model parameters

$M_B = 5.2795$ GeV	$M_D = 1.869$ GeV	$M_\pi = 0.1396$ Gev
$m_b = 4.8$ GeV	$m_d, m_u = 0.25$ GeV	$m_c = 1.6$ GeV
$a_B = 0.55$	$a_D = 0.46$	$a_\pi = 0.33$

For pseudoscalar to pseudoscalar transitions the covariant decomposition of this current reads [8]:

$$J^\mu(\mathbf{p}'_D, \mathbf{p}_B) = \left[(p_B + p_D) - \frac{m_B^2 - m_D^2}{q^2}\, q\right]^\mu F_1(Q^2, s) + \frac{m_B^2 - m_D^2}{q^2}\, q^\mu F_0(Q^2, s)\,. \tag{7}$$

The physical consequences of wrong cluster properties, inherent in the Bakamjian-Thomas construction, become obvious in this decomposition. The form factors cannot be choosen such that they are functions of the squared 4-momentum transfer at the W-meson vertex, they also depend on Mandelstam s, i.e. the invariant mass squared of the whole neutrino-B-meson (or equivalently electron-D-meson) system. This does not spoil Poincaré invariance of the scattering amplitude, it just means that the WBD-vertex is also influenced by the presence of the scattering lepton. The Mandelstam-s dependence may also be interpreted as a frame dependence of the $W^*B \to D$ subprocess. We consider two extreme cases, namely the minimum value of s necessary to reach a particular $q^2 < 0$ and $s \to \infty$. The first choice corresponds to the Breit frame (BF), the second to the infinite-momentum frame (IF).

Using simple harmonic-oscillator wave functions with the oscillator parameters and masses given in Tab. 1 we obtain the results that are plotted in Figs. 2 and 3 for the $B \to D$ and, in addition, also for the $B \to \pi$ transition, respectively.[1] Whereas the differences between IF and BF are small for F_1, they can be sizeable for F_0.

Semileptonic Meson Decay

In the time-like momentum transfer region these form factors can be measured in semileptonic weak decay processes. Theoretically it is straightforward to adapt our relativistic multichannel approach such that one can deal with decay processes like $B \to De\bar{\nu}_e$. Working in the velocity-state representation the decaying B-meson has to be at rest. For this kinematical situation it is, however, known from front-form calculations [11] that non-valence contributions leading to, so called, "Z-graphs" may become important. A similar observation can be made for space-like momentum transfers, if the form factors are calculated in the Breit frame. For kinematical reasons Z-graphs are, however, suppressed for space-like momentum transfers, if the form factors are calculated in the infinite-momentum

[1] The physical meson masses are the PDG values [9]. The constituent quark masses and wave-function parameters are taken from a corresponding front-form calculation [10].

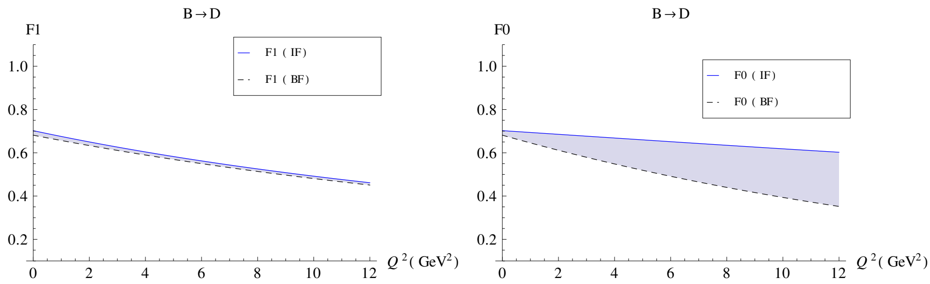

Fig. 2. The weak form factors F1 and F0 for the $B \to D$ transition in the space-like region as functions of $Q^2 = -(p_B - p'_D)^2$. The solid and dashed lines refer to calculations in the infinite momentum frame and the Breit frame, respectively. The shaded area indicates the frame dependence caused by the violation of cluster separability.

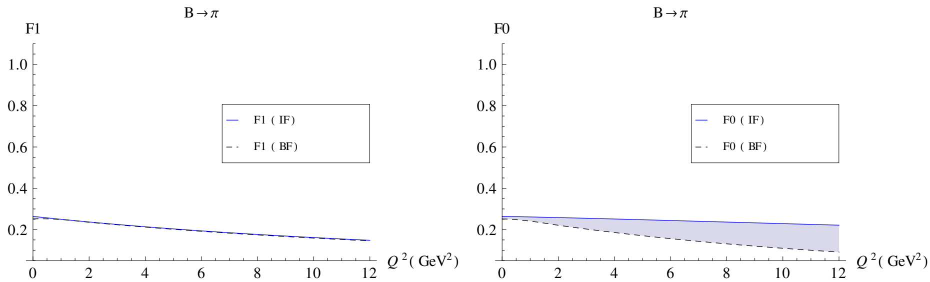

Fig. 3. Same as in Fig. 2, but for the $B \to \pi$ transition.

frame. In order to avoid problems with Z-graphs it is thus suggestive to take the form factor expressions obtained in the IF for space-like momentum transfers and continue them analytically to time-like momentum transfers by the replacement $Q \to iQ$. This is done in Figs. 4 and 5 for the $B \to D$ and $B \to \pi$ transitions, respectively (solid lines). In these figures the results of a direct decay calculation [7] (Z-graphs absent) are also shown (dashed lines). The differences may be considered as an estimate of the size of Z-graph (or "quark-pair") contributions. From the considerations just made it is clear that the solid line should be closer to experiment than the dashed one. This is indeed the case. What one knows experimentally is the slope of F_1 at zero recoil as measured in $B \to D e \bar{\nu}_e$ decays. It agrees approximately with the value which we get from our analytic continuation, whereas the decay calculation provides a much smaller value.

Outlook

Our next task is now the explicit inclusion of Z-graphs in the decay calculation. A typical Z-graph contribution to the $B \to D\, e\, \bar{\nu}_e$ decay is shown in Fig. 6. The non-valence contribution is easily accommodated within our coupled-channel approach, but in addition one has to say, how the $c\bar{c}$-pair is created. We plan to use a simple $^3P_0$ pair-creation model [12]. In this way we hope to achieve a

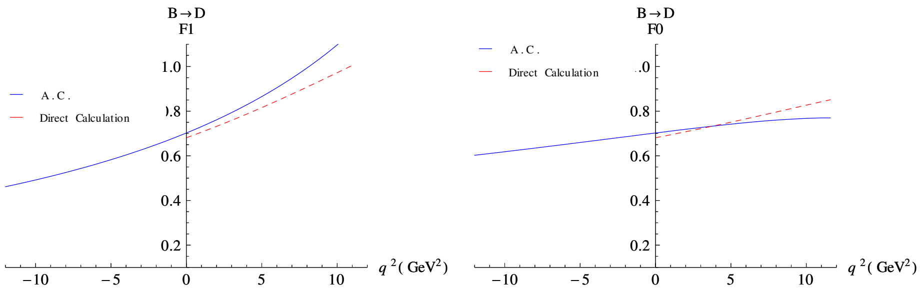

Fig. 4. The weak form factors F1 and F0 for the B $\longrightarrow$ D transition for space- and time-like momentum transfers. The solid line refers to the analytic continuation from space- to time-like momentum transfers by making the replacement Q $\longrightarrow$ iQ in the IF result. The dashed line is the outcome of a decay calculation in the B rest frame [7].

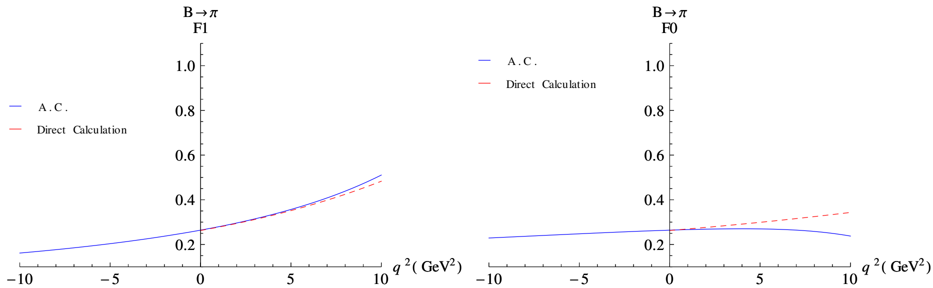

Fig. 5. Same as in Fig. 4, but for the B $\longrightarrow$ π transition.

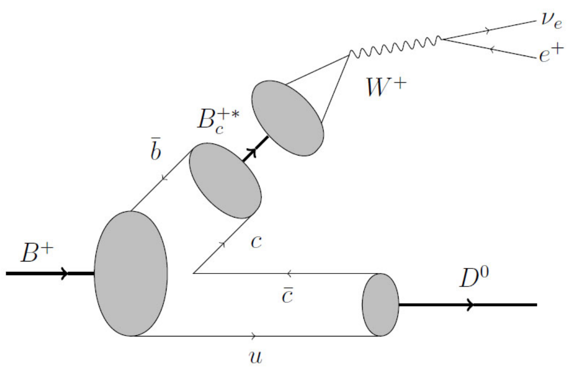

Fig. 6. Z-graph contribution to the 1*W*-exchange amplitude of the semileptonic B $\longrightarrow$ D e $\bar{\nu}_e$ decay

more quantitative estimate of Z-graph contributions in weak meson decay form factors.

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